Eratosthenes and Us, It Just Keeps Going and Going and Going...

Introduction

"From this project I learned that I can be good at math and science and now I even like them." – Liza

"At first I did not understand anything but later when I talked to Maya I understood better. Then Liliana explained it to me again and after I heard it two or three more times it began to make sense. I just had to hear it over and over again until it finally sunk in." – Cierra

"As for my participation in this project was involuntary, but I am grateful for such wonderful opportunity." – Eli

In the spring of 2000 we completed our third use of a unit based upon Eratosthenes' measurement of the size, or more accurately, circumference of the Earth. The genesis of this project was when one of us, Paul, attended a workshop on getting scientists involved in K–12 education. The workshop emphasized hands on activities and a constructivist approach (*see sidebar on constructivism*). At that time his eldest daughter had just started the 5th grade at Glenarden Woods Elementary School, in Glenarden, Maryland. At this school, each year the 5th and 6th grades study one of three ancient cultures (Ancient Egypt, Ancient China, or Medieval Europe) in a year–long thematic unit that involves language arts, social studies, and art. But, at the time, integration of math and science into this unifying theme was very limited. Since these cultures have an astronomical heritage, (but for that matter so does practically every culture), the accidental coincidence of the workshop and his daughter's schooling suggested his involvement with the school in helping bring in hands–on astronomy projects relevant to that year's chosen culture into the classroom.

We cast about for an astronomy project to do, and we eventually decided to duplicate Eratosthenes' measurement of the size of the Earth (*see sidebar on Eratosthenes*) based upon a suggestion given to us by Dr. Dave Dearborn via his connections at the Center for Archeoastonomy. We decided to do the project for several reasons:

- Even though Eratosthenes lived much later in time than the students were studying (he lived and experimented in the Ptolemaic era while the unit focused on the time of the Pharaoh's), it did tie into the Egyptian theme of the unit.

- The students had also studied Columbus and Marco Polo in a social studies unit. Columbus used a size of the world much smaller than what Eratosthenes calculated and some erroneous information based upon *The Travels of Marco Polo* to convince himself and others (notably the necessary financial backers) that he could sail from Europe to Japan without running out of supplies. Thus, the project had a connection to this unit and to today; how do we decide to fund exploration and science projects?

- The experiment is a great activity by itself for the students to learn and apply science process skills. It gave them a chance to collect and process data on a real life problem. They had to: assess the data quality, communicate their findings with others, and use the data they got from their teammates to come up with their final calculations.

Here we relate our experience the second time we did the project in 1998. This year was unique in that we were funded by a NASA IDEAS (Initiative to Develop Education through Astronomy and Space Science) grant to support curriculum development and a teachers' workshop.

Second Year

Our team was composed of a NASA scientist (Paul), sixth grade teachers in Glenarden, Maryland, two fifth grade teachers in Amherst, Massachusetts (Liliana and Tamara) and a Massachusetts high school teacher of home schooled students at Pathfinder Learning Center. In addition to Paul, Karl Martini, a professor at Amherst College was another scientist the Amherst teachers worked with. As a team, and as individuals, we all felt comfortable accepting our own strengths and limitations. Paul would often ask for the teachers' help on how to introduce a concept to the students. The teachers shared with Paul the students' scientific and mathematical problems, as well as their own, to have his guidance and knowledge to better help the students. The exchange among all members of the team was rich in content and context.

The first order of business was to draw up a lesson plan. For this we drew in part upon lesson plans in the Project Astro book: *The Universe at Your Fingertips,* produced by the Astronomical Society of the Pacific. While *The Universe at Your Fingertips* does not have a specific activity that duplicates Eratosthenes work, it does have activities that are useful (B6 and

C2 cover understanding shadow length and movement, C3 covers Eratosthenes' reasoning, and M1 relates Eratosthenes to Columbus). Liliana helped us to focus the plan along constructivist lines and presenting the material to the students as a problem to be solved by them. We emphasize that our lesson plan was a working plan for us. We were not afraid to modify it according to the level of the students, for instance, the high school students attacked the problem at their level, use of trigonometric functions, planetary motion, etc. or to modify it as went along.

One key part of the project is that the students need to know the Euclidean geometry fact that opposite interior angles formed by a line intersecting two parallel lines are equivalent. However, we did not tell the students this fact. Instead, we guided the students in a discovery process via construction i.e. drawing parallel lines and then drawing an intersecting line and measuring the angles. They also drew non–parallel lines and an intersecting line to discover the converse, that if the angles are not the same then the two lines are not parallel.

The teachers never told the students how Eratosthenes used opposite interiors angles to measure the circumference of the earth, thus they had to discover, or rediscover, how he used it as a tool. This made learning geometry meaningful and relevant to students rather than being taught in isolation as it is generally done in schools. In Amherst, the students were measuring the length of shadows at solar noon without yet being aware of how Eratosthenes had used this information. Tamara asked the students what they were doing and they explained they were measuring the shadows. The teacher then asked the two students, "Do you have any idea how Eratosthenes might have used this information?" The students explained that they thought that the tomato stake was the line crossing the parallel lines and that the sun's rays were the parallel lines in the Euclidean geometry lesson they had done the week prior. Tamara had never told the students what they "needed" to know, the students evolved their understanding on their own. The teacher, delighted, told them to continue thinking in this way, that they were "Totally on the right track." The students were actively involved, piecing the parts together to understand how Eratosthenes used shadows, angles and parallel lines to determine the circumference of the earth.

Evolving one's knowledge in this way is a powerful way to learn. The material presented has stayed with at least one student, Will, to this day. Will's mother recently related the following story to us. Will, an eighth grader now, was in math class when his teacher announced that he was going to talk about Euclid and how to measure opposite angles. Will told him that he knew how to do that and he even knew how to use them in "real life too." The teacher showed disbelief but played along with him. He asked him, "When did you learn this?" Will answered that it was in fifth grade. The teacher told him to show him what he knew. Will proceeded to show him not just about geometry but how to use it to measure the circumference of the earth. The teacher was incredulous that a fifth grade student had learned this in class, understood it and recalled it three years later.

Because we encouraged the students to question what they were doing and why they were doing it we engaged in many discussions with them and among ourselves. In Amherst, the big question for the students was, are the Sun's rays really parallel? This originated from a student who was quite adamant that the sun's rays were not parallel. This challenged the teachers, especially when, via consultation with Paul, they learned that the student was correct, the Sun's rays are not exactly parallel. The teachers then shared what they learned with the students and all gained a deeper understanding of Eratosthenes' process through this student initiated exchange. Interestingly, while the students eventually understood that since the Sun is so very far from the Earth we could assume its rays to be parallel for the project, they did not question how could Eratosthenes know this? (He knew because of previous work by Aristarchus of Samos on the Earth–Moon distance and the Earth–Sun distance.)

For the Glenarden students the major challenge was with making the measurement. We have found through the years that while the students can grasp the theory behind the measurement,; it can be hard for them to make the required careful measurements of shadow length. We also had a data anomaly in that the final data set of solar zenith angle was bimodal. Both modes were a normal distribution, but while one was centered on the correct value the other was centered 3 degrees away. This opened the doors to a "teachable" moment for the students about when and how do you average data? (The average of the all of the data was, of course, in between the modes where there was no reported value! A sure tip off that something is wrong) This in turn lead to a "teachable" moment for Paul as he realized that to him, a scientist, average means reducing the random error in a measurement, to elementary students average means average performance, as in a teacher averaging their test scores to get their grade for the semester.

With the IDEAS funds we gave a teacher's workshop and developed materials that we have used and have been used by others. A video, produced by Tamara, was made, demonstrating our collaboration and how the project was put in practice in the classroom. This video has been broadcast on the Amherst PBS several times, shown at national conferences, and used at the University of Massachusetts in the Constructivist Educator Training

Program (CETP) to train teachers on how to teach science and math in a constructivist class. A lesson plan/guide was developed and has been shared at the above conferences and workshops and others. The set of lesson plans includes student work to give samples of what the project looks like in the classroom to encourage teachers to reproduce the project.

Conclusion

In one sense, there is no conclusion to this project as it is still going on and we are still going on together. Be that as it may, there are some things that we have noticed about pursing such a project as this.

The project has enriched everyone who has been involved with it, scientists, teachers and students beyond their initial expectations. The challenges we encountered were motivating and allowed us to grow as professionals and students. We all learned about ancient science and mathematics. The students had the opportunity to be science detectives as they rediscovered how to apply the same concepts Eratosthenes used to measure the circumference of the Earth. The constructivist approach gave the students the responsibility for their learning and the freedom to think critically and creatively. In the process of evolving their understanding and applying the mathematics and physics concepts, the students developed the ability to think logically. For example, students questioned that the Sun's rays were parallel, the need to average the data, and why Eratosthenes's size was not the same as today's value. Teachers, with the aid of the scientists, worked together to understand these concepts and created lesson plans to teach the concepts the best way possible.

We feel the students benefited from having an integrated unit that included science, math, social studies and art. We made it relevant to them and put it in an historical–scientific context. From this, the students appreciated the input of ancient science and scientists to our current knowledge. They learned how to use angles in real life application, something lacking in elementary education. The students were encouraged to use information and concepts learned in math and apply them in science, thus helping them to make transferences of concepts learned in different areas and to apply them to concrete problems. Students also felt motivated to think and question, intelligently, all the possibilities. When they graduated from sixth grade the great majority acknowledged that the Eratosthenes project was the best and most influential project they had done. One unexpected benefit for the students was an increase in self–confidence. Figuring out the geometry behind the measurement on their own and being able to measure the circumference of the Earth to within 10% of the correct value with tomato stakes makes a big and lasting impression.

We worked closely as a team and were able to show this to our students. The benefits of group work became clear to them when they saw how teachers reached out to the scientists and vice versa. The students also took this project very seriously because a "real" scientist was involved.

This project has had an effect on the teachers that has been carried beyond the classroom. They have trained other teachers and encouraged them to do this unit. They have held workshops at local and national levels (NSTA and NCTM conferences) promoting the project and encouraging collaborations between scientists and educators. One of the teachers had local astronomers come to her class and demonstrate how telescopes were made and persuaded local amateur astronomers to hold a night star show for students and their families to enjoy and learn. Liliana has become involved with NASA as an educator for the Solar System Educator Program (SSEP).

Teachers were grateful to have a scientist and vice versa. Teachers found in Paul and Karl sounding boards to ask questions and check understanding. Teachers want to teach accurately and understand in–depth the concepts. They feel more secure when the information comes directly from a working scientist. It was important that teachers feel free and safe to openly talk with the scientists and that their questions were not going to be taken as "dumb" questions. Scientists should be sensitive in talking with non–scientists on how to explain concepts in non–technical language but maintaining the high level and sophistication that the concepts demand. Teachers do not want to be talked down or be given dumbed down explanations. Paul was also grateful to work with teachers. Paul was able to see the project take shape in ways he did not imagine could be done. The teachers gave Paul the pedagogical theory behind the reasons for teaching in certain ways by addressing the audience's age, knowledge and interests.

The benefits of explaining intricate concepts to teachers in simpler terms have been substantial benefits to Paul. He recognized that in order to explain certain concepts to the teachers he needed to have them clear and organized himself in order for others to follow. There is nothing like having to teach a topic in order to learn it.

There are, however, the inevitable sacrifices and difficulties, no matter how surmountable they may be. Time, scheduling, and weather were our "challenges" (i.e. enemies) with time being the worst. The amount of work that the teachers and scientists placed in it was significant. Scientists may feel that teaching in schools or collaborating with teachers will rob them of precious research hours. It is a realistic aspect of this collaboration. However, scientists interested in outreach should know that these projects are short lasting, from four to 10 weeks, but the benefits for the teachers and the students are priceless. Scheduling constraints are also problematic on the teachers' side. The project needs to fit the time of the year so that testing and other scheduled activities are not in conflict. In this project we were also weather dependent and the students needed to be available to go outside for an hour to measure the shadows. We worked together to find solutions to these scheduling difficulties. The teachers, as much as possible, taught their units in an integrated way giving more flexibility to the times and days they could use for instruction. For example, angles were taught during math, Christopher Columbus during social studies.

Even though the team had not met prior to doing the project, the distance between Paul in Maryland and the teachers in Massachusetts was not a problem to our collaborationWe did most of our communication via e–mail and phone. At the end of the project, Paul went to visit the Amherst students and teachers. It was a great experience to finally see each other face to face.

This project was indeed special, but not unique. The same project or the development of similar projects can be reproduced in any place. What is needed are scientist/s, teacher/s and students. They should all have in common the desire to learn from each other, explore new ideas, let anybody be able to lead the project into new directions, communicate without fear or insecurity about shortcomings or lack of understanding and support each other. We have discovered new aspects of ourselves through this collaboration that we would have never known otherwise. Unlike Eli our participation was voluntary, but like Eli, we are grateful for such wonderful opportunity.

"This project taught me that math and science are fun and that I am good at them." – Liza

"I want to be an astronomer when I grow up." – Chelsea

"This was the best project I've ever done." – Joel

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Sidebars

Eratosthenes

Eratosthenes (276–197 BC) was born in Cyrene (now Shahhat, Libya). After studying in Alexandria and Athens he became the director of the Library in Alexandria. (The Library at Alexandria, while functioning as what we think of as a library today, also had elements of what we think of as a research laboratory.) He is best known for his accurate calculation of the Earth's circumference, within 15% of the modern day value, and this work is often referenced in introductory astronomy textbooks. One of the great beauties of his observation and measurement is the combination of clever insight, or the "Aha moment" and careful measurement. The "Aha moment" is in his realization that since the rays of the Sun striking the Earth can be taken as parallel, that on any given day the solar zenith angle at solar noon of a point, A, on the Earth is the same size as one with its vertex at the center of the Earth with one ray pointing to point A and the other pointing directly to the Sun (i.e. the point on the Earth where the Sun is at the zenith, the overhead point O on diagram). The careful measurement part is in measuring this angle and the distance between O and A. Eratosthenes knew that at the summer solstice the noon Sun was reflected in a well dug at Syene (modern Aswan). Thus, this was the overhead point. Eratosthenes then determined via shadow measurements that the Sun as observed at Alexandria was south of the vertical by about 1/50th of a full circle. The Earth's circumference would thus be 50 times the north–south distance between the sites.

What is Constructivism?

Constructivism is a theory about acquisition of knowledge and learning process, not a teaching technique. "Learning from this perspective is understood as a self–regulated process of resolving inner cognitive conflicts that often become apparent through concrete experience, collaborative discourse, and reflection." (*Brooks and Brooks, The Case For Constructivist Classroom. 1993*). Students come to class with an understanding of how the world works. When they encounter new experiences, the students synthesize and assimilate them based on their prior knowledge. The students adjust their prior understanding with their current one by formulating new rules. Students engage with each other and the task at hand as independent and active learners. They experience and evolve the concept through the activity and their own questions. As they discuss and experience their discoveries, they share their thinking and listen to each other. Via oral or written expression, the students are always reflecting on what they know and what they need to know. This metacognitive process, done as individuals and in group, is a continuous checking point of students' self assessment.

Thus, students are active learners who consciously or unconsciously build their own new understanding based on their prior knowledge. In science, students come to class with a preexisting perception of natural phenomena, which they use to make sense of new situations. Taking advantage of students' prior information, they and their teachers can develop new concepts and foment solid insights and learning as well as correcting misconceptions.

In a constructivist class students are actively engaged whether they are interacting with each other or self–reflecting. Students are encouraged to work in groups, listen to each other and be heard, as they reflect on their ideas and that of their friends, they become flexible thinkers. Students are exposed to different, methods, ways of thinking, approaches and formulating questions. They are constantly encountering new disequilibrium wherefore they revise their schemata and reformulate new concepts.

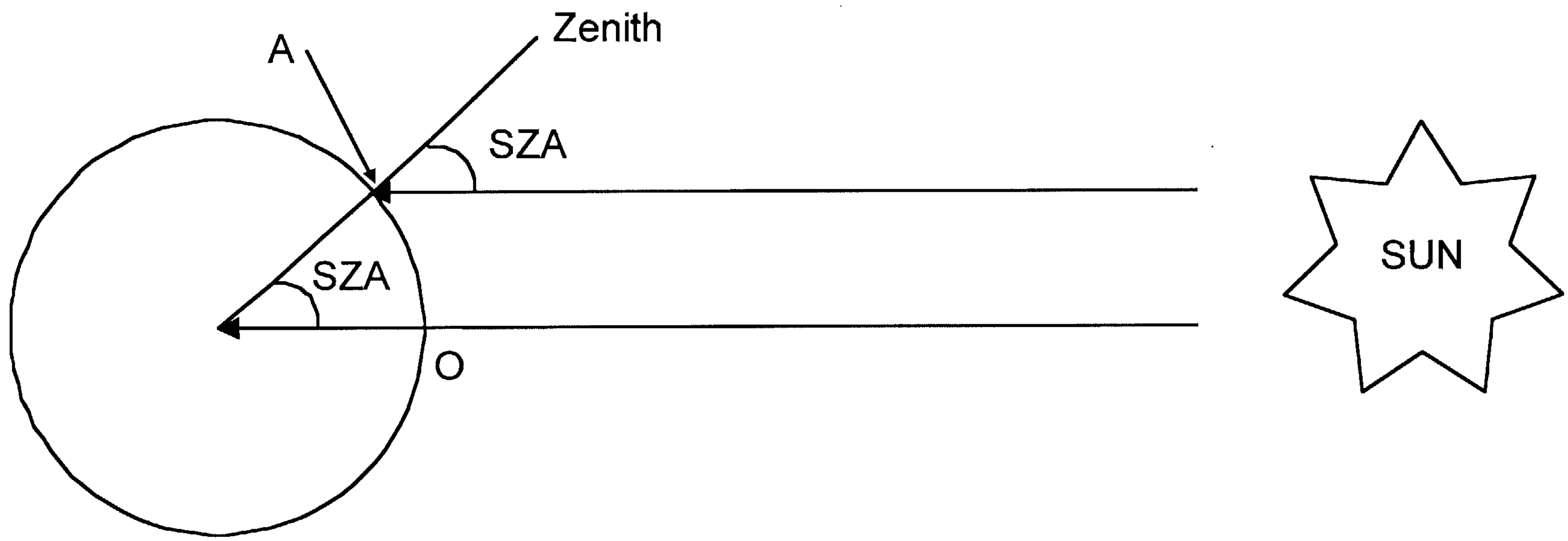
The activities and/or deliberations are different at each group and self–directed making the classroom abuzz with discussions, movement and hard work. The activities and focus of the discussions are designed by the students according to what they want to explore at that moment creating a high level of interest, engagement and motivation. If the teacher is the designer of the group activity it is usually based on the students' needs maintaining still the high level of student interest and commitment. This group model encourages critical thinking skills, deeper and authentic learning, and participation on the part of the students.

Even though the students are self–directed, they are bound by a curriculum that gives focus to the constructivist class. One misconception about constructivism is that children do what they want and therefore dictate the curriculum to be taught. This could not be further from reality. The curriculum is ever present in the teacher's mind, leading her/his planning and actions throughout the lessons. This facade of chaos is apparent to outsiders but not to the classroom teacher. The teacher observes how the students are approaching a problem, the kind of questions they are asking and how they answer them, what concepts are solid and which ones are still evolving. The teacher assesses what the students needs are and how to best help them in order for them to evolve their concept. In this respect the students influence the planning of the lessons and oftentimes shape the curriculum; but they do not dictate it. The role of the constructivist teacher is significant in that she/he has to be aware of the students at all times in order to set up the projects and plan the next lessons based on what students were working on the prior the class. The educator has clear what fundamental concepts she/he wants the students to learn, as established by the classroom curriculum, and steers the students to remember the questions they set out to answer. The educator refocuses the students, suggests a new direction, or encourages the students' selected path. The teacher is more of a coach, a designer and a facilitator than a presenter of "knowledge" and information.

The constructivist teacher organizes the curriculum around conceptual clusters from which the students develop their understanding. In more traditional settings, the teacher may "give" the students what the students "need" to learn and often these are a series of facts isolated from each other and the learner. The student is passive in that he/she takes what it's been given as a series of must-be-memorized essentials. In a constructivist class the activities the students do are relevant to them and the unit they are learning, i.e. they have to use the knowledge acquired as a tool to solve the larger question they are trying to answer. Geometry, science, history, language arts are tools that students use when they need them in meaningful ways.

In more traditional classrooms, students ask questions with the expectation of being given the answer. Their question is more of curiosity for a fact than investment. In a constructivist class the students' questions are very different. Oftentimes, they will ask a question of which they expect no answer from the teacher but are an indication of their own reflection and future plans. The teacher too asks questions not always expecting an immediate answer but to motivate student critical thinking and reflection. Students and teacher have an even intellectual exchange in which both are learning and discovering new elements together.

The bond between student and teacher in a constructivist class is strong. What links the classroom community together is having a common goal; we are learning and exploring together. The teacher is as much a learner as the students are teachers. There is basic respect for each other's opinions and knowledge. The students feel acknowledged and their sense of importance equals their contributions to the class. The teacher feels free to express her/his limitations and appreciate when the students teach her/him something she did not know, thus making a contribution in the teacher's growth. The classroom experience becomes a two way street in which all are responsible for each other and oneself.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