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Nucleation of Quantized Vortices from Rotating Superfluid Drops

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A. Objectives

The long-term goal of this project is to study the nucleation of quantized vortices in helium II by investigating the behavior of rotating droplets of helium II in a reduced gravity environment. The objective of this ground-based research grant was to develop new experimental techniques to aid in accomplishing that goal. The development of an electrostatic levitator for superfluid helium, described below, and the successful suspension of charged superfluid drops in modest electric fields was the primary focus of this work. Other key technologies of general low temperature use were developed and are also discussed below.

B. Background and introduction

The levitation of liquid helium drops provides a means of studying a number of interesting phenomena, for which the presence of rigid surfaces bounding the fluid needs to be avoided. One fundamentally important example is the nature of quantized vortex nucleation in Helium II, resulting from spin-up of the superfluid. It is understood that this process is masked by the growth of "remnant" vorticity pinned on surface imperfections of an ordinary container. On the other hand, a levitated drop of Helium II has no pinning sites and therefore provides a volume free of residual vortices. Such a drop, then, can offer an opportunity to observe the intrinsic nucleation of vortices generated by fluid rotation and to compare this process, for example, with the nucleation due to moving ions in a superfluid.

In general, the conditions for which different states of matter nucleate as a relevant control parameter is varied can be complex and the proper variable difficult to single out. The intrinsic nucleation of quantized vortices in helium II is an example of a nucleation problem in which very clean experiments can be done, owing to the fact that helium at low temperatures is easily free of contaminants and can be made nearly isotopically pure.

To better understand better the motivation for nucleation experiments utilizing rotating levitated drops it is at first instructive to consider the well known problem of vortex nucleation in a cylindrical bucket of helium II rotating at angular velocity $\Omega$ (see Figure 1). Helium II is the phase of liquid helium below 2.176 K which exhibits superfluidity. Hydrodynamically, it acts as if it were an interpenetrating mixture of a normal, viscous, fluid and a superfluid, having no viscosity or entropy. If the two fluid model of helium II were correct in its simplest form, a bucket of helium II at low temperatures, being predominantly superfluid, would simply not come into rotation in a rotating bucket experiment. The free surface, for example, would not be curved in rotation.

It has been known, however, for many years, however, that a bucket of helium II rotates entirely classically because of the existence of quantized vortex lines. The situation in a rotating bucket when vortex lines are present is shown in Figure 1: They form a uniform array whose vorticity $2\Omega$ just matches the vorticity of the normal component of helium II. The problem...
arises when one tries to discover from where these vortices come from. Many lines of evidence suggest they must come from the boundaries of the bucket. However, it is not hard to show that because of image effects, the nucleation of a vortex line of any substantial length from the walls of a bucket is opposed by an almost insuperable energy barrier. The conclusion is that the vortex lines must come from some pre-existing source, probably vortices trapped by pinning sites on the walls (Awschalom and Schwarz, 1984). When this kind of situation prevails, one refers to "extrinsic nucleation". While extrinsic nucleation is undoubtedly important, it is not as fundamental as the "intrinsic nucleation" problem where vortex lines appear when before none were present.

![Figure 1](image1.png)

Figure 1. Quantized vortex lines in a rotating bucket of helium II. The appearance of these vortices is assumed to be an example of extrinsic nucleation.

This research program was designed to understand how the first quantized vortex lines appear in a rotating drop of helium II.

The only nucleation situation investigated so far in which remnant vortices are unlikely to exist is ion motion at low temperatures. Muirhead, Vinen and Donnelly (1984) have calculated the barrier to intrinsic nucleation of an initial vortex loop from an ion at low temperatures, and have argued that penetration of the barrier comes by quantum mechanical tunneling of the loop. The height of the barrier has been confirmed experimentally by Hendry et al. (1988).

Another example of a situation in which intrinsic nucleation would likely occur is in a freely suspended drop of Helium II, which would presumably have no pinning sites for trapping remnant vortices. Therefore we expect its rotation might also lead to the intrinsic nucleation of vortices at some critical angular velocity well above that needed in the case of the rotating bucket.

While other recent work investigating helium drop levitation has utilized dielectric or diamagnetic properties of helium (Weilert, et al. 1995, 1996), we proposed to study vortex dynamics in a drop that is levitated by purely electrostatic means, both to minimize shape distortion due to high magnetic and electric fields and as a robust alternative to levitation techniques being developed elsewhere. Purely electrostatic levitators can, in principle, be used for drops up to 1mm in diameter in normal gravity.
The force balance for electrostatic levitation is given simply by \( mg = qE \), in terms of the mass \( m \), and charge \( q \) of a drop acted upon by gravity \( g \) in a region of opposed electric field strength \( E \). Electrically charged and levitated drops have been investigated over the last century (Rayleigh, 1882; Wilson, 1929; Adornato and Brown, 1983), with much recent general interest related to positioning in space-materials-processing applications (Rhim, et al. 1985). Perhaps the most well-known electrostatic levitation device was that used by Millikan (1924) to determine the electronic charge. Indeed, an apparatus described below can be considered a variation of that device, where it should be noted that Millikan necessarily studied considerably less massive droplets having few units of charge.

A complication of our approach is the well known fact that an electrostatic approach requires active control as a consequence of Earnshaw's theorem (Earnshaw, 1839) which states that electric charges cannot be stably bound in an electrostatic potential well. In essence, the requirement for restoring forces in three dimensions implies a non-zero divergence of the electric field. Below, we discuss the design of an electrostatic device having lateral restoring forces and requiring only active control vertically.

C. Task Significance and justification for future zero gravity experiments

1. With electrostatic levitation, larger drops without shape distortion can be positioned in micro-g. From consideration (Adornato and Brown) of the limiting cases of zero field (Rayleigh limit) and zero charge (Taylor limit), the maximum radius of a levitated helium drop can be estimated as \( r_{\text{max}} = (\sigma / \rho g)^{1/2} \) where \( \sigma \) is the surface tension of the liquid helium, \( \rho \) is its density and \( g \) is the acceleration due to gravity. Constants of order unity have been omitted. While this is about 0.5 mm for Helium II drops in normal gravity, it could be increased orders of magnitude in orbit, depending on the acceleration noise level. Shape distortion is minimized by using smaller confining fields than required, e.g., for diamagnetic or dielectric based levitation.

2. The effect of surface charges on vortex nucleation can be tested by comparing it with similar experiments conducted using uncharged drops. In micro-g, either neutral or charged drops may be positioned in a weak magnetic bottle created by a pair of small solenoids.

3. In microgravity, arbitrarily shaped seed particles can be introduced into the drops without the limitation of neutral buoyancy. This opens up the possibility of a wide variety of light-scattering experiments to study flow patterns within the drops.

D. Relationship of the overall task area to NASA objectives

The droplet experiments constitute not a single experiment, but a class of experiments which should stimulate and justify implementation of new flight equipment and capabilities in the future. It is the coincidence of new developments in helium seeding technology (Donnelly, et al. 2001) and application of advanced light scattering techniques (Du, et al) which may open an entirely new field of study for container-less superfluids.
E. Progress during the grant period

Superfluid helium drops have been successfully levitated at the University of Oregon using an electrostatic levitator shown schematically in Figure 2. Here, the two horizontal plates are about 6 cm in diameter, and are separated by a 2-cm tall cylindrical ring electrode. This ring electrode has small window areas removed for viewing and illumination. In this scheme, the drops are injected upwards into the trap, so that the initial field strength is close to that needed for levitation. The tip electrode is shown inserted into a small hole in the bottom plate. The original intent was a pulsed variant of an electric fountain effect discussed by Niemela (1997), albeit with positive currents. As it turned out, the best results were obtained by having the tip completely out of the bath. Applying a large positive voltage pulse resulted in numerous charged drops of various sizes being repelled upwards. This pulse was produced by a custom-designed generator (Ion Physics Corp.) which could deliver single pulses of duration 1µs or less and of variable height 4-10kV. The charging was accomplished by a point-plane gap corona discharge rather than field ionization so a sharp field tip was really not necessary. This made the process much more robust, since it is difficult to maintain sharp tips under these conditions for the duration, say, of any single flight experiment. The ring electrode was added to provide a horizontal centering force and to assist in positioning the drops.

The effect of this additional electrode can be simply understood in the case where it is grounded, along with the bottom horizontal plate, while the top plate is at some negative potential $V_0$. The potential in a trap of height $H$ and radius $R$ then has the form (Elliot, 1993):

$$\varphi(r, z) = 2V_0 \sum_{m=1}^{\infty} \frac{J_0\left(\gamma_{0m} \frac{r}{R}\right) \sinh\left(\gamma_{0m} \frac{z}{R}\right)}{\gamma_{0m} J_1(\gamma_{0m}) \sinh\left(\gamma_{0m} \frac{H}{R}\right)}$$

Various equi-potential lines corresponding to Eq. 1 are plotted in Figure 3. From this figure it is evident that there are centering components to the electric field that increase in relative magnitude with radial distance from the cell center and in absolute magnitude with the vertical field.

The voltage to the top plate was easily adjustable by using a knob-operated variable voltage divider on the output of the power supply. Charged drops were trapped by visually focusing on an individual drop on the video monitor and continuously adjusting the top plate voltage to keep the drop in view. Drops whose charge-to-mass ratios differed from the one being followed were then lost from the trap. Drops were typically stabilized in about 10 seconds horizontally and the top plate voltage could then be more finely adjusted to fix the drop vertically. On the other hand, by deliberately over- and under-compensating the vertical field,
the drops could be made to oscillate straight up and down with fairly large amplitude with no discernible lateral movement, indicating field uniformity. In its present rudimentary form, the trap has no automatic position-sensing scheme, although that technology can be borrowed almost directly from the laboratory work of W.K. Rhim and colleagues at JPL.

It was found that the charge-to-mass ratio proved to be very stable at a temperature of about 1.85K, and this is where most experiments were done. Indeed, it was possible to levitate drops without any discernible changes in the required field for long periods of time. The longest such suspension was for 20 minutes before losing visual contact with the drop. On the other hand, at temperatures closer to 2K (but still below the lambda point) there was a noticeable loss of charge from the drops, so that the levitation field required continual increases. This may reflect the added thermal energy of the ions and their subsequent loss through evaporation from the drop surface. Below 1.85K the charging method was somewhat less efficient, owing to the decrease in density of the gaseous medium.

Figure 2. Levitation apparatus for stable levitation of 100-150 micrometer drops. A point electrode is mounted from the bottom and connected to a high voltage pulse generator. A cylindrical ring electrode surrounds the gap.
Figure 3. Equipotential lines for the cylindrical electrode model (Eq. 1) of the ring levitator. Height $z$ and radius $r$ are plotted in terms of the overall height $H$ and radius $R$.

Most of the drops easily captured by the above method were of order 100 micrometers in diameter. The average levitation field required was about $23,500 \text{ V/m}$ (i.e., $-470 \text{ V}$ on the top plate with the bottom plate grounded). Correspondingly, the charge-to mass ratio of the levitated drops is deduced to be $4.17 \times 10^4 \text{ C/kg}$, meaning there are about $2 \times 10^5$ positive ions per drop. For a surface charge layer this corresponds to about $6.3 \times 10^{15}$ ions/m$^2$, which is roughly an order of magnitude smaller than the maximum charge density calculated from the Rayleigh limit for a drop of surface area $A$:

$$\left(\frac{Q}{A}\right)_{\text{max}} = \left(\frac{4\varepsilon_0 \sigma}{r}\right)^{1/2}$$

where $\sigma$ is the surface tension, and $\varepsilon_0$ is the permittivity of free space. For a 100-micrometer diameter drop Eq. (5) yields a maximum charge density of $9.3 \times 10^{13}$ ions/m$^2$.

The maximum size for stable electrostatically levitatable helium drops can be calculated (Adornato and Brown, 1983; Millikan, 1924) by considering the limiting cases of zero field (Rayleigh limit), and zero charge (Taylor limit). Specifically, Adornato and Brown (1983) give a locus of stability points in the dimensionless $\bar{E} - \bar{Q}$ phase space which smoothly connect these limiting values, where

$$\bar{E} = E \left(\frac{\sigma}{4\pi\varepsilon_0 r}\right)^{-1/2}$$

and
with $E$ and $Q$ the dimensional electric field and charge respectively. Balancing gravitational and electric forces, the radius of a levitated drop is given by

$$
\tilde{r} = \frac{\sigma}{\rho g}
$$

where $\rho$ is the density of the drop. Substituting the maximum value of the product $E\tilde{Q}$ on the stability curve ($\sim 4.4$) in Eq.(8) gives the maximum radius of an electrostatically levitated drop as

$$
[\frac{3\sigma E\tilde{Q}}{4\pi\rho g}]^{1/2}
$$

which is about 0.5 mm for Helium II drops in normal gravity. This is indeed very close to what is observed in these experiments. It is easy to see from Eq. 6 that micro-gravity conditions will greatly alleviate this limitation.

Finally, several modifications were made to the apparatus in order to have more control on the position and motion of the stably levitated drops. Another ring electrode was constructed out of two half-circles so that a translational field could be superimposed onto the vertical levitation field. It turned out to be relatively easy to move the drops from side to side as desired or diagonally by adjusting both lateral and vertical fields separately. By applying a time-varying lateral field, the drop was observed to oscillate about its mean position at the frequency of the applied voltage. This oscillation of the drop had no significant effect on the stability of its levitation, and demonstrates the possibility to induce rotation of the drop via time-varying lateral electric fields. Thus, the required features of an apparatus to rotationally induce vorticity in container-less superfluids without sizable shape distortion have been achieved to a great extent.

Theory

Finally, we mention that Bauer et al. (1995) in related theoretical work at Oregon, have numerically investigated the behavior of a single quantized vortex line in a spherical drop of liquid helium at temperatures below the lambda transition. The results of this additional investigation at Oregon show that it is possible to contain a vortex segment in a spherical drop of liquid helium by rotating the drop and that there is a critical angular momentum below which the drop cannot support the existence of the vortex, thereby confirming the conjecture that the drop can have an initial vortex-free configuration.

The dielectric constant of liquid helium

Due to a paucity of reliable information in the literature, it was undertaken in the course of this research to design an experimental program to precisely and accurately measure...
the temperature dependence of the dielectric constant of liquid helium, particularly in confined samples, and to this end a novel device of potentially general low temperature use was designed and built. In fact, we have developed a micronic-gap capacitive transducer, with in-situ control and measurement of the gap height in a parallel plate configuration, for determining liquid helium properties from the dielectric constant. Such quantities include the thermodynamic response functions: heat capacity $C_p$, isobaric thermal expansion coefficient, or isothermal compressibility. We were able to rule out many actuators for obtaining in-situ variation, measurement and control of a micron-size gap from sub-micron levels over a range of at least two orders of magnitude. The work entailed a search for the optimum actuators. Piezoelectric crystals were deemed unreliable and had insufficient travel to be useful at 2K. A possible solution was Strontium Titanium Oxide (STO) which was reported to have an enormous piezoelectric effect at low temperatures. After some investigation, we found that the original work was in error (as the authors later confirmed) and thus there are still no piezoelectric materials fully suited for our task. Magnetostrictive actuators, developed at JPL, have the disadvantage of needing large electrical currents. Finally, we have settled on a variation of a common loudspeaker to produce large plate movements at cryogenic temperatures and with small currents. These voice-coil actuators have a stroke of 75 micrometers at 200mA at both room temperature and at 4K. We can further improve these by reducing the magnet-coil gap, increasing the number of turns, optimizing the core, etc. Furthermore, the actuators are only 1 cm tall, compared with the piezoelectric stacks that were over 4 cm tall. This is crucial in terms of fitting these devices into low temperature cryostats. The main disadvantage is that without active control these actuators are more susceptible to vibrations than their piezoelectric counterparts. We have, however, perfected an optical feedback mechanism for controlling the plate separation as reported in two NASA workshops. With this new system one can now easily span the range from confined to bulk behavior in a single system with non-varying surface conditions.

The gap itself is formed by two highly polished fused silica plates, the lower one movable and the upper one fixed. Coarse height and tilt adjustments may be made before the cell is sealed via 3 screws; in operation, the gap is determined and maintained parallel by a feedback system of 3 servo-actuators. The feedback is provided by measurements of interference fringes produced by 3 Fabry-Perot interferometers, one above each actuator. A thick Au film on the upper surface of the lower (movable) plate serves as one of the Fabry-Perot mirrors and as one of the capacitor electrodes. On the upper plate are the partially reflecting Fabry-Perot mirrors and a set of electrodes. To maintain parallelism and correct spacing (gap), the light from a 670 nm laser beam is split into 3 beams. Each beam then passes through a diplexer, which is simply a Cr/Au mirror with a pinhole. The light travels through the multimode fiber optics and is focused onto the interferometer mirrors. The beams then reflect back into the fiber optics. When they return to the diplexers, most of the light reflects into photo diodes. This signal is used to control the current to the coils. Details are reported in various publications cited below [see, e.g., More, T., Dax, C., Niemela, J., Ihas, G. "A New Low temperature Device for High Resolution, In-Situ Measurement and Control of Submicron Gaps", *Journal of Low Temperature Physics* **121**(5/6): 825-830, December 2000].

To summarize this: a capacitor has been designed whose gap may be varied without need for disassembly, while maintaining a very precise level of parallelism. Furthermore, the gap may be changed while the apparatus is cold and filled with helium. One advantage of this design is
that the capacitor can be parked with a very small gap. This small gap acts as a filter to keep dust out while the capacitor is being transported to, and mounted on, the cryostat. Other advantages also make this new capacitor design very exciting. For instance, the surfaces (capacitor plates) defining the helium sample are the same for all measurements, something never achieved before with differing gap widths (crucial for high precision finite size effect measurements). There are no non-helium materials between the electrodes, i.e., spacers, avoiding problems with temperature-dependent materials masking the measurements of dielectric constant.

Due to its generic utility, there are other exciting scientific possibilities can be pursued with this device as well. For instance, with the gap wide open one may pool a helium film of some thickness on the lower plate. This film would then have one boundary condition determined by a solid surface and the other exposed to vacuum. After measuring the dielectric constant as a function of reduced temperature, the bottom plate can be moved so that the top plate contacts the helium with a gap equal to the free film thickness just investigated. The second set of measurements allows the direct determination, for the first time, of the nature of the vacuum/film interface.

This is ongoing work, and has involved the direct participation of undergraduate students. A paper with their contributed authorship has been submitted to the Journal of the Optical Society of America. We are pleased that our undergraduates were able to participate in actual laboratory experiments and the experience will be invaluable in furthering their careers in science.

F. Summary

In summary, we have succeeded in developing an implementing, for the first time, a purely electrostatic levitator for macroscopic superfluid helium drops. With additional electrodes we have been able to position the drops on independent axes and can oscillate them spatially- the precuser to rotation. With recent advances in low temperature light scattering techniques (Du, et al; Donnelly, et al.) the ultimate goal of rotation induced vortex nucleation in a pure superfluid drop is within reach. Furthermore, to facilitate our knowledge of the temperature dependence of the dielectric constant in confined samples on the order of the smallest drop diameters, we have developed and tested under this grant a Fabry-Perot device capable of in-situ variations in gap height at superfluid temperatures. This device will be of general utility in many phases of low temperature work, particularly in the study of finite-size effects near the normal-superfluid phase transition. All devices are described in more detail in the journal articles and proceedings listed at the end of this document below.

G. Bibliography

H. Undergraduate and graduate students

There were two undergraduate students who participated in this research and gained valuable research experience in the process: Sy Stange and Murat Alp. Gregory Bauer earned a Ph.D. during the course of this grant working on theoretical aspects of the problem.
I. Post-doctoral students and visitors

One post-doctoral student (Tamar More) was supported on this grant. Gary Ihas (UF Gainesville) was a visitor to Oregon during 1999 and directly assisted (at no cost) on the project.

J. Outreach

Two Lecture-Demonstrations related to this project were given at a local elementary school by J. Niemela (Edgewood Elementary School, Eugene, OR Grades 3-4, 1996. NASA’s role in basic scientific research was also represented by J. Niemela at a local high school (South Eugene high school) during a Career Day activity (May, 2001).

K. Publications and presentations resulting from this work

Presentations


Journal publications


MSAD Task book