APPLICATION OF MONTE-CARLO ANALYSES FOR THE MICROWAVE ANISOTROPY PROBE (MAP) MISSION

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ABSTRACT – *The Microwave Anisotropy Probe (MAP) is the third launch in the National Aeronautics and Space Administration's (NASA's) a Medium Class Explorers (MIDEX) program. MAP will measure, in greater detail, the cosmic microwave background radiation from an orbit about the Sun-Earth/Moon L_2 Lagrangian point. Maneuvers will be required to transition MAP from it's initial highly elliptical orbit to a lunar encounter which will provide the remaining energy to send MAP out to a lissajous orbit about L_2. Monte-Carlo analysis methods were used to evaluate the potential maneuver error sources and determine their effect of the fixed MAP propellant budget. This paper will discuss the results of the analyses on three separate phases of the MAP mission – recovering from launch vehicle errors, responding to phasing loop maneuver errors, and evaluating the effect of maneuver execution errors and orbit determination errors on stationkeeping maneuvers at L_2.*

KEYWORDS: Monte-Carlo, L2, Lagrange Point, phasing loops, lissajous orbit

INTRODUCTION

The Microwave Anisotropy Probe (MAP) is the third launch in the National Aeronautics and Space Administration's (NASA's) a Medium Class Explorers (MIDEX) program. The goal of the MAP mission is to measure, in greater detail, the cosmic microwave background radiation (the radiant heat left over from the Big Bang) as a follow-up to the successful Cosmic Background Explorer (COBE).

MAP launched on June 30th, 2001 from the Eastern Range on a Boeing Delta-II 7425 expendable launch vehicle (ELV). Following the burnout of the Delta third-stage, MAP was separated into a highly elliptical orbit (HEO) with an inclination of 28.7° and an orbit energy of -2.6 km^2/s^2. The MAP trajectory design dictated that the spacecraft would remain in this HEO for three or five loops, depending on the launch date (the June 30th launch utilized three phasing loops). During this time, perigee maneuvers were required to increase the orbit energy, raise apogee to the lunar distance, and alter the orbit period to properly time the lunar gravity assist – hence the name phasing loops. The lunar gravity assist was used to propel MAP on a three month trip out to the Sun-Earth/Moon L_2 Lagrangian point – 1.5 million km from the Earth in the anti-Sun direction. From this vantage point, MAP is free from environmental disturbances that could interrupt science observations. Periodic stationkeeping maneuvers, roughly three months apart, will be required to maintain MAP's lissajous orbit around L_2 for its nominal two-year mission. Figure 1 shows the MAP trajectory for the June 30th launch. In this picture, we see the three phasing loops, the lunar swingby, and the travel out to L2, where a single orbit is shown. Over the course of the mission design phase, it became desirable to perform statistical analyses of the different phases of the fixed ΔV budget (due to a propellant tank filled to capacity). The Monte-Carlo analysis method was chosen to perform these analyses for MAP.

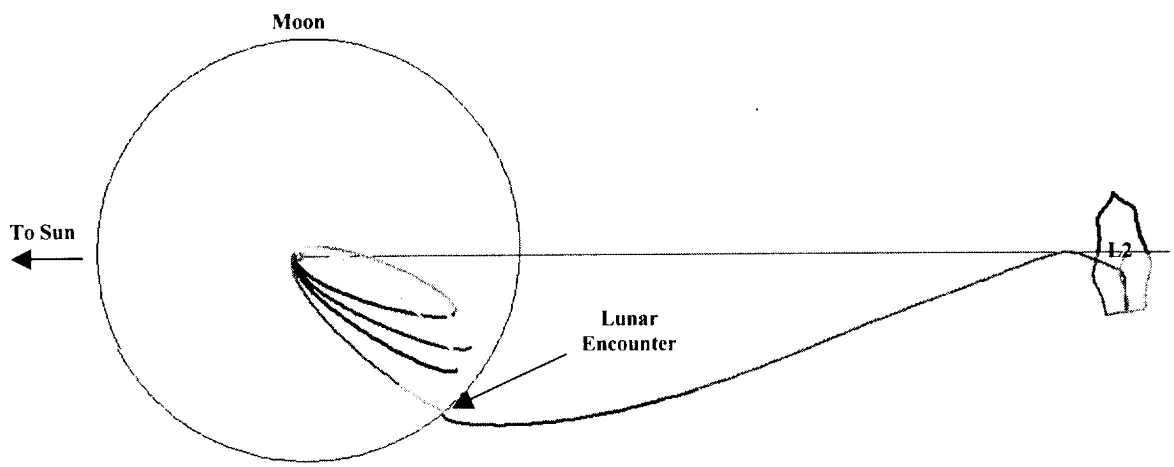

Figure 1: MAP Trajectory for June 30, 2001 Launch (Shown in Sun-Earth Rotating Coordinates)

METHODOLOGY

In Monte-Carlo analyses, random variables are identified and assigned probability distributions. For this analysis, an "active" Monte-Carlo mode was utilized. In the active mode, the random variables are sampled from the assigned Gaussian distributions and the trajectory is forward propagated until a desired boundary condition is met (e.g. the lunar encounter). This method is "active" because maneuvers can be retargeted to achieve the desired conditions at the boundary. Note that maneuvers used to retarget the trajectory cannot be used as random variables and vice versa. In general, the trajectory connecting the initial state to the desired conditions on the boundary will differ from the nominal case so mission constraints (e.g. shadows, min/max maneuver size, sun-angle on spacecraft, etc.) must be monitored during the forward propagation. Repeating this process using different values of the random variables allows statistics to be collected and analyzed.

Several different tools were used to complete this analysis. Analytical Graphics Inc.'s (AGI) Satellite Took Kit (STK) module Astrogator was used as the primary trajectory design tool for MAP analysis. In STK/Astrogator, the MAP mission sequence was built with successive propagation steps from launch to the lunar gravity assist (with the appropriate stops at the apogees and perigees along the way) and out to L2. Impulsive maneuvers were allowed at all perigees to be used as control variables for targeting the lunar encounter. Apogee maneuvers were inserted only as a control variable to keep subsequent perigee altitudes above the minimum allowable value of 500 km. All maneuvers at apogee and perigee were constrained to have only a tangential component (i.e. parallel to the velocity vector). The other main tool used was Mathwork's MATLAB. MATLAB was used as both a driver for the Monte-Carlo simulations and as means to collect and plot the results. As the Monte-Carlo driver, MATLAB interfaced with STK/Astrogator through AGI's Connect module, which allows STK commands to be executed through a socket connection. Other tools used included *FreeFlyer* (from *a.i. solutions, inc.*) and a "phasing loop calculator", a program that analytically computes potential perigee maneuver combinations needed to target a lunar encounter.

These tools were used in a variety of ways to perform Monte-Carlo analyses on the errors incurred from the launch vehicle, the execution errors incurred when performing a maneuver, and the orbit determination errors expected during operations at L2.

LAUNCH VEHICLE ERROR ANALYSIS

An important consideration in determining the viability of any given launch day is by determining the impact which launch vehicle errors have with respect the ΔV budget. The historical approach (used on previous missions WIND, Clementine, SOHO, ACE, etc.) to circumvent these difficulties was to assume that the overwhelming error source was the error in the magnitude of the transfer trajectory insertion (TTI) maneuver and that the required correction ΔV varied monotonically with the magnitude of this error. This "end-of-box" approach then required only two additional trajectories to be run for each launch date: one for a $+3\sigma$ TTI magnitude error and one for a -3σ TTI magnitude error. Although judged adequate on previous missions, questions from a MAP peer review panel prompted a re-examination of the validity of the end-of-box approach for MAP. At that point, it was decided that some form of Monte-Carlo analysis was needed to statistically determine the maximum amount of ΔV that was needed to correct for the ELV errors. In determining this value, we would ensure that a viable MAP trajectory could be obtained under any combination of possible pointing and energy errors (over the range from -3σ to $+3\sigma$) at TTI. In principle, the impact of the launch vehicle errors can then be determined by modeling the upper-stage injection with a random error sampled from a provided covariance matrix and then retargeting and optimizing the phasing-loop maneuvers to achieve a fuel-optimal trajectory that meets all the mission requirements. However, no tool is currently available that can fuel-optimize these types of phasing-loop, lunar gravity assist trajectories – especially when we have to consider turning apogee maneuvers on/off as they are needed. In an attempt to model the ELV error, a Monte Carlo analysis of the stability of the phasing loops with respect to the Delta-II dispersions in magnitude and pointing was performed. For the simulations, the magnitude error (11.6 m/s, 3σ) and pointing error ($2°$, 3σ) were modeled as Gaussian distributed errors. To account for the need to include an A1 maneuver on some of the trials, the first phasing-loop (TTI through P1) was modeled numerically in ***FreeFlyer®***. The simulation conditionally targeted an A1 perigee-raising maneuver when needed. A patched conic approximation, known as the phasing-loop calculator, was used to analytically estimate the minimum ΔV distribution across all of the perigees (done in MATLAB). The individual ΔV's for each maneuver (shown schematically in Figure 2) were kept for each trial for later statistical analysis. Following TTI, maneuvers at perigee are used to absorb the launch vehicle errors and phase into the lunar gravity assist while the apogee maneuver are needed to ensure that perigee altitudes remain above a safe minimum. MAP's ΔV budget dictated that the sum of all of the phasing loop maneuvers must be less than 70 m/s. A single 3-loop block was chosen for analysis in order to validate the end-of-box method.

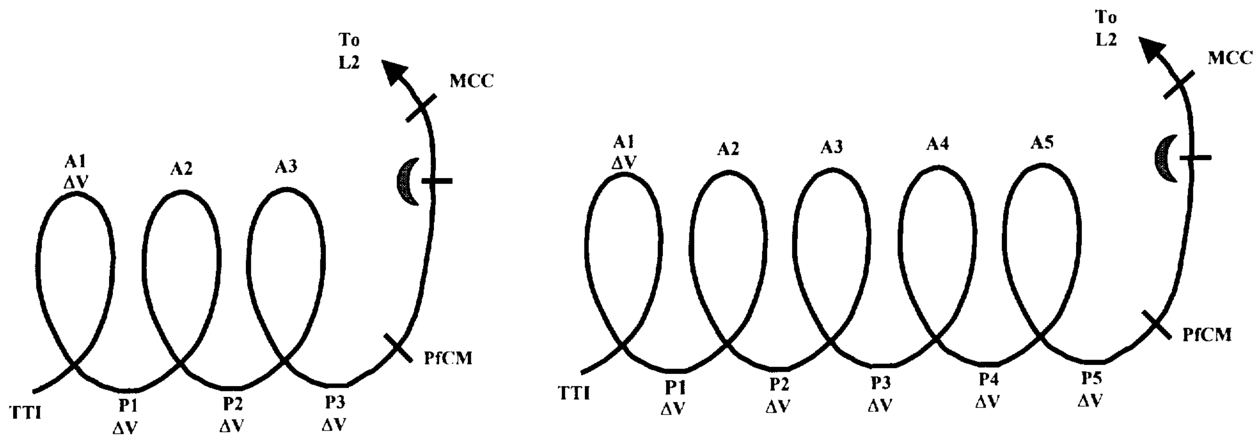

Figure 2: Schematics for MAP 3-Loop and 5-Loop Options

Figure 3 shows the Monte Carlo results for 250 trials for the May 3rd 3-loop case. The resulting mean and 3σ values of the distribution very closely match the end-of-box results for the same May 3rd launch date. In addition, we found that these results changed by a negligible amount when we turned the pointing error off and only modeled the magnitude error. The average ΔV costs from the Monte Carlo study were 28 m/s with a 3σ width of 11 m/s compared to the nominal cost of 27 m/s with an end-of-box width of 12 m/s. Note that different pairs of maneuvers (P1-P3, P1-P2, etc.) were used with different trials in an attempt to minimize the total ΔV. In particular, the nominal case had a zero ΔV cost associated with P2 while various trials showed values of P2 ranging up to (in magnitude) approximately 25 m/s.

Table 1 shows the results for the May 3-loop launch block. In general, the results of 3-loop (250 trials) cases show good correlation with corresponding "end-of-box" results for mid-block cases (May 3-5). Discrepancies at the end of the launch block most likely signal that the patched-conic approximation is no longer valid (due to strong solar or lunar perturbations, etc.).

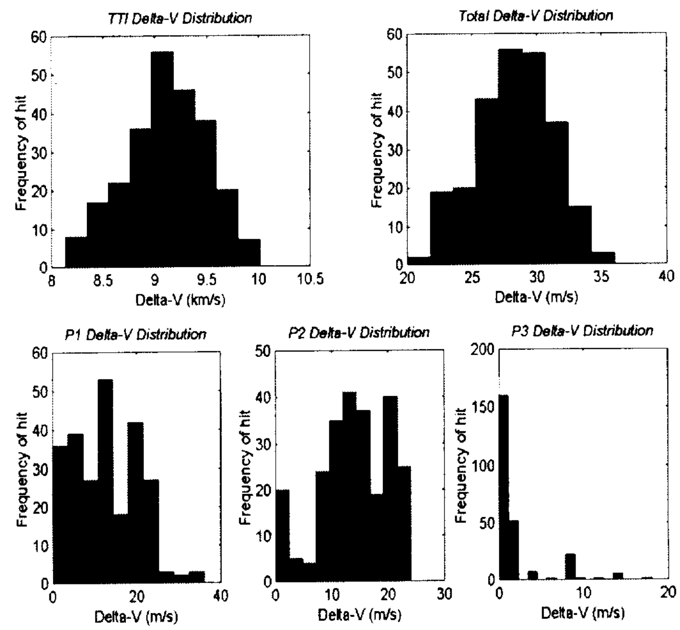

Figure 3: Monte-Carlo Histograms for 05/03/2001 Launch Case

Table 1: Launch Vehicle Monte-Carlo Results for MAP's May Launch Block (3-Loop)

Date	End-of-Box			Monte-Carlo			Apogee Maneuver? (YES | NO)
	ΔV_{avg} (m/s)	$3\sigma\ \Delta V$ (m/s)	ΔV_{TOTAL} (m/s)	ΔV_{avg} (m/s)	$3\sigma\ \Delta V$ (m/s)	ΔV_{TOTAL} (m/s)	
May 2nd	26.6	15.0	41.6	27.6	11.4	39.0	YES
May 3rd	26.5	12.4	38.9	28.3	11.1	39.4	NO
May 4th	28.2	8.8	37.0	28.3	11.5	39.8	NO
May 5th	29.5	8.9	38.4	27.9	9.7	37.6	NO
May 6th	28.5	11.6	40.1	28.9	6.8	35.7	NO
May 7th	28.3	17.2	45.5	34.3	14.0	48.3	YES

The Monte-Carlo results from this study of a single MAP launch block were sufficient to show the validity of the "end-of-box" method. They furthermore showed the ability to find viable launch days that met the MAP ΔV budget limit of ≤ 70 m/s in the phasing loops.

PHASING LOOP MANEUVER ERROR ANALYSIS

In this phase of the analysis, a maneuver execution error study was performed to examine the effects of several random error sources (in particular, thruster performance and attitude) on planned finite maneuvers. Monte-Carlo simulations were performed to model thruster efficiency errors ($\pm$ 5%, 3σ) and pitch errors ($\pm$ 5°, 3σ) during the maneuvers at the first perigee (P1) and the final perigee (P_f) in the phasing loops. This method was applied to several launch cases (including both 3- and 5-loop scenarios) in order to validate the 5% execution error allocation in the ΔV budget. Table 2 shows the four launch cases that were examined. For each launch day, there existed a set of maneuvers needed to target the lunar encounter. In all cases, there were perigee maneuvers at the first and last perigees and apogee maneuvers were only used to ensure a safe minimum perigee of 500 km. Included in Table 2 are the maneuvers required for a nominal launch and for the "end-of-box", ±3σ launch vehicle errors. Each column will be represented with a separate Monte-Carlo simulation for it's respective P1 and Pf maneuvers, a total of 24 simulations.

Table 2: Launch Days & Maneuvers Used for Error Analysis Study

Maneuver	04/18/2001 (5-loop)			05/04/2001 (3-loop)			06/30/2001 (3-loop)			07/16/2001 (5-loop)		
	Nom	+3σ	-3σ	Nom	+3σ	-3σ	Nom	+3σ	-3σ	Nom	+3σ	-3σ
A1	6.9	5.4	7.9	0.0	0.0	0.0	0.0	0.0	2.9	5.2	2.4	7.1
P1	11.4	-0.2	24.4	15.5	-4.7	32.7	20.7	-5.9	35.8	11.4	-1.2	23.5
A2	13.8	13.4	13.7	12.6	19.1	7.9	0.0	0.0	0.0	0.0	3.9	0.0
Pf*	6.9	5.4	7.9	0.0	0.0	0.0	10.0	22.6	5.7	26.7	26.4	29.6

* Pf occurs at Perigee 3 for a 3-loop case and Perigee 5 for a 5-loop case

Once the random errors were applied in each case, it became necessary to correct back to some nominal trajectory. The MAP trajectory design team used B-Plane parameters at the lunar encounter as their targets of choice. As a refresher, the B-Plane is the plane perpendicular to the incoming asymptote of the approach hyperbola and is a common method used for targeting gravity assists. [5] For this analysis, a combination of targets was used - B•R and C3 energy. The B•R value is the normal component of the B-vector – the swingby distance above or below the lunar orbit plane. B•R was an important indicator in defining the phase of the final lissajous orbit at L2. The Earth-referenced C3 energy value at the MCCM point (7 days after the lunar encounter) was used as a target to ensure that the correct amount of energy was received from the swingby. [6]

The P1 execution errors were analyzed in the following manner. First, the impulsive maneuver was transformed into a finite maneuver using with MAP's hydrazine blowdown propulsion system. MAP utilized 4, 1-lb thrusters during its perigee maneuvers. At this point, a MATLAB script was executed which sampled random thrust efficiency and pitch errors consistent with the prescribed 3σ values. The script communicated these maneuver errors to STK/Astrogator through a socket connection and STK's Connect module. Astrogator was then used to re-target the P2 and Pf maneuvers to ensure that a viable swingby was achieved. After convergence, data was collected on the new maneuver sizes and the data was stored in MATLAB. This process continued until a sample size of 1000 trials was achieved. The results are presented in Table 3. The Monte-Carlo trials showed that the thrust efficiency is the dominant factor in the executions error. We see this in the strong linear correlation between ΔV error and thrust efficiency in Figure 4. The relationship between the ΔV error and the attitude error is much less correlated. This is reasonable as a 5° pitch error only causes a cosine loss of less than 0.5%. Figure 5 shows the ΔV cost as a percent of the P1 maneuver magnitude. It is interesting to note that the ΔV cost in a 3-loop case is much less (roughly half) than the cost in a 5-loop case. Seeing these results, it appears that the line item carrying a 5% penalty for execution errors appears to be too conservative for the 3-loop cases.

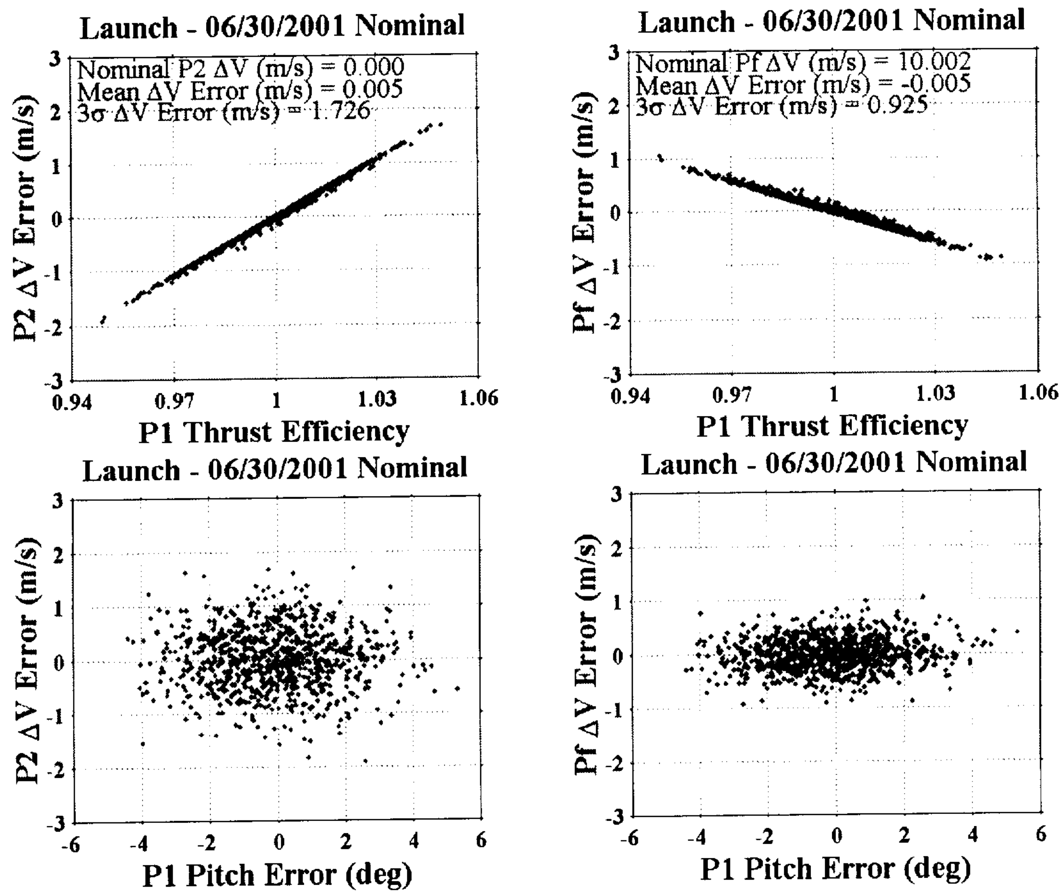

Figure 4: Results of P1 Execution Error Analysis for June 30, 2001 Launch Case

Table 3: P1 Execution Error Results (ΔV in m/s)

Maneuver	04/18/2001 (5-loop)			05/04/2001 (3-loop)			06/30/2001 (3-loop)			07/16/2001 (5-loop)		
	Nom	+3σ	-3σ	Nom	+3σ	-3σ	Nom	+3σ	-3σ	Nom	+3σ	-3σ
P2 ΔV Error	0.6	NA*	1.5	1.4	0.4	2.7	1.7	0.5	2.9	0.6	0.1	1.2
Pf ΔV Error	0.1	NA*	0.1	0.7	0.2	1.4	0.9	0.3	1.6	0.1	0.0	0.5
ΔV Cost	0.7	NA*	1.6	2.1	0.6	4.1	2.6	0.8	4.4	0.7	0.1	1.7
ΔV Cost (% of P1 ΔV)	6	NA*	6	13	13	12	13	14	11	6	6	7

* Error analysis not performed because the small maneuver size (0.2 m/s)

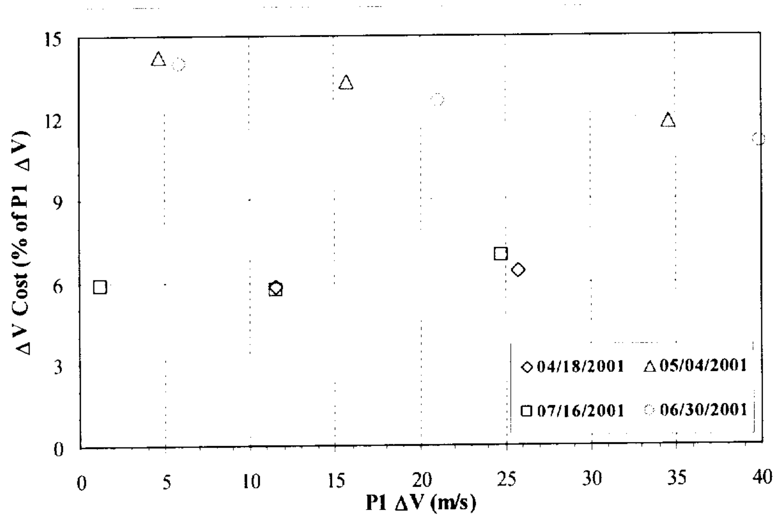

Figure 5: Graph of P1 Execution Errors as Function of P1 Magnitude

The Pf execution error analysis was performed in a similar manner as the P1 analysis. In this case, the Pf maneuver was perturbed, using the same error sources, and it was up to a Pf correction maneuver (PfCM) to ensure that the targets were met for a proper gravity assist. The PfCM is nominally planned to occur 18 hours after Pf. At the time, the PfCM was kept to 15 m/s in size – roughly 50% of the largest allowable Pf maneuver. In order to keep a "square" targeting profile (2 control variables with 2 constraints), it was necessary to use both the tangential and normal components of a ΔV maneuver at PfCM. Again, Monte-Carlo simulations were performed on each launch date/launch vehicle error combination (Table 4).

Table 4: Pf Execution Results (ΔV in m/s)

	04/18/2001 (5-loop)			05/04/2001 (3-loop)			06/30/2001 (3-loop)			07/16/2001 (5-loop)		
Maneuver	**Nom**	**+3σ**	**-3σ**	**Nom**	**+3σ**	**-3σ**	**Nom**	**+3σ**	**-3σ**	**Nom**	**+3σ**	**-3σ**
PfCM ΔV_T	2.3	2.3	2.1	3.4	4.9	1.8	2.6	4.5	1.3	4.4	5.1	4.0
PfCM ΔV_N	2.9	2.4	2.8	2.5	3.7	1.4	0.4	0.8	0.2	1.6	1.7	1.7
PfCM ΔV	3.7	3.4	3.5	4.2	6.1	2.3	2.6	4.6	1.4	4.7	5.4	4.3
PfCM ΔV (% of Pf ΔV)	26	25	26	33	32	29	26	20	24	17	20	14

It appears from the results that keeping the PfCM to a 15 m/s ceiling is very conservative again. All of the simulations yielded PFCM's that were much less than 35% of the Pf maneuver (Figure 6). In fact, one of the data points shows that a 30 m/s Pf maneuver, the July 16th -3σ case, yields only a 4.3 m/s maneuver (less than 15% of the size of Pf). A straight linear fit of this data shows that PfCM is roughly 21% of the Pf maneuver (Figure).

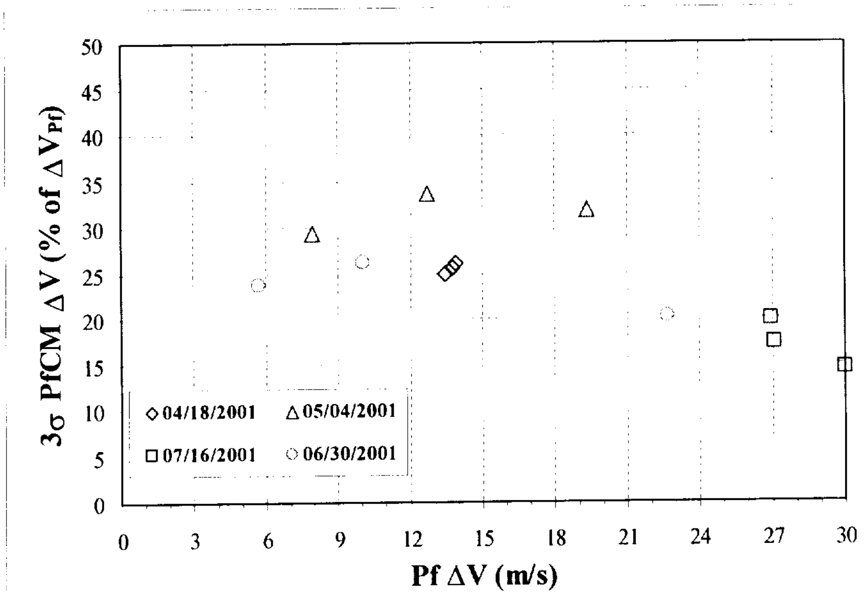

Figure 6: Graph of 3σ PfCM vs. Pf

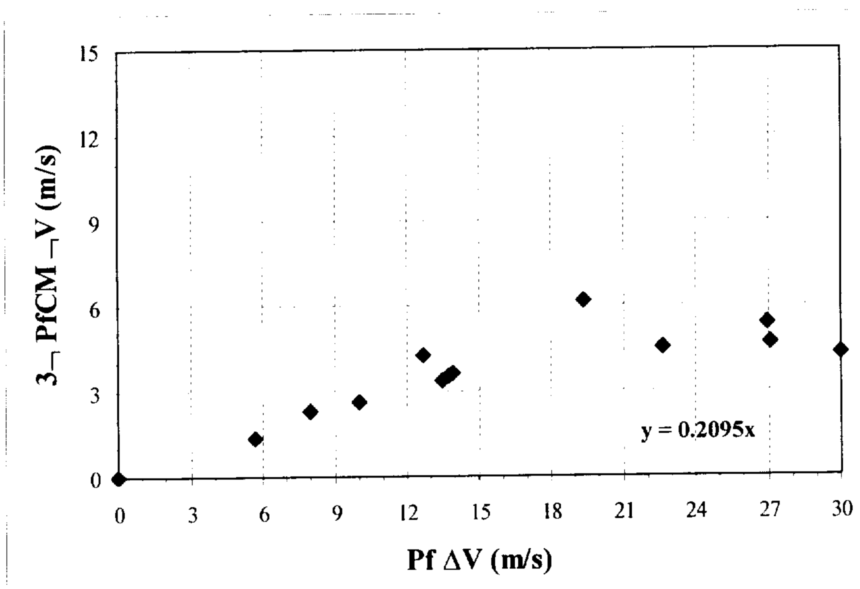

Figure 7: Linear Fit of PfCM from Pf

The phasing loop Monte-Carlo analysis was very beneficial in that it allowed us to update several ΔV budget items. We discovered a difference in the P1 execution error between the 3-loop and the 5-loop launch options. Also, it was determined that we were being very conservative in estimating the size of the Pf correction maneuver.

L2 STATIONKEEPING ERROR ANALYSIS

MAP will be required to perform periodic stationkeeping maneuvers in order to maintain its orbit about L2. For the stationkeeping analysis, it became apparent that the orbit determination errors were the primary error source. The objective of this phase of the Monte-Carlo analysis was to determine an attainable error budget for station keeping maneuvers, given the budgeted ΔV (95 cm/s per maneuver, 4 maneuvers per year). A Monte Carlo analysis consisting of eighteen different cases were run, using various values of position and velocity uncertainty. A subset of these runs was made using initial Lissajous states whose epochs corresponded to three different locations along the Lissajous orbit; the nominal case, 45 days after the nominal case, and 90 days after the nominal case. This was done to examine the effects of the station keeping maneuvers at various points along the lissajous orbit. Each Monte Carlo run consisted of one hundred trials.

The two tables below summarize the results from the Monte Carlo analysis. Both tables list the 3σ total fuel costs (the mean ΔV plus 3 times the standard deviation) for the simulated stationkeeping maneuver for a given position and velocity uncertainty as well as the location of the initial state on the Lissajous orbit. Table 5 results were run using a fixed value of 5km for the position uncertainty while varying the velocity uncertainty. Keeping in mind that the fuel budget for each station-keeping maneuver for this study is 95 cm/sec; the "Total" values must be below this to be considered acceptable. Referring to Table 5, a velocity uncertainty of up to 3.5 cm/sec can be tolerated and still meet the fuel budget (ΔV values for the 4 cm/sec case exceed the 95 cm/sec budget). Data is also presented for the other two Lissajous states. These results reveal that the location along the Lissajous orbit, at which the maneuvers are executed, does not significantly affect the ΔV costs.

Table 5: L2 Results - Vary Velocity Uncertainty, Position Uncertainty = 5 km

Velocity Uncertainty	Nominal state	45 days later	90 days later
3.0 cm/sec	ΔV_{avg} = 23 cm/sec	ΔV_{avg} = 22 cm/sec	ΔV_{avg} = 24 cm/sec
	$\Delta V_{3\sigma}$ = 51 cm/sec	$\Delta V_{3\sigma}$ = 53 cm/sec	$\Delta V_{3\sigma}$ = 55 cm/sec
	ΔV_{TOT} = 74 cm/sec	ΔV_{TOT} = 75 cm/sec	ΔV_{TOT} = 79 cm/sec
3.5 cm/sec	ΔV_{avg} = 27 cm/sec	ΔV_{avg} = 23 cm/sec	ΔV_{avg} = 26 cm/sec
	$\Delta V_{3\sigma}$ = 63 cm/sec	$\Delta V_{3\sigma}$ = 58 cm/sec	$\Delta V_{3\sigma}$ = 62 cm/sec
	ΔV_{TOT} = 90 cm/sec	**ΔV_{TOT} = 81 cm/sec**	**ΔV_{TOT} = 88 cm/sec**
4.0 cm/sec	ΔV_{avg} = 32 cm/sec	ΔV_{avg} = 31 cm/sec	ΔV_{avg} = 33 cm/sec
	$\Delta V_{3\sigma}$ = 74 cm/sec	$\Delta V_{3\sigma}$ = 63 cm/sec	$\Delta V_{3\sigma}$ = 72 cm/sec
	ΔV_{TOT} = 106 cm/sec	ΔV_{TOT} = 94 cm/sec	ΔV_{TOT} = 105 cm/sec

Once an acceptable level of the velocity uncertainty was determined, a similar study was performed in order to determine if the stationkeeping maneuvers were sensitive to the position error. In Table 6 we see the results of these simulations where the velocity uncertainty was held constant at 3.5 cm/s while the position uncertainty was varied. This portion of the analysis reveals the fact that velocity is indeed the major contributor to the fuel cost and that changing the position uncertainty has little effect on the results.

Table 6: L2 Results - Vary Position Uncertainty, Velocity Uncertainty = 3.5 cm/s

Position Uncertainty	Nominal state	45 days later	90 days later
2.0 km	ΔV_{avg} = 24 cm/sec	ΔV_{avg} = 27 cm/sec	ΔV_{avg} = 27 cm/sec
	$\Delta V_{3\sigma}$ = 55 cm/sec	$\Delta V_{3\sigma}$ = 54 cm/sec	$\Delta V_{3\sigma}$ = 62 cm/sec
	ΔV_{TOT} = 79 cm/sec	ΔV_{TOT} = 81 cm/sec	ΔV_{TOT} = 89 cm/sec
3.5 km	ΔV_{avg} = 27 cm/sec	ΔV_{avg} = 27 cm/sec	ΔV_{avg} = 24 cm/sec
	$\Delta V_{3\sigma}$ = 60 cm/sec	$\Delta V_{3\sigma}$ = 54 cm/sec	$\Delta V_{3\sigma}$ = 54 cm/sec
	ΔV_{TOT} = 87 cm/sec	ΔV_{TOT} = 81 cm/sec	ΔV_{TOT} = 78 cm/sec
5.0 km	ΔV_{avg} = 27 cm/sec	ΔV_{avg} = 23 cm/sec	ΔV_{avg} = 26 cm/sec
	$\Delta V_{3\sigma}$ = 63 cm/sec	$\Delta V_{3\sigma}$ = 58 cm/sec	$\Delta V_{3\sigma}$ = 62 cm/sec
	ΔV_{TOT} = 90 cm/sec	ΔV_{TOT} = 81 cm/sec	ΔV_{TOT} = 88 cm/sec

Through this analysis, it was determined that the orbit determination uncertainty of 5.0 km in position and 3.5 cm/s in velocity would satisfy the stationkeeping requirements for MAP.

CONCLUSION

Monte-Carlo analysis methods were used for three separate phases of the MAP mission in order to validate mission requirements. In the launch phase, the analysis showed that the ΔV budget could absorb the launch vehicle errors while being able to obtain a viable trajectory. More importantly, the results help to prove the validity of the "end-of-box" approach to analyzing ELV errors. The phasing loop analysis helped to correct assumptions about the P1 maneuver execution error. The early 5% assumption was appropriate for the 5-loop launch cases but proved to be too small for the 3-loop cases. A 13% error was finally budgeted for the 3-loop launch cases. The ΔV budget was further refined with the news that the PfCM was much small that was previously thought. The analysis provided an equation to estimate the PfCM as 21% of the size of the Pf maneuver. Finally, the stationkeeping analysis helped to determine the maximum acceptable orbit determination velocity errors.

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