

**GEOSTATISTICAL METHODS FOR DETERMINATION OF ROUGHNESS,
TOPOGRAPHY, AND CHANGES OF ANTARCTIC ICE STREAMS
FROM SAR AND RADAR ALTIMETER DATA**

NASA Polar Research Program, Project NAGW-3790 / NAG 5-6114

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— Final Report —

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Notice: A report to this grant had been written and sent to NASA, but was lost, likely in the transition of the offices to Arlington, VA. Since the report cannot be relocated, in the following there is a new report.

SUMMARY

The central objective of this project has been the development of geostatistical methods for mapping elevation and ice surface characteristics from satellite radar altimeter (RA) and Synthetic Aperture Radar (SAR) data. The main results are an Atlas of elevation maps of Antarctica, from GEOSAT RA data, and an Atlas from ERS-1 RA data, including a total of about 200 maps with 3 km grid resolution.

Maps and digital terrain models are applied to monitor and study changes in Antarctic ice streams and glaciers, including Lambert Glacier / Amery Ice Shelf, Mertz and Ninnis Glaciers, Jutulstraumen Glacier, Fimbul Ice Shelf, Slessor Glacier, Williamson Glacier and others.

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REFERECES

LIST OF PUBLICATIONS RESULTING FROM PROJECT NAGW-3790 / NAG 5-6114

APPENDIX: REPRINTS OF PUBLICATIONS

(1) INTRODUCTION: MAPPING AND MODELING OF THE ANTARCTIC ICE SHEET

The cryosphere plays a key role in the unstable global climate system. The polar ice caps and the Greenlandic inland ice shield are sensitive to changes in temperature (Huybrechts 1993). A collapse of the West Antarctic Ice Sheet could cause as much as 6 m sea-level rise (Bindschadler 1991). Discussion of instabilities of the Antarctic Ice Sheet was put forward as early as 1962 by J. Hollin. The mechanisms that may lead to ice-sheet collapse have been investigated and modeled in many studies, but are still a matter of debate (e.g., Alley and Whillans 1991). Earlier work culminated in conclusions of catastrophic consequences (Hughes 1973; Mercer 1978; Thomas 1977), whereas later modeling work showed that such catastrophic behavior is unlikely (Van der Veen 1985; Muszynski and Birchfield 1987). Scenarios for dynamic instabilities (ice-creep instabilities or even surging of the East Antarctic Ice Sheet) have been discussed (Clarke et al. 1977; Schubert and Yuen 1982). Huybrechts (1993) concludes from a simulation that most of the variability in Antarctic ice mass and hence in sea level results from changes in the West Antarctic Ice Sheet, whereas the East Antarctic Ice Sheet seems to be robust to temperature changes. Changes in the East Antarctic Ice Sheet are also discussed in Colhoun (1991). The hypothesis that the stability of an ice sheet grounded below sea level depends on the stability of its marine ice shelves (e.g., Mercer 1978; Thomas and Bentley 1978; Lingle 1984) indicates a need to study the Antarctic ice shelves. That rapid retreat does occur in present times is documented by the examples of catastrophic retreat of Columbia Glacier, Alaska (Meier and Post 1987) and of break-up of Wordie Ice Shelf, Antarctic Peninsula (Vaughan 1993) (both located in warmer climates). Ice streams moving 10 to 100 times as fast as the adjacent ice result in instability points in the dynamic system of an ice sheet. A prediction based on any model, however, can only be as good as the information on which the model is based. Many studies suffer from the fact that they are simulations lacking adequate data support. Satellite observations provide an efficient source of information for remote areas, and for large parts of Antarctica they are the best information presently available - once we understand how to use it right. One problem with investigations of the Antarctic ice mass is the lack of accurate topographic maps for large parts of the continent.

The widely used Antarctic glaciological and geophysical folio edited by Drewry (1983) contains maps of a small scale only. A topographic map of the Filchner-Ronne-Schelfeis based on satellite images and ground-based geodetic surveys was recently published by Sievers et al. (1993).

Most satellite payload yields images. Analysis of image data has many useful applications. Images of Antarctic ice streams have been compiled using AVHRR (Absolute Very High Resolution Radiometer) data from a NOAA (National Oceanic and Atmospheric Administration, USA) satellite (Bindschadler and Scambos 1991) with a 1-km spatial resolution. Images of higher resolution are obtained by the Synthetic Aperture Radar (SAR) data, which have become available to the scientific community through ERS-1/2, JERS-1 and RADARSAT (ESA 1992a,b, 1993; Canadian Space Agency et al. 1994). A major difficulty with the analysis of SAR data is that quantitative analysis is not directly possible. One promising avenue in that direction is the application of interferometry, a technique that exploits the phase differences of two images, but at the same location, possible in the rare situation of very close repeat of the ground tracks (Goldstein et al. 1993) and good correlation of the images to be compared (Zebker and Villasenor 1992). The best-known application is the extraction of the velocity of the ice (Goldstein et al. 1993). If no movement occurred and the environment did not change between the times of collection of the two images, it is possible to compute topography from pairs of SAR images using interferometry. There is ongoing work on construction of elevation maps from SAR stereo images, but that has yet to be completed. Examples of applications of

interferometry are restricted to date to the study of smaller regions, and SAR images can only be collected for 10 minutes per revolution. The technique is not suitable for mapping large areas of the Antarctic ice, leaving ample necessity for altimetry-based mapping.

The best data source for topographic mapping from satellite is altimetry. The first satellite carrying an altimeter became operational in 1978 (SEASAT). Together with data from the GEOSAT Geodetic Mission (1985-86) and the Exact Repeat Mission (1987-89) and data from ERS-1 (1992-96) and ERS-2, almost a 20-year record of altimeter data is available. This makes altimeter data the type of data most suited for the study of changes on a regional or continental scale for length of record. One disadvantage of studying Antarctica by satellite data is that the orbital coverage of the previously mentioned satellites does not extend to the poles.

Geostatistical analysis of satellite radar altimeter data may be utilized to construct maps of 3-km-by-3-km resolution of areas several 100 km large, which have a high accuracy (Herzfeld et al. 1993, 1994). Bamber (1994) produced a map of Antarctica (north of 82° S) from ERS-1 altimeter data with 20-km grids. Limitations of this map are the lower resolution and the fact that the map is only reliable in areas with a slope of less than 0.65° (Bamber 1994). By total area most of Antarctica is flatter than 0.65°, but the steeper regions include the dynamically important ice streams and outlet glaciers.

The geostatistical method (cf. Herzfeld et al. 1993) facilitates calculation of maps of higher accuracy and including steeper areas, but is computationally more intensive. The need for higher resolution is not well met if all of Antarctica is shown on one map sheet. An alternative is to construct an atlas, which in turn requires specific cartographic considerations.

The central task of this project has been the calculation of an atlas of Antarctica, consisting of DTMs and maps with 3 km resolution, from GEOSAT and ERS-1 radar altimeter data, along with development of the necessary processing tools and geostatistical methods; and resulting in glaciologic applications.

(2) REPORT: PREVIOUS WORK AND OBJECTIVES

Work under this project built on the development of geostatistical estimation (interpolation / extrapolation) methods and numerical implementation, specifically for the analysis of satellite radar altimeter data, resultant from my work under a previous NASA grant, for Lambert Glacier / Amery Ice Shelf (Herzfeld, Lingle and Lee, 1993, 1994). This method facilitates construction of digital terrain models with 3 km grid distance, high accuracy (50 m elevation accuracy on Lambert Glacier), and detection of the grounding line. Our result of a 10 km advance of Lambert Glacier (by change of grounding line position) settled the at the time open question of advance or retreat of Lambert Glacier (based on several cross-over analyses). Our result could also be confirmed from cross-over analysis (Lingle et al., 1994).

The original proposal NAGW-8790 (this project) had two central themes: (a) application of the geostatistical method to selected Antarctic glaciers and ice streams (Lambert Glacier, West Antarctic Ice Streams) and monitoring these glaciers; (b) development of a method to analyze SAR data. Following a request by NASA Polar Program manager Dr. Robert Thomas, the first objective was extended to evaluate all available radar altimeter data with the geostatistical method and produce maps for all of Antarctica (north of the limit of satellite RA coverage, 72.1° S for GEOSAT, 82.1° S for ERS-1), and the second objective was largely dropped (but see section (5.1) on geostatistical classification for results).

Mapping all of Antarctica with individual maps was a monumental task.

(3.) GEOMATHEMATICAL TOOLS FOR RADAR ALTIMETER DATA ANALYSIS

Geomathematical tools that were needed and developed for this project and are now available for other satellite data evaluation include:

- geostatistical interpolation (with search routines adapted for RA tracks)
- track-error correction routines
- atlas-mapping scheme
- TRANSVIEW tool

(3.1) Data acquisition and correction

A direct data connection between the Ice Sheet Altimetry Group at NASA GSFC (Dr. H.J. Zwally, Dr. J. DiMarzio and coworkers) and my group was set up for transfer of radar altimeter data. When ERS-1 data became available, our group was also instrumental in testing the necessary new correction algorithms and writing routines to identify bad tracks. Data processing by the Ice Sheet Altimetry Group includes: using the method of Martin and others (1983) for retracking, Goddard Earth Model (GEM) T2 orbits (Marsh and others 1983) for data reduction, and applying corrections for atmospheric effects and solid earth tides as described by Zwally and others (1983), slope corrections as described by Brenner and others (1983) and water-vapor corrections. After obtaining Ice Data Records (IDR) data sets, those points with retracked and slope-corrected data were retained. For each map sheet, a track plot is constructed to investigate coverage and ensure that coverage by retracked and slope-corrected data is sufficient. This was the case for all map sheets. After this processing, "bad" tracks with elevation (a) much lower than the surrounding area, or (b) of about constant small (50 m) difference to the surrounding area remained in several ERS-1 maps. An algorithm was developed to identify and remove these bad-track data. Elevation is given with reference to the WGS84 ellipsoid.

(3.2) Atlas mapping for continent-wide coverage

(3.2.1) Atlas concept

Rather than inverting all the Antarctic radar altimeter data onto a grid to produce a single map covering Antarctica (with, of course, a hole for the area south of the limit of radar altimetry coverage), we use an atlas mapping scheme. This improves resolution, facilitates mapping of detailed structures, and reduces distortion due to cartographic projection, which is particularly severe for high latitudes and large areas in most algorithms.

An atlas in the sense of differential analysis (Holmann and Rummler, 1972, p.63) is a set of maps that (i) covers a given area completely (that is, each point in the area is contained in at least one map); (ii) projections restricted to areas that appear on two (adjacent) maps (subsets of two maps) are identical on the intersection. In cartography, only property (i) is required for an atlas, and the neighbourhood relationships need to be matched between sheets.

A useful projection for mapping at high latitude and in atlas form is the Universal Transverse Mercator Projection (UTM) (Snyder, 1987), which results in an orthogonal coordinate system with coordinates in meters. Sufficient overlap of adjacent sheets is convenient for the user of the atlas and necessary to ensure that each point of Antarctica is contained in at least one map despite of the distortion of the map edges introduced by the projection algorithm.

(3.2.2) TRANSVIEW tool

One of the oldest problems in mapping the Earth is the definition of projections of the Earth's surface onto a two-dimensional map sheet. For mapping purposes, the Geoid is commonly approximated by a sphere or ellipsoid. Desirable properties of map projections are conservation of area (equal-area projection), of distances (equal-distance projection), of angles (equal-angle or conformal projection), which are mutually exclusive when mapping on a plane, and projection to a rectangular coordinate system (for examples see Hake, 1982; Snyder, 1987). Because it is not possible to satisfy all of these conditions, some projections have been defined that do not fulfill any conditions exactly, but a combination of them approximately (Hake, 1982; Snyder, 1987). For series of topographic maps in countries with a long tradition in mapping, algorithms have been designed to construct maps constituting an atlas.

For mapping Antarctica, however, we had to design a much-needed tool to convert, match and visualize UTM coordinates and geographic coordinates, to satisfy the Atlas mapping conditions.

From the viewpoint of interpolation of irregularly distributed data onto a regular grid, an orthogonal coordinate system facilitates the algorithm and saves computation time. The latter is especially important if distance-dependent measures are used, such as in inverse-distance weighting or in geostatistical methods.

Distance may be calculated on the sphere or on the ellipsoid (cf. Moritz, 1980; Torge, 1980), but this requires transformations at each step of the interpolation algorithm which usually is dependent on the number of points squared. In comparison, the number of essential operations for coordinate transformation depends only linearly on the number of points. Methods involving the covariance function or the variogram (kriging, least-squares prediction; cf. Ferzfeld, 1992) would require estimation of the structure function over the ellipsoid which would be troublesome. It is apparent that coordinate systems with orthogonal coordinates in meter units are thus extremely convenient for interpolation purposes.

Common practice is not to change coordinate systems, but to simply use geographic coordinates, which is unproblematic for small areas. For large areas, neglect of the coordinate transformation results in a severe distortion of the spatial structure in the data. (Recall that 1° latitude is always about 111 km, but 1° longitude is cosine of latitude times 111 km; so, at 60° North/South it is only 0.5 times 111 km or 55.5 km.)

The distortion is particularly severe for mapping at high latitude. The Arctic and Antarctic are usually treated separately in one map using the polar stereographic projection. Typically, such maps of polar regions are at a small scale and do not show much detail. The importance of the polar system in the Earth's global systems and its role in 'global change' have become increasingly recognized. A useful projection algorithm for mapping at all latitudes is the Transverse Mercator projection. A Mercator projection is defined by a cylinder that is tangent to the Earth and a mapping to orthogonal coordinates. For the (common) Mercator projection, the tangent circle is the Equator, for the Transverse Mercator projection, the tangent circle is a meridian (called the central meridian of the projection). The advantage of the Transverse Mercator

projection is that all latitudes are mapped with the same distortion. The disadvantage is that areas far away from the central meridian are strangely distorted. The solution provided by the UTM system is to rotate the cylinder around the Earth in steps of 6° . Zone 1 corresponds to central meridian 177° W. The standard central meridians are at 3° (for $0^\circ - 6^\circ$), 9° (for $6^\circ - 12^\circ$), 15° (for $12^\circ - 18^\circ$) etc. The central meridian is projected with a factor of 0.9996, lines of true scale are approximately parallel and lie approximately 180 km east and west of the central meridian. The border meridians are projected slightly lengthened, for example at 50° latitude with a factor of 1.00015. The projection is defined everywhere except 90° away from the central meridian. In the UTM scheme, the projection is chosen such that the central meridian is mapped to East coordinate 500,000, units are in meters; the North coordinate along the central meridian is in meters from the equator (along the ellipsoid). UTM is conformal and close to an equal-area projection, it has orthogonal coordinates in meters. The UTM projection, for instance, does not satisfy condition (ii) of an atlas, if two adjacent maps belong to two different central meridians; however, this defect may be compensated for by overlaps between neighboring map sheets.

The objective of our program TRANSVIEW is to provide a tool to calculate and visualize

- (a) the shape of a map that is rectangular in geographic coordinates when transformed to UTM coordinates
- (b) the amount of distortion for any map sheet on the Earth
- (c) the largest map that is rectangular in UTM coordinates and inscribed in a given map that is rectangular in geographic coordinates
- (c) the overlap necessary to map a large area in individual sheets using the UTM projection, and
- (d) to provide a system that works also for Antarctica.

TRANSVIEW works for any rectangular area on the Earth. The only restriction is that the area needs to be located entirely on the Northern or on the Southern hemisphere. The UTM projection is defined relative to an appropriate central meridian (3° W, 9° W, 15° W, etc., uneven multiples of 3°). For map areas that do not contain a central meridian of the UTM projection, an appropriate meridian needs to be determined by the user of the program. The latter is of particular importance for mapping small areas. The program discerns the location of the map relative to the central meridian and automatically selects the appropriate case for the transformation (see Figure 1). All possible cases are given in Figure 1. The algorithm is given in Herzfeld et al. (1999) and available from the website of the International Association for Mathematical Geology (<http://www.iamg.org/>). An example of transformation and visualization is given in Figure 2.

(3.2.3) Result: Definition of map sheets for the Antarctic Atlas Projects

GEOSAT ATLAS: For the GEOSAT Atlas, the rows and sheet tiling are defined as follows: rows: $72.1^\circ - 67^\circ$; $68^\circ - 63^\circ$; $64^\circ - 60^\circ$. Maps in row $72.1^\circ - 67^\circ$ are 546 km (183 gridnodes) E-W and 543 km (182 gridnodes) N-S. Maps in row $68^\circ - 63^\circ$ are 666 km (223 gridnodes) E-W and 531 km (178 gridnodes) N-S. Latitude 72.1° South marks the poleward limit of coverage of altimeter data from the Seasat and Geosat satellites. The size of sheets for the Antarctic atlas is defined as follows: 16° longitude span, 2° overlap on each side, 12° offset from one map to the next, and 1° overlap at top and bottom.

ERS-1 ATLAS: Map sheets of the ERS atlases are designed to match those of the GEOSAT atlas. The Antarctic is divided into rows of map sheets ($63^\circ - 68^\circ$ S, $67^\circ - 72.1^\circ$ S, $71^\circ - 77^\circ$ S, $75^\circ - 80^\circ$ S, $78^\circ - 81.5^\circ$

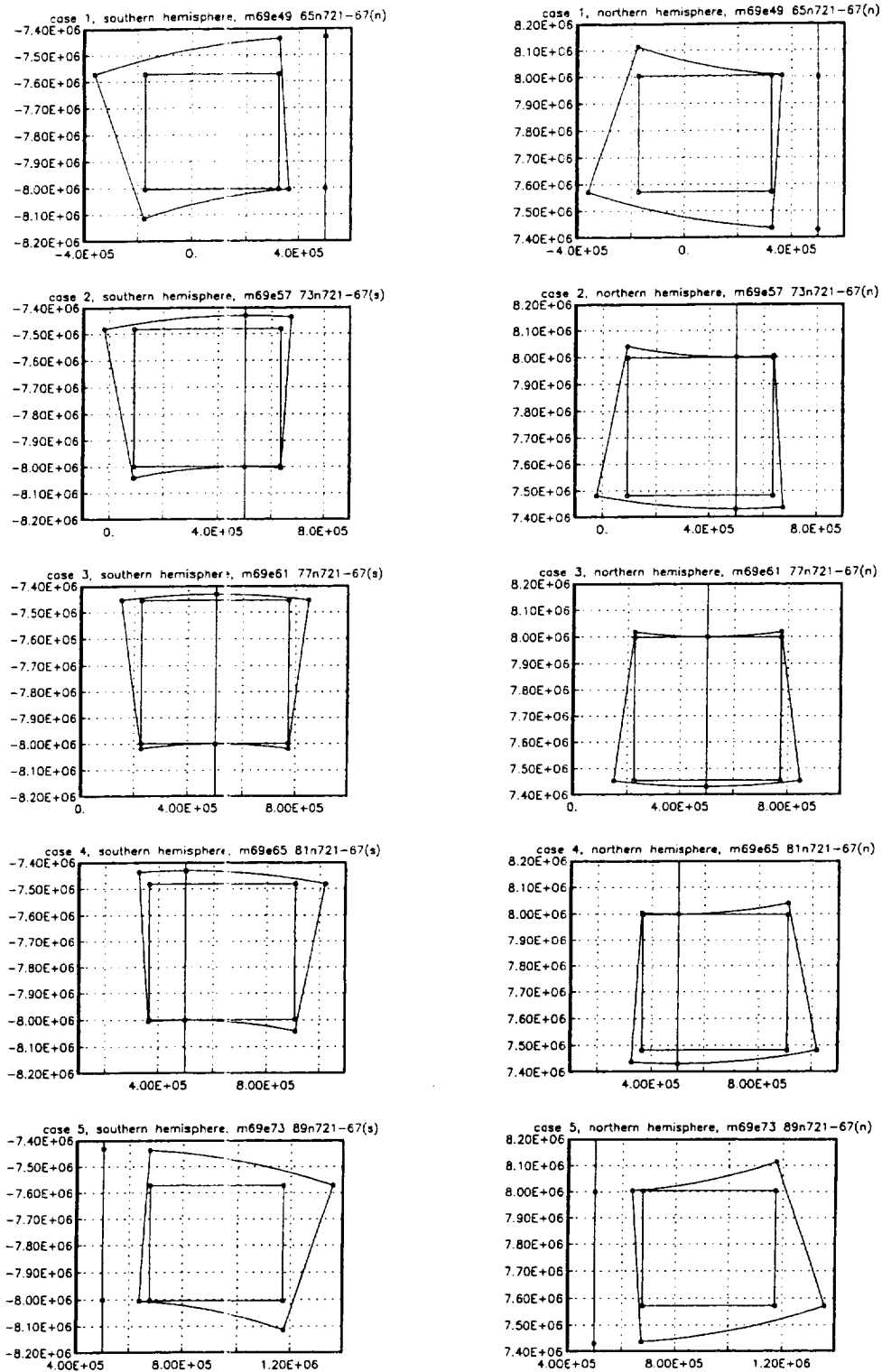


Fig. 1. Cases of possible location of ar area that is rectangular in geographic coordinates relative to central meridian. Program TRANSVIEW distinguishes automatically among these cases. Plots in UTM coordinates; solid circles: nominal map area that is rectangular in geographic coordinates; open circles: inscribed map area that is rectangular in UTM coordinates; vertical line: central meridian of the UTM projection (mapped to 500,000).

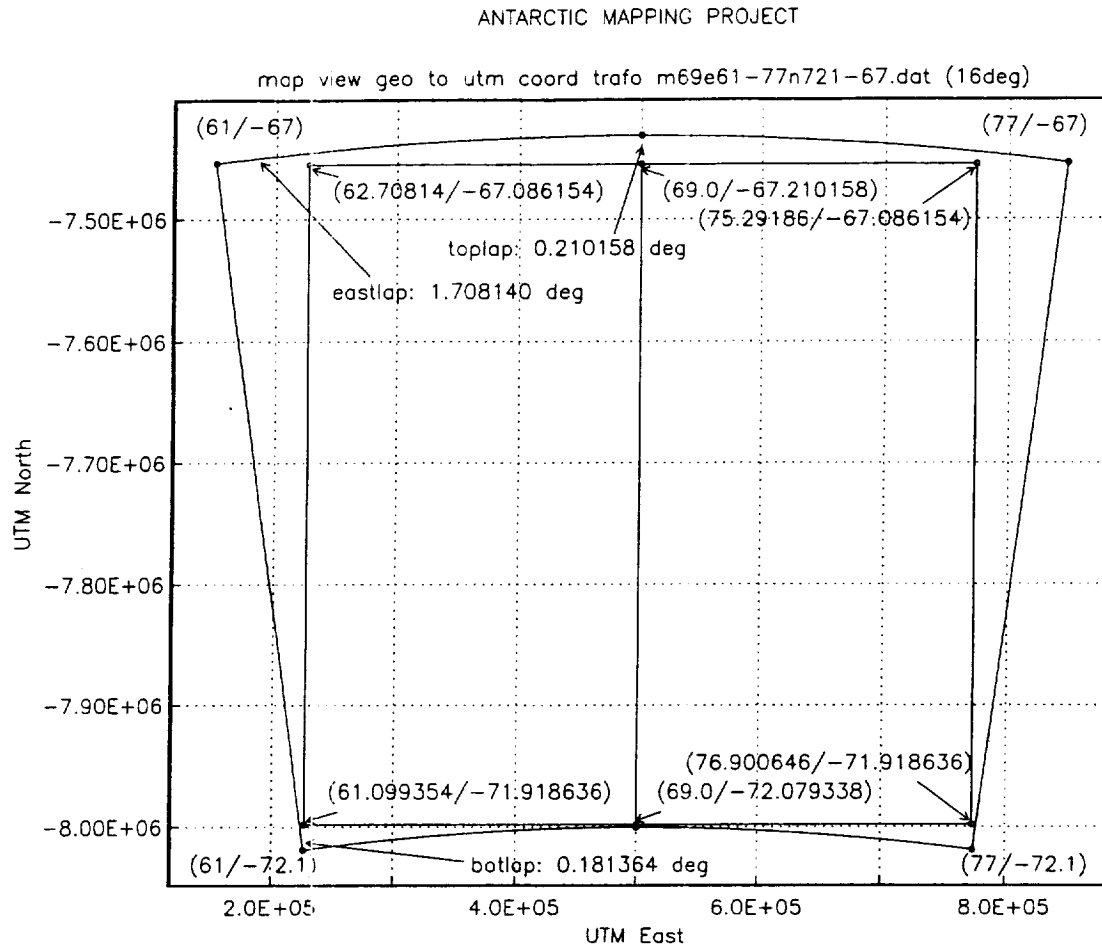


Fig. 2. Example of transformation and visualization using TRANSVIEW, for area 61°–77° E/67°–72.1° S, using central meridian 69° E (UTM zone 42) and map size of 16° east–west and 5.1° north–south. Curvilinear rectangle marked by filled circles delineates map edge of area originally rectangular in geographic coordinates after transformation to UTM system. Straight rectangle marked by empty circles delineates edges of inscribed UTM map. Annotated coordinates in brackets are backtransformed geographic coordinates; eastlap, toplap, botlap are minimal overlaps with adjacent sheets in degrees required for gap-free coverage in east–west, north and south direction, respectively

S). In rows $63^\circ - 68^\circ$ S and $67^\circ - 72.1^\circ$ S maps are of 16 degrees longitude nominal size and overlap 2 degrees on each side, so each map is offset against the next one by 12 degrees longitude. In rows $71^\circ - 77^\circ$ S, $75^\circ - 80^\circ$ S, and $78^\circ - 81.5^\circ$ S maps are of 36 degrees longitude nominal size, overlap is 6 degrees on each side, offset of two adjacent maps is 24 degrees longitude. The map names (e.g. m69e61-77n67-721) give central meridian (69°) and extent of the nominal map area (61° to 77° E, 72.1° to 67° S (the minus in the north coordinate was dropped in the file names for ease of reading)). The true map area is then the maximal area contained in the nominal map area with the property that it has straight edges in the UTM coordinate system and the grid coordinates are multiples of 3000 (Herzfeld and Matassa, 1999). All maps in one row have the same size and grid coordinates.

(3.3) Geostatistical methods for elevation mapping

Interpolation of corrected altimeter elevation data is carried out using geostatistical methods, in this application, ordinary kriging. "Kriging" comprises a family of interpolation/extrapolation methods based on the least-squares optimization principle and usually introduced in a probabilistic framework (Matheron, 1963; Journel and Huijbregts, 1978). However, it can be considered as a numerical interpolation technique only (Herzfeld, 1992a). Kriging consists of two steps: 1. analysis of spatial structures (variography), and 2. estimation, interpolation and extrapolation (kriging).

(3.3.1) Effect of variogram models on elevation mapping

In the first step, a variogram is calculated according to

$$\gamma(h) = \frac{1}{2n} \sum_{i=1}^n [z(x_i) - z(x_i + h)]^2 \quad (1)$$

where $z(x_i)$, $z(x_i + h)$ are measurements at locations x_i , $x_i + h$, respectively, inside a region D , and n is the number of pairs separated by the vector h . The residual variogram is

$$res(h) = \gamma(h) - \frac{1}{2}m(h)^2 \quad (2)$$

where

$$m(h) = \frac{1}{n} \sum_{i=1}^n [z(x_i) - z(x_i + h)] \quad (3)$$

is the drift component. The experimental variograms are calculated in distance classes from the measurements, in our case from the radar altimeter (RA) data, then are fitted by analytical variogram models. A variogram model describes the type of transition from the strong covariation between closely neighbouring samples to weaker covariation of samples farther apart. The variogram model is characterized by its function type, which has to meet the positive-definiteness criterion to ensure existence and uniqueness of the solution of the kriging system. For mapping altimeter data of ice surfaces, a Gaussian variogram model with a relatively high nugget effect is used, because

(1) the ice surface at 3 km resolution is smooth, and the Gaussian model is the model of a mean-square differential process,

(2) altimeter data at this resolution have a high signal-to-noise ratio, and the spacing of tracks influences the variogram.

The function of the Gaussian variogram model is

$$\gamma(h) = C_0 + C_1(1 - \exp(-\frac{3h^2}{a^2})) \quad (4).$$

The nugget effect, C_0 , is the residual variance of resampling in the same location for altimeter data, it also contains contributions caused by surface morphology at a resolution higher than the resolution of the altimeter footprint (called the support). The total sill, $C_0 + C_1$, is close to the total variance of the data. The concept of a regionalized variable, which is fundamental in geostatistics, perceives that closely neighbouring measurements have a higher covariance than measurements spaced farther apart. For distances increasing from the support to the range parameter a in equation (4), the variogram increases and the regionalization effect decreases; for distances beyond the range, the regionalized variable theoretically behaves like a random variable (probabilistically speaking), and data with a distance larger or equal to the range all have the same influence on the interpolated value (numerically speaking).

For GEOSAT data

$$\begin{aligned} C_0 &= 250m^2 \\ C_1 &= 343m^2 \quad (5) \\ a &= 18000m; \end{aligned}$$

for ERS-1 data

$$\begin{aligned} C_0 &= 43m^2 \\ C_1 &= 18m^2 \quad (6) \\ a &= 16000m. \end{aligned}$$

The noisiness of the data is captured by the ratio n_0 of the nugget effect C_0 to the total sill $C_0 + C_1$ in the following equation:

$$n_0 = \frac{C_0}{C_0 + C_1} \quad (7).$$

The nugget effect ratio n_0 is 0.422 for GEOSAT data and 0.705 for ERS-1 data, so ERS-1 data are noisier. The variograms were fitted to data from an ice stream around the grounding zone (Lambert Glacier). To avoid edge effects between maps of the same atlas, the same variogram is applied throughout any one atlas.

The effect of using a variogram with a higher nugget effect ratio is that contour lines are smoother. This is correct, because noisiness in the data does not warrant to pick up too many details everywhere.

Selection of the variogram, however, does not only depend on the data analysis, but also on the mapping purpose. For monitoring of changes from one year to another, the same variogram needs to be used for both years, to facilitate comparison. This has been applied in the study of Lambert Glacier/Amery Ice Shelf (Herzfeld and others, 1997).

To improve the representation of ice surface structures in terrain models of individual glaciers, regional variations of the variogram need to be taken into account. To this extent, classes of ice with homogeneous surface properties such as floating ice tongues, grounded glacier ice, steep margins, mountainous terrain, and almost flat inland ice are characterized by specific variogram functions, which may then be used in the interpolation.

(3.3.2) Kriging

The kriging method called "ordinary kriging" (universal kriging of order 0) is better suited for interpolation of RA data than universal kriging of a higher order, because the drift parts modelled by a polynomial component in the higher order universal kriging methods is likely to create artefacts in the gaps.

In ordinary kriging, the value $z_0 = z(x_0)$ at a node x_0 is estimated by

$$Z_0^* = \sum_{i=1}^n \alpha_i Z_i \quad \text{with} \quad \alpha_i \in IR \quad (8)$$

with data $z_i = z(x_i)$ at locations x_i ($i = 1, \dots, n$) in a neighbourhood of the grid node x_0 . In the Antarctic atlas mapping, the 4 nearest neighbours in each quadrant were used in the interpolation of any given grid node. The coefficients are determined such that the estimation error has minimum variance and the estimation is unbiased, which requires a condition

$$\sum_{i=1}^n \alpha_i = 1 \quad (9).$$

A solution of the kriging system is obtained using the variogram model specified earlier. (For a derivation of the kriging system, see Herzfeld 1992a,b.) This is carried out for every grid node in the map area. A 3 km grid size is chosen so that the resultant maps or terrain models can be used in glaciodynamic investigations.

(4.) GLACIOLOGIC RESULTS AND APPLICATIONS

(4.1) GEOSAT and ERS-1 Atlas of Antarctica

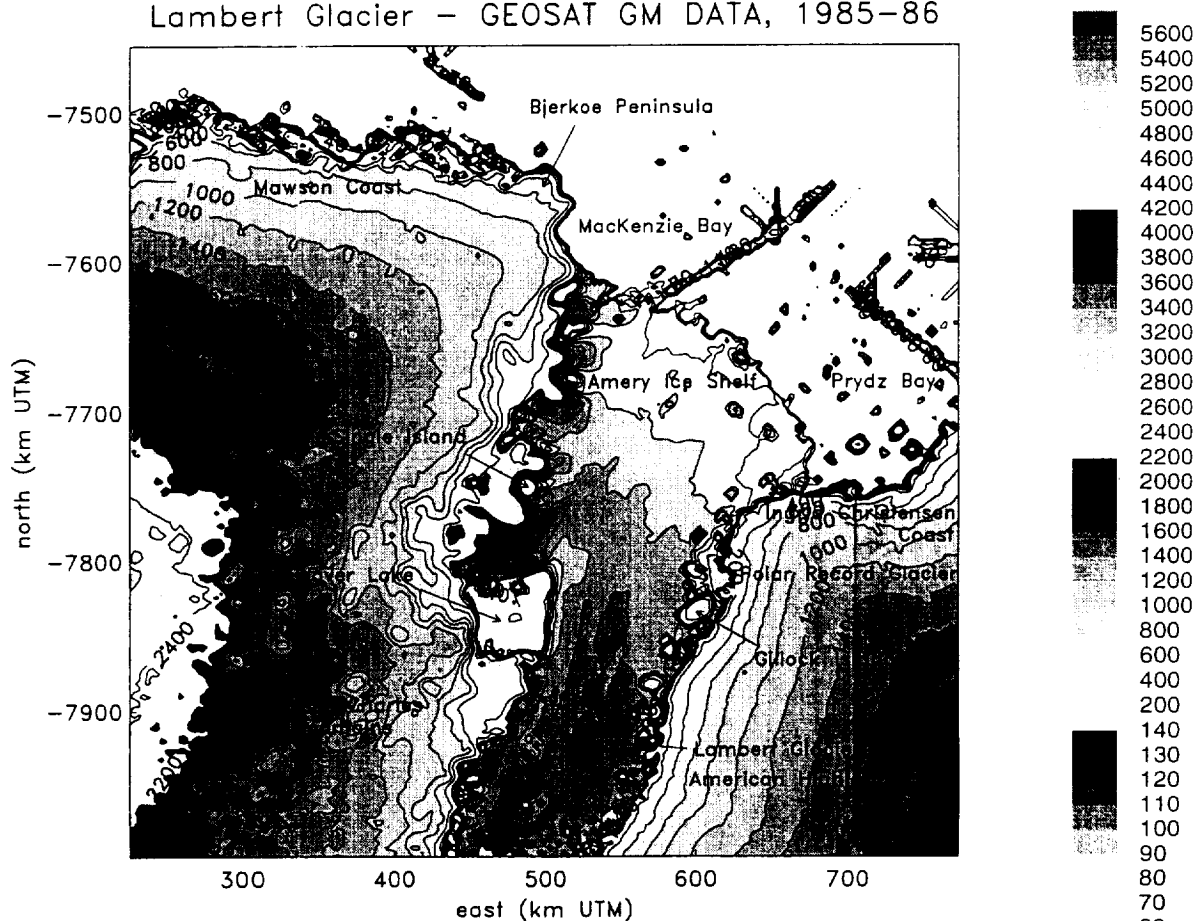
The GEOSAT Atlas was produced from 1985–86 GM data, the ERS-1 Atlas from 1995 GM data. Work on the ERS-1 Atlas was continued after completion of the work under this grant. Both atlases contain a total of over 200 individual maps (some are given in Figures 3–8) (see also Herzfeld and Matassa, 1999; Herzfeld et al., 2000a).

The Atlas DTMs are available through the website of the National Snow and Ice Data Center (<http://www.nsidc.org/>).

Quality of maps: Maps are available as either contoured maps or digital terrain models. For the study of individual glaciers, it should be noted that much more information is contained in the Atlas DTMs than is visible in the Atlas maps. For example, Williamson Glacier: Williamson Glacier flows into Moscov University Ice Shelf at a location east of Law Dome, as seen on the ERS-1 atlas map m117e109-125n63-68 "Sabrina Coast" (Fig. 5a in Herzfeld et al., 2000a). The details in the digital terrain model may be investigated, as shown in the map of Williamson Glacier in Figure 5b in Herzfeld et al. (2000a). The resolution of the detail map is the same (3 km grid) as that of the entire atlas. This demonstrates that more information is contained in the digital terrain model than is obvious from a plot of the maps at atlas scale. The models may be used for geophysical study, or for monitoring smaller outlet glaciers of the Antarctic Inland Ice. Not much information other than satellite data is available for this part of East Antarctica.

So, there are numerous glaciologic applications possible. A few are given in the sequel.

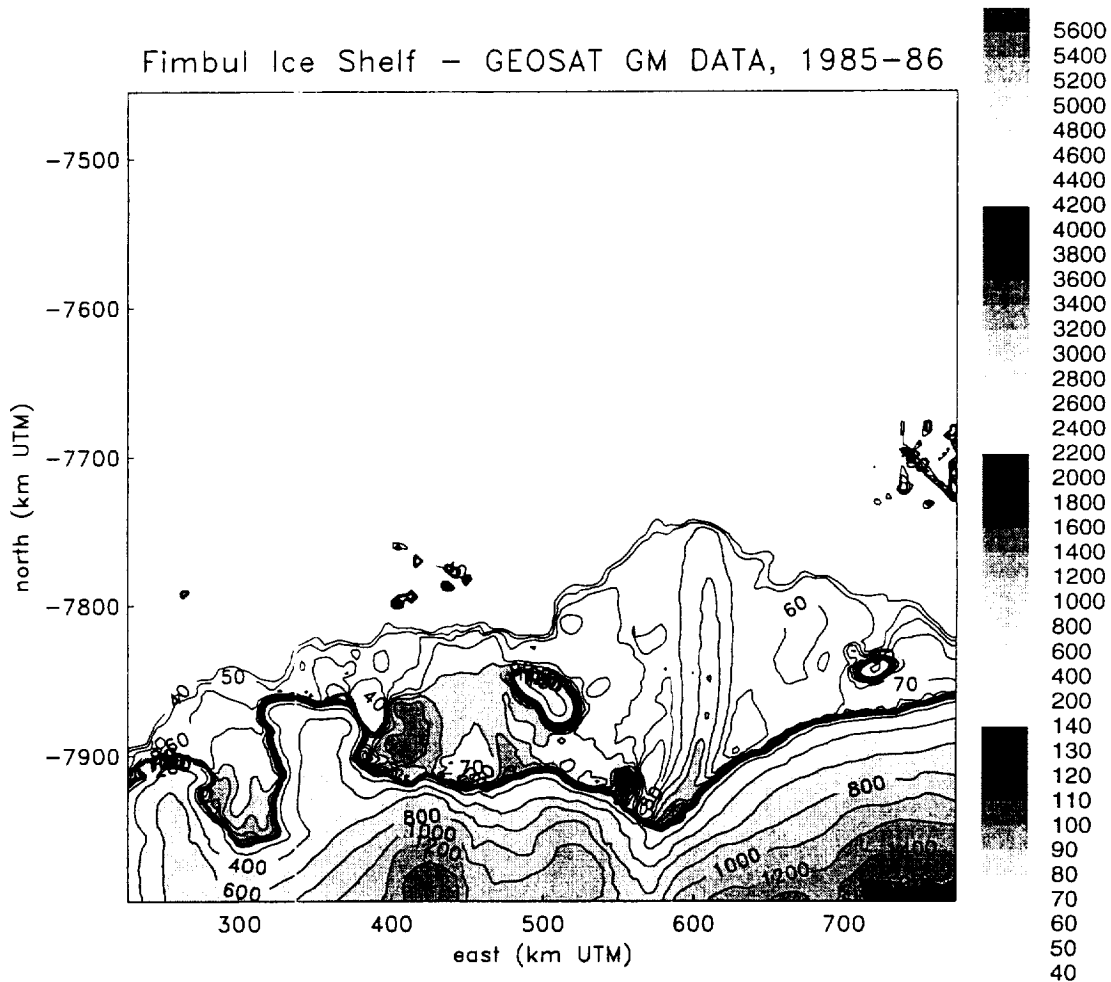
Lambert Glacier - GEOSAT GM DATA, 1985-86



e61-77n67-721, WGS84, Gaussian variog., central mer. 69, slope corrected, scale 1:5000000, 970723

Fig.3

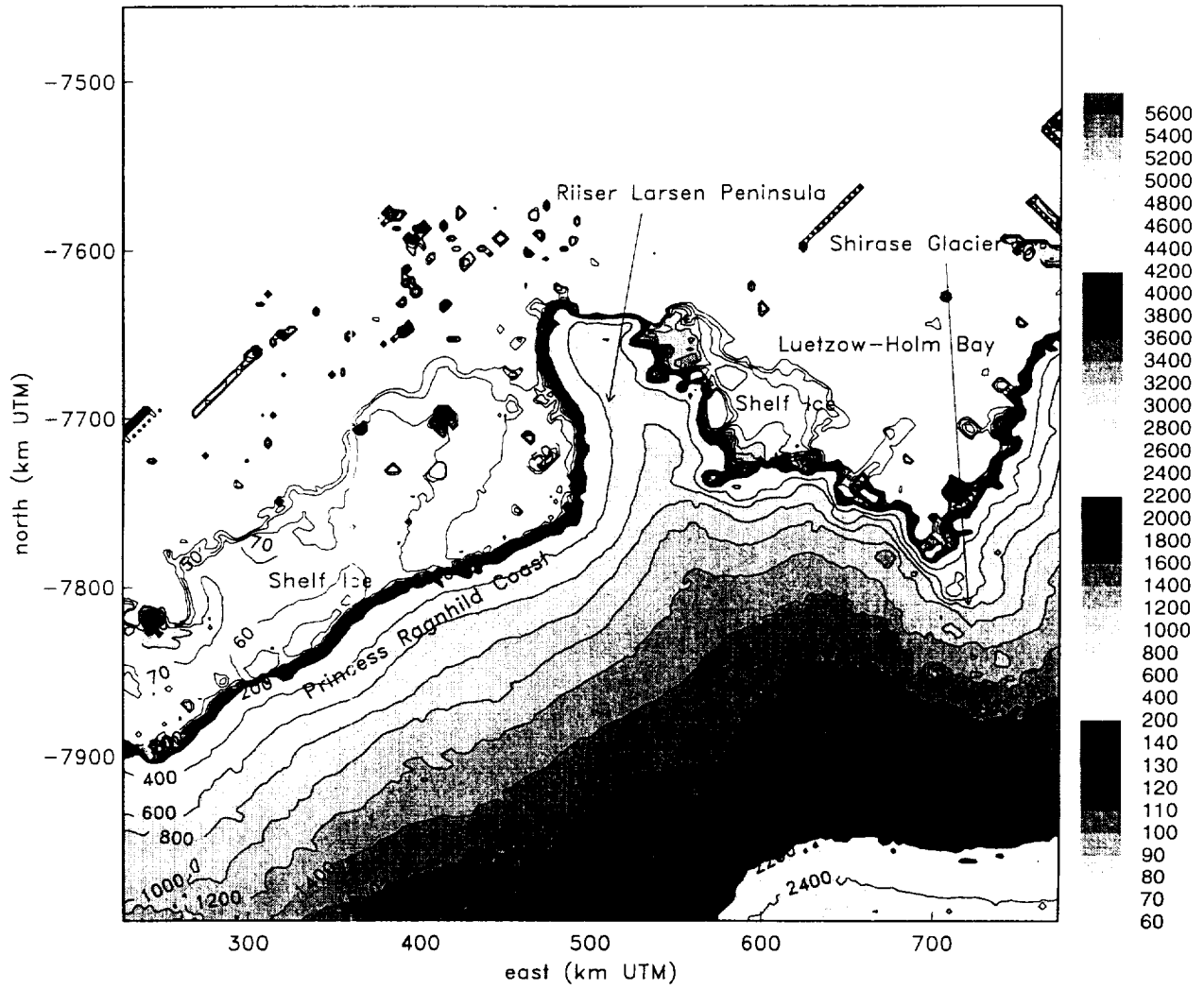
Fimbul Ice Shelf - GEOSAT GM DATA, 1985-86



e11W-5n67-721, WGS84, Gaussian variog., central mer. 3W, slope corrected, scclle 1:5000000, 970723

Fig. 4

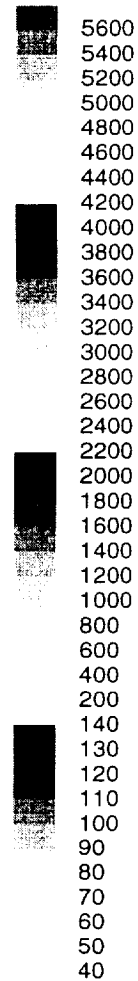
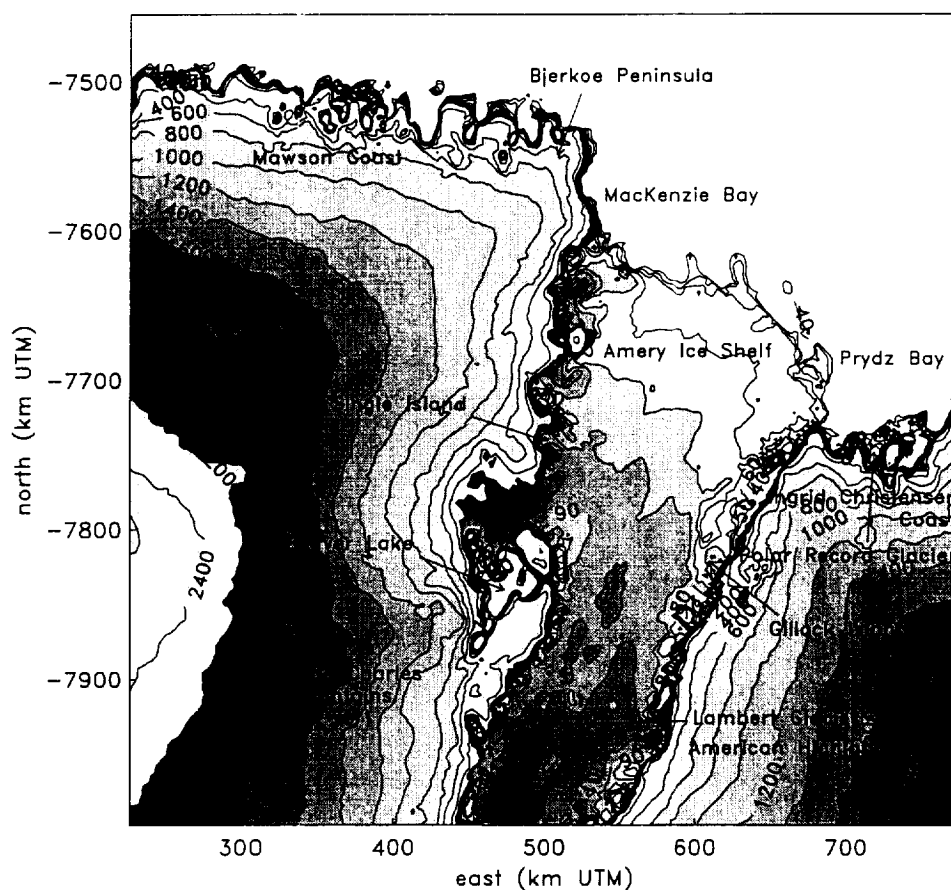
Riiser-Larsen Peninsula - GEOSAT GM DATA, 1985-86



WGS84, Gaussian variog., prime mer. 33, m33e25 41n67 721, 960703

Fig 5

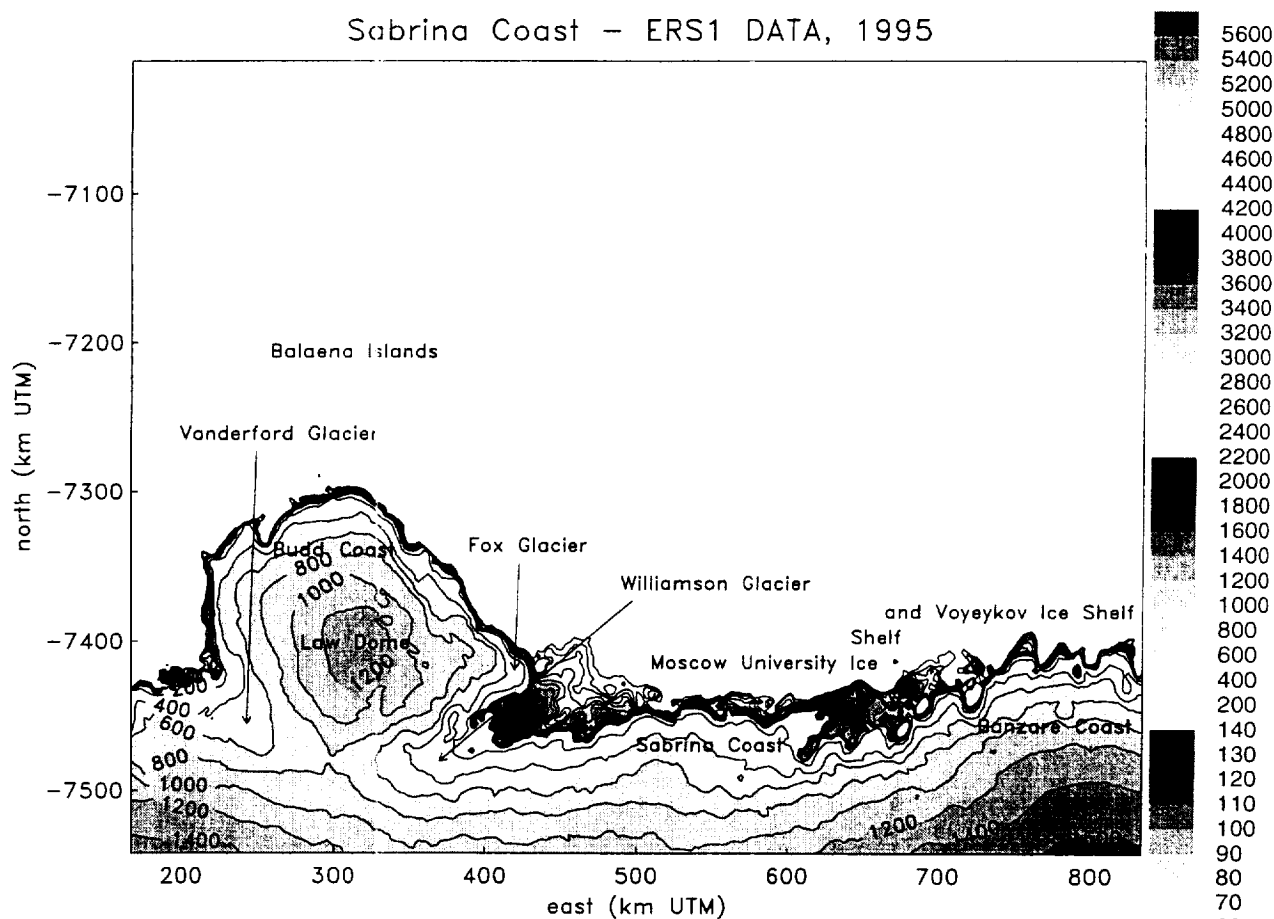
Lambert Glacier - ERS1 DATA, 1995



e61-77n67-721, WGS84, Gaussian variog., central mer. 69, slope corrected, scclle 1:5000000, 970727

Fig 6

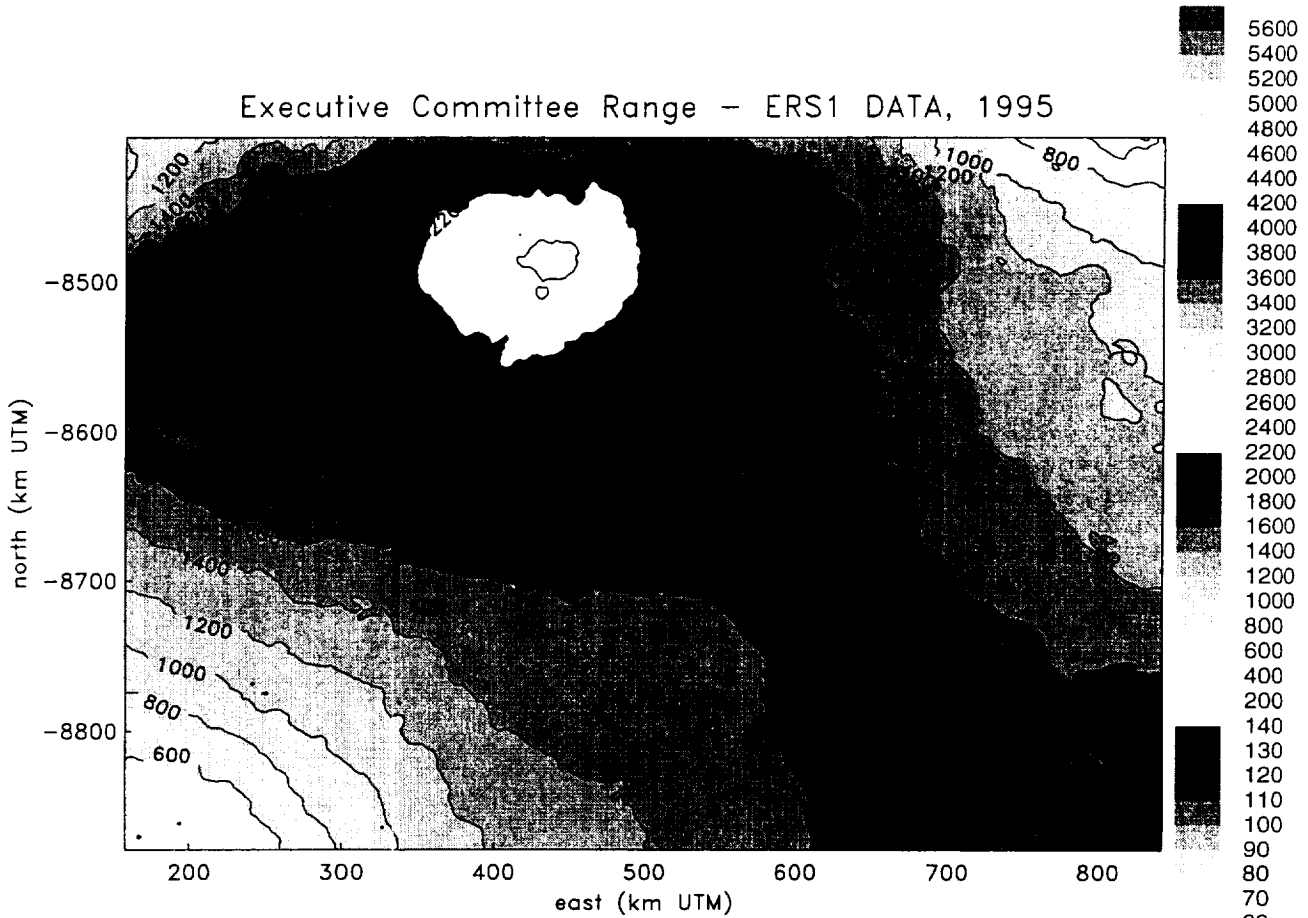
Sabrina Coast - ERS1 DATA, 1995



e109-125n63-68, WGS84, Gaussian variog., central mer. 117, slope corrected, scale 1:5000000, 970725

Fig. 7

Executive Committee Range - ERS1 DATA, 1995



e219-255n75-80, WGS84, Gaussian variog., central mer. 237, slope corrected, scale 1:5000000, 971105

Fig.8

(4.2) Presentation of ice surfaces with reference to Geoid models

In the previous sections, elevations referenced to the WGS84 ellipsoid have been employed. When studying changes, the reference surface is unimportant. Comparison with published studies and field work may be facilitated by maps referenced to a geoid, although for the Lambert Glacier area elevations are rarely given in the literature. Referencing to a geoid, however, involves geodetic problems of geoid determination, which is difficult in areas of poor availability of gravity data. Mathematical methods for geoid determination are described in Moritz (1984), Tscherning (1984), and Engels and others (1993).

We used two recently published geoids, "Goddard Earth Model" GEM-T3 (Lerch and others, 1992) and "Ohio State University Model" OSU91A (Rapp, 1992, 1994), kindly made available to us by NASA GSFC and by R. H. Rapp (pers. comm., Feb. 1995). Lambert Glacier maps referenced to GEM-T3 and OSU91A are given in Figures 3F and 3G in Herzfeld et al. (1997), respectively. Comparison of the two geoid models in the area of our maps shows significant differences: The GEM-T3 surface gradually slopes in a northeasterly direction, the OSU91A surface has a more complex topography with a hill (Herzfeld et al., 1997, 1998). According to R. H. Rapp (pers. comm., Feb. 1995), both models are based on the same set of data of only fair reliability (mean anomalies in 1° by 1° cells), therefore differences between the two models should be attributed to philosophical differences in the interpolation method rather than warrant a geophysical interpretation. For the OSU91A model, global average in estimation of the total undulation error is 0.57 m, but 2 m for land area with no surface gravity data (Rapp, 1992).

On the other hand, our ellipsoid-referenced topographic maps can be used for terrain correction in the inverse gravimetric problem, and thus may facilitate improvements of the geoid in this poorly constrained region. For purposes of field study and comparison to the literature both geoid-referenced maps are sufficiently accurate, as well as the WGS84-referenced maps when a constant of about 20 m is subtracted from the elevation values (the approximate difference between ellipsoid and geoid in the map area).

(4.3) Monitoring Lambert Glacier / Amery Ice Shelf system

Lambert Glacier / Amery Ice Shelf is the Antarctic ice-stream/ice-shelf system with the longest record of satellite RA data coverage, starting with SEASAT (1978), simply because it is located largely north of 72.1° S. By analyzing time series of satellite-altimetry-based digital terrain models, we studied changes in Lambert Glacier. Data for austral winters 1978, 1987, 1988, 1989 were used and various types of correction studied (total of 17 maps) (Herzfeld et al., 1997). When ERS-1 data became available, monitoring of Lambert Glacier was continued with 1992 maps and a 1995 map (Matassa et al., 1996; Herzfeld and Matassa, 1999).

A detailed discussion of sensitivity of the results to slope correction, frequency filtering, and mathematical method and statistical algorithms for calculation of changes is given in Herzfeld et al. (1997). As a result it was found that data that are neither slope-corrected nor frequency-filtered are best suited for monitoring changes.

The purpose of calculating elevation changes is to answer the following glaciological questions:

(GQ1) Time distribution of changes: How is the general advance of the grounding line between Seasat and Geosat times (Herzfeld, Lingle, and Lee, 1994) and overall increase in surface elevation, deduced from crossover analyses (Lingle and others, 1994) distributed in time?

A general increase in elevation for the entire glacier/ice-shelf system is derived from averaging elevations of the DTMs. This is consistent with results from crossover analysis (Lingle and others, 1994) and the advance of the grounding line deduced from DTMs (Herzfeld, Lingle, and Lee, 1994). The absolute differences between Seasat and Geosat data, however, are lower than the fluctuations between the three Geosat years 1987, 1988, and 1989, with 1988 values either too high or too low to match the trend (see Table 1). These may be due to interannual variation, or more likely, to undetected problems with the altimeter, because apparent interannual elevation changes indicated by the 1988 figures are rather large. With available data, however, we have no means to exclude a sudden change.

(GQ2) Location breakdown: Is the glacier homogeneously increasing in elevation?

In summary, a significant increase in elevation is observed for the Amery Ice Shelf and the grounding zone (for all outline locations and both averaging algorithms tested), whereas average surface elevations of the rougher and topographically more varied grounded Lambert Glacier may have decreased (but the decrease is not sufficiently supported by the data).

It is concluded that the glacier advanced about 10 -12 km between Seasat and Geosat ERM times (taken for all or each of 1987, 1988, 1989), and that the apparent interannual variability between 1987, 1988, and 1989 is high relative to the change observed for the 10-to-12-year interval.

Partington and others (1987) identify the position of several points along the grounding line. Their map, however, does not show any coordinates nor scale, which makes an exact comparison impossible. Their northernmost grounding point is in the same general area (south of Beaver Lake and Jetty Peninsula) as the grounding zone on our Seasat-data-based map (Fig. 3A in Herzfeld et al., 1997), the other points are up to about 80 km farther south. Budd, Corry, and Jacka (1982) identify a grounding point from a break in surface slope at UTM 485,600E/-7902,900S (3 km south of their point T4), this grounding point is located in the southern part of the grounding zone identified on our Seasat-data-derived map (Fig. 3A in Herzfeld et al., 1997). This comparison might indicate a small advance between the time of the Australian expeditions (1968-1971) and 1978. Brooks and others (1983) find a break-in-slope 43 km south of the grounding point of Budd, Corry, and Jacka (1982); however, Brooks and others (1983) remark that their altimeter data processing does not allow to identify the grounding line. According to Partington and others (1987) most estimates in the literature indicate a net accumulation, except McIntyre (1985). How much of the observed elevation changes and the advance of the grounding line is due to mass increase cannot be determined from the available data. A more detailed survey of the velocity field of Lambert Glacier/ Amery Ice Shelf may contribute to answering related questions.

The dynamics of the Amery Ice Shelf and the ice streams that feed into it may be complex. The fast-flowing ice discharges at about 1.2 km/yr into the shelf, only about one third of the discharge is contributed by (lower) Lambert Glacier (Budd, Corry, and Jacka, 1982). Wellmann (1982) finds geomorphologic evidence that Fisher Glacier, one other tributary of the Amery Ice Shelf, may be surging. Budd (1966) reports a 50 km retreat of the ice shelf front between 1955 and 1965. Consequently, trends in elevation changes need not necessarily coincide for the lower Lambert Glacier and the Amery Ice Shelf. This same observation is made in our analysis.

Brooks and others (1983) conclude from a comparison of Seasat data and data published by Budd, Corry, and Jacka (1982) that Lambert Glacier may have retreated between 1968-1971 and 1978, while Charybdis

Glacier (another tributary) may have surged. However, a visual comparison between the Seasat-altimetry-derived map by Brooks and others (1983) and our Seasat-based map (Fig. 3A in Herzfeld et al., 1997), using the 20 m-mean offset between the Geoid and the WGS84 ellipsoid (cf. Figs. 3E and 3F in Herzfeld et al., 1997), shows that the shapes of the contour lines match well in the floating part and that absolute differences are within 5 meters, but differences in morphology exist in the grounded part of Lambert Glacier (except for a general high on the western side of Lambert Glacier, and a low on the eastern side, common to both evaluations). The differences are a consequence of the different approaches to evaluation. Brooks and others (1983) used an inverse-distance gridding algorithm and contoured by hand, based on elevations from orbits adjusted into a common ocean surface. These results indicate that some of the differences in different studies may be due to different processing algorithms and data references and do not permit a glaciodynamic interpretation.

Reasons for apparent rapid changes are difficult to determine from remote sensing information only. There is a possibility that a change in the dynamic system of the glaciers feeding into Amery Ice Shelf occurred. Thickening of the lower part of the system and thinning of the upper part, as indicated by the spatial breakdown in our analysis, is typical for surges. The hypothesis of a dynamical event is mentioned here for sake of completeness, as the evidence from altimeter data alone is not sufficient for such a conjecture. The hypothesis is found in the literature (Wellmann, 1982; Brooks and others, 1983), but largely without supporting data.

(4.4) Investigations of other glaciers

Other Antarctic glaciers and areas studied and described in some detail include:

- Mertz and Ninnis Glacier Tongues (Herzfeld and Matassa, 1997)
- Riiser-Larsen Peninsula (Herzfeld and Matassa, 1999)
- Prince Olav Coast (Herzfeld and Matassa, 1999)
- Mawson Coast West (Herzfeld and Matassa, 1999)
- Ingrid Christensen Coast (Herzfeld and Matassa, 1999)
- Pennell Coast (Herzfeld and Matassa, 1999)
- Napier Mountains (Herzfeld and Matassa, 1999)
- Knox Coast (Herzfeld and Matassa, 1999)
- Sabrina Coast (Herzfeld and Matassa, 1999)
- Graham Land, Antarctic Peninsula (Herzfeld and Matassa, 1999)
- Slessor Glacier (Herzfeld et al., 2000a)
- Fimbul Ice Shelf, Jutulstraumen Glacier (Herzfeld et al., 2000a)
- Williamson Glacier (Herzfeld et al., 2000a)

(5.) WORK ON SAR DATA AND GEOSTATISTICAL SURFACE CLASSIFICATION

(5.1) Geostatistical Surface Classification

The difference between geostatistical interpolation and geostatistical classification is explained in Herzfeld (1999). As a consequence of the shift in focus of the proposal from classification method development to Antarctic-wide RA data analysis, and due to the delay in availability of RADARSAT SAR data, not much effort could be expended under this project to the development of the geostatistical surface classification method for SAR data. Nevertheless, I give a brief summary of results on geostatistical classification and include those publications, in which the partial support through this grant is acknowledged. Through some of my other work, the principles of geostatistical classification have by now advanced to a considerable level, and some roots of these ideas go back to work under NASA grant NAGW-3790/NAG5-6114.

Geostatistical estimation (interpolation/extrapolation) is a known method, adapted by myself to the evaluation of RA data. Geostatistical classification now summarizes a suite of methods developed by myself for various classification projects in geophysics and glaciology.

While interpolation utilizes the primary information in the data, a geostatistical surface classification method is developed to derive secondary information from elevation and backscatter data. Based on quantitative properties of the variogram, elements of surface structures are used for mapping and segmentation of an area into provinces homogeneous in surface characteristics.

A critical issue in the analysis of satellite Synthetic Aperture Radar (SAR) data is the availability of ground truthing to distinguish between intensity variations caused by subscale-resolution geophysical variability and noise, and to determine small-scale sources of variations in backscattering. During the 1993-1995 surge of Bering Glacier, Alaska, GPS-located video data were collected from small aircraft and analyzed systematically with the geostatistical ice surface classification system ICECLASS. The objectives are to (a) help understand the relationships between ice velocities, surface strain states, and progression of deformation processes during the surge, and (b) provide a technique for surface classification based on video data, SAR data, or image data in general.

For principles of surface classification, see Herzfeld (1998) (also images of Bering Glacier during the surge). A connectionist-geostatistical system for classification is given in Herzfeld et al. (1996). Application of classification to SAR data is the objective of Herzfeld et al. (2000b).

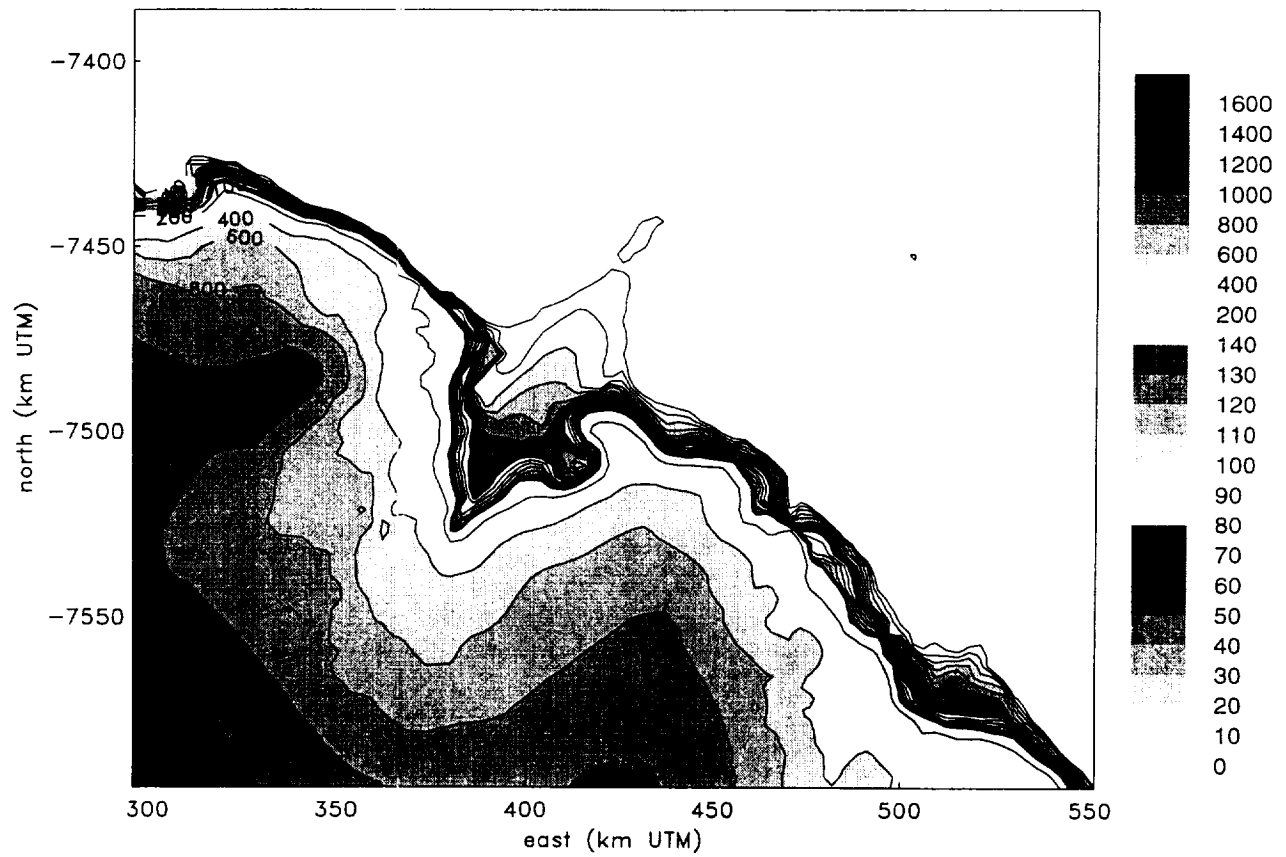
(5.2) Mertz and Ninnis Glacier Tongues — Comparison of results from SAR and RA data analysis

A comparison of quantitative information obtained from SAR and RA data gave surprising results for Mertz and Ninnis Glaciers, Antarctica (Herzfeld and Matassa, 1997).

Mertz and Ninnis Glaciers are both glaciers with long tongues that extend into the ocean and fluctuate considerably in length. Results from the GEOSAT Antarctic Atlas DTMs were compared to results from SAR data (Wendler et al., 1996). Mertz and Ninnis Glacier maps were expanded from Atlas maps, and details of slope and length of the tongue were measured. The detail maps showed a surprising amount of detail (which was later confirmed by geologists working in the area; G. Kleinschmidt, pers. communication).

Mertz Glacier is at UTM 360,000 - 440,000 E / 7,525,000 - 7,440,000 S. Its drainage is a broad valley that is

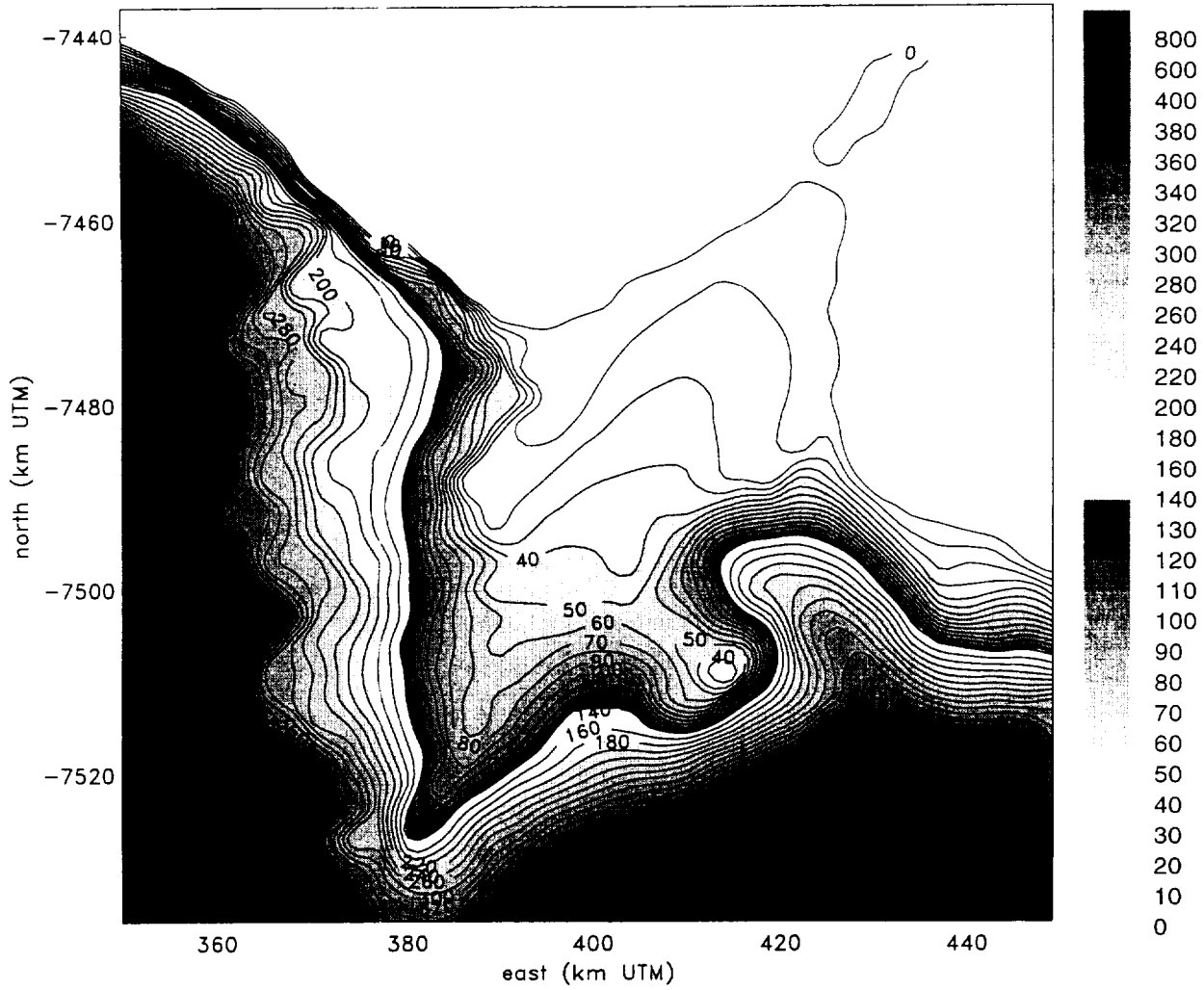
Mertz Glccier - GEOSAT GM DATA, 1985-86



WGS84, Gaussian variog., central mer. 147, m147e142 1485n665 685, 970424

Fig. 3

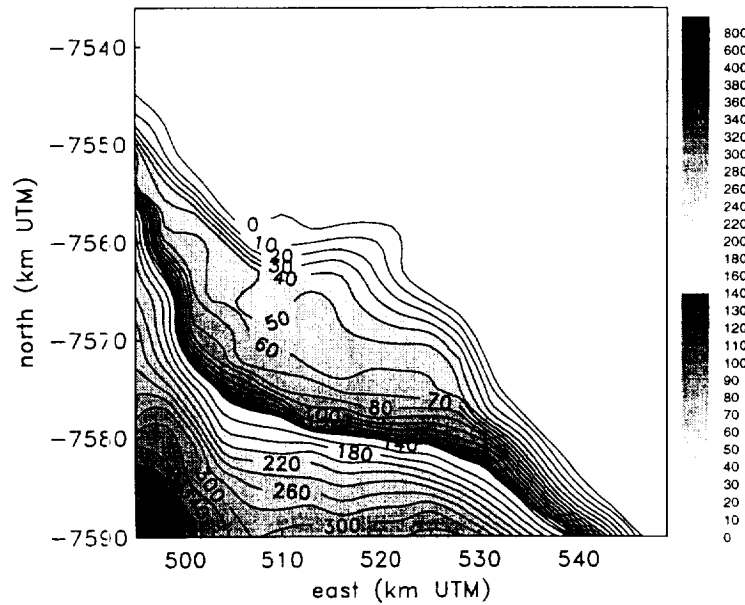
Mertz Glacier - GEOSAT GM DATA, 1985-86



WGS84, Gaussian variog., central mer. 147, 970416

Fig. 10

Ninnis Glacier - GEOSAT GM DATA, 1985-86



WGS84, Gaussian variog., central mer. 147, 970422

Fig. 11

distinguishable at the southern map boundary. The sides of the western valley are transected by erosional features. At its head, the glacier is fed by a narrow valley, approximately 4 km wide. The distance from the 80 m contour line at the head to the ice front is 85 km. In the eastern arm of Mertz Glacier there is a 20 m overdeepening, centered at 415,000 / -7510. This is located below the steepest part of the valley walls. This is not an interpolation error, because the contours are really smooth (see Figures 9-11).

Mertz Glacier is about 27 km wide. The glacier tongue appears to extend about 45 km seaward of the coastline (see discussion below). The grounding line of Mertz Glacier is probably located in the vicinity of the 40-50 m contour. The 60 m contour still exhibits the indentation of the valley that continues subglacially from the valley leading to the head of the glacier, while the 50 m contour does not. The 40 m and 50 m contours show the signs of the eastern side valley. At 30 m, the tongue is definitely floating. Slopes to 2° were calculated accurately.

Ninnis Glacier is at UTM 500,000 - 540,000 E / 7,550,000 - 7,580,000 S. The glacier tongue appears to extend about 10-15 km seaward from the coastline (the coastline being identified by the steep gradient of contours). Ninnis Glacier lies below a steep cliff, at 90 m above WGS 84 and lower. The entire extension from the break in slope at the foot of the cliff to the 0 m contour line is 20 km. [Notice that the 0 m contour does not coincide with sea level, because the reference level is the ellipsoid.] Ninnis Glacier is about 35 km wide. Ninnis Glacier does not really have a "tongue" anymore (as opposed to 1913) as noted by Wendler et al. (1996).

Discussion on accuracy and reliability. Mapping of atlas type and careful analysis of the spatial structure and the distribution of noise levels in radar altimeter data allows us to extend the limits of use of altimeter data for mapping. Bamber (1994) produced a map of Antarctica (from ERS-1 altimeter data) with 20-km grids, reliable only in areas with a slope of less than 0.65° according to the author. The drainages of Mertz Glacier and the Mertz and Ninnis Glacier area are considerably steeper than that. Accuracy depends on topographic relief, as calculated in Herzfeld et al. (1993, 1994), with submeter accuracy on ice streams. However, the accuracy of a kriged map in areas of high topographic relief refers to the pointwise error rather than to the error of the "mean" surface elevation mapped. The "pointwise error" is the probable difference between a point on the map (estimated surface elevation) and a radar-altimetry-derived surface elevation: it depends on the averaging process of kriging. Shape and elevation of the surface at the Mertz Glacier drainage and on the glacier itself, however, are more accurate than inferred from the noise calculation, which is best conceived from inspection of the maps. The contour lines are smooth on the resolution of the 3-km grid. Smooth contour lines indicate a continuous or differentiable surface function with low error; little islands and edging contour lines indicate a rougher surface function with higher errors. That means the maps are reliable also in areas of steep terrain. An area with high relief is located on the western side of Mertz Glacier. Notice that the grid spacing of the subarea enlargements is the same 3 km as for the large map, and that a wealth of information becomes only available in the enlargement. Mertz Glacier Tongue appears to extend about 40 km seaward of the coastline, Ninnis Glacier Tongue about 20 km.

Neither satellite altimetry nor kriging are tools designed to track the location of an ice cliff. Because of the effect described in Thomas and others (1983) and Partington and others (1987) (snagging of altimeter) the location of the ice edge is systematically wrong, differently so in descending and ascending orbits. Thomas and others (1983) attempted to trace the ice edge from altimeter data designing a technique not used here. In the Ice Data Record the ice edge is in the middle of both errors. Kriging employs a moving window

averaging procedure, which results in smoothing of steep edges. Consequently, the exact location of the ice edge cannot be inferred from altimetry, but the cliffs may be identifiable as a sequence of dense coverage. Comparing our map with the images in Wendler and others (1996), however, we note that the results of kriging altimeter data are surprisingly good. On their map in fig. 2, Mertz Glacier Tongue extends about 80 km off the (averaged) coastline, Ninnis Glacier Tongue 20-30 km. (Also, $1993 - 85 = 8$ years, 0.9 km per year for 1962-1993, 8×0.9 km = 7.2 km or 8×1.2 km = 9.6 km advance since 1985/86, according to Wendler and others (1996) assuming constant rate of advance.)

This indicates that, while SAR images are superior to altimetry-based maps for location of the ice edge, changes in advance/retreat can still be monitored from satellite-altimeter-derived digital terrain models, which are available for a longer time span (since 1978, Seasat mission). SAR imagery shows surface features in the floating ice tongue. The slope and elevation of the glacier surface are lost in the grey-shaded image. Additional information on ice flux can be derived from the surface elevation in the altimetry-based DTMs, calculated for the drainage basins.

Another advantage of radar altimeter data over SAR data is that records have been available since the 1978 Seasat mission. A maximal amount of information may be expected from a quantitative combination of SAR and radar altimeter data.

In general, radar altimeter data and SAR data can be combined to study changes in position, surface morphology and elevation of Antarctic ice streams and glaciers. The advantage of RA data over SAR data is their longer record in time (since 1978), more frequent repeat, and lesser data volume, which facilitates frequent collection and rapid evaluation for large areas.

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- (16) HERZFELD, U.C., Applications of geostatistics to glaciology from centimeter to continental scale, Proceedings, Forum for Research on Ice Shelf Processes (FRISP) Workshop, Münster, 5.-7. Juli 2000

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