Is Asteroid 951 Gaspra in a Resonant State with Its Spin Increasing Due to YORP?

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POPULAR SUMMARY

Gaspra, the first asteroid ever to be photographed up close by a spacecraft, is a small asteroid only 6 km in radius, rotating with a 7 hour period, and orbiting in the asteroid belt at 2.2 AU from the Sun. From its known shape and spin axis orientation and assuming a uniform density, Gaspra's axial precession period turns out to be nearly commensurate with its orbital precession period, which leads to a resonance condition with consequent huge variations in obliquity (obliquity being the tilt of Gasptra's equator with respect to its orbital plane. How did the asteroid get into this resonance? One way is for the orbit to change over time. However, computing its orbit for the last 3 million years indicates its orbit is highly stable.

A more likely explanation is the YORP effect (YORP stands for Yarkovsky-O'Keefe-Radzievskii-Paddack). In the YORP effect Gaspra absorbs sunlight and reradiates it in the infrared. Due to its highly irregular shape, the reradiation causes the asteroid to spin faster, and for its obliquity to change. It may be that Gaspra started out in a nonresonant state, fell into the resonance thanks to YORP, and has been temporarily trapped there ever since. Eventually, though, Gaspra is expected to wander out, also thanks to YORP.

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Abstract. Asteroid 951 Gaspra appears to be in an obliquity resonance with its spin increasing due to the YORP effect. Gaspra, an asteroid 5.8 km in radius, is a prograde rotator with a rotation period of 7.03 hours. A 3 million year integration indicates its orbit is stable over at least this time span. From its known shape and spin axis orientation and assuming a uniform density, Gaspra's axial precession period turns out to be nearly commensurate with its orbital precession period, which leads to a resonance condition with consequent huge variations in its obliquity. At the same time its shape is such that the Yarkovsky-O'Keefe-Radzievskii-Paddack effect (YORP effect for short) is increasing its spin rate. The YORP cycle normally leads from spin-up to spin-down and then repeating the cycle; however, it appears possible that resonance trapping can at least temporarily interrupt the YORP cycle, causing spin-up until the resonance is exited. This behavior may partially explain why there is an excess of fast rotators among small asteroids. YORP may also be a reason for small asteroids entering resonances in the first place.

1. Introduction

The Yarkovsky-O'Keefe-Radzievskii-Paddack effect (YORP for short), which produces torques due to sunlight heating up an irregularly shaped body, can alter the spin of meteoroids on geologically significant timescales [*Radzievski*, 1954; *O'Keefe*, 1976; *Paddack* 1969, 1973; *Paddack and Rhee*, 1975; *Komarov and Sazanov*, 1994; *Sazanov*, 1994]. The YORP effect has recently been applied to the rotation and obliquity of small asteroids [*Rubincam*, 2000]. (Obliquity is the axial tilt with respect to the orbital plane). Asteroid 951 Gaspra appears to be in such a state, perhaps accounting for its short spin period of 7.03 hours. We conjecture that a significant fraction of the prograde rotation population of asteroids may be in such states. These states may help explain to some extent the observed excess of fast rotators among small asteroids [*Harris*, 1996].

2. Orbit integration

In order to understand the spin of 951 Gaspra, an asteroid 5.8 km in radius, we first integrated the planets' orbits (except Pluto) plus that of Gaspra. We integrated backwards in time for 3 million years using an eleventh-order Cowell predictor-corrector scheme with a step size of 1.25 days. All moons, including the Earth's moon, were lumped in with their planets, so that the planets as well as Gaspra were assumed to be point-masses. The ephemeris for the planets was J2000, while the position of Gaspra was taken from *Yeomans et al.* [1993]. Gravitation was the only force acting on the bodies. Relativity was included, but not Yarkovsky forces, which would perturb a large body like

Gaspra only by a small amount [*Öpik*, 1951; *Peterson*, 1976; *Rubincam*, 1995; *Afonso et al.*, 1995; *Farinella and Vokrouhlicky*, 1999; *Bottke et al*, 2000]. The mass of Gaspra is unknown [*Belton et al.*, 1992; *Thomas et al.*, 1994]; we assumed a nominal density of 3 g cm^{-3}. All km-sized bodies whose average densities are known have values lower than this [e.g., *Rubincam*, 2000], but any departure from the assumed density has only a tiny effect on Gaspra's orbit about the Sun.

The results of the orbit integration are shown in Figure 1. The inclination I of and node Ω Gaspra's orbit is with respect to the fixed J2000 ecliptic. The orbit was stable for the entire 3 million year run. We then performed a time series to extract the major frequencies in the orbit, so that the Keplerian elements K can be expressed in the simple form

$$K \quad = \quad K_0 + K'_{avg}t + \sum_{i=1}(K_i^C \cos \sigma_i t + K_i^S \sin \sigma_i t) \tag{1}$$

where K is the orbital eccentricity e, inclination I, or node Ω. In this equation, K_0 is a constant, K'_{avg} is the secular rate of change with time t, while K_i^C and K_i^S are the coefficients of the cosine and sine terms which have an angular frequency of σ_i. See Table 1. The coefficients can probably be derived from the theory of *Brouwer and van Woerkom* [1950], but no attempt is made to do that here. In what follows below the semimajor axis a is assumed to be fixed at a value of 2.20967 AU, since it varied little in the integration.

3. Spin equations

We then used the above analytical expressions together with the equations governing angular momentum

$$\frac{d(H\hat{\mathbf{s}})}{dt} = \frac{3 J_2 G M_S}{2(1-e^2)^{3/2} a^3} (\hat{\mathbf{n}} \bullet \hat{\mathbf{s}})(\hat{\mathbf{n}} \times \hat{\mathbf{s}}) + \mathbf{T}_{\mathbf{YORP}} \qquad (2)$$

to determine the evolution of the asteroid's spin state. The first term gives the gravitational torque averaged over one revolution about the Sun [e.g., *Goldreich*, 1966; *Ward*, 1974], while the second is the YORP torque. Here

$$\hat{\mathbf{n}} = (\sin I \sin \Omega)\,\hat{\mathbf{x}} - (\sin I \cos \Omega)\,\hat{\mathbf{y}} + (\cos I)\,\hat{\mathbf{z}}$$

where $\hat{\mathbf{n}}$ is the unit vector normal to the orbital plane, with $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$,and $\hat{\mathbf{z}}$ being the unit vectors of the fixed J2000 ecliptic coordinate system; $\hat{\mathbf{s}}$ the unit vector along the positive spin axis. Gaspra's obliquity Θ is given by $\cos \Theta = \hat{\mathbf{n}} \bullet \hat{\mathbf{s}}$. Also, $H = \lambda M_G R_G^2\, \theta'$ is the magnitude of Gaspra's rotational angular momentum, where θ' is the angular rotational speed of Gaspra about its axis, while $\lambda = C/M_G R_G^2$ is Gaspra's condensation factor with C being the moment of inertia, M_G Gaspra's mass, and R_G the radius of a sphere equivalent to the volume of Gaspra. Further, J_2 is the asteroid's gravitational flattening, M_S is the mass of the Sun, G is the universal constant of gravitation, and t is time; a primed quantity means time derivative. Gaspra was assumed to be homogeneous in density, as seems to be appropriate for small solar system objects [e.g., *Rubincam et al.*, 1995].

The YORP torque is given by

$$\mathbf{T}_{\text{YORP}} = T_{\Theta} \frac{\hat{\mathbf{s}} \times (\hat{\mathbf{s}} \times \hat{\mathbf{n}})}{|\hat{\mathbf{s}} \times (\hat{\mathbf{s}} \times \hat{\mathbf{n}})|} + T_{\Phi} \frac{\hat{\mathbf{n}} \times \hat{\mathbf{s}}}{|\hat{\mathbf{n}} \times \hat{\mathbf{s}}|} + T_z \hat{\mathbf{s}} \quad (3)$$

where T_Θ changes the obliquity, T_Φ changes the precession angle Φ, and T_z changes the spin rate, with the rotation axis $\hat{\mathbf{s}}$ being the body-fixed z axis of the asteroid. These torques come about from sunlight being absorbed and reemitted as infrared radiation. The infrared photons which leave the surface carry momentum; by action-reaction they kick the body. An irregularly shaped body which has a "windmill factor" will be kicked asymmetrically, giving rise to the torques [[e.g., *O'Keefe*, 1976; *Rubincam*, 200], which can change both the spin and the tilt; see Figure 2. The precessional torque T_Φ is assumed to be zero because the torques are averaged over one revolution; averaging essentially circulizes the orbit, for which this torque is zero.

The highly irregular Gaspra has a significant windmill factor. The asteroid's shape was modeled as a 12th degree and order spherical harmonic surface, and Gaspra was assumed to be a black body which emits like a Lambertian radiator, with the torques having the simple form

$$T = \sum_{k=0}^{N} A_k \Theta^k \quad (4)$$

[*Rubincam*, 2000]. Moreover, Gaspra was taken to be a rigid body, and in principal axis rotation, and its initial spin state was taken to be $\Theta = 71.86°$, $\Phi = 117°$, with a rotation period of 7.03 hours [*Belton et al.*, 1992; *Thomas et al.*, 1994].

4. Spin state integration

We used an eleventh order Cowell predictor-corrector scheme with a 0.05 year step size to integrate (2) over 15×10^6. This is much longer than the 3 million year integration of Gaspra's orbit, and it was simply assumed that (1) held over the longer time interval.

We first integrated (2) without the YORP torques. It turned out that for the chosen parameters and initial conditions was in a resonant obliquity state, i.e., the precession of the orbit was comparable to the precession rate of the spin axis; this is shown in Figure 3 (bottom). For comparison, Figure 3 (top) shows the obliquity oscillations for Gaspra for the same initial Θ and Φ, but with a rotation period of 6 hours, so that Gaspra is far from resonance.

Next we integrated (2) with the YORP torques turned on and increased by a factor of 10 to speed up their effect on the spin state of the asteroid, and integrated for 15 million years. Gaspra remained remained in its resonant state. We also turned off the first term in (2), which gives rise to the obliquity oscillations, and integrated again over the same time interval with the Runge-Kutta scheme described in *Rubincam* [2000] using 0.5×10^6 year step sizes.

The results of both integrations are shown in Figure 4. The full YORP calculation, which includes the obliquity oscillations, is shown as discrete points; circles represent the first third of the data (5×10^6 years), x's the next third, and squares the final third. The horizontal axis position of each point gives the "average" obliquity over a 0.05 $\times 10^6$ interval, where "average" is defined as the value of Θ in (2) which gives the change in rotation rate over the interval; this definition reduces the scatter on the plot somewhat. The vertical axis position gives the rotation speed in cycles day^{-1}. The solid line gives the YORP evolution over 15×10^6 years (the same time period as the discrete points) ignoring the obliquity oscillations completely, as was done in [*Rubincam*, 2000]. The initial value for the obliquity was 42° for this integration, since this is the average value over the first oscillation when YORP is turned off and the gravitational torque alone remains.

The two integrations shown in Figure 4 are are dramatically different: the solid curve (no obliquity oscillations) trends to the right so that the asteroid, after speeding up for a while, starts to slow its rotation, moving down and to the right on the graph. On the other hand, the discrete points of the full YORP calculation (obliquity oscillations included) trend to the left and up, just the opposite trend of the solid curve, so that the asteroid continues to speed up.

5. Discussion

The YORP torques were increased by a factor of 10, in order to speed up the integration. Because the YORP torques scale like R^2 [*Rubincam*, 2000], this is equivalent to integrating an asteroid 1.8 km in radius with the shape assumed here.

Choosing values for J_2 which differ by a few per cent from the nominal value and integrating the spin equations again can give trajectories in which Gaspra is not in a resonant state. Hence the time evolution of the obliquity is sensitive to initial conditions and chosen parameters. What indicates that Gaspra is presently in a resonant state instead of a nonresonant one is the unlikelihood of finding it so near the resonance; most other values for spin rate and obliquity give nonresonant states.

Since 951 Gaspra, the first asteroid to be examined, was found to possibly be in a resonant state, it seems likely many other small asteroids are also in such states. *Skoglöv* [1998] has shown that asteroids can enter and leave such resonant states without the aid of YORP; a near-Earth asteroid such as 433 Eros changing its semimajor axis would be an example [e.g., *Michel et al.*, 1996]. However, given the apparent stability of Gaspra's orbit, it may be that it entered its resonant state thanks to YORP, and it may be that many small asteroids enter such states due to this mechanism.

Moreover, small asteroids tend to show an excess of fast and slow rotators [*Harris*, 1996]. W. F. Bottke (private communication, 1998) suggested that YORP somehow might deplete the center population of a collisionally evolved Maxwellian distribution of rotators to the extremes. The YORP mechanism outlined above may provide at least a partial answer: those which do not encounter the obliquity resonance slow down via the YORP cycle [*Rubincam*, 2000], while prograde rotators which do encounter it speed up, interrupting the cycle. Ultimately Gaspra and other asteroids in similar states would be expected to leave the resonance due to YORP's continued changing of the spin rate.

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Table 1

The constants used in the analytical expression of the orbital elements.

K	K_0	K'_{avg}	i	σ_I	K_i^C	K_i^S
e	0.15445	0	1	0.0000208°	-0.0779	-0.0521059
			2	0.0001397°	0.000773	-0.005682
Ω	115.93900°	0.0099742°yr^{-1}	1	0.00115200°	-11.4943°	-17.0411°
			2	0.002664°	1.14886°	13.499°
			3	0.009972°	-7.04923°	15.5026°
I	5.29169°	0	1	0.000324°	0.08529°	0.14400°
			2	0.001476°	-0.26707°	-0.15530°
			3	0.002628°	1.04115°	-0.06062°
			4	0.009972°	1.35545°	0.59856°

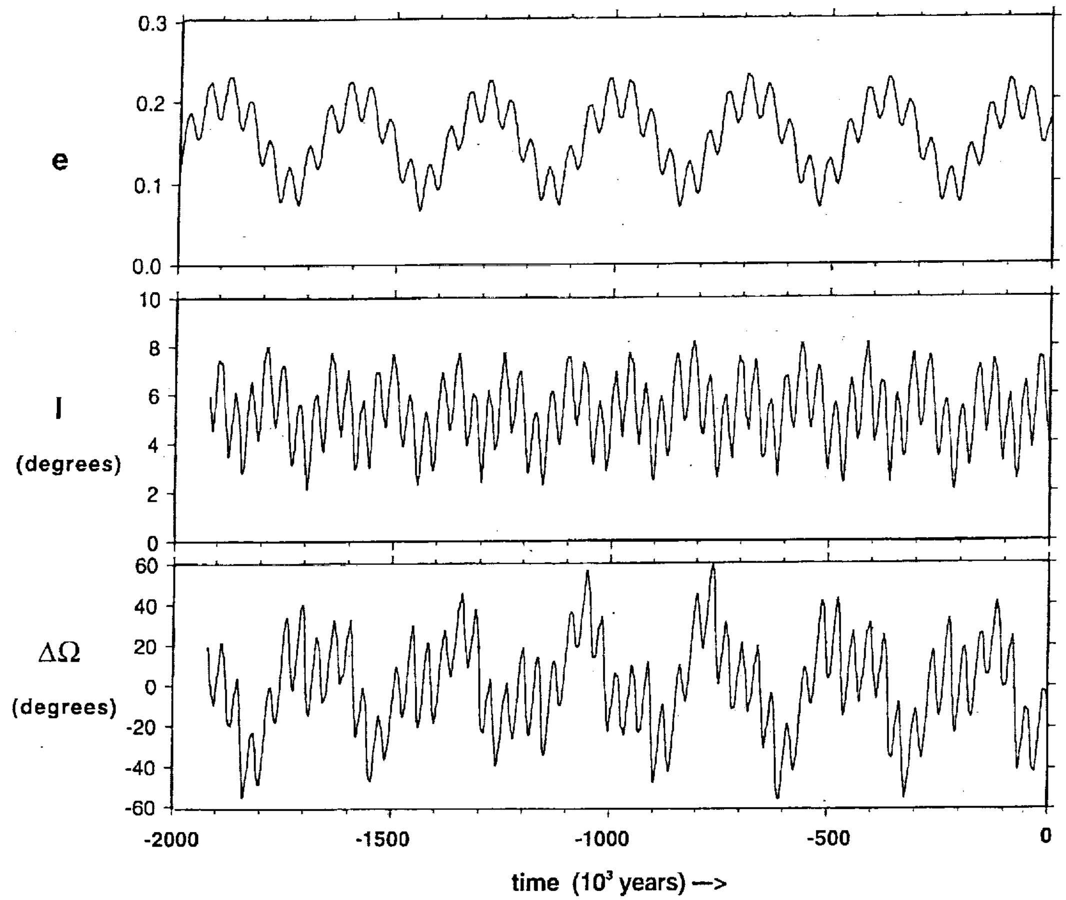

Figure 1

Orbital eccentricity e, inclination I, and nodal variation $\Delta\Omega$ for asteroid 951 Gaspra for the last 3×10^6 years. The node position is $\Omega = \Omega_0 + \Omega' t + \Delta\Omega$, where $\Omega' = -0.0099742°\text{yr}^{-1}$.

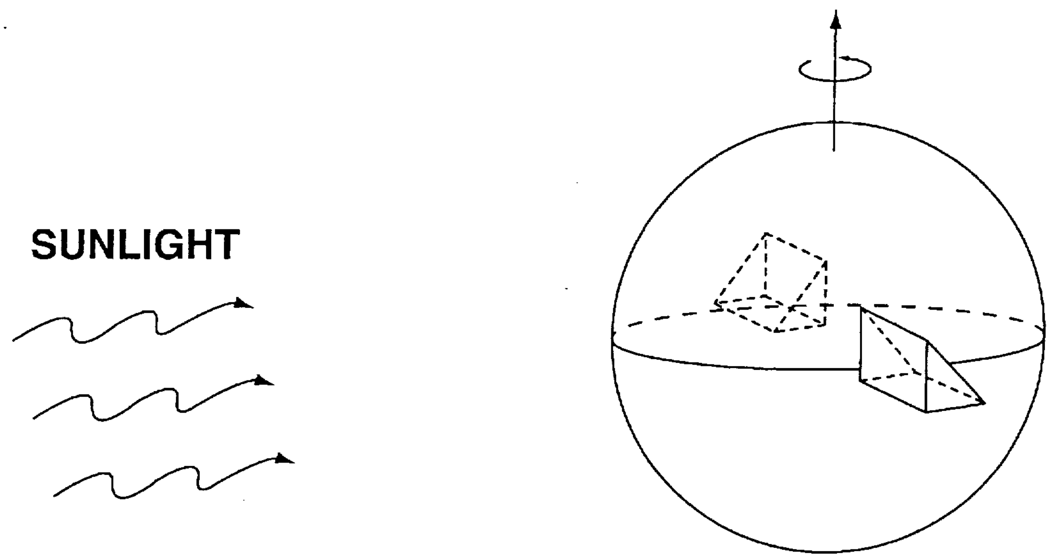

Figure 2

An asymmetric asteroid modeled as a sphere with two wedges attached to its equator. The asteroid is assumed to be a blackbody, so that the sunlight falling upon it is reradiated in the infrared. The sphere gives no torques, but the wedges do; this configuration increases the rotation rate.

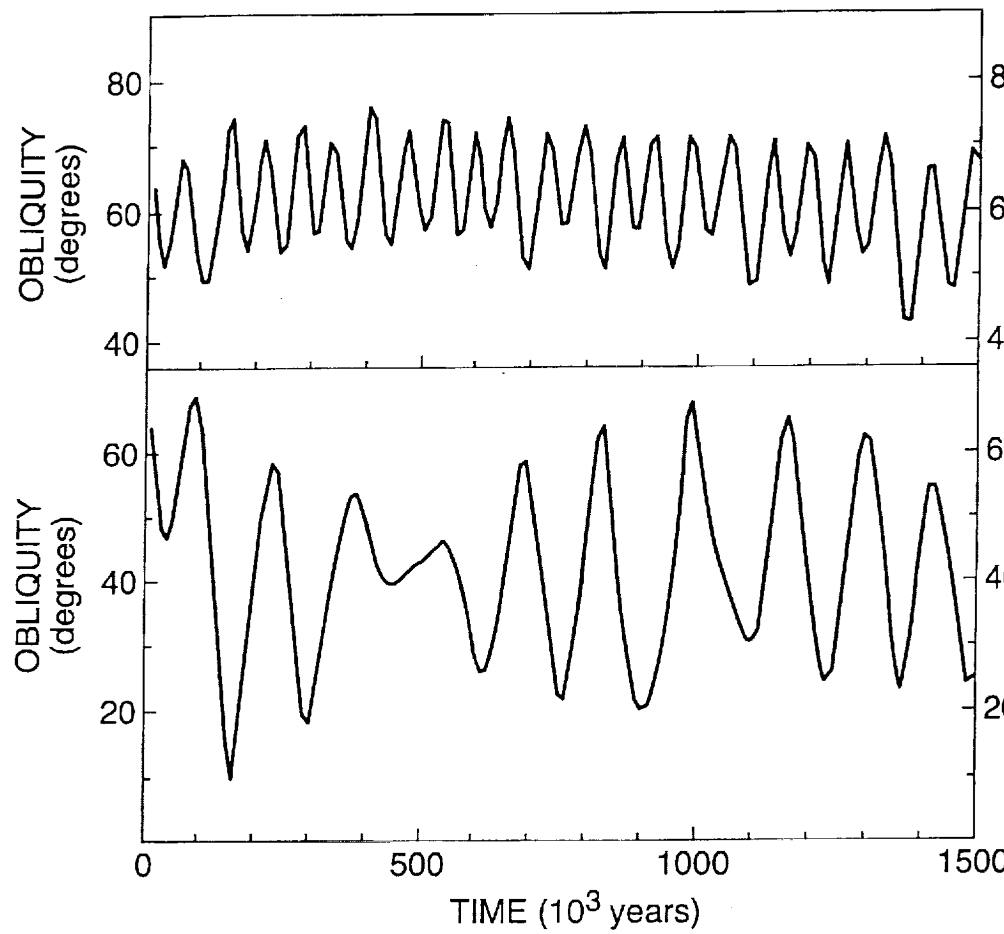

Figure 3

Obliquity Θ versus time for a hypothetical rotation period of 6 hours (top) and the actual rotation period of 7.03 hours (bottom). Time moves forward to the right. Both integrations start with the initial values of $\Theta = 71.86°$ and $\Phi = 117°$. The top graph is far from resonance, while the bottom is in resonance. These calculations assume that Gaspra is homogeneous. No YORP torques were included.

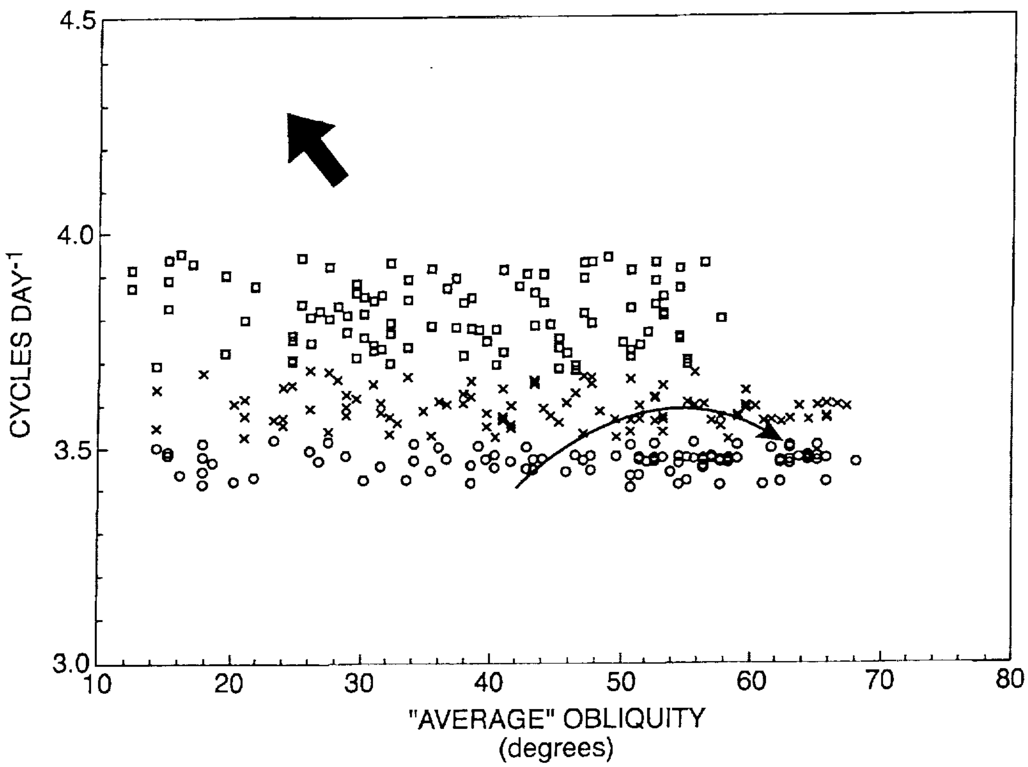

Figure 4

"Average" obliquity in degrees vs. rotation speed in cycles day^{-1} for 15×10^6 years. The full YORP calculation, which includes the obliquity oscillations, is shown as discrete points; circles represent the first third of the data, x's the next third, and squares the final third. Each point gives the "average" obliquity over a 0.05×10^6 interval, where "average" is defined as the value of Θ in eq. 2 which gives the change in rotation rate over the interval. The solid line gives the YORP evolution for 15×10^6 years ignoring the obliquity oscillations completely. The initial value for the obliquity was 42° for this integration, since this is the average value over the first oscillation when YORP is turned off and the gravitational torque alone remains. The solid curve moves to the right as time marches forward; the discrete points move up and to the left. All of the YORP torques were increased by a factor of 10 to speed up the integration.