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Anil V. Kantak
Monika J. Danos
Jet Propulsion Laboratory, Pasadena, California

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1.0 Introduction

Currently, quite a few missions are being studied to send satellites to the outer and inner planets and their moons of the solar system; a large percentage of these missions will have a landed element. NASA's Origins program, Solar System Exploration Program and Sun Earth Connection (SEC) program, etc., will have a variety of spacecrafts to various solar system planets and their moons to sample and analyze the related atmospheres as well as the soil once the lander lands on the body. These sampling missions may involve a lander element sampling the atmosphere by performing experiments while descending into the atmosphere or a rover collecting samples to return to Earth or a station for experimentation on the planet surface. In either of these cases, the pertinent data generated will have to be sent to the Earth through a communication link. Communications with the lander during the Entry, Decent and Landing (EDL) phases of a mission is of paramount importance. This article explores a particular method of passing through the atmosphere while communicating with the ground station (DSN station) before landing an instrument package (the lander) on the surface of the planet or moon of interest.

Whether the future mission desires to land an instrument package on the surface of the extra-terrestrial body under investigation or it plans to study the body's atmospheric content as well as its properties, the best scenario is use of either a parachute or a balloon or some variation thereof. Specially in the case of missions designed for measuring the atmospheric contents of the body, the needed measuring instruments must be placed in the atmosphere of the planet and they need to be there for some appreciable amount of time for the experiment to be completed and data to be generated. In this case a balloon or a parachute is ideal because these devices remain in the atmosphere long enough for the experiments, and also move around with the winds of the planet and hence samples of the atmosphere may be obtained from various latitudes and longitudes of the body. Many times the free flying balloon may go higher or lower in the planet's atmosphere along with the thermals in the atmosphere and this allows for more detailed measurements. In any case, it is very convenient to put the measurement instruments in a gondola suspended from the balloon/parachute and let it descend into the atmosphere of the planet while taking data. Another advantage of a balloon/parachute is that the lander package may be delivered to the surface with minimum shocks imparted to the package: i.e., a soft and more or less controlled landing can be managed.

2.0 Direct Communications to Earth Station or Relay Links?

The data generated by the instruments and the experiments must be communicated to the ground station either directly or via a relay link. Both methods have advantages and drawbacks : a tradeoff must be made to select a particular method.

The direct communications from the balloon/parachute gondola to Earth station can be effected using a large enough antenna (mostly a large parabolic reflector) on the gondola and carrying a large enough battery contingent to supply the needed power to the communications system. However, the power required to communicate with the Earth station (DSN) is, of course, dependent upon the data rate generated and needed to be transmitted as well as the data fidelity (Bit Error Rate, BER) needed along with the range (distance) between the planet of interest and Earth. More often than not, the communications system for this link becomes too heavy and bulky for the balloon or the parachute to carry and also provide a proper decent rate through the atmosphere of the planet to be useful for the atmospheric sampling and experimentation.

Many times the balloon or the parachute is deployed on a body that was visited before and communications orbiters are already present around the body in known orbits. This is especially true for Mars. In any case, even if there is no communications orbiter present at the time of deployment of the parachute or the balloon, the stage that brought the lander assembly to the desired heavenly body can be appropriately designed and scheduled to be used as a communications relay orbiter to serve the parachute/balloon relay link. The orbiter will also be needed to relay the data generated by the landed package to the Earth station. Even when the landed package may have equipment to establish a direct communications to Earth, the orbiter relay link may serve as a backup communications link or the direct and relay links may be worked together to reduce the load on the direct communications link.

3.0 Requirements and Assumptions of the Com System

The exact requirements on the communications system may vary according to the science requirements of the mission. However, following general requirements may be levied on the communication system.

- A large enough data rate to transmit all the data taken in the atmosphere within the time available during parachute/balloon descent.

- The orbiter will have a system for tracking coarse/fine position location of the balloon to minimize antenna pointing loss towards the balloon/parachute carrying the landed element.

- Proper frequency selection commensurate with the atmospheric absorption losses.

- The antenna on the balloon/gondola should not require pointing, reducing the electronic equipment necessary for the balloon/parachute operation. Thus, the antenna used on the balloon/parachute must essentially be an omni antenna.

- It will be assumed that the balloon/parachute will be in the orbital plane of the orbiter. This assumption not only will reduce the three dimensional problem to a simpler two dimensional problem but it will provide the worst also case analysis in terms of the range between the orbiter and the balloon/parachute. The telecom system must be designed for the worst case range.

- It will be assumed that the orbiter satellite is in a circular orbit at a particular altitude from the planet's surface. This assumption is most often true for a lander element carrying mission.

Initially, the carrier ship to the planet or a moon of a planet will carry the balloon in a deflated manner. The balloon-carrying canister will be released at a particular instant either by remote control from the ground or by autonomous operation of the carrier satellite. This canister will have all the equipment for the atmosphere sampling and the landed experimentation. It will also have electronics, such as an altimeter, pressure sensors etc., which will help to decide the altitude at which the balloon must be inflated. As the balloon is inflated, the canister is discarded. Once the balloon is inflated using the appropriate gas, it will float and go up and down dependent on the atmospheric conditions.

4.0 Theory

Figure 1 shows a balloon and the attached gondola descending on a planet's/moon's surface that has appreciable atmosphere. The gondola will be carrying the instruments necessary to measure the atmospheric content and perform any experiments. Along with the instruments, the gondola will also carry the communications equipment to establish the link between the balloon and the orbiter and send the data to the desired receiver. For the case when the landed experimentation is required, the gondola will also carry the lander.

As was mentioned before, the communications receiver in this scenario can be the ground station on the Earth. However, in this article it is assumed that the balloon or the parachute communicates with the orbiter that is already in its designated known orbit. Even though the word balloon is used in the following write up, the analysis is equally valid for the parachute.

This analysis assumes that the antenna used on the balloon/parachute gondola is a dipole/monopole antenna with the antenna placed vertically. The antenna may be placed below the gondola, above the parachute or balloon, or on a special mounting attached to the balloon assembly in some convenient manner. The particular placement of the antenna may depend upon the shape of the balloon/parachute and the placement of the communications transponder or transceiver in the gondola. Large distances between the antenna input terminal and the transponder/ transceiver will be avoided to reduce the line losses which is a major component of the losses in the communications assembly. A salient assumption in this analysis is that the fabric of the balloon/parachute will be such that it will of be totally transparent to the frequency of communication: i.e., even if there is shadowing of the communications antenna by the balloon/parachute material while communicating with the orbiter, the loss induced due to the shadowing will be essentially 0 dB. This assumption may be true for some frequency band more than for other frequency bands. Hence, a judicious selection of the balloon/parachute material will be assumed. Figure 2 shows only the dipole antenna in a vertical fashion. All the pertinent parameters necessary to evaluate the performance of the communications system are defined below.

A is the position of the gondola in which the telecommunications system with the dipole antenna is housed. As explained before, the antenna is assumed to be placed vertically, so that the toroidal antenna pattern of the antenna will be used for the analysis to follow.

AP is the altitude of the balloon or parachute (more precisely, of the antenna) at any time instant. Let this altitude be defined as $\triangleq h_b$. Also let $c \triangleq r_p + h_b$.

OP is radius of the planet or the moon around which the orbiter is in a circular orbit $\triangleq r_p$ = OE.

$\angle$BAJ is the angle from the antenna pattern maximum direction. Let this angle be defined as $\triangleq \alpha$. It would be good to see from the geometry of the figure that the following angles are equal in extent : $\angle BAJ = \angle CAJ = \angle HAE = \angle HAF$.

S is the position of the orbiter satellite in its orbit. It is assumed that the parachute or the balloon is in the orbital plane of the satellite when the communications between those two takes place. $\angle SOL \triangleq \theta_s$ gives the position of the satellite with respect to the defined x axis.

D is the limiting visibility position of the satellite from the antenna: i.e., line AED is tangential to the planet at the point E. Thus, the orbiter will not be able to "see" the balloon/parachute below this point.

PM Is the altitude of the orbiter satellite in a circular orbit $\triangleq h$.

OM Is the magnitude of the radius vector to the satellite $\triangleq a = r_p + h$.

Figure 1 A balloon with its gondola descending on a planet's surface.

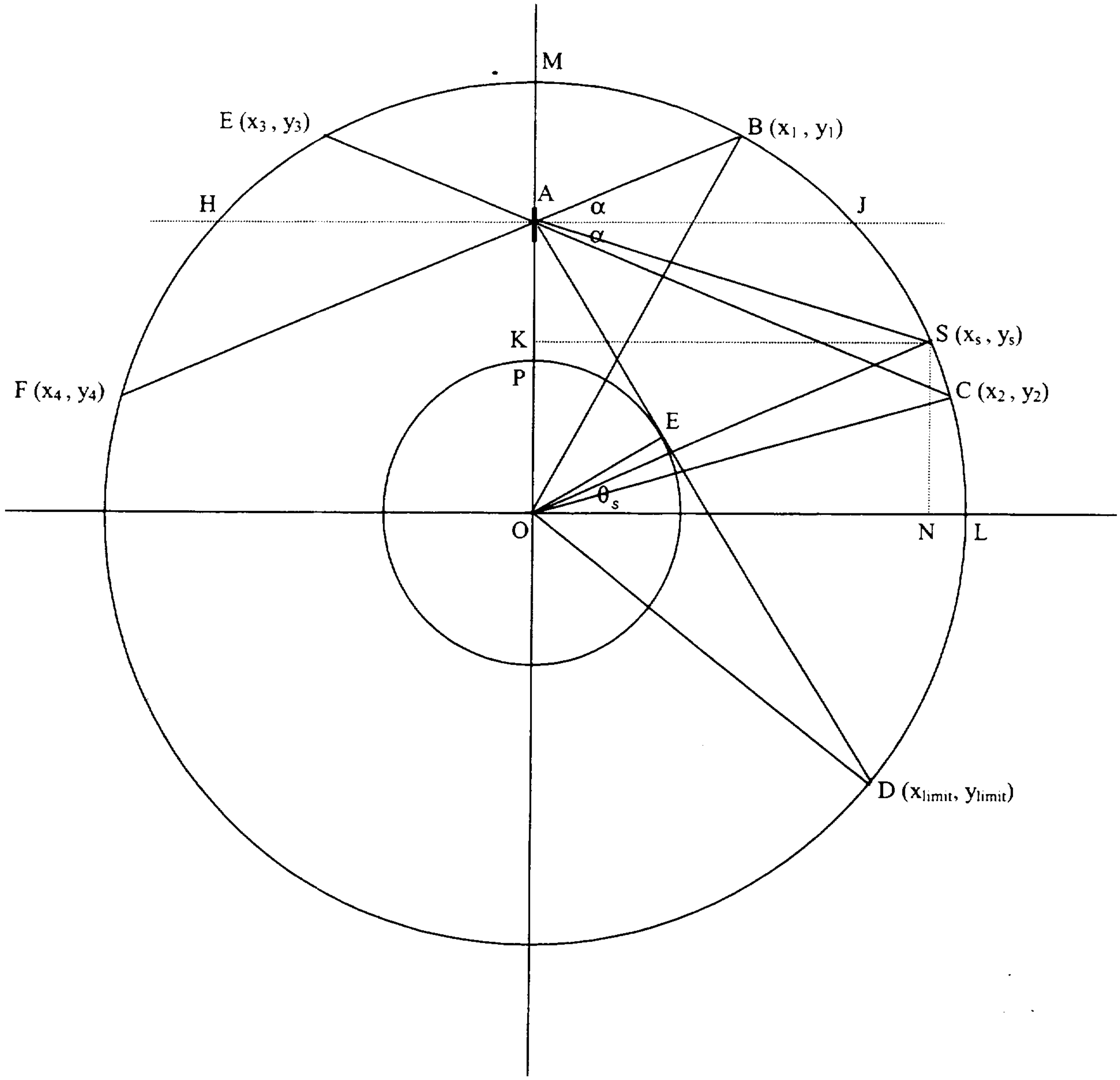


Figure 2. Balloon/Parachute descent geometry.

It will be assumed that the telecommunications system is housed in a gondola that is attached to the balloon or the parachute. In Figure 1, the letter A indicates the position of the gondola and the antenna that is used for communications with the orbiter. At this time, it will be assumed that the antenna used is the dipole antenna. Also, it will be assumed that the parachute or the balloon is present in the orbital plane of the satellite.

The object here is to compute the satellite visibility time for a given minimum gain from the given position of the antenna. Towards that end, we will first compute the coordinates of the points B, C, E, and F, which are shown in the figure generated by the appropriate lines at an angle α to the maximum of the dipole antenna direction AJ or AH. The coordinates of the points B and F are obtained by the solution of the following simultaneous nonlinear equations:

$$y = m x + c$$

$$y^2 = -x^2 + a^2 \tag{4.1}$$

Where the quantities 'c' and 'a' are defined in terms of the satellite orbital geometry and the balloon/parachute altitude as shown above. The parameter m is defined as:

$$m = \mathrm{Tan}(\alpha) \tag{4.2}$$

Similarly, the coordinates of the points C and E are obtained by the solution of the following simultaneous equations.

$$y = -m x + c$$

$$y^2 = -x^2 + a^2 \tag{4.3}$$

The results of the solution of these equations are given below.

Point $B(x_1, y_1)$:

$$x_1 = -\left(\frac{m\,c}{1 + m^2}\right) + \sqrt{\left(\frac{m\,c}{1 + m^2}\right)^2 + \frac{a^2 - c^2}{1 + m^2}}$$

$$y_1 = \left(\frac{c}{1 + m^2}\right) + m\left(\sqrt{\left(\frac{m\,c}{1 + m^2}\right)^2 + \frac{a^2 - c^2}{1 + m^2}}\right) \tag{4.4}$$

Point $C(x_2, y_2)$:

$$x_2 = \left(\frac{m\,c}{1 + m^2}\right) + \sqrt{\left(\frac{m\,c}{1 + m^2}\right)^2 + \frac{a^2 - c^2}{1 + m^2}}$$

$$y_2 = \left(\frac{c}{1 + m^2}\right) - m\left(\sqrt{\left(\frac{m\,c}{1 + m^2}\right)^2 + \frac{a^2 - c^2}{1 + m^2}}\right) \tag{4.5}$$

Similarly one can obtain the coordinates of points E and F. The results obtained are given below.

Point $E(x_3, y_3)$:

$$x_3 = \left(\frac{m\,c}{1 + m^2}\right) - \sqrt{\left(\frac{m\,c}{1 + m^2}\right)^2 + \frac{a^2 - c^2}{1 + m^2}}$$

$$y_3 = \left(\frac{c}{1 + m^2}\right) + m\left(\sqrt{\left(\frac{m\,c}{1 + m^2}\right)^2 + \frac{a^2 - c^2}{1 + m^2}}\right) \tag{4.6}$$

Point $F(x_4, y_4)$:

$$x_4 = -\left(\frac{m c}{1 + m^2}\right) - \sqrt{\left(\frac{m c}{1 + m^2}\right)^2 + \frac{a^2 - c^2}{1 + m^2}}$$

$$y_4 = \left(\frac{c}{1 + m^2}\right) - m\left(\sqrt{\left(\frac{m c}{1 + m^2}\right)^2 + \frac{a^2 - c^2}{1 + m^2}}\right) \tag{4.7}$$

The angle α provides the angle from the boresite of the antenna (the maximum gain direction) for which the communications between the satellite and the gondola can be sustained. The gain of the dipole antenna at the angle α from its boresite can be computed from the gain function of the dipole antenna given below in Equation (4.8). In that equation, l is the total length of the dipole and λ is the wavelength of the transmission frequency (velocity of light)/Frequency.

$$G(\alpha) = \eta_{eff} \frac{2\left[\dfrac{\operatorname{Cos}\left(\frac{\pi l}{\lambda}\operatorname{Cos}\left(\frac{\pi}{2} \pm \alpha\right)\right) - \operatorname{Cos}\left(\frac{\pi l}{\lambda}\right)}{\operatorname{Sin}\left(\frac{\pi}{2} \pm \alpha\right)}\right]^2}{\displaystyle\int_{\beta=0}^{\beta=\pi}\left[\dfrac{\operatorname{Cos}\left(\frac{\pi l}{\lambda}\operatorname{Cos}(\beta)\right) - \operatorname{Cos}\left(\frac{\pi l}{\lambda}\right)}{\operatorname{Sin}(\beta)}\right]^2 \operatorname{Sin}(\beta)\, d\beta} \tag{4.8}$$

The denominator of the above equation is the normalization factor that must be computed before the gain predicted by the antenna may be used. The normalization factor is a function of the dipole length and the transmission wavelength. Figure 3 plots the value of this factor as a function of the ratio of dipole length to the transmission wavelength (h/r_p) multiplied by π. Thus, for a half wave dipole, the normalization factor is 1.22. A polynomial approximation to the denominator can be devised and may be used, as is done below in equation (4.9), to facilitate the computation of equation (4.8). The loss of precision by using the approximation is only very little and, hence, can be used with confidence. Thus, the formula given in Equation (4.9) now can be used to

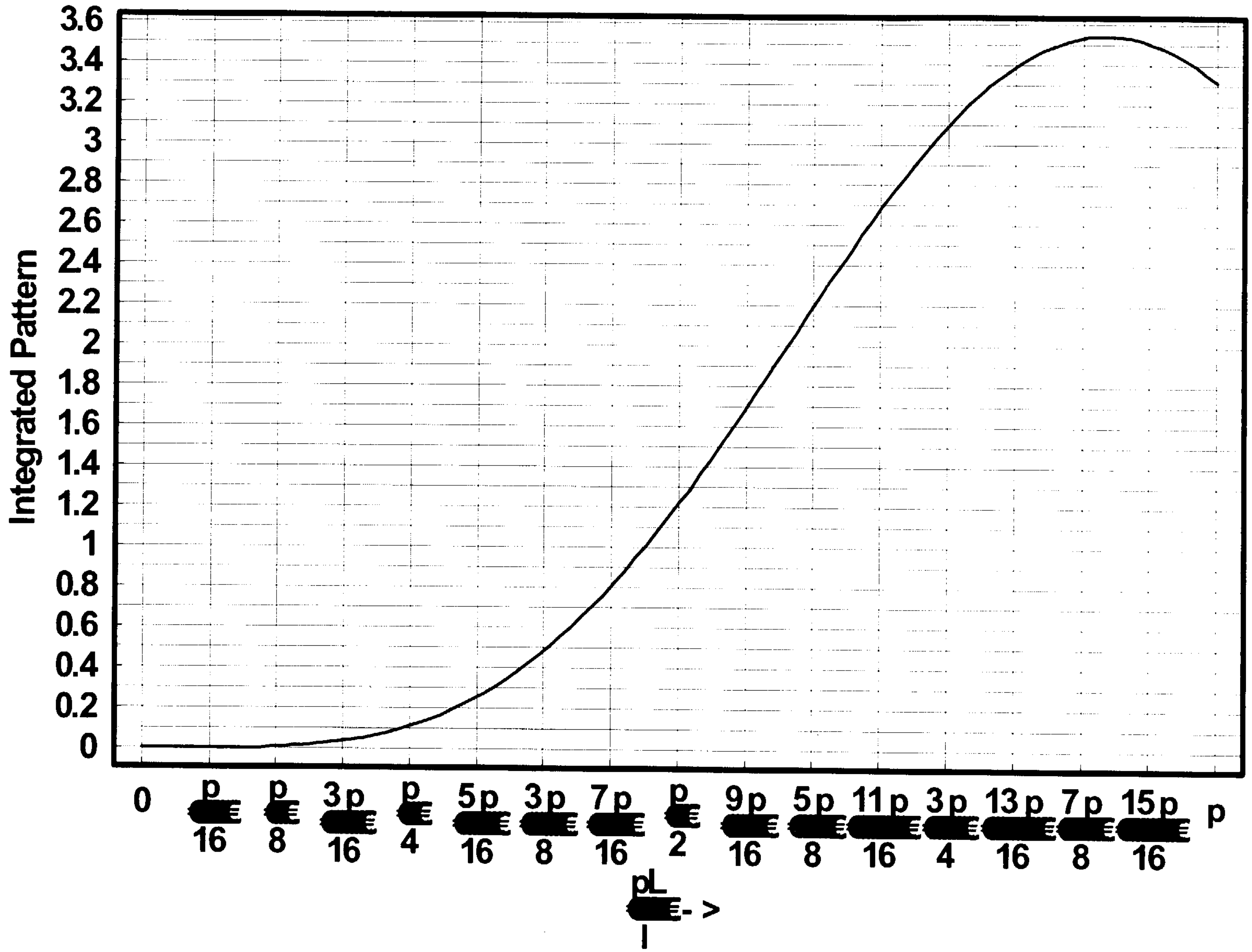


Figure 3 Normalizing factor of equation (4.8).

compute the gain of the dipole antenna in the pointing direction α. It should be noted that the parameter η_{eff} used in equations (4.8) and (4.9) is the antenna efficiency and is in no way connected with the central angle η

$$G(\alpha) = \frac{2\,\eta_{eff}\left[\dfrac{\mathrm{Cos}\left(\dfrac{\pi l}{\lambda}\,\mathrm{Cos}\left(\dfrac{\pi}{2}-\alpha\right)\right)-\mathrm{Cos}\left(\dfrac{\pi l}{\lambda}\right)}{\mathrm{Sin}\left(\dfrac{\pi}{2}-\alpha\right)}\right]^2}{-0.000781104-0.000921702\left(\dfrac{\pi l}{\lambda}\right)+0.0926785\left(\dfrac{\pi l}{\lambda}\right)^2-0.369984\left(\dfrac{\pi l}{\lambda}\right)^3+0.892439\left(\dfrac{\pi l}{\lambda}\right)^4-0.389927\left(\dfrac{\pi l}{\lambda}\right)^5+0.04814\left(\dfrac{\pi l}{\lambda}\right)^6} \tag{4.9}$$

5.0 Visibility of Orbiter for a Gain Angle of the Balloon Antenna

Once it is established that the communications link will be sustained for the given set of system parameters and the transmit antenna gain G (α), one can compute the visibility time of the satellite for this condition: i.e., the time for which the satellite will remain within the angle α above and below the boresite of the dipole antenna (see Figure 2). It is the time for which the satellite stays within the arc BC of the circle that is proportional to the central angle $\angle$ COB in the same figure. The angle $\angle$ COB can be computed as follows:

$$\angle \mathrm{COB} \triangleq \eta = \mathrm{Cos}^{-1}\left[\frac{\underline{\mathrm{OB}}\cdot\underline{\mathrm{OC}}}{|\underline{\mathrm{OB}}|\,|\underline{\mathrm{OC}}|}\right] \tag{5.1}$$

Where $\underline{OC}$ is the vector from O to C, and $\underline{OB}$ is the vector from O to B in figure 2. Using the coordinates of C, B and O (0,0) given in figure 2 one may reduce the equation (5.1).

$$\eta = \cos^{-1}\left[\frac{x_1 x_2 + y_1 y_2}{\sqrt{x_1^2 + y_1^2}\,\sqrt{x_2^2 + y_2^2}}\right] \tag{5.2}$$

Using equations (4.4) and (4.5), $\angle$ COB may be evaluated in terms of m, c, and a as follows:

$$x_1 x_2 = \left(\frac{m\,c}{1 + m^2}\right)^2 + \frac{a^2 - c^2}{1 + m^2} - \left(\frac{m\,c}{1 + m^2}\right)^2 = \frac{a^2 - c^2}{1 + m^2}$$

$$y_1 y_2 = \left(\frac{c}{1 + m^2}\right)^2 - m^2\left[\left(\frac{m\,c}{1 + m^2}\right)^2 + \frac{a^2 - c^2}{1 + m^2}\right] = \frac{c^2 - m^2 a^2}{1 + m^2} \tag{5.3}$$

Thus,

$$x_1 x_2 + y_1 y_2 = \frac{a^2 - c^2}{1 + m^2} + \frac{c^2 - m^2 a^2}{1 + m^2} = \left(\frac{1 - m^2}{1 + m^2}\right) a^2 \tag{5.4}$$

Noting that,

$$|\underline{OC}| = \sqrt{x_1^2 + y_1^2} = a \quad \text{and} \quad |\underline{OB}| = \sqrt{x_2^2 + y_2^2} = a \tag{5.5}$$

Using equation (5.4) and (5.5) into equation (5.2) we have

$$\eta = \cos^{-1}\left[\frac{x_1 x_2 + y_1 y_2}{\sqrt{x_1^2 + y_1^2}\,\sqrt{x_2^2 + y_2^2}}\right] = \cos^{-1}\left[\frac{1 - m^2}{1 + m^2}\right] \tag{5.6}$$

Now using the definition of 'm' from equation (4.2), we finally arrive at

$$\eta = \mathrm{Cos}^{-1}\left[\frac{1 - m^2}{1 + m^2}\right] = \mathrm{Cos}^{-1}\left[\frac{1 - \mathrm{Tan}^2(\alpha)}{1 + \mathrm{Tan}^2(\alpha)}\right] \tag{5.7}$$

Using some trigonometric identities, this reduces to the following equation.

$$\eta = \mathrm{Cos}^{-1}\left[\mathrm{Cos}^2(\alpha)\left(\frac{\mathrm{Cos}^2(\alpha) - \mathrm{Sin}^2(\alpha)}{\mathrm{Cos}^2(\alpha)}\right)\right] = \mathrm{Cos}^{-1}\left[\mathrm{Cos}(2\,\alpha)\right] = 2\,\alpha \tag{5.8}$$

Thus, the visibility angle η is computed to be 2α, the total antenna angle described previously (see Figure 2). Before one computes the visibility time of the satellite from the balloon or parachute antenna for any angle less than α from the boresite of the antenna main lobe, one must consider the limiting case of the visibility.

In the limiting case, the value of the angle $\alpha = \alpha_{limit}$ will be such that the line AC in Figure 2 will be tangential to the planet's surface such as the line AD. The angle α_{limit} can be computed rather easily from the geometry and is given below..

$$\alpha_{limit} = \frac{\pi}{2} - \left[\frac{\pi}{2} - \mathrm{Cos}^{-1}\left(\frac{r_p}{c}\right)\right] = \mathrm{Cos}^{-1}\left(\frac{r_p}{c}\right) \tag{5.9}$$

$$\text{Let} \quad m_{limit} = \mathrm{Tan}(\alpha_{limit}) = \frac{\sqrt{c^2 - r_p^2}}{r_p} = \sqrt{\left(\frac{c}{r_p}\right)^2 - 1} \tag{5.10}$$

The coordinates of the point $D(x_{limit}, y_{limit})$ can be calculated as follows:

$$x_{limit} = \left(\frac{m_{limit}\, c}{1 + m_{limit}^2} \right) + \sqrt{ \left(\frac{m_{limit}\, c}{1 + m_{limit}^2} \right)^2 + \frac{a^2 - c^2}{1 + m_{limit}^2} }$$

$$y_{limit} = \left(\frac{c}{1 + m_{limit}^2} \right) - m_{limit} \left(\sqrt{ \left(\frac{m_{limit}\, c}{1 + m_{limit}^2} \right)^2 + \frac{a^2 - c^2}{1 + m_{limit}^2} } \right) \tag{5.11}$$

Using these coordinates, one may compute the central limiting angle defined as η_{limit} , as follows:

$$\eta_{limit} = Cos^{-1}\left[\frac{\underline{OD} \bullet \underline{OB}}{|\underline{OD}|\,|\underline{OB}|} \right] \tag{5.12}$$

Where D is the vector from O to D and $\underline{OB}$ is the vector from O to B in figure 2. Using the coordinates of D, B and O (0,0) given in Figure 2, one may reduce equation (5.12). Noting that,

$$|\underline{OD}| = \sqrt{x_{limit}^2 + y_{limit}^2} = a \quad \text{and} \quad |\underline{OB}| = \sqrt{x_1^2 + y_1^2} = a \tag{5.13}$$

$$\eta_{limit} = Cos^{-1}\left[\frac{x_1\, x_{limit} + y_1\, y_{limit}}{a^2} \right] \tag{5.14}$$

Thus, at a particular altitude of the balloon or the parachute, the central angle used for the time of visibility will be given by:

$$\eta = 2\alpha \qquad \text{For} \quad \alpha \le Cos^{-1}\left(\frac{r_p}{c} \right)$$

$$\eta = \eta_{limit} \qquad \text{For} \quad \alpha > Cos^{-1}\left(\frac{r_p}{c} \right) \tag{5.15}$$

The total visibility time i.e., the time for which the orbiter is visible to the balloon/parachute assembly can be calculated using the following formula. It should be noted that the period of the low altitude satellites is generally small compared to the period of higher altitude satellites, hence, the balloon or the parachute will easily see the entire traverse of the satellite.

$$\text{Satellite Period} = 2\pi \frac{a^{3/2}}{\sqrt{\mu}} \quad \text{(Sec)}$$

$$\text{Visibility Period} = \text{Satellite Period}\left(\frac{2\eta}{2\pi}\right) = 2\eta \frac{a^{3/2}}{\sqrt{\mu}} \quad \text{(Sec)} \tag{5.16}$$

Where the parameter μ is the gravity constant of the planet (known as Kepler's constant). The values of μ are given for many planets in Table 1 below.

Table 1. A table of planetary constants.

Planet	Equitorial Radius (km)	Orbital Semi Major Axis (AU)	Orbital Eccentricity ε	Mean Solar Distance 10^6 (km)	Kepler's Const, μ km^3/sec^2
Sun	696000	-----	-----	-----	1.327×10^{11}
Mercury	2487	.3871	.2056	57.9	2.232×10^{4}
Venus	6187	.7233	.0068	108.1	3.253×10^{5}
Earth	6378	1.000	.0617	149.5	3.986×10^{5}
Mars	3380	1.524	.0934	227.8	4.305×10^{4}
Jupiter	71370	5.203	.0482	778	1.268×10^{8}
Saturn	60400	9.519	.0539	1426	3.795×10^{7}
Uranus	23530	19.28	.0514	2868	5.820×10^{6}
Neptune	22320	30.17	.0050	4494	6.896×10^{6}
Pluto	7016	39.76	.2583	5896	3.587×10^{5}

Figure 4 shows a plot of orbiter total visibility angle at the balloon/parachute as a function of the normalized orbiter altitude. It should be noted that the central visibility angle predicted by equation (5.15) is the visibility angle of the orbiter at the balloon/parachute until the orbiter reaches the zenith direction of the balloon/parachute. Hence, the total visibility angle will be twice that predicted by the equation (5.15) to account for the visibility during the rise of the orbiter up to the zenith of the balloon and then from zenith to setting point of the orbiter. This factor of 2 is already taken into account in Figure 4.

Figure 4 is drawn for the normalized balloon/parachute altitude of 0.01: i.e., the balloon/parachute altitude divided by the radius of the planet equals 0.01 = 1% value. The parameter of the figure that changes from curve to curve is the angle α that is specified in terms of the antenna pattern. It is specified in terms of the dB down from the maximum of the antenna pattern. Thus, the 0.2 dB down curve in Figure 4 is drawn for the angle α at which the antenna power gain is 0.2 dB down from the maximum of the antenna power pattern.

As an example, the total central visibility angle for the normalized balloon/parachute altitude of 0.01 with the 3 dB down from the antenna maximum (boresite) is about 62 degrees when the normalized orbiter altitude is 0.2 = 20%. It should be noted that the figure is valid only for the normalized balloon/parachute altitude of 0.01. A series of curves must be drawn for various values of the normalized balloon/parachute altitude.

Figure 5 shows a plot of orbiter total visibility angle at the balloon/parachute as a function of the normalized balloon/parachute altitude. This figure is similar to the Figure 4; However, the independent variable is now the normalized balloon/parachute altitude. In this figure too, the visibility is the total visibility angle, including the factor of 2 explained in the Figure 4 description.

Figure 5 is drawn for the normalized orbiter altitude of 0.1: i.e., the orbiter altitude divided by the radius of the planet equals to 0.1 = 10% value. The parameter of the figure that changes from curve to curve is the angle α that is specified in terms of the antenna pattern, as in the case of Figure 4.

As an example, the total central visibility angle for the normalized orbiter altitude of 0.1 with the 3 dB down from the antenna maximum (boresite) is about 63 degrees when the normalized balloon/parachute altitude is 0.02 = 2%. It should be noted that the figure is valid only for the normalized orbiter altitude of 0.1. A series of curves must be drawn for various values of the normalized orbiter altitude. Figure 6 plots a three dimensional graph for the total central visibility angle (including the factor 2) as a function of the normalized orbital altitude and normalized balloon/parachute altitude. In both the axis, the normalization factor is the radius of the planet. This figure is harder to read compared to the Figure 3 and Figure 4 graphs due to its 3 dimensional nature; however, the advantage is that both the normalized orbiter altitude as well as the normalized balloon/parachute altitude can change simultaneously.

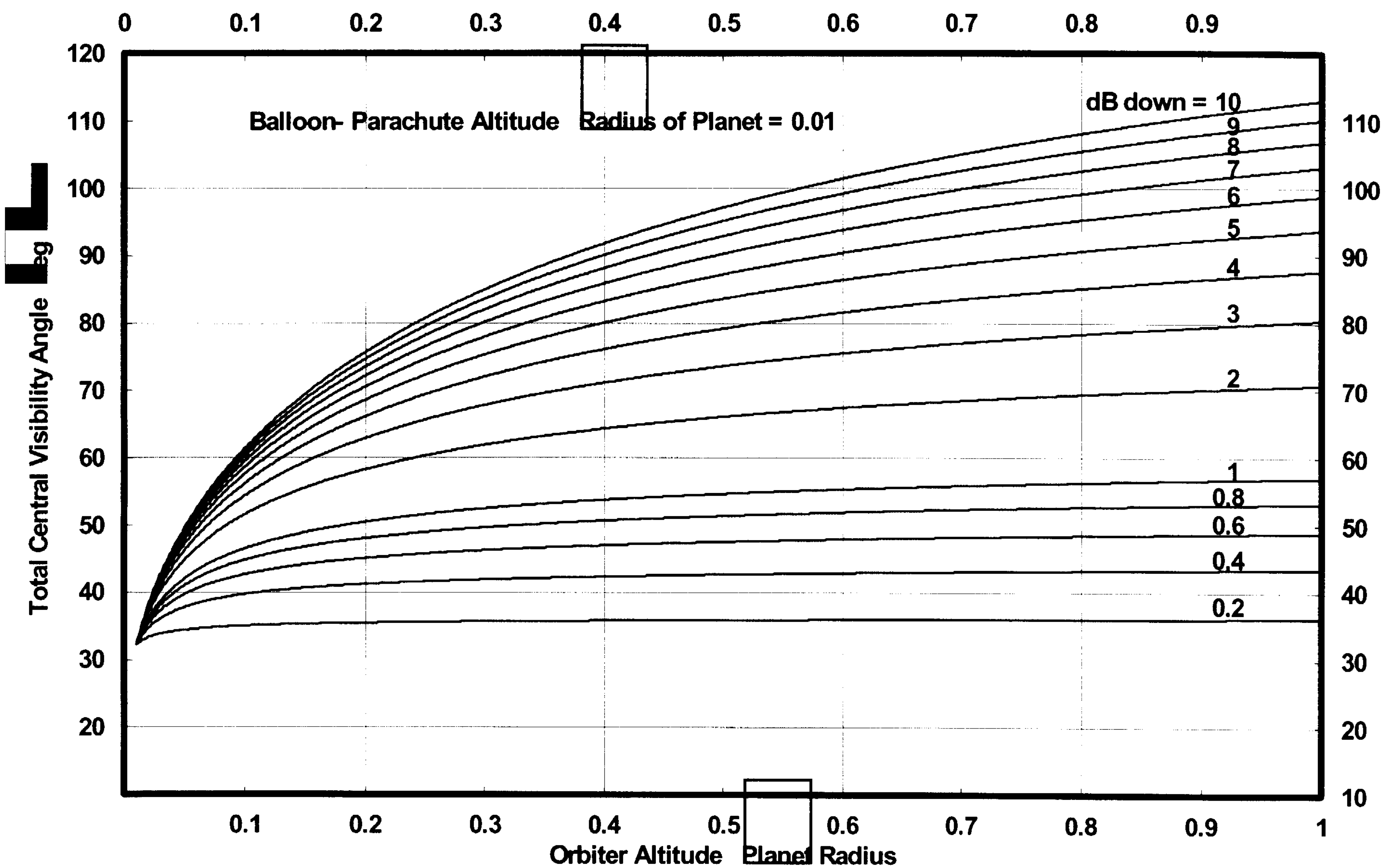


Figure 4. Orbiter total visibility angle as a function of normalized orbiter altitude.

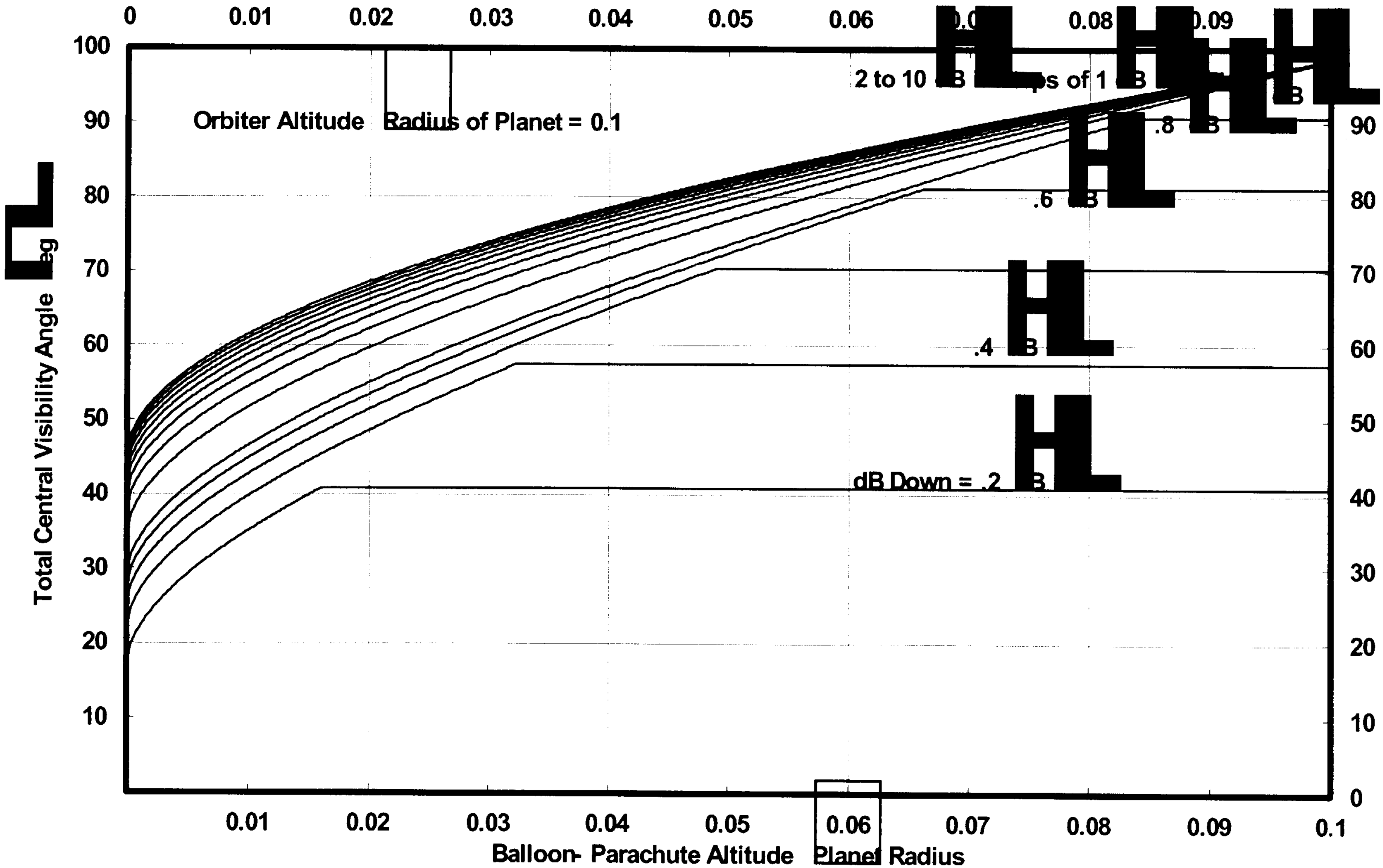


Figure 5. Orbiter total visibility angle as a function of normalized Balloon/Parachute altitude.

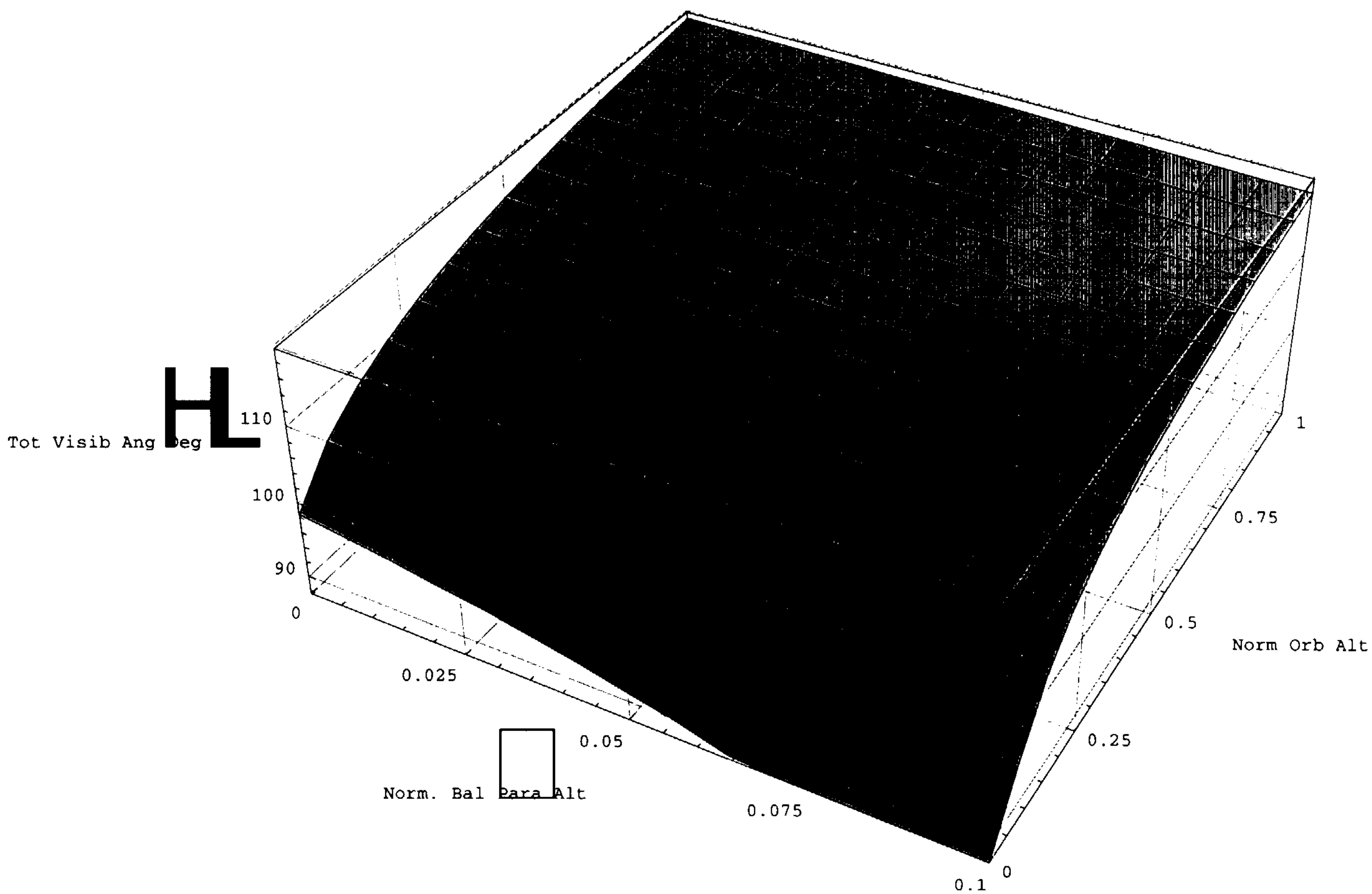


Figure 6. Total visibility angle as a function of normalized balloon altitude and normalized orbiter altitude.

Another disadvantage of the 3 dimensional representation is that the surface drawn in Figure 6 can be drawn for only one dB down number. Figure 6 is drawn for the angle α computed for 3 dB down condition.

Once the total visibility central angle is obtained using the above curves, one needs some method to convert it into the actual visibility time of the orbiter to the balloon/parachute assembly. The orbital period of the orbiter will naturally depend upon the planet's gravity constant, μ, and the orbiter altitude above the surface of the planet. Equation (5.16) given above shows the orbiter's orbital period and is reproduced below with some modifications.

$$\text{Satellite Period} = 2\pi \frac{a^{3/2}}{\sqrt{\mu}} \quad \text{(Sec)} \tag{5.17}$$

and

$$\text{Visibility Period} = \text{Satellite Period} \cdot \left(\frac{2\eta}{2\pi}\right) = 2\eta \frac{a^{3/2}}{\sqrt{\mu}} \quad \text{(Sec)}$$

$$= \underbrace{(2\eta)}_{T_{\text{Visibility Angle}}} \times \underbrace{\left(\frac{\pi}{180}\right)\left(\frac{r_p^{3/2}}{\sqrt{\mu}}\right)\left(1 + \frac{h}{r_p}\right)^{3/2}}_{V\left(\frac{h}{r_p}\right)} \quad \text{(Sec)}$$

$$= T_{\text{Visbility Angle}} \times V\left(\frac{h}{r_p}\right) \quad \text{(Sec)} \tag{5.18}$$

Equation (5.18) connects the total visibility central angle, $2\eta = T_{\text{Visibility Angle}}$, with the actual visibility time in seconds. It will be very useful to compute the visibility times of the orbiter from the balloon/parachute for any planet. Figure 7 plots the function $V\left(\frac{h}{r_p}\right)$ for the solar system planets; and Figure 8 does the same for the solar system moons of interest. To compute the visibility time, the following procedure is to be followed.

1. Using either Figure 4 or Figure 5, determine the total visibility angle $2\,\eta = T_{\text{Visibility Angle}}$ depending on the available parameter values of the orbiter and the balloon/parachute. Note down the $\frac{h}{r_p}$ value used in this determination.

2. Using either Figure 7 or Figure 8 depending upon which planet or moon the orbiter is located at, for the value $\frac{h}{r_p}$ noted down in the above step, note down the value of $V\left(\frac{h}{r_p}\right)$.

3. Multiplying the value of the total visibility angle from step 1 with the value of $V\left(\frac{h}{r_p}\right)$ from step 2 produces the visibility time in seconds for the conditions used in step 1 and 2.

6.0 Useful Gain of Balloon Antenna and Orbiter S/C Range

The antenna gain directed towards the position of the satellite and the range between the antenna and the satellite will be calculated next. This gain and the range will be used to compute the link budget for the balloon/parachute telecom system.

$$\text{Position of the antenna} = A(0\,,\,h_b)$$

$$\text{Position of the Spacecraft} = S\big(a\,\text{Cos}(\theta_s)\,,\,a\,\text{Sin}(\theta_s)\big) \tag{6.1}$$

The range of the satellite, R, from the antenna of the parachute/balloon antenna for a given position of the satellite will be given by the following expression:

$$\text{Range} \triangleq R = \sqrt{a^2 + c^2 - 2\,a\,c\,\text{Sin}(\theta_s)} \tag{6.1}$$

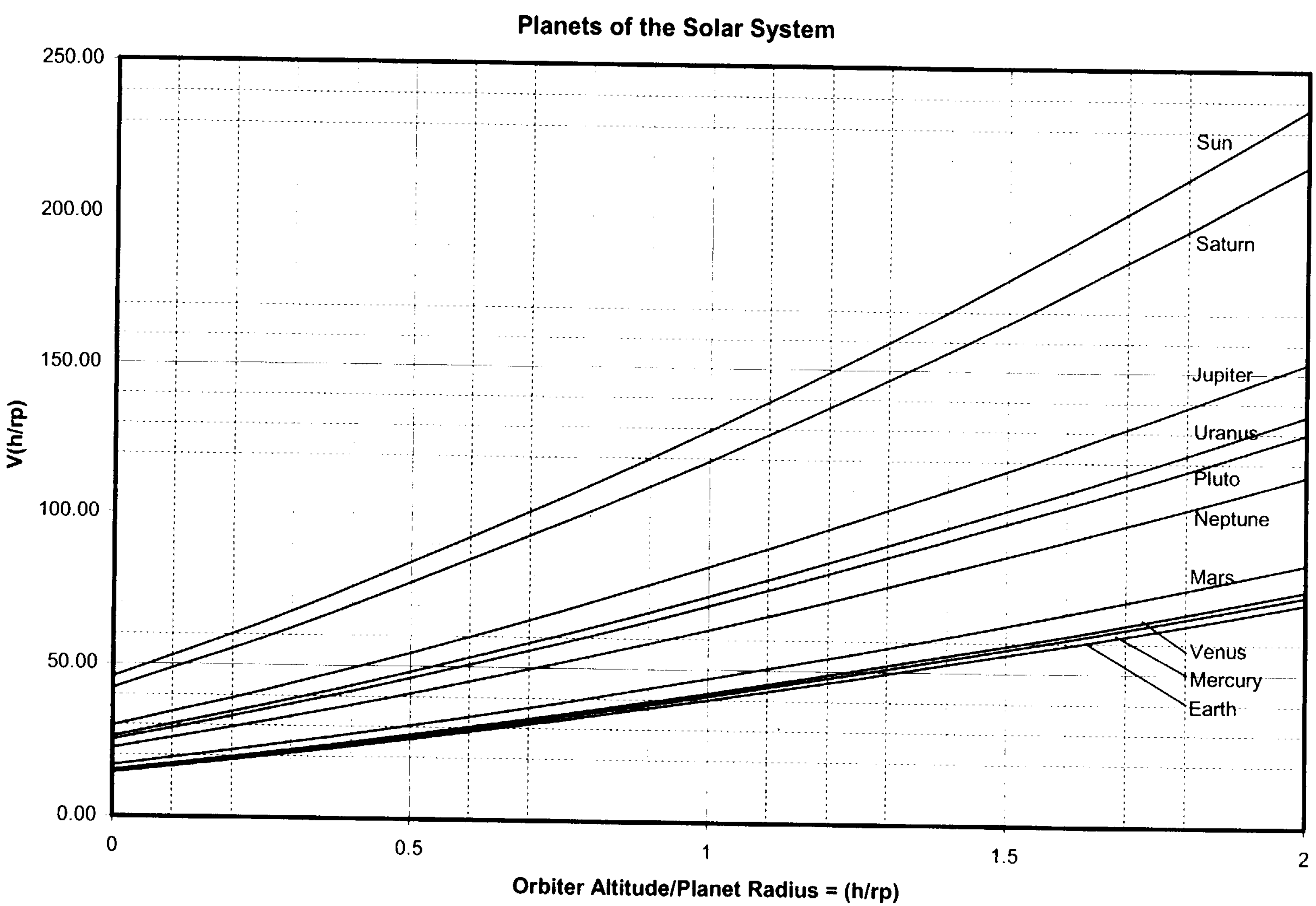


Figure 7. A plot of $V\left(\frac{h}{r_p}\right)$ function for various solar system planets.

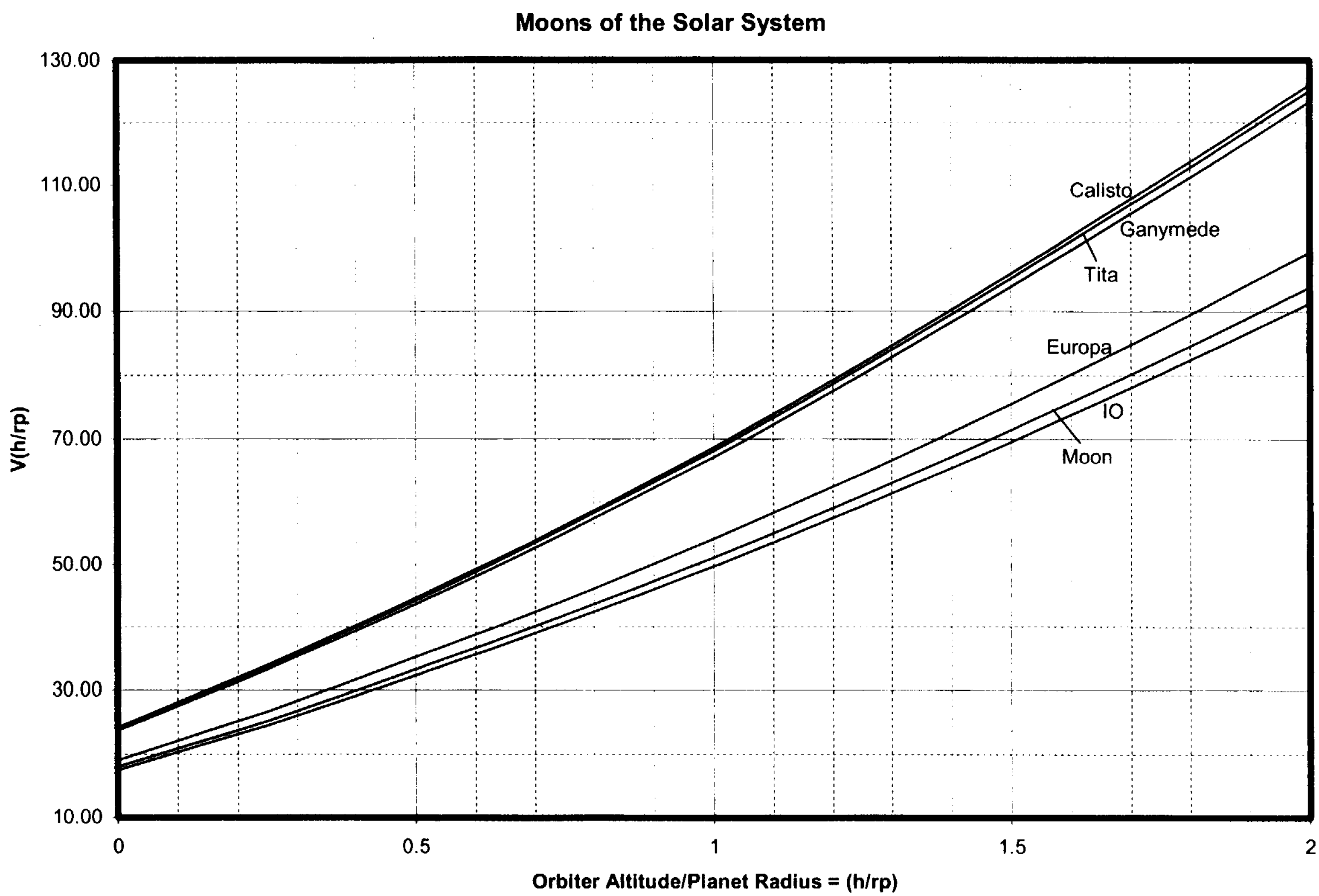


Figure 8. A plot of $V\left(\frac{h}{r_p}\right)$ function for various solar system moons.

Many times a normalized range will be necessary in the theory to follow. The normalized range is defined as the actual range divided by the radius of the planet/moon around which the orbiter is in a given orbit. This division allows to normalize the balloon/parachute altitude as well as the orbiter altitude as shown below in equation (6.2).

$$\text{Range Normalized} \triangleq RN = \frac{R}{r_p} = \sqrt{\left(1 + \frac{h}{r_p}\right)^2 + \left(1 + \frac{h_b}{r_p}\right)^2 - 2\left(1 + \frac{h}{r_p}\right)\left(1 + \frac{h_b}{r_p}\right)\mathrm{Sin}(\theta_s)} \tag{6.2}$$

The gain of the balloon/parachute antenna and the range to the satellite can be used in a telecommunications link budget to calculate the space loss for the link budget and ultimately, the sustainable bit rate by the link. Using the right triangle KAS in Figure 2, we obtain:

$$\angle KAS = \mathrm{Sin}^{-1}\left[\frac{a\,\mathrm{Cos}(\theta_s)}{R}\right] \tag{6.3}$$

To compute the gain of the antenna at a given position of the satellite we need the following expression:

$$\theta = \pi - \angle KAS = \pi - \mathrm{Sin}^{-1}\left[\frac{a\,\mathrm{Cos}(\theta_s)}{R}\right] \tag{6.4}$$

For computing the useful gain for the communications link the following quantities are computed:

$$\mathrm{Cos}(\theta) = \mathrm{Cos}\left\{\pi - \mathrm{Sin}^{-1}\left[\frac{a\,\mathrm{Cos}(\theta_s)}{R}\right]\right\} = -\left(\frac{c - a\,\mathrm{Sin}(\theta_s)}{R}\right) = \left(\frac{a\,\mathrm{Sin}(\theta_s) - c}{R}\right) \tag{6.5}$$

Similarly, the sine of the angle may be found using the following formula:

$$\mathrm{Sin}(\theta) \;=\; \mathrm{Sin}\left\{\pi \;-\; \mathrm{Sin}^{-1}\left[\frac{a\,\mathrm{Cos}(\theta_s)}{R}\right]\right\} \;=\; \frac{a\,\mathrm{Cos}(\theta_s)}{R} \tag{6.6}$$

The gain of the antenna directed towards the satellite will then be obtained by the formula given in equation (6.7) below. Note that the angle θ used to compute the gain of the antenna in equation (6.7) is a function of the satellite position θ_s, altitude of the balloon/parachute h_b, and the orbital radius of the spacecraft a. The relationship between the angle θ and the satellite position θ_s is defined in the equations (6.5) and (6.6).

$$G(\theta) = \frac{2\eta\left[\dfrac{\mathrm{Cos}\left(\frac{\pi l}{\lambda}\,\mathrm{Cos}(\theta)\right) - \mathrm{Cos}\left(\frac{\pi l}{\lambda}\right)}{\mathrm{Sin}(\theta)}\right]^2}{\begin{array}{r}-0.000781104 - 0.000921702\left(\frac{\pi l}{\lambda}\right) + 0.0926785\left(\frac{\pi l}{\lambda}\right)^2 - 0.369984\left(\frac{\pi l}{\lambda}\right)^3 \\ + 0.892439\left(\frac{\pi l}{\lambda}\right)^4 - 0.389927\left(\frac{\pi l}{\lambda}\right)^5 + 0.04814\left(\frac{\pi l}{\lambda}\right)^6\end{array}} \tag{6.7}$$

$$\triangleq \; f\left(\frac{\pi l}{\lambda}\right)\left[\frac{\mathrm{Cos}\left(\frac{\pi l}{\lambda}\,\mathrm{Cos}(\theta)\right) - \mathrm{Cos}\left(\frac{\pi l}{\lambda}\right)}{\mathrm{Sin}(\theta)}\right]^2 \tag{6.8}$$

Where the function $f\left(\frac{\pi l}{\lambda}\right)$ is given by the following formula:

$$f\left(\frac{\pi l}{\lambda}\right) = \frac{2\,\eta_{eff}}{-0.000781104 - 0.000921702\left(\frac{\pi l}{\lambda}\right) + 0.0926785\left(\frac{\pi l}{\lambda}\right)^2 - 0.369984\left(\frac{\pi l}{\lambda}\right)^3 + 0.892439\left(\frac{\pi l}{\lambda}\right)^4 - 0.389927\left(\frac{\pi l}{\lambda}\right)^5 + 0.04814\left(\frac{\pi l}{\lambda}\right)^6} \tag{6.9}$$

The link from the balloon/parachute dipole antenna to the orbiter spacecraft naturally depends upon many parameters; however, the only parameters that are dependent on the balloon/parachute altitude are the gain of the transmitting antenna (the dipole) and the range from the transmitting antenna to the orbiting spacecraft, designated by R. As this altitude changes, the range R becomes a function of time. This analysis assumes all the other parameters of this scenario constant. This is not really true in actual practice. The atmospheric loss parameter for example, does not remain constant and depends upon the actual length of the path through the atmosphere. We will assume for the sake of the first approximation that such parameters have a definite value for the computation of the link between the parachute/balloon and the orbiter but the values does not change while the satellite is in motion. The major change in the link budget is due to the change of the range R; the change of the gain of the antenna due to the satellite position change.

The range R gives the space loss given by the following well-known equation:

$$\text{Space Loss} = \left(\frac{\lambda}{4\,\pi\,R}\right)^2$$

$$\text{Space Loss Using Normalized Range} = \left(\frac{\lambda}{4\,\pi\,(RN)}\right)^2 \tag{6.10}$$

Where the normalized range, RN, is defined in equation (6.2). The antenna gain pattern provides the useful gain of the antenna in the spacecraft direction. Using the above equations, the value of the gain directed to the spacecraft multiplied by the space loss is given by the following formula:

$$\begin{matrix}\text{Space}\\ \text{Loss}\end{matrix} \times \begin{matrix}\text{Useful}\\ \text{Gain}\end{matrix} = \left(\frac{\lambda}{4\pi R}\right)^2 \left[f\left(\frac{\pi l}{\lambda}\right) \left\{ \frac{\mathrm{Cos}\left[\left(\frac{\pi l}{\lambda}\right)\left(\frac{a\,\mathrm{Sin}(\theta_s) - c}{\sqrt{a^2 + c^2 - 2\,a\,c\,\mathrm{Sin}(\theta_s)}}\right)\right] - \mathrm{Cos}\left(\frac{\pi l}{\lambda}\right)}{\frac{a\,\mathrm{Cos}(\theta_s)}{\sqrt{a^2 + c^2 - 2\,a\,c\,\mathrm{Sin}(\theta_s)}}} \right\} \right]^2 \tag{6.11}$$

Equation (6.11) provides the effect of the space loss and the useful gain of the balloon/parachute directed towards the orbiter spacecraft. This is a useful formula; however, sometimes it is helpful to convert this formula into the quantities normalized by the radius of the planet. Using the normalized values of the parameters 'a' and 'c' (dividing the parameters by radius of the planet) along with the normalized value of the range RN defined in equation (6.2) we obtain an equivalent formula given below:

$$\begin{matrix}\text{Space Loss}\\ \text{Using Normalized}\\ \text{Range Value}\end{matrix} \times \begin{matrix}\text{Useful}\\ \text{Gain}\end{matrix} = \left(\frac{\lambda}{4\pi\,(RN)}\right)^2 \left[f\left(\frac{\pi l}{\lambda}\right) \left\{ \frac{\mathrm{Cos}\left[\left(\frac{\pi l}{\lambda}\right)\left(\frac{\left(1+\frac{h}{r_p}\right)\mathrm{Sin}(\theta_s) - \left(1+\frac{h_b}{r_p}\right)}{(RN)}\right)\right] - \mathrm{Cos}\left(\frac{\pi l}{\lambda}\right)}{\frac{\left(1+\frac{h}{r_p}\right)\mathrm{Cos}(\theta_s)}{(RN)}} \right\} \right]^2 \tag{6.11a}$$

It should be noted that the gain function now appears as a function of the orbiter satellite position angle θ_s. Thus, as the orbiter satellite moves around in its orbit, a different value of the parachute/balloon antenna gain will be directed towards it and, of course, the parachute/balloon to the orbiter range changes too. Equations (6.11) and (6.11a) incorporate effects of range change as well as antenna useful gain directed towards the orbiter into one formula that may be used for the link analysis of the link mentioned above.

Generally, the altitude of the parachute or the balloon when they are deployed from the entry vehicle depends upon the experiments to be performed on the way to the planet's surface or while going around the planet, as the case may be. If the planet is a gaseous planet, there is another question about the gaseous absorption of the radiated waves such as the ammonia gas absorbs the most and the absorption is proportional to the square of the link frequency. Thus, this consideration may force the telecom engineer to use the UHF frequency because this has the minimal gaseous absorption. However, the UHF has disadvantage in that the telecom equipment is larger in weight and volume and the gains of the antennas of the link are lower which is not made up by the lower space loss due to the lower frequency.

As the orbiter spacecraft goes around in its orbit, there is a limiting angle at which it just starts seeing the balloon and goes to the zenith of the balloon and then repeats the same in a reverse order. This should be quite easily seen in the Figure 2. One needs the angle θ_s when the orbiter just rises over the horizon of the balloon above the surface of the planet. This is the limiting angle θ_{sLimit} and given by the following expression:

$$\text{Limiting } \theta_s \text{ angle} = \theta_{sLimit} = \frac{\pi}{2} - \mathrm{Cos}^{-1}\left(\frac{r_p}{r_p + h_b}\right) - \mathrm{Cos}^{-1}\left(\frac{r_p}{r_p + h}\right)$$

$$= \frac{\pi}{2} - \mathrm{Cos}^{-1}\left(\frac{1}{1 + \frac{h_b}{r_p}}\right) - \mathrm{Cos}^{-1}\left(\frac{1}{1 + \frac{h}{r_p}}\right) \tag{6.12}$$

Neglecting the surface diffraction, equation (6.11) indicates that the balloon/parachute dipole antenna gain received by the orbiter must follow the following rule.

$$\text{If } \theta_s \geq \theta_{sLimit} \Rightarrow \text{Usable Antenna Gain} = \text{Gain Pattern } (\theta_s)$$

$$\text{If } \theta_s < \theta_{sLimit} \Rightarrow \text{Usable Antenna Gain} = 0 \text{ (not dB!)} \tag{6.13}$$

It should be noted that the limiting value of the spacecraft location angle θ_s depends on the balloon altitude and the orbital radius of the orbiter; hence, as the balloon/parachute changes its position with respect to the surface of the planet, the limiting spacecraft location angle also changes.

Figure 9 plots the useful gain of the antenna directed towards the orbiter from the balloon/parachute dipole antenna as a function of the spacecraft location angle. All the parameter values are given in the graph. The plot starts from a limiting value of the spacecraft location angle described above because below this angle the orbiter cannot see the balloon/parachute antenna. The orbiter altitude to planet radius ratio is held constant at 0.5: i.e., the spacecraft altitude is half the planet's radius. The parameter of the graph is the normalized altitude of the balloon or the parachute, h_b/r_p. This parameter is varied from 0 to 0.5 in the particular steps shown in the figure. The useful gain of the dipole antenna located on the balloon/parachute is reasonably good for the orbiter till the orbiter position angle of about 65 degrees because for the parameter values given for the plot, the orbiter spacecraft stays within the (+2 to –3 dB) region of the antenna. At the time when the orbiter reaches the position angle of 65 degrees, the antenna gain directed towards the orbiter falls off rapidly. By the time the spacecraft location angle becomes about 80 degrees, depending on the parameters of the orbit and altitude selected, the gain falls below –5 dB from the maximum.

Figure 10 is similar to Figure 9 in that it also plots the useful gain of the antenna directed towards the orbiter from the balloon/parachute dipole antenna as a function of the spacecraft location angle. However, the parameter of these curves is the normalized orbiter altitude: i.e., the orbiter altitude divided by the planet's radius. The balloon/parachute altitude to planet radius ratio is held constant at 0.05, i.e., the spacecraft altitude is 5% of the planet's radius. This parameter is varied from 0.1 to 1 in the particular steps shown in the figure 10. The plot starts from the limiting value of the spacecraft position angle depending upon the other parameter values selected. The figure shows that the useful gain of the balloon/parachute dipole antenna remains between the (0 to 2 dB) range for about 70 degrees orbiter spacecraft location angle depending upon the parameter value selected. At the time when the orbiter reaches the position location angle of 75 to 80 degrees, the antenna gain directed towards the orbiter falls off rapidly, and by the spacecraft location angle of about 80 degrees, the gain falls below –5 dB of the maximum.

Figure 11 plots the normalized range between the orbiter and the balloon/parachute. The parameter of the graph is the normalized altitude of the orbiter. The normalized balloon/parachute altitude is held constant to a value of 0., 01. All the normalizations are carried out using the planet radius r_p as the basis. The figure shows that for any normalized altitude of the orbiter, as the spacecraft position angle increases fro 0 to 90 degrees, the range between the orbiter and the balloon decreases. The actual range may easily be obtained by simply multiplying the value read off from the graph by the planetary radius.

$$\text{Actual Range (km)} = \begin{pmatrix} \text{Value Read} \\ \text{From Figure 11} \end{pmatrix} \times \text{Planet Radius (km)} \tag{6.14}$$

The maximum normalized range is a function of the normalized balloon altitude and the normalized orbiter altitude. In fact, the maximum normalized range may be computed using the following formula:

$$\frac{R_{max}}{r_p} = \frac{\sqrt{c^2 - r_p^2} + \sqrt{a^2 - r_p^2}}{r_p} = \sqrt{\left(1 + \frac{h_b}{r_p}\right)^2 - 1} + \sqrt{\left(1 + \frac{h}{r_p}\right)^2 - 1} \tag{6.15}$$

The maximum range occurs when the line joining the balloon/parachute antenna center to the focal point of the orbiter antenna becomes tangential to the planet surface. This equation is plotted in Figure 12 as a function of the normalized balloon/parachute altitude. The figure shows that for any normalized orbiter altitude, after an initial small range of normalized balloon/parachute altitude, the maximum range increases almost linearly. To obtain the actual maximum range, following equation may be used:

$$\text{Actual Maximum Range (km)} = \begin{pmatrix} \text{Value Read} \\ \text{From Figure 12} \end{pmatrix} \times \text{Planet Radius (km)} \tag{6.16}$$

Equations (6.14) and (6.16) provide the means of finding actual quantities when needed for further analysis.

Figures 13 and 14 plot the space loss using the normalized range given in Equation (6.15), only in Figure 13 a S-Band (2450 MHz) frequency is used and Figure 14 uses a UHF (401.6) MHz frequency. It should be remembered that the number read from these two figures are not the actual space loss, because the normalized range is used instead of the actual range.

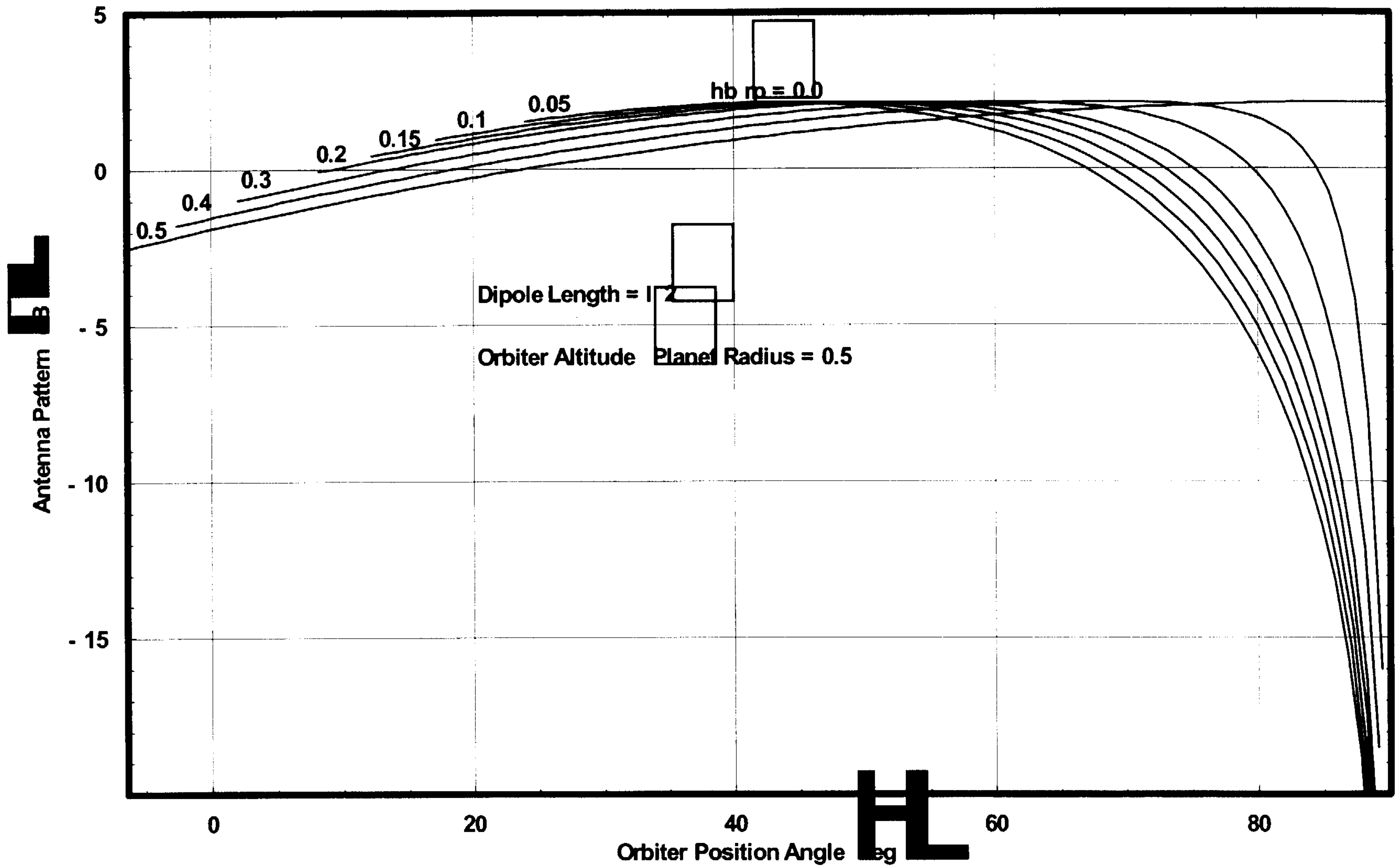


Figure 9. Antenna pattern as a function of orbiter position angle with variable hb/rp.

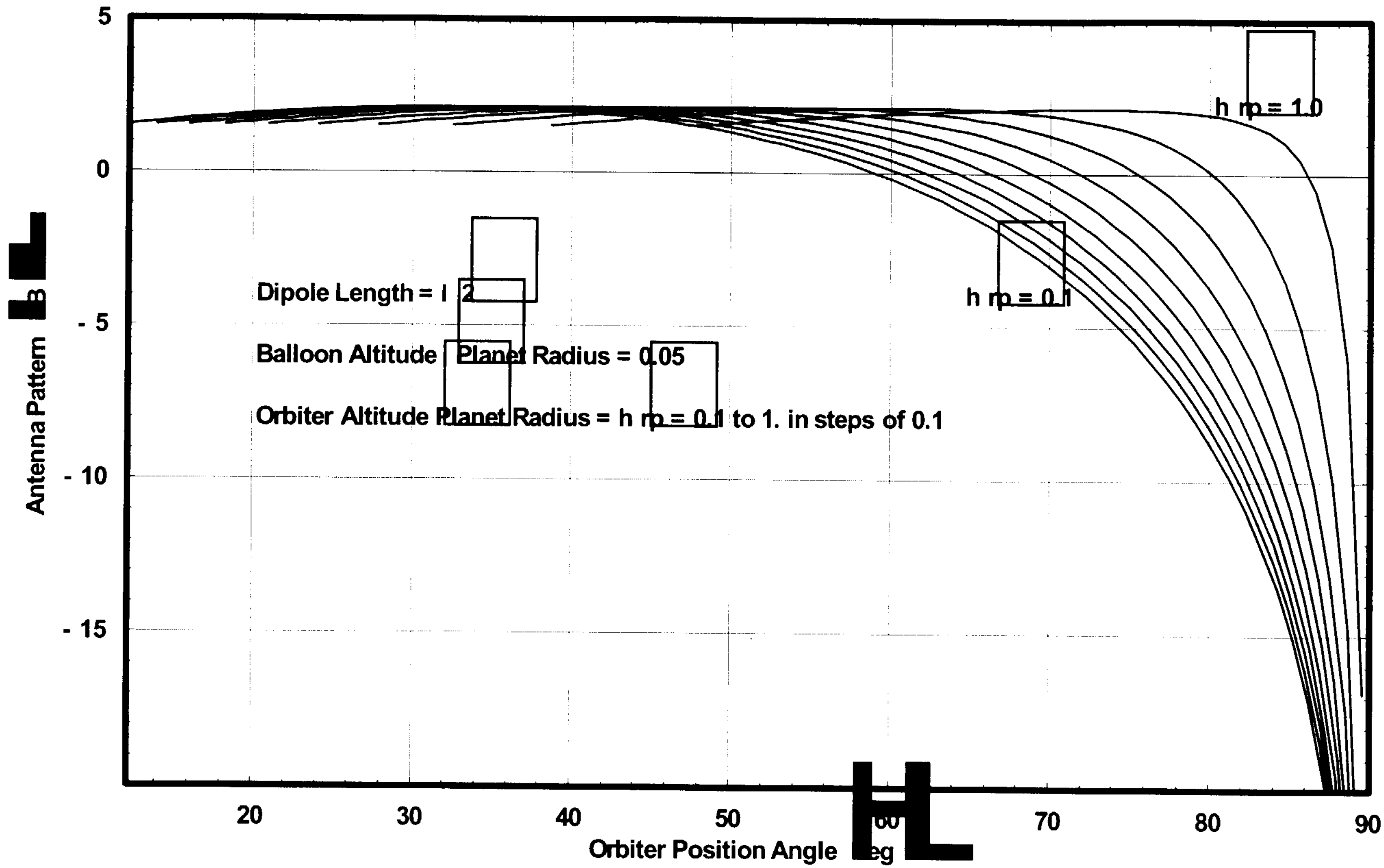


Figure 10. Antenna pattern as a function of orbiter position angle with variable h/rp.

To obtain the actual dB loss due to the range, use the following formula:

$$\text{Actual Space Loss (dB)} = \begin{pmatrix} \text{dB Value Read From} \\ \text{Figure 13 or Figure 14} \end{pmatrix} - \left[(\text{Planet Radius})^2\right]_{(\text{dB})}$$

(6.17)

To help computation of the actual space loss, Figure 15 is provided. This figure plots the $\left[(\text{Planet Radius})^2\right]_{(\text{dB})}$ as a function of the planet radius and shows the points for the known planets and moons. The same figure may be used if the planetoid radius is known.

As an example of the use of the equation (6.17), suppose a mission sends a spacecraft in the Martian atmosphere and the spacecraft that brought the balloon becomes the orbiter for the balloon link. Suppose the orbiter altitude to Mars radius ratio is equal to 0.1. It is desired to compute the worst case of space loss when the balloon altitude to Mars radius is 0.01 for a UHF link (frequency 401 MHz) between the balloon and orbiter.

Figure 12 shows that the worst-case value of the space loss is about –20 dB when the orbiter position angle is about 58 degrees. Thus, we have obtained the value for the first quantity of the right hand side of the equation (6.17). Next, the table in the Figure 15 shows that for Mars the y-axis reads 70.58 dB. This is the second factor in equation (6.17) to evaluate the space loss. The space loss for this example is obviously equal to:

$$\text{Actual Space Loss} = -20 - 70.58 = -90.58 \quad (\text{dB})$$

(6.18)

It will be desirable to see the effect of the useful antenna gain directed towards the orbiter and the space loss simultaneously on the link. Figure 16 and Figure 17 plot the two quantities multiplied together in terms of decibels for the S-Band frequency and for UHF frequency. These figures follow the shape of Figure 13 and 14 for most of the part because the loss due to the range is much too large for the antenna gain to counteract effectively. However, as the spacecraft position angle becomes larger at which the balloon/parachute antenna gain falls off much rapidly compared to the reduction of space loss magnitude due to the reduced range, the product also falls of rapidly. It should be noted that the dipole antenna is designed for the UHF and S-Band separately: i.e., the same antenna is not used for both frequencies. Both the figures use the normalized range while

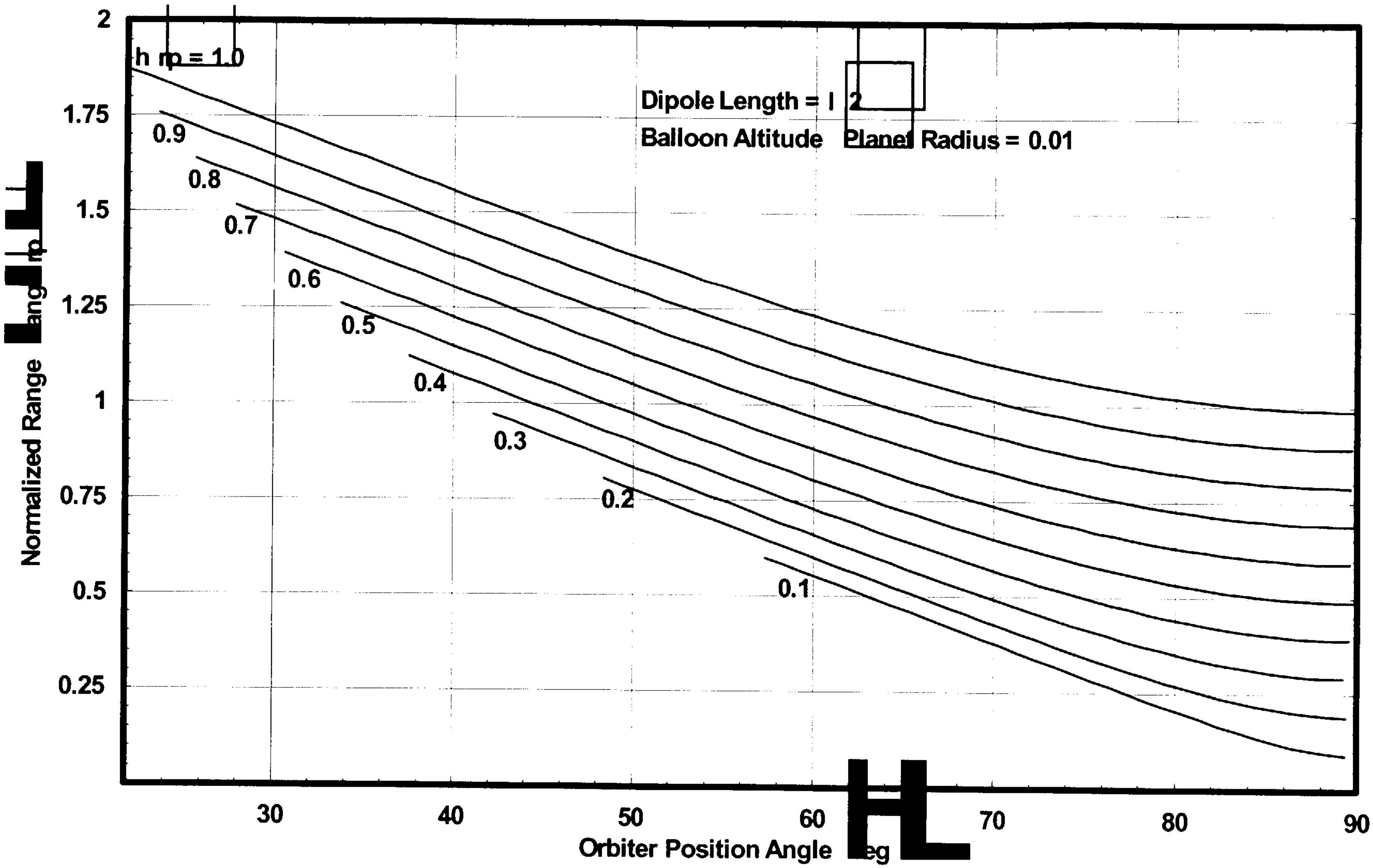


Figure 11. Plot of normalized range as a function of the orbiter position angle.

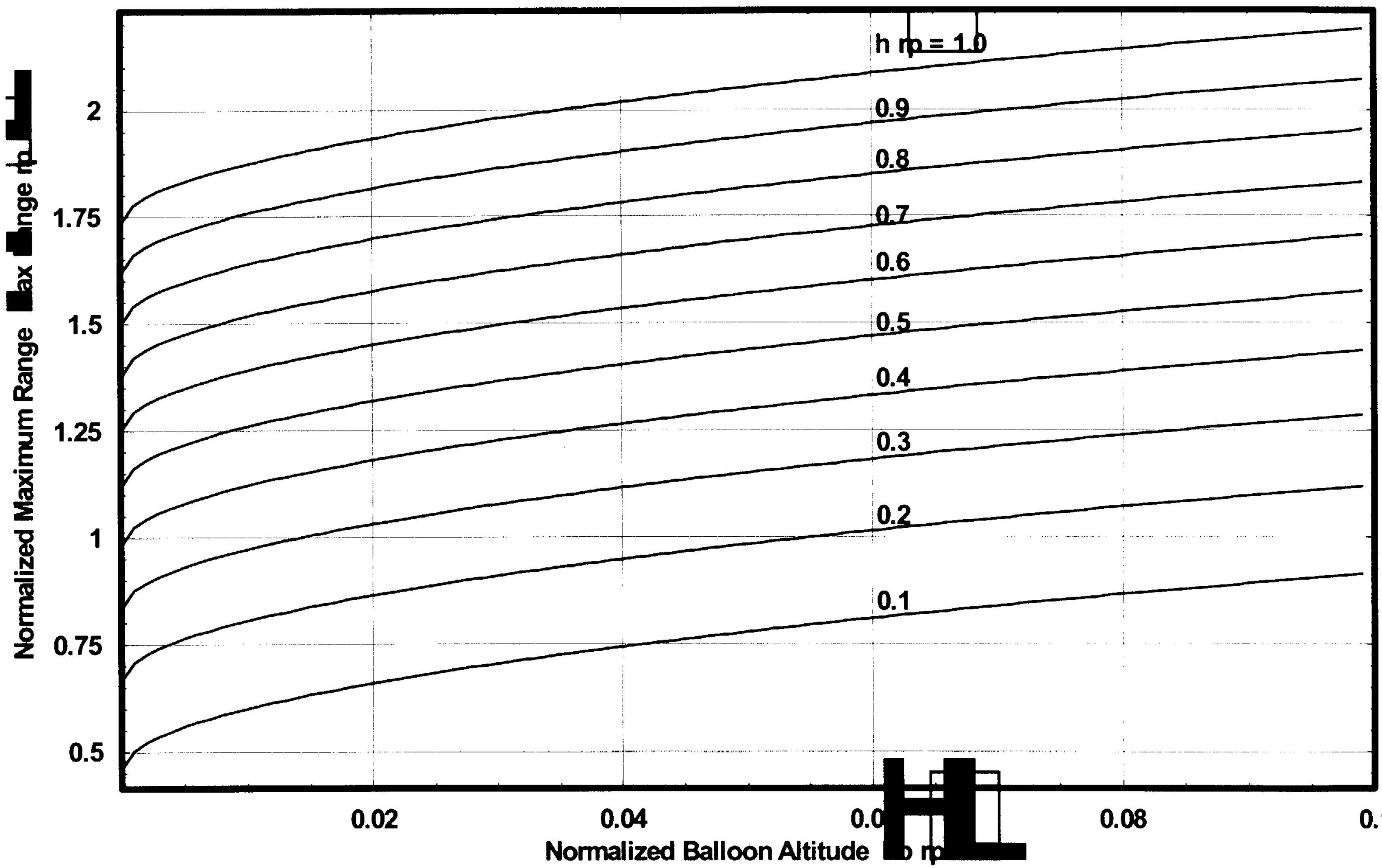


Figure 12. Plot of Maximum normalized range as a function of normalized balloon/parachute altitude.

calculating the space loss; hence, to get actual numbers from these figures, one needs to follow the procedure outlined in equation (6.17) above: i.e., use the following equation.

$$\text{Actual Total Loss (dB)} = \begin{pmatrix} \text{dB Value Read From} \\ \text{Figure 16 or Figure 17} \end{pmatrix} - \left[(\text{Planet Radius})^2\right]_{(\text{dB})} \tag{6.19}$$

The value of second factor in equation (6.19) is obtained from the Figure 15, as was done before. For the example set up for showing the use of equation (6.17), the actual total loss, i.e., the space loss and the antenna gain in dB can be calculated using Figures 15 and 17. For the orbiter altitude to Mars radius ratio of 0.1 and balloon altitude to Mars radius ratio of 0.01 the Figure 17 gives the worst-case value of total loss of about –34 dB. This is the value for the first quantity of the right side of equation (6.19). Figure 15 provides the value of 70.58 dB. Thus the total value of the actual total loss can be found by the following equation.

$$\text{Actual Total Loss} = -34 - 70.58 = -104.58 \quad \text{(dB)} \tag{6.20}$$

7.0 Computation of Sustainable Bit Rate for Balloon – Orbiter Link

To compute the bit rate of a communications link between the balloon/parachute and the orbiter spacecraft, one needs to define additional link parameters. The parameters will characterize the balloon/parachute and the orbiter telecommunications systems. Parameters include system losses for both the telecom system, antenna pointing loss for the orbiter receiving antenna, and the system noise temperature for the receiving system etc.

It should be realized that this is not an attempt to make a complete link budget for the link between the balloon/parachute and the orbiter; rather, the results will be sufficiently accurate only for a quick bit rate analysis. The parameters mentioned above and some more are given in the following Table 2 and these will be used to derive the further results.

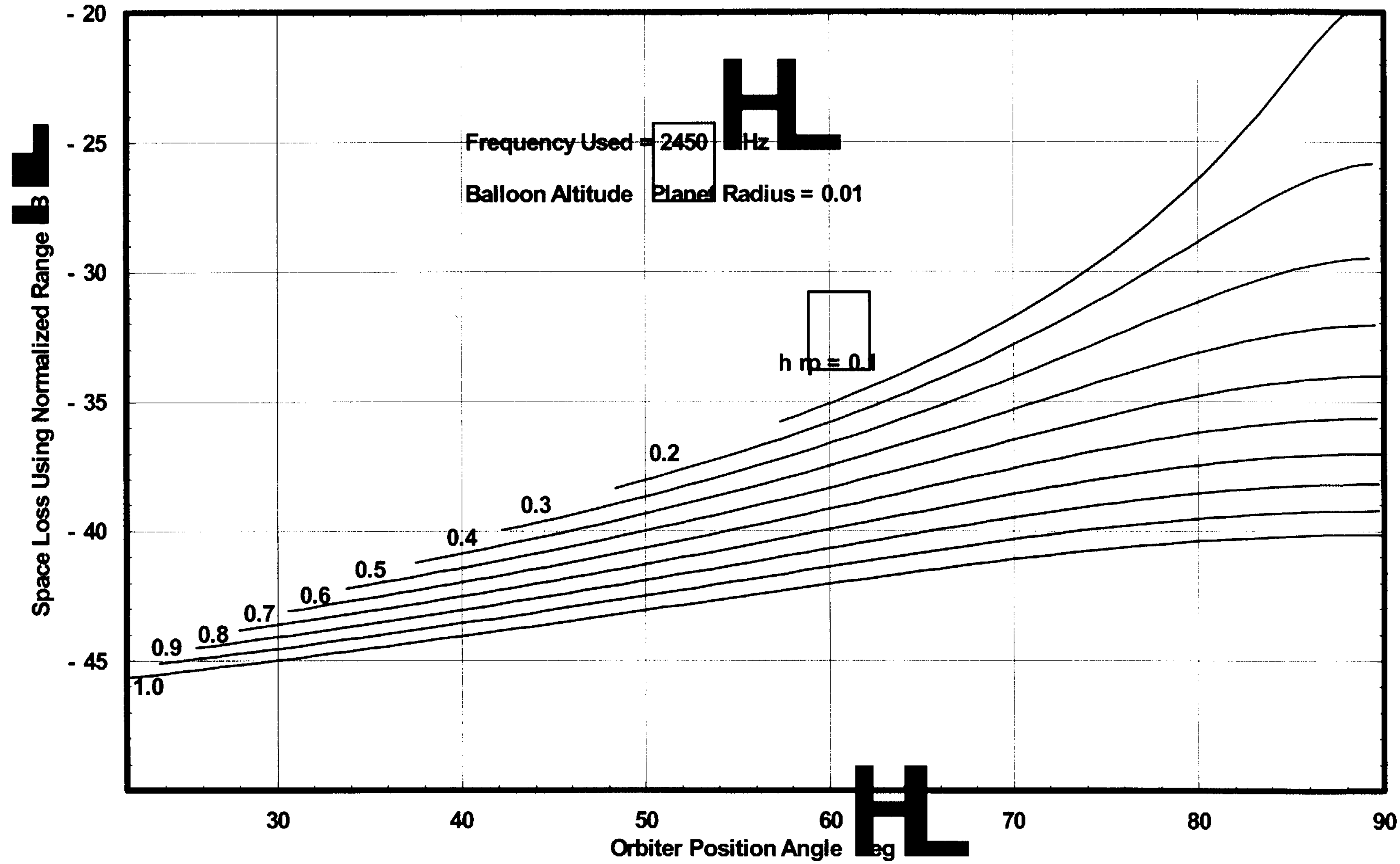


Figure 13. Plot of space loss (S-Band) using the normalized range as a function of orbiter position angle.

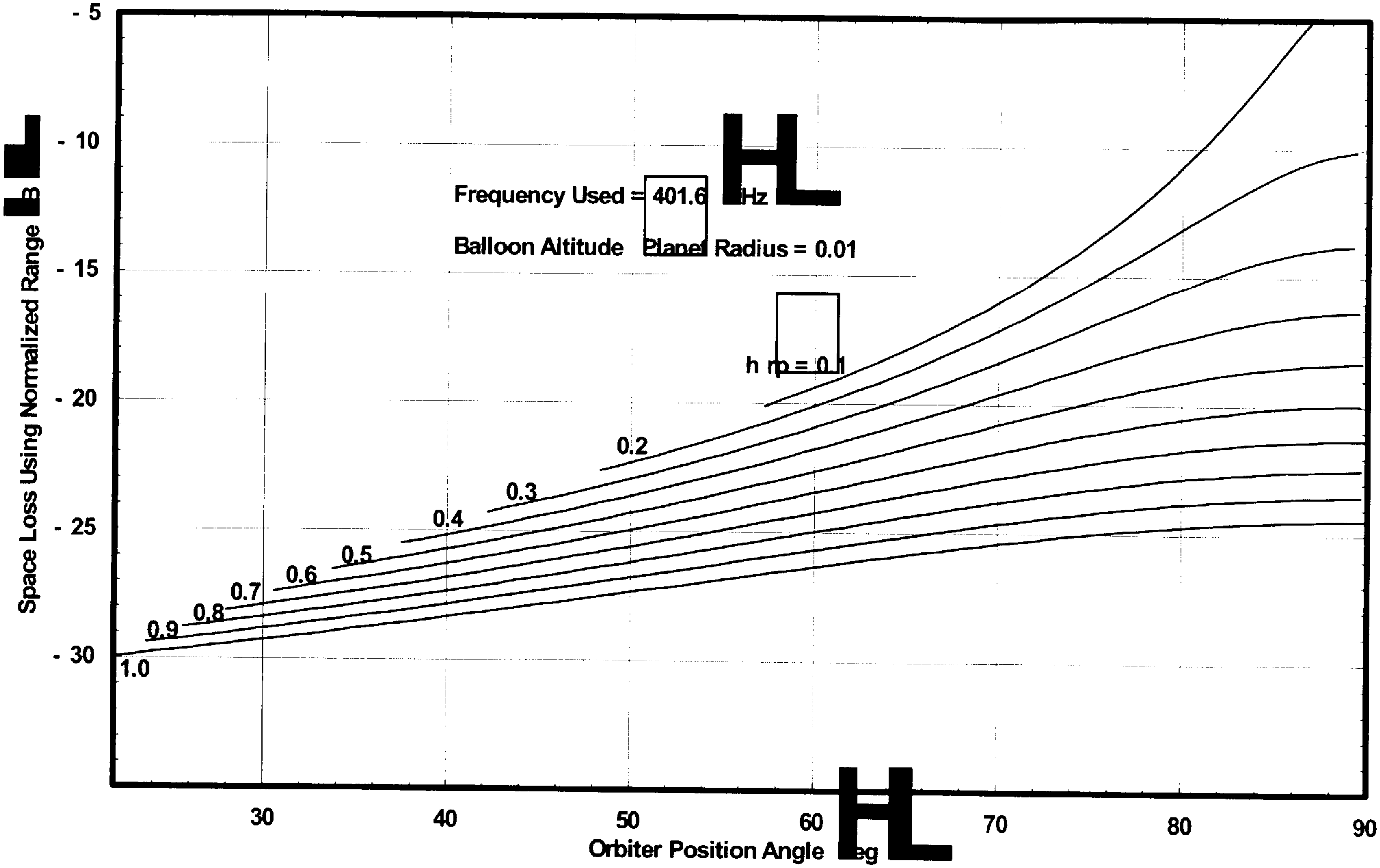
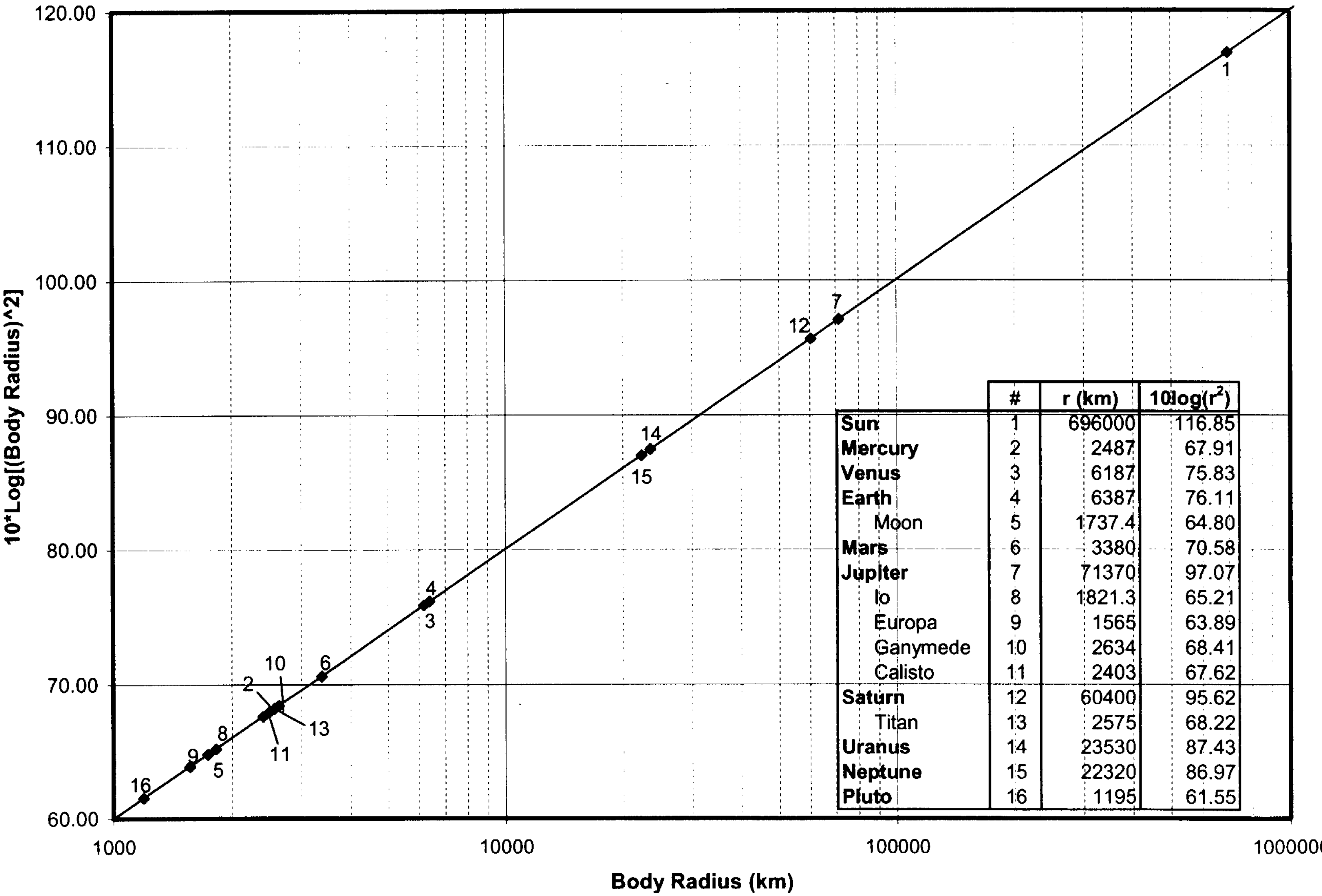


Figure 14. Plot of space loss (UHF) using the normalized range as a function of orbiter position angle.



	#	r (km)	10log(r^2)
Sun	1	696000	116.85
Mercury	2	2487	67.91
Venus	3	6187	75.83
Earth	4	6387	76.11
Moon	5	1737.4	64.80
Mars	6	3380	70.58
Jupiter	7	71370	97.07
Io	8	1821.3	65.21
Europa	9	1565	63.89
Ganymede	10	2634	68.41
Calisto	11	2403	67.62
Saturn	12	60400	95.62
Titan	13	2575	68.22
Uranus	14	23530	87.43
Neptune	15	22320	86.97
Pluto	16	1195	61.55

Figure 15. Plot of body radius squared in dB.

Table 2. Balloon/parachute to orbiter link parameters

Frequency Used	2250 MHz (S-Band)
Balloon or parachute altitude/planet radius	0.01
Orbiter altitude/planet radius	0.1
Balloon or parachute antenna	Dipole with Length = $\lambda/2$
Balloon or parachute transmit system losses	2 dB
Transmit system error correcting code	R=1/2, k=7 Convolutional Code
Eb/N0 requirement for a BER = 10^{-5}	4.46 dB
Transmitted RF power	1 W
Transmitted signal format	BPSK – Suppressed Carrier
Orbiter receiving antenna	Parabolic reflector.
Orbiter receiving antenna efficiency	55%
Receiving system pointing and system losses	3 dB
Receiver system noise temperature	400 K = 26.02 dB
Boltzmann's constant	-228.6 dB/K
Carrier Tracking	Assume carrier locked
Data margin of the link	0 dB

Using these parameters the following 6 figures (Figure 18 to Figure 23) were generated. These figures plot the normalized bit rate in dB units as a function of the orbiter spacecraft position in its orbit for a particular orbiter spacecraft receive antenna diameter. The data margin is assumed to be 0 dB because the best possible bit rate is desired. JPL/Mission standards may require a different data margin and, hence, the curves must be modified accordingly to obtain the allowable bit rate values.

It should be noted that since the balloon/parachute antenna has a length of = $\lambda/2$ meters regardless of the frequency of use i.e., the length of the dipole changes with the frequency and, consequently, the gain of the dipole antenna will be the same for any frequency. With this assumption the figures 18 to 23 **do not** change with the change of frequency. The change in the space loss is cancelled by the change of gain of the receiving parabolic reflector antenna (constant) while the transmitting dipole antenna maintains the same gain due to the change in its length depending upon the frequency. It should be noted that these figures are drawn assuming that the link starts as soon as the orbiter becomes visible to the balloon/parachute.

Following is an example of the use of the figures. Suppose one needs to find the bit rate possible for the conditions given in the above table with a receiver antenna of 1-meter diameter. This implies that one must use Figure 21 in the 18

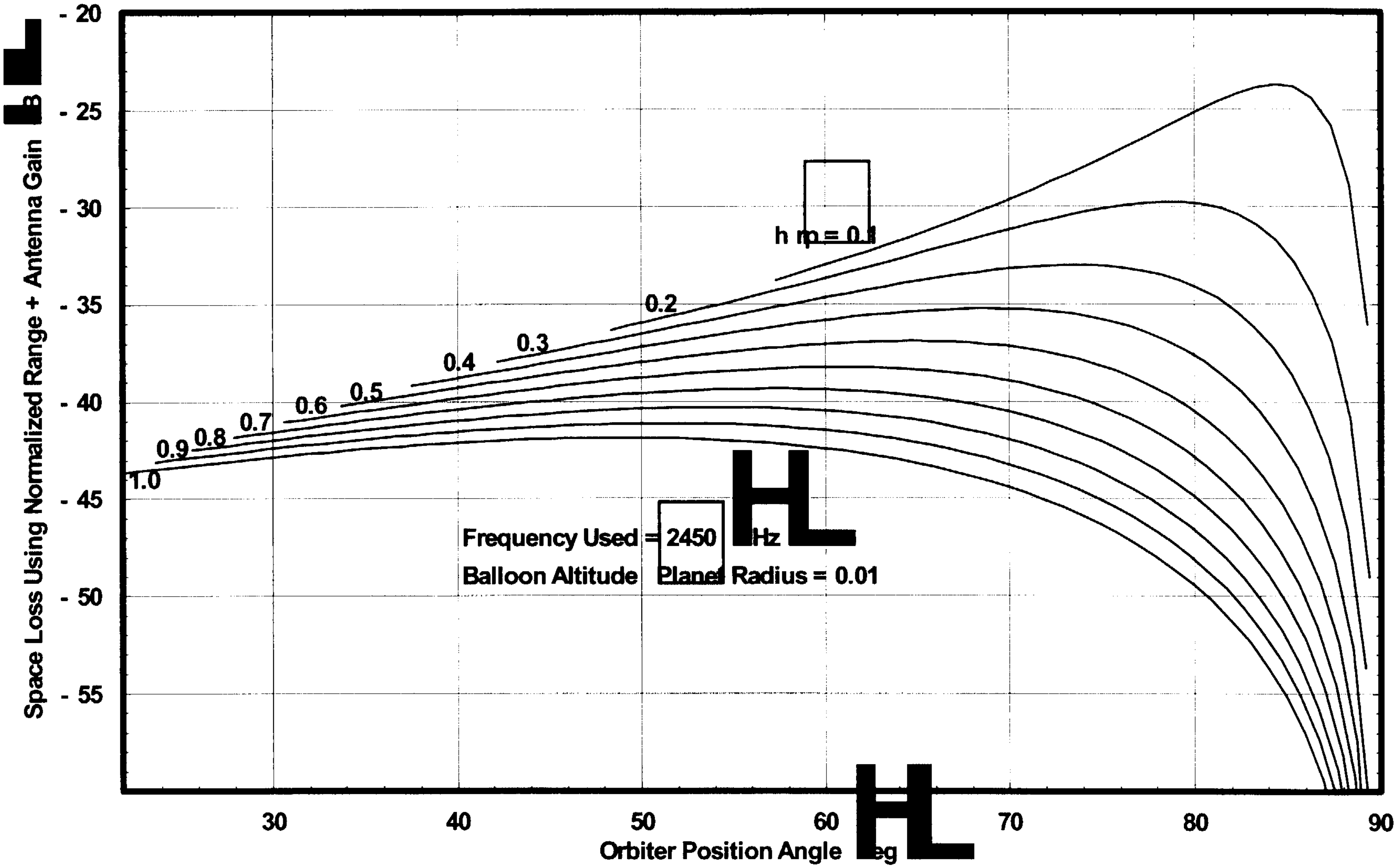


Figure 16. Plot of S-Band space loss using the normalized range + antenna gain as a function of orbiter position.

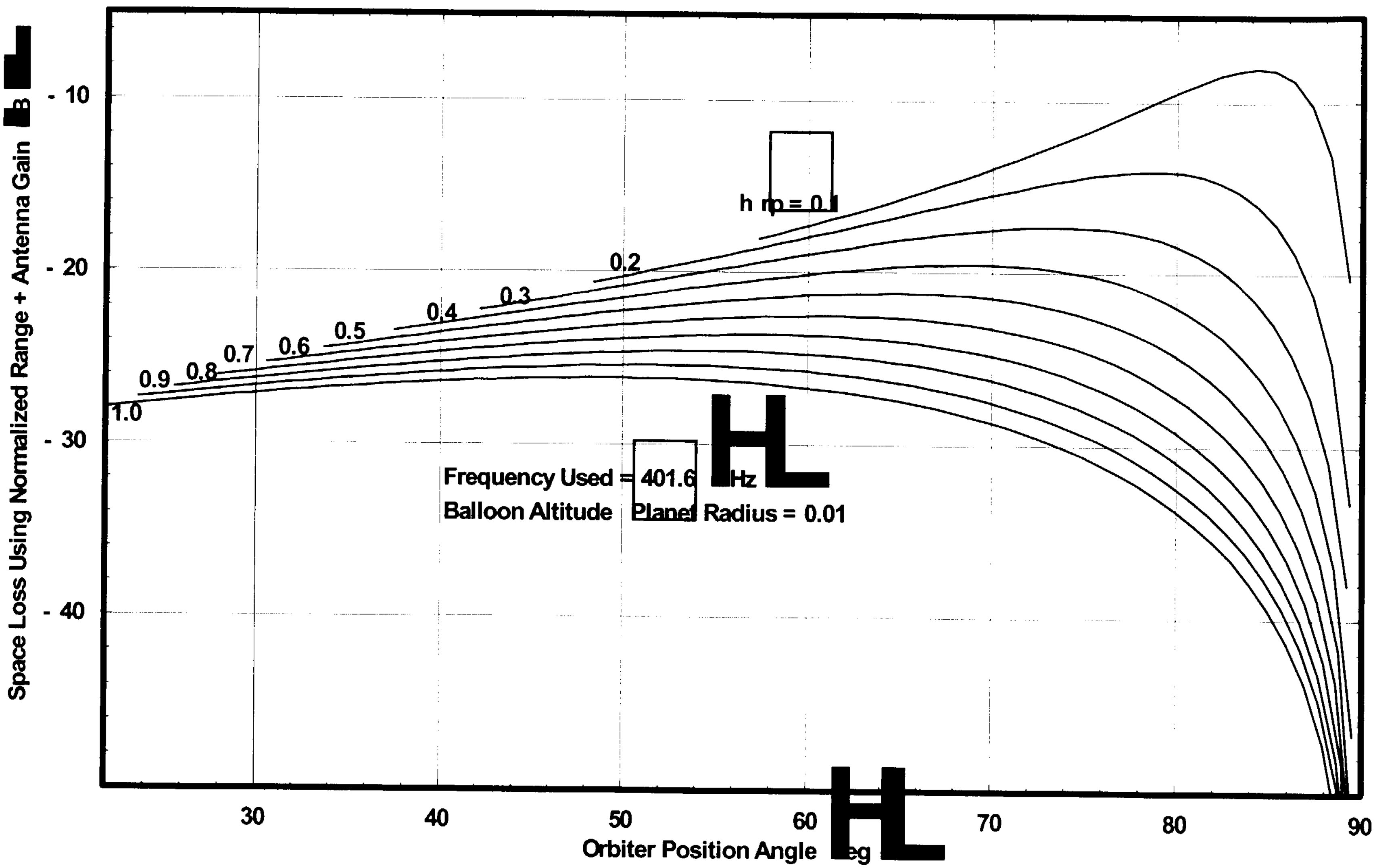


Figure 17. Plot of space loss (UHF) using the normalized range + antenna gain as a function of orbiter position.

to 23 Figures. From this figure, for the balloon altitude to the planet ratio of 0.01 and for the orbiter altitude to planet radius ratio of 1, we select the highest curve. For the orbiter position angle to be 70 degrees, the normalized bit rate is read to be 154 dB. To obtain the real world quantities, we need to add to the number just obtained the normalization constant in dB. This is done using the following equation.

$$\text{Actual Data Rate (dB)} = \left(\begin{array}{c} \text{dB Bit Rate Value} \\ \text{From Figures 18 to 23} \end{array} \right) - \left[(\text{Planet Radius})^2 \right]_{(\text{dB})} \tag{7.1}$$

If we assume that the planet is Mars, Figure 15 provides the value of 70.58 dB for the normalization constant (the second bracket in Equation 7.1); and this value must be subtracted from 154 dB. Thus,

$$\text{Actual Data Rate} = 154 - 70.58 = 83.42 \text{ dB} \tag{7.2}$$

Thus, the possible maximum bit rate may as large as $10^{8.342}$ bits per second or approximately 220 Mbps. It should be noted that as the parameter values change, it is only a simple matter to see the effect of the changed values on the sustainable data rate. As an example, if the radiated RF power is reduced to 10 mW, because the original computations used 1 W transmitted power to produce the curves, the current bit rate will be changed to 220 * (10/1000) Mbps, or 2.2 Mbps. Another example could be: if a 3 dB data margin is desired then the actual data rate in equation (7.2) should be changed to 83.42 – 3 = 80.42 dB and this equals $10^{8.042}$ or with a 0 dB margin this will support about 110 Mbps and if a 6 dB data margin is desired then it will support 55 Mbps data rate etc. Note that this computation for the bit rate supportable by the link is done only at one point on the orbiter's position: namely 70 degrees. For any other position location angle, the calculations must be repeated.

8.0 Computation of Data Volume Received on Orbiter.

To compute the total data gathered per pass, one needs to write the equation (6.11a) as a function of time. This, in turn, implies that the spacecraft position angle θ_s be converted into time. This can be done easily considering that the orbiter is in a circular orbit and hence the spacecraft speed remains constant throughout its flight around the parent body. The minimum spacecraft location angel, θ_{sLimit} that was defined in the equation (6.12) (which is reproduced below for convenience) is the starting point for the time, 't': i.e., t=0. The progress of time is formulated as follows.

$$\text{Limiting } \theta_s \text{ angle} = \theta_{sLimit} = \frac{\pi}{2} - \text{Cos}^{-1}\left(\frac{r_p}{r_p + h_b}\right) - \text{Cos}^{-1}\left(\frac{r_p}{r_p + h}\right)$$

$$= \frac{\pi}{2} - \text{Cos}^{-1}\left(\frac{1}{1 + \frac{h_b}{r_p}}\right) - \text{Cos}^{-1}\left(\frac{1}{1 + \frac{h}{r_p}}\right) \tag{8.1}$$

Let

$$\text{Satellite Period} \triangleq T_p = 2\pi \frac{a^{3/2}}{\sqrt{\mu}} \quad \text{(Sec)} \tag{8.2}$$

The time required for the spacecraft position angle, θ_s, to change from θ_{sLimit} to the current position, θ_s, can be written down easily because of the uniform speed of the circular orbit satellite in its flight path.

$$\theta_s - \theta_{sLimit} = \left(\frac{2\pi}{T_p}\right) t = 2\pi \underbrace{\left(\frac{t}{T_p}\right)}_{t_n} \triangleq 2\pi t_n$$

$$\theta_s = 2\pi t_n + \theta_{sLimit} \tag{8.3}$$

Where t_n is the normalized time. After substituting equation (8.3) into (6.11a) we obtain the following equation.

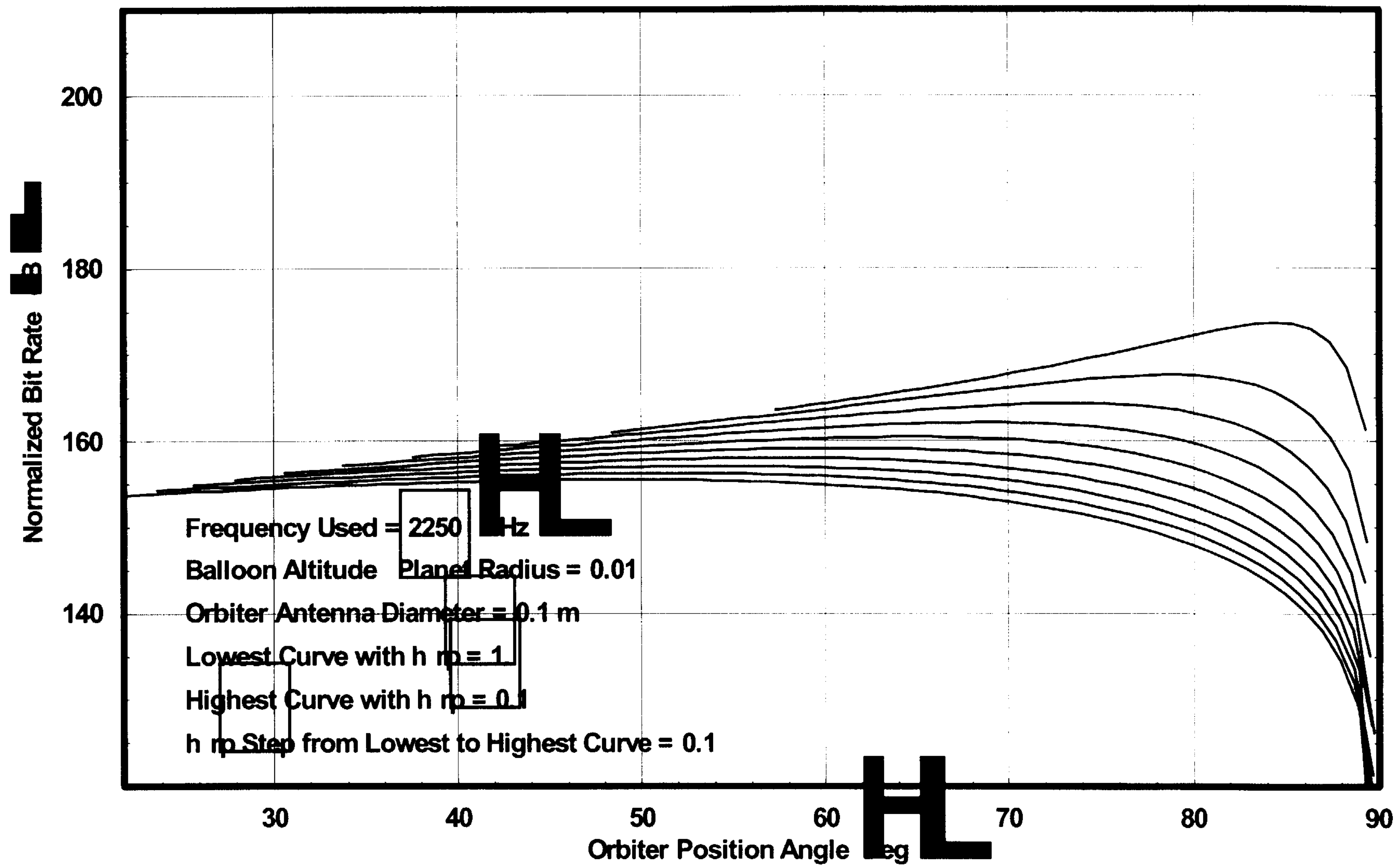


Figure 18. Sustainable bit rate for the balloon/parachute to orbiter link with orbiter antenna = 0.1 m diameter.

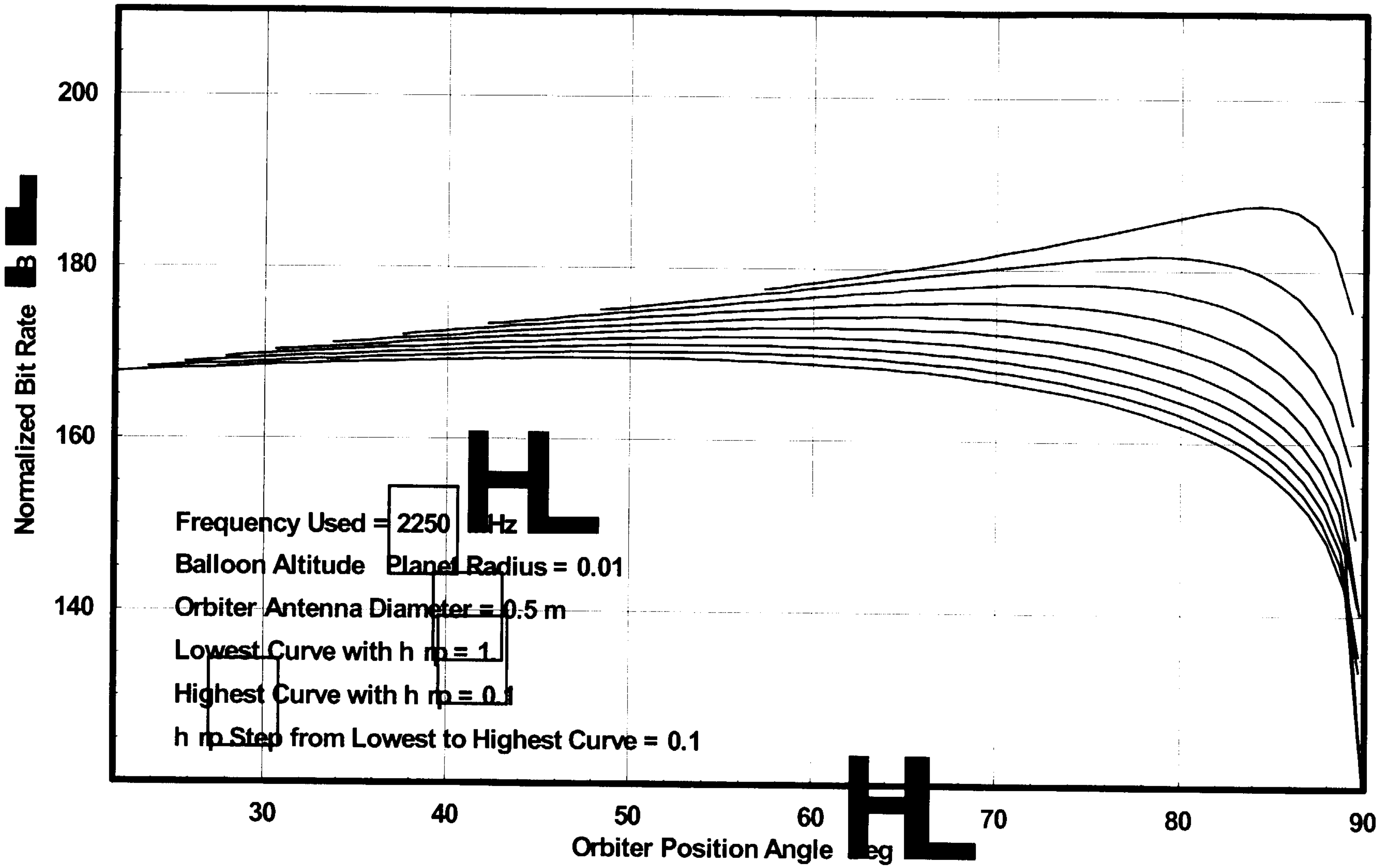


Figure 19. Sustainable bit rate for the balloon/parachute to orbiter link with orbiter antenna = 0.5 m diameter.

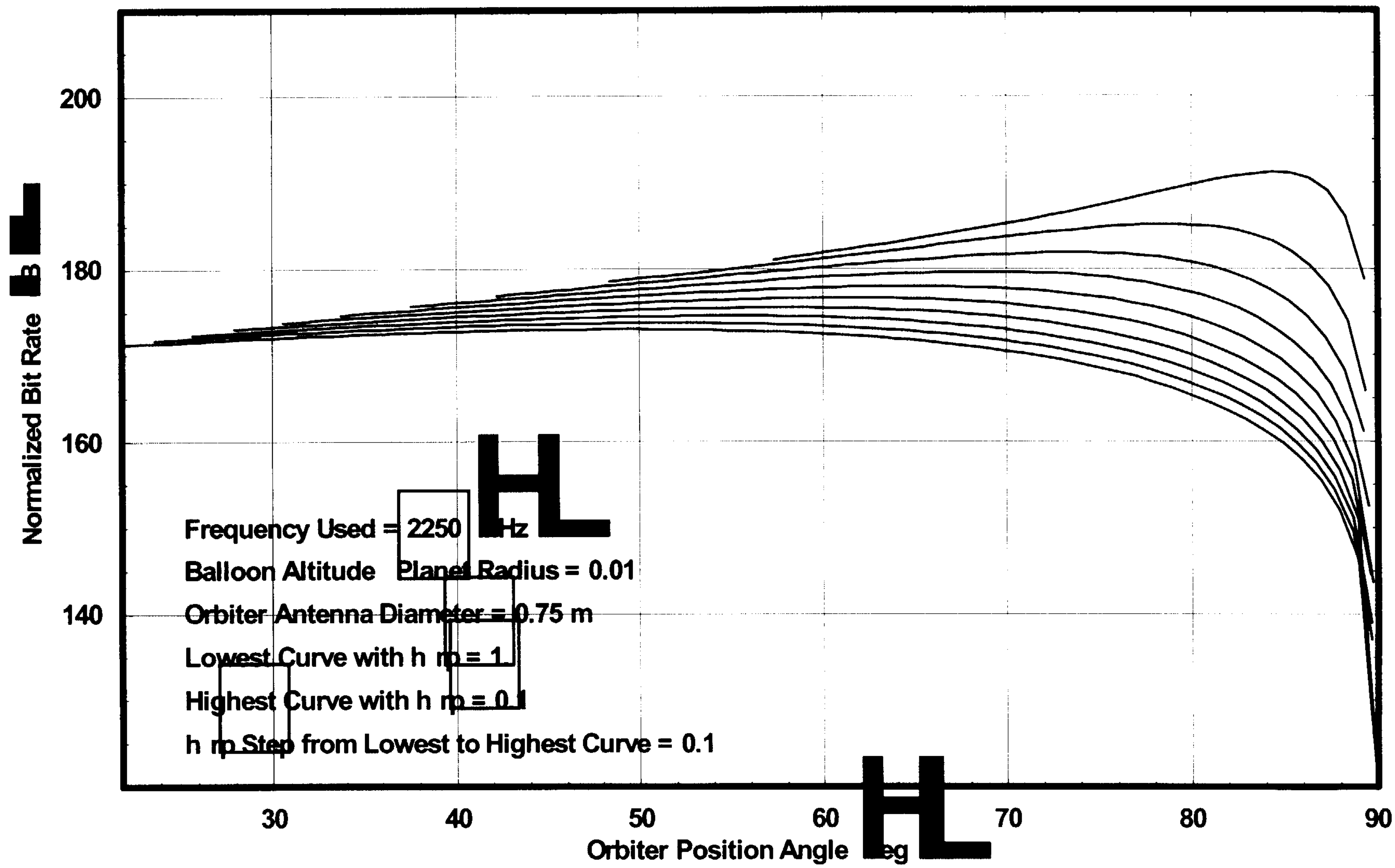


Figure 20. Sustainable bit rate for the balloon/parachute to orbiter link with orbiter antenna = 0.75 m diameter.

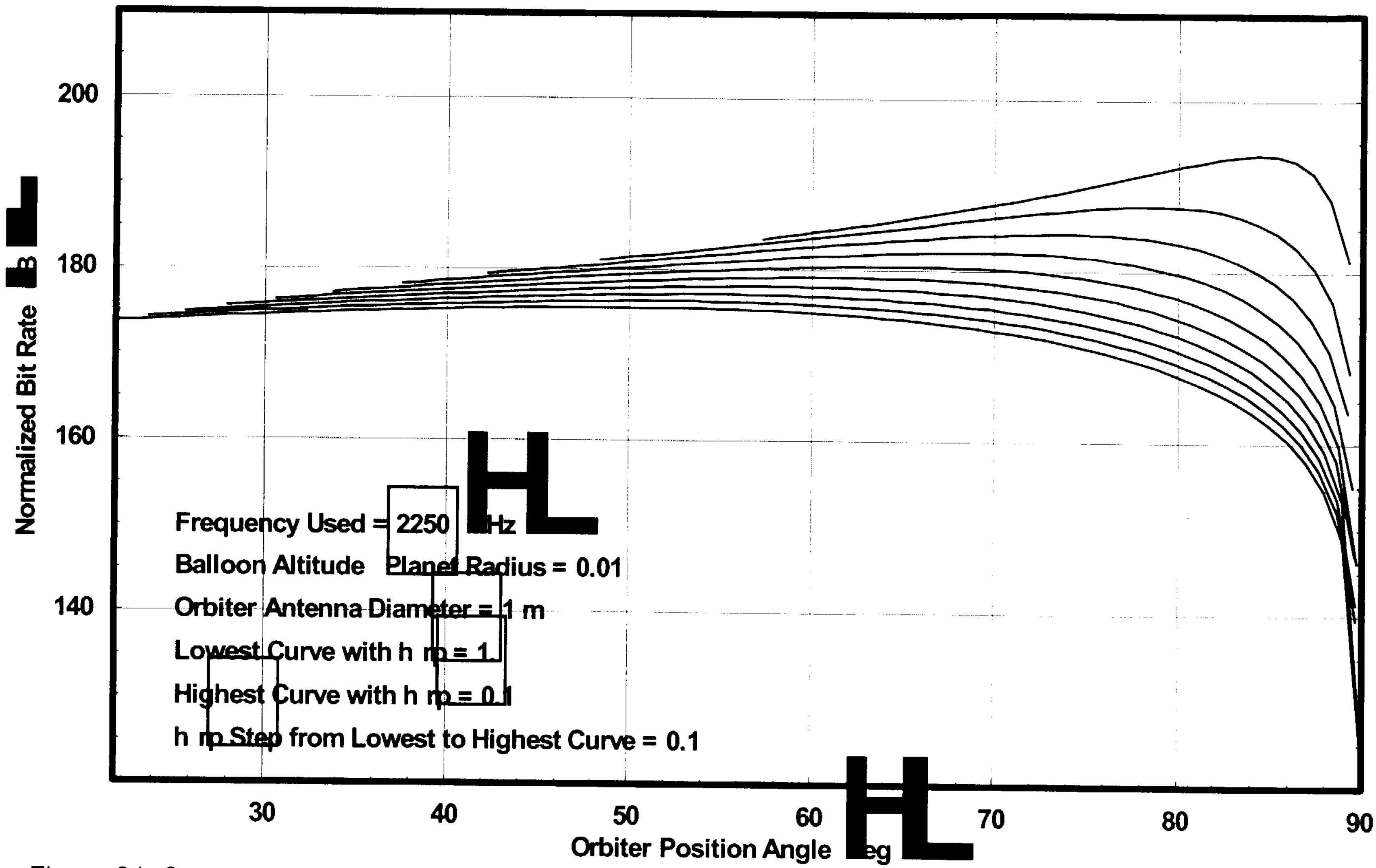


Figure 21. Sustainable bit rate for the balloon/parachute to orbiter link with orbiter antenna = 1.0 m diameter.

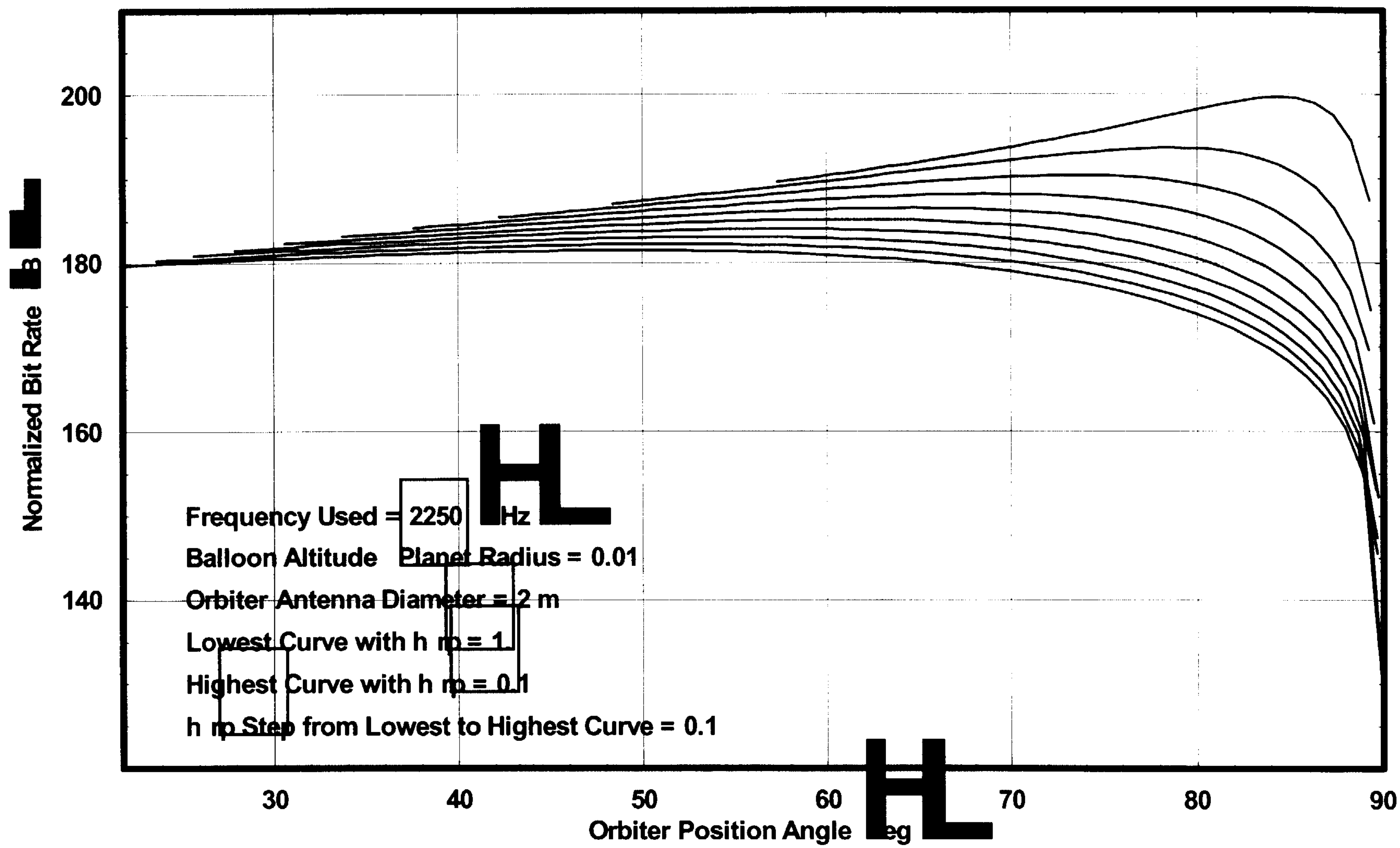


Figure 22. Sustainable bit rate for the balloon/parachute to orbiter link with orbiter antenna = 2.0 m diameter.

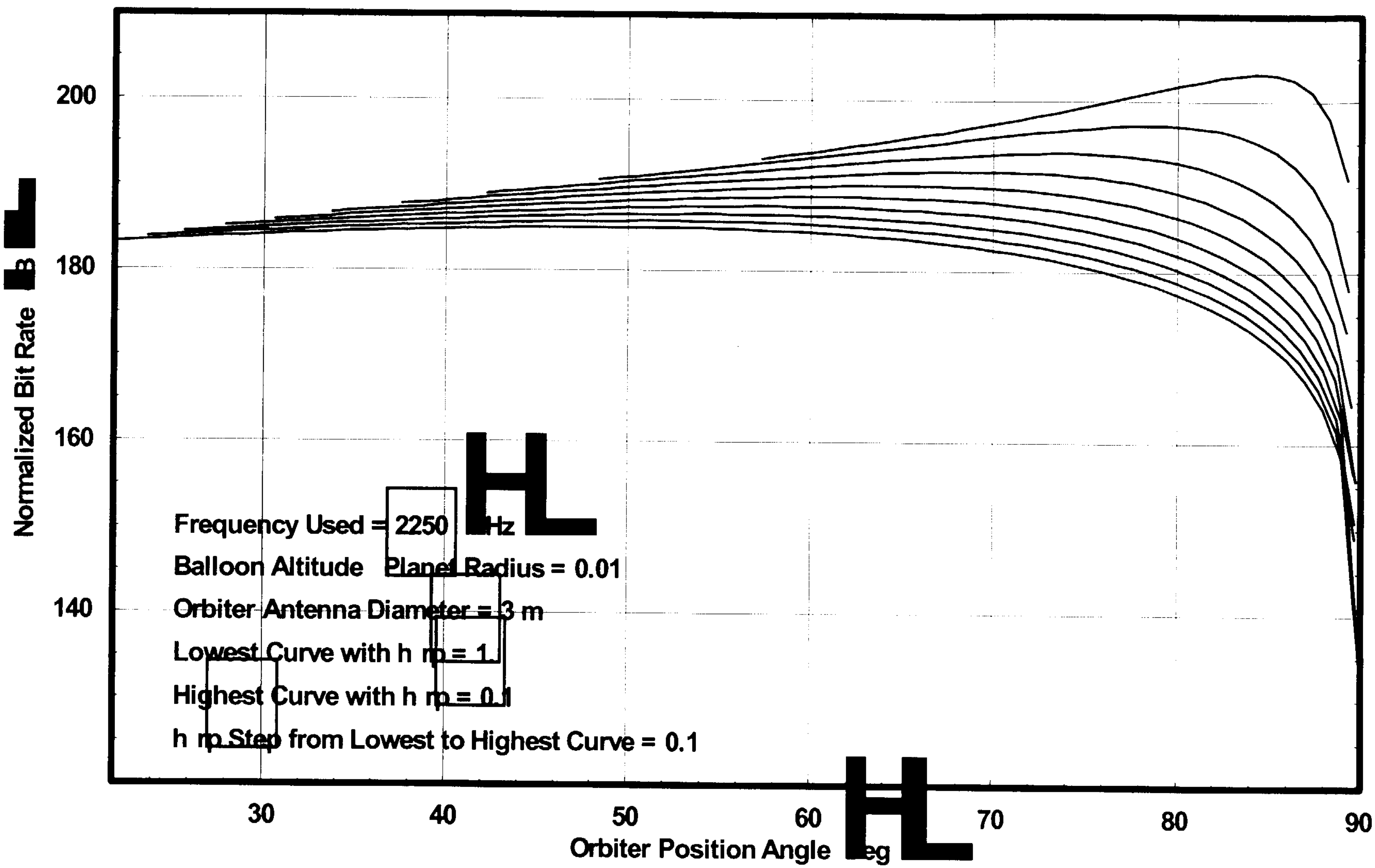


Figure 23. Sustainable bit rate for the balloon/parachute to orbiter link with orbiter antenna = 3.0 m diameter.

$$= \left(\frac{\lambda}{4\pi\,(RN)}\right)^2 \left[f\left(\frac{\pi l}{\lambda}\right) \left\{ \frac{\left[\cos\left[\left(\frac{\pi l}{\lambda}\right)\left(\frac{\left(1+\frac{h}{r_p}\right)\sin(2\pi\,t_n+\theta_{sLimit}) - \left(1+\frac{h_b}{r_p}\right)}{(RN)}\right)\right] - \cos\left(\frac{\pi l}{\lambda}\right)\right]}{\frac{\left(1+\frac{h}{r_p}\right)\cos(2\pi\,t_n+\theta_{sLimit})}{(RN)}} \right\} \right]^2$$

(8.4)

The normalized time, t_n, was defined in the equation (8.3); however, its connection with the body around which the orbiter is stationed is given below.

$$t_n = \frac{t}{T_p} = \frac{t}{2\pi\,\frac{a^{3/2}}{\sqrt{\mu}}} = r_p^{3/2}\left(\frac{\sqrt{\mu}}{2\pi}\right)\left(\frac{1}{1+\frac{h}{r_p}}\right)^{3/2} t$$

(8.5)

With g (t_n) = equation (8.4), and using balloon/parachute to orbiter link parameter values of Table 2 one may write down the equation for the total bits generated in a pass of the orbiter. It should be noted that we have assumed (as was done previously) that the velocity of the balloon/parachute is negligible with respect to the visibility time of the orbiter. In that case, the total visibility time would be twice the time the satellite needs to go from θ_{sLimit} to $\pi/2$.

$$\text{Amount of Bits Gathered} = 2\int_0^{\frac{1}{4}-\frac{\theta_{sLimit}}{2\pi}} g(t_n)\,10^{\left(\frac{GL}{10}\right)}\,dt_n$$

(8.6)

Where the parameter GL is defined using the parameters defined in the Table 2 as follows:

$$GL = - \text{Orbiter System Loss (dB)} - \text{Balloon System Loss (dB)} - \text{Reqd. EbON0 (dB)} + \text{Atmospheric Loss (dB)} + 10 * \text{Log}_{10}(\text{PowerTransmitted}) \text{ (dB)} + \text{Orbiter Antenna Gain (dB)} + 228.6 - 10 * \text{Log}_{10}(\text{Teq}) \tag{8.7}$$

One important assumption about the orbiter antenna is that the antenna pointing of the orbiter antenna is done perfectly. One possible way this can be done is the proper use of the balloon/parachute position knowledge with respect to the orbiter. Another way of achieving the same thing is by providing a pilot beacon on the balloon/parachute that always transmits and the orbiter senses this pilot and orients its antenna towards that direction automatically.

Equation (8.6) is plotted in Figure 24 as a function of the ratio orbiter altitude/radius of the planet, (h/r_p). The figure shows that the bits accumulated by the orbiter becomes gradually less as the parameter h/r_p increases. This is due to the fact that as the altitude of the orbiter increases, the space loss increases and at the same time the antenna gain directed towards the orbiter decreases.

The amount of bits gathered using equation (8.6) are the data using the normalized quantities for the space loss and the orbiter period. Thus, to use the number obtained from equation (8.6), one needs to multiply a constant given below:

$$\begin{matrix}\text{Normalization} \\ \text{Constant}\end{matrix} = \frac{2\pi}{\sqrt{r_p \mu}} \left(1 + \frac{h}{r_p}\right)^{3/2} \tag{8.9}$$

This constant will be different for different planets and their moons. Figure 25 shows the normalization constant in dB for each planet and major moons of the solar system. Following is an example of computation of the total data bits received from the balloon/parachute at the orbiter. Suppose the balloon is descending in Venus's atmosphere and the orbiter is at an altitude of 0.5 Venus radii, i.e., $h/r_p = 1/2$. Also the antenna carried by the orbiter is the parabolic reflector antenna of diameter 0.25 m (25 cm) and according to our previous assumptions, is automatically pointed to the balloon/parachute position. This automatic pointing of the orbiter antenna implies automatic tracking of the balloon/parachute as the relative positions between the balloon/parachute and the orbiter changes.

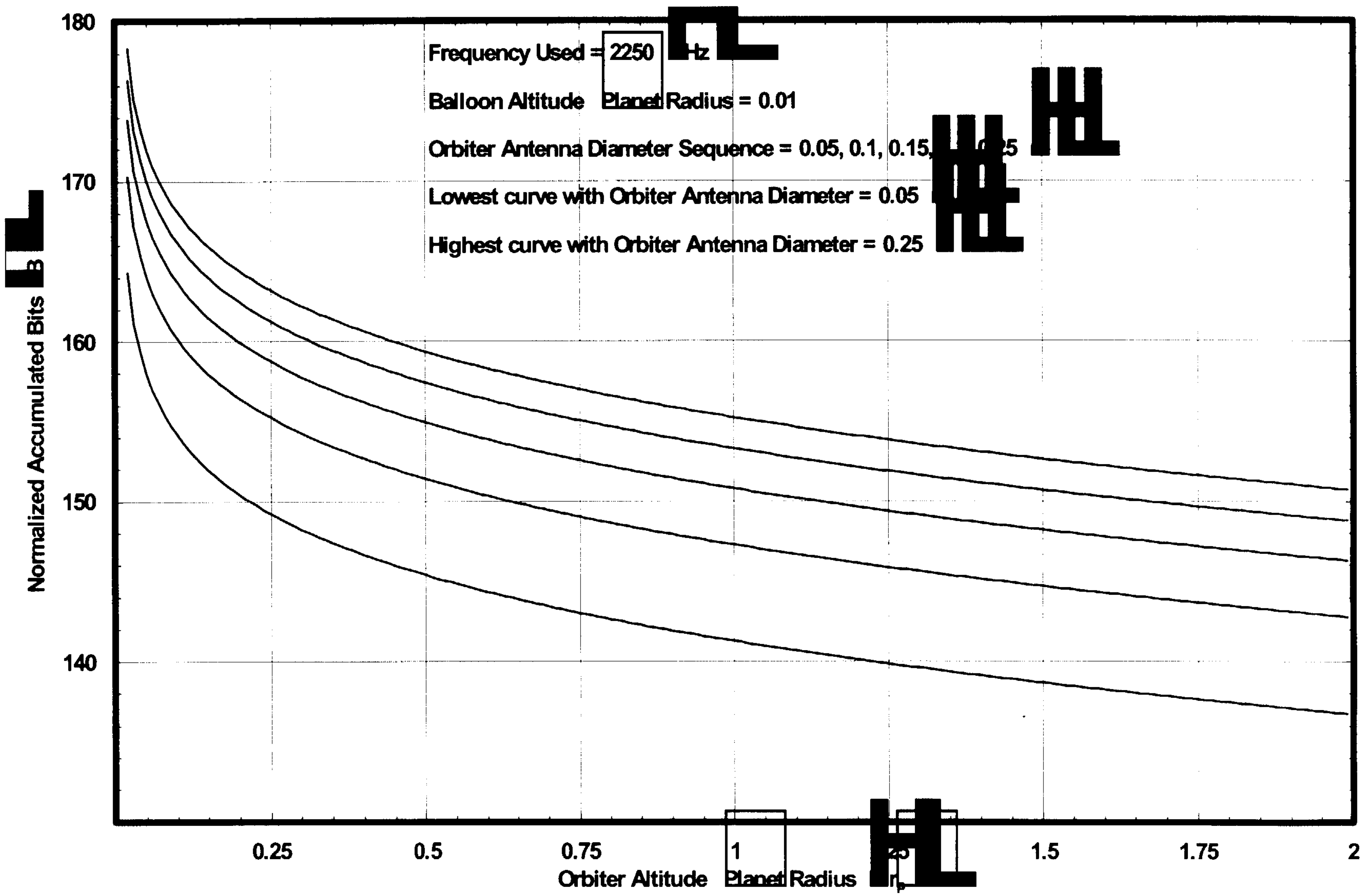


Figure 24. Normalized accumulated bits per pass of the orbiter.

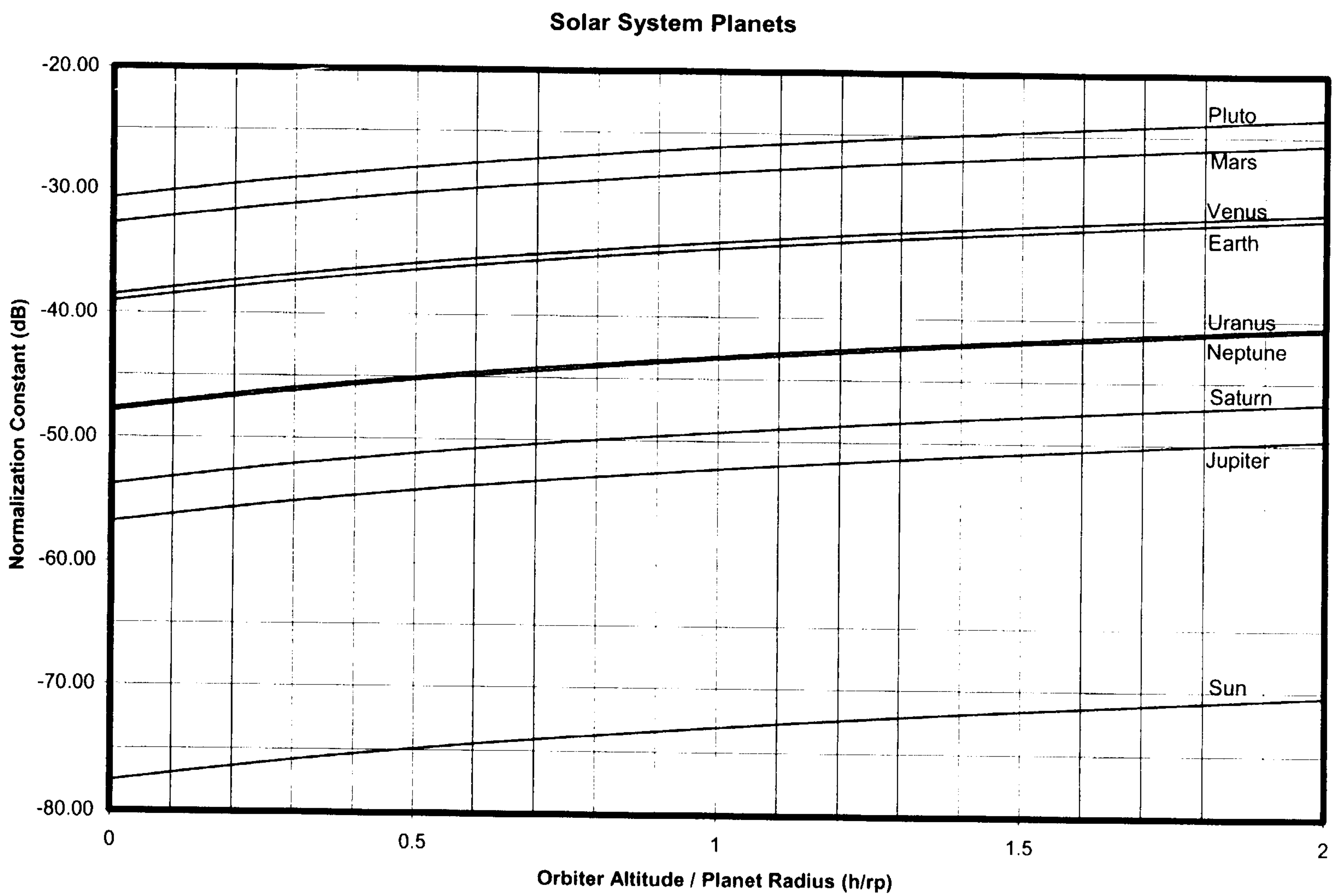


Figure 25. Constant needed to obtain the data bits gathered for the solar system planets.

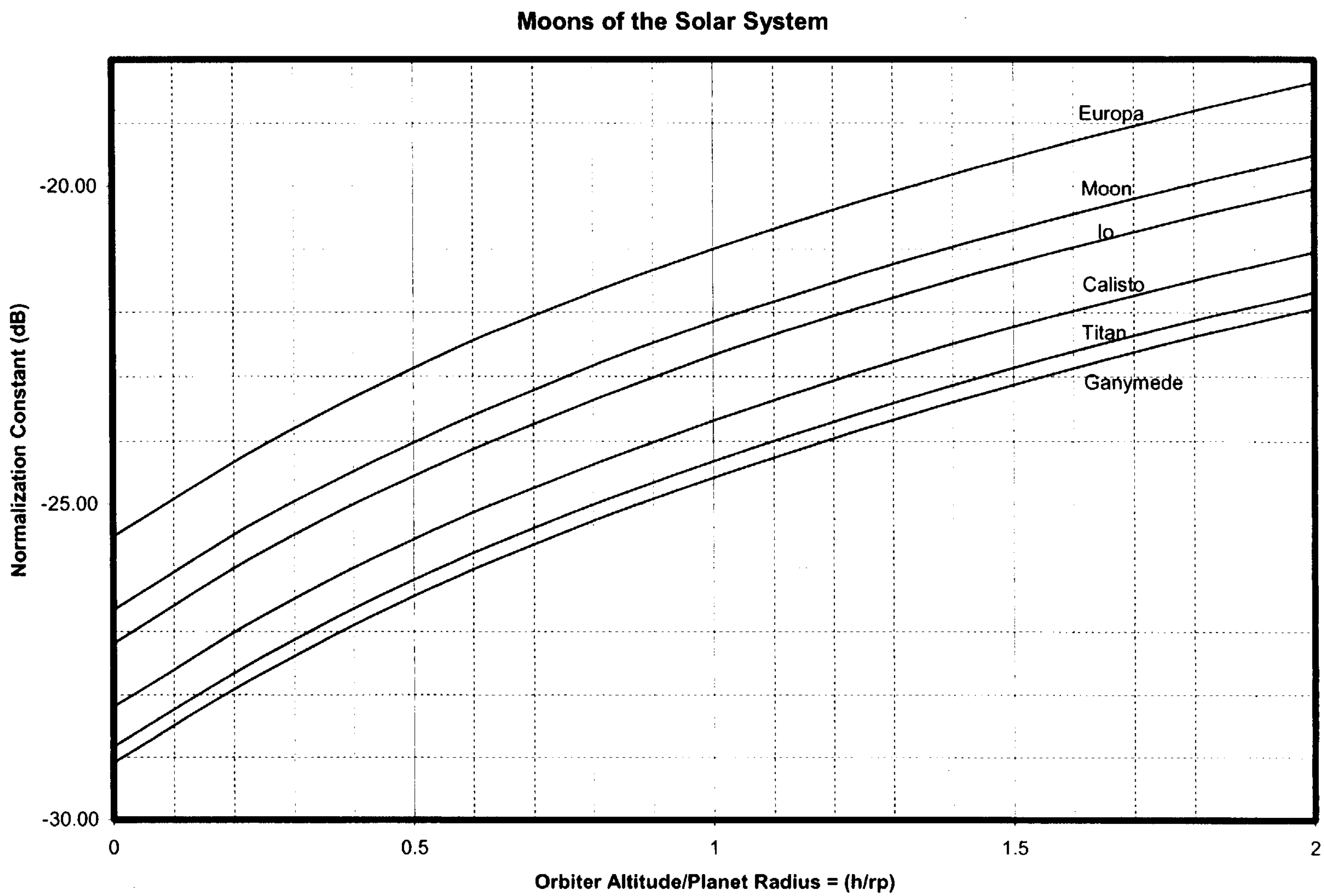


Figure 26. Constant needed to obtain the data bits gathered for the solar system planet moons.

With all the assumptions given in Table 2 for the link between the balloon/parachute held valid, the total number of normalized bits accumulated by the orbiter is read from Figure 24 to be 152 dB. To convert this into the real data collected, Figure 25 gives for the h/r_p of 0.5 and for the planet Venus a value of the normalization constant of –36 dB. Thus, the total actual data gathered by the orbiter at 0.5 h/r_p equals 152 – 36 = 116 dB, or equal to $10^{11.6}$ bits. It should be noted that if the actual values used in the link budget between the orbiter and the balloon is different than the assumptions, the results obtained may be scaled appropriately to get the correct results. For instance, if the actual transmitted RF watts is, as an example, 1 mW, then, since the assumptions that produced the figure 25 has the transmitted RF watts to be 1 W, the true value of the total data gathered by the orbiter will be $10^{11.6} \times 10^{-3} = 10^{8.6}$ bits etc.

9.0 Balloon/parachute Oscillations

It is conceivable that the balloon or the parachute carrying the telecom and science package may execute oscillations while descending to the body's (planet or the moon as the case may be) surface. This is mainly due to the presence of the atmosphere as well as the atmospheric winds. The above analysis has ignored this problem; i.e., the analysis is valid for a perfectly still atmosphere through which the balloon or the parachute is descending. In a sense, this is the 'mean' or the 'average' result of the case with oscillations and a new theory must be developed to fathom the details of the oscillatory case. This is attempted below.

Assumptions for the oscillations

- The oscillations will be small in nature (e.g., the end-to-end oscillations will not cover more than 40 degrees from its vertical position).

- The balloon/parachute assembly does not move considerably in the vertical direction during each oscillation period.

- The oscillations described by the balloon/parachute assembly may be approximated by the classical pendulum motion: i.e., all the mass of the assembly is concentrated in its gondola and the length of the pendulum string is equal to the height of the inflated balloon/parachute assembly.

- For the first degree of approximation, it will be assumed that the atmosphere does not put a damping force on the oscillations (this is not true in actual practice; however, the results obtained by using this assumption may not be too different from the actual results).

Oscillation Theory

Figure 27 shows the balloon/parachute as a pendulum; the necessary parameters to describe the motion are shown in the figure.

We will start with the well-known classical equation describing the motion of a pendulum given below. It is assumed that the balloon/parachute executes vibrations with the center of the balloon/parachute shown in the figure as the pivot. Following parameters are assumed.

The distance between the pivot and the center of gondola mass	L (m)
Mass of the oscillating gondola	M (kg)
Rotational inertia of the gondola around the pivot	ML^2
Angel of oscillation	$\phi(t)$
Planet's gravitational acceleration	g_p (m/sec^2)

Equation of pendulum:

$$M L^2 \left(\frac{d^2[\phi(t)]}{dt^2} \right) = -M g_p L \operatorname{Sin}[\phi(t)] \tag{9.1}$$

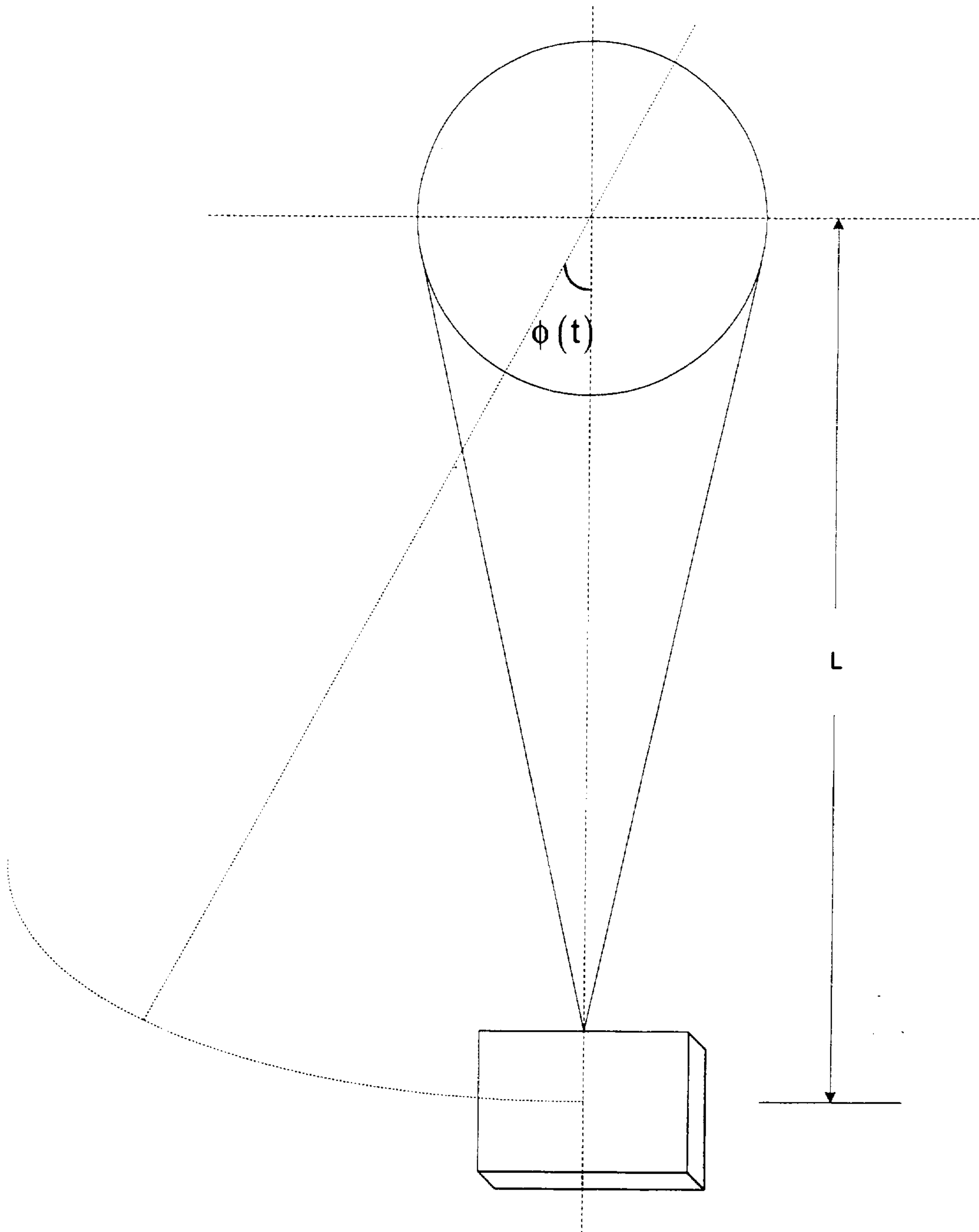


Figure 27. Balloon/parachute as a pendulum.

Hence,

$$\frac{d^2[\phi(t)]}{dt^2} = -\left(\frac{g_p}{L}\right)\mathrm{Sin}[\phi(t)] \tag{9.2}$$

The solution to this differential equation is not very simple and is very difficult to use. As was mentioned above, in the assumptions that the total oscillation angle is below 40 degrees (which is a very practical assumption), the approximation $\mathrm{Sin}(x) \cong x$ may be used. Using this approximation, equation (9.2) reduces to the one shown in (9.3) and now a simple harmonic solution does exist.

$$\frac{d^2[\phi(t)]}{dt^2} = -\left(\frac{g_p}{L}\right)\phi(t) \tag{9.2}$$

The solution to this linear differential equation is easy to find and is given below, where $\phi(0)$ and $\phi'(0)$ are the initial conditions of the balloon/parachute oscillation scenario.

$$\phi(t) = \phi(0)\,\mathrm{Cos}\left[\left(\sqrt{\frac{g_p}{L}}\right)t\right] + \phi'(0)\,\mathrm{Sin}\left[\left(\sqrt{\frac{g_p}{L}}\right)t\right] \tag{9.3}$$

Figure 28 plots $\phi(t)$ given by the equation (9.3) for L = 10m , 20m, and 30 m for Earth. One may notice that as the length of the pendulum increases, the period also increases.

Even though the angle $\phi(t)$ is with respect to the vertical line passing through the pivot, considering the dimensions of the orbital elements and the altitude of the balloon/parachute compared to the dimensions of the balloon, the vertical antenna will be assumed to be tilting by the same angle in both directions. The points $B(x_1, y_1)$ and $C(x_2, y_2)$ must be changed to accommodate the pendulum like motion of the balloon/parachute.

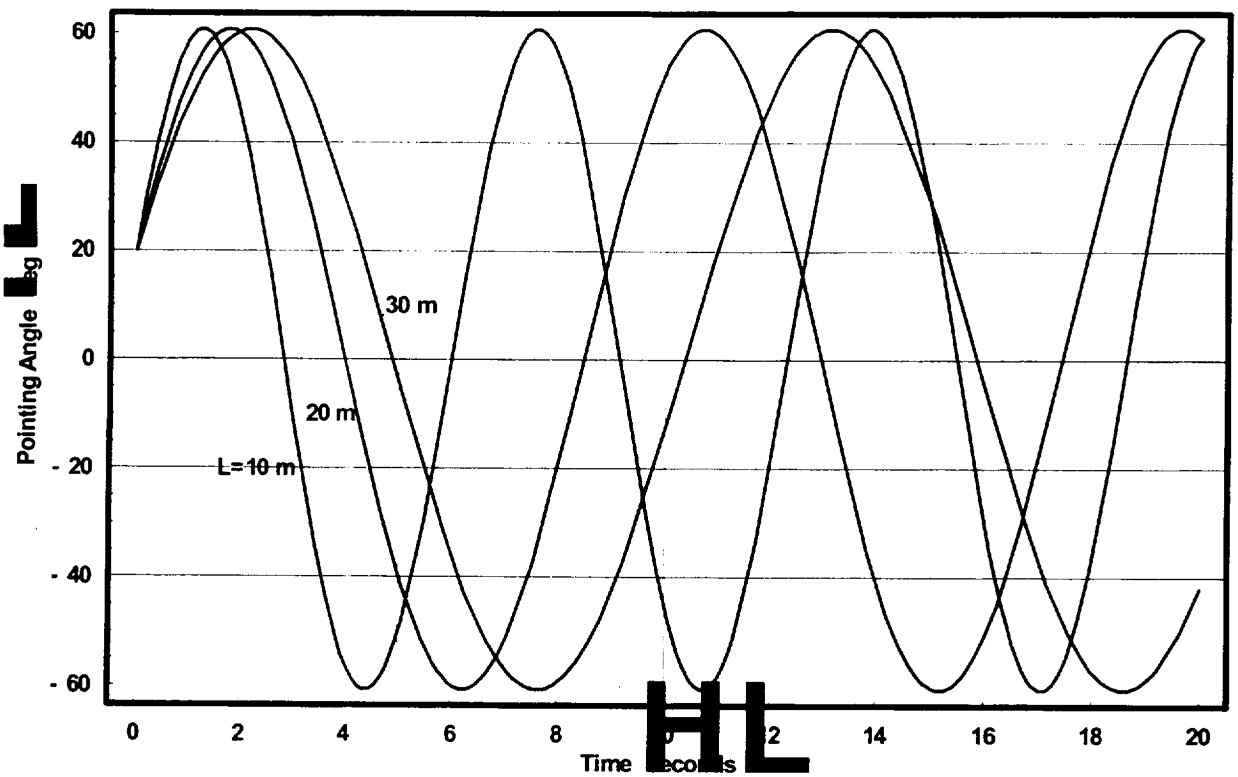


Figure 28. Balloon/parachute pendulum motion.

The point $B'(x_{1'}(t), y_{1'}(t))$ is the point $B(x_1, y_1)$ in Figure 2, including the pendulum - like displacement of the radiating antenna and similarly, the point $C'(x_{2'}(t), y_{2'}(t))$ is the point $C(x_2, y_2)$ in Figure 2, including the pendulum-like displacement of the radiating antenna. These points are given by the following formulas:

$$x_{1'}(t) = -\left(\frac{m'(t)\,c}{1 + (m'(t))^2}\right) + \sqrt{\left(\frac{m'(t)\,c}{1 + (m'(t))^2}\right)^2 + \frac{a^2 - c^2}{1 + (m'(t))^2}}$$

$$y_{1'}(t) = \left(\frac{c}{1 + (m'(t))^2}\right) + m'(t)\left(\sqrt{\left(\frac{m'(t)\,c}{1 + (m'(t))^2}\right)^2 + \frac{a^2 - c^2}{1 + (m'(t))^2}}\right)$$

(9.4)

Similarly, one may compute the second point:

$$x_{2'} = \left(\frac{m'(t)\,c}{1 + (m'(t))^2}\right) + \sqrt{\left(\frac{m'(t)\,c}{1 + (m'(t))^2}\right)^2 + \frac{a^2 - c^2}{1 + (m'(t))^2}}$$

$$y_{2'} = \left(\frac{c}{1 + (m'(t))^2}\right) - m'(t)\left(\sqrt{\left(\frac{m'(t)\,c}{1 + (m'(t))^2}\right)^2 + \frac{a^2 - c^2}{1 + (m'(t))^2}}\right)$$

(9.5)

Where,

$$m'(t) = \mathrm{Tan}(\alpha + \phi(t))$$

(9.6)

The angle α provides the angle from the boresite of the antenna (the maximum gain direction) for which the communications between the satellite and the gondola can be sustained, as before.

The visibility of the satellite may start from the satellite position (x_{limit}, y_{limit}) to the zenith of the balloon/parachute (see Figure 2). The visibility due to the other side of the satellite orbit beyond zenith is the same as the side under consideration and, hence, can be taken into account by doubling the visibility time later on (see the assumptions). Thus one is interested in the angle $\angle$ AOD in the figure. This angle is obtained by the following formula:

$$\angle AOD = \mathrm{Cos}^{-1}\left[\frac{\underline{OA} \cdot \underline{OD}}{|\underline{OA}|\,|\underline{OD}|}\right] = \mathrm{Cos}^{-1}\left[\frac{0\,x_{limit} + c\,y_{limit}}{c\,a}\right] = \mathrm{Cos}^{-1}\left[\frac{y_{limit}}{a}\right]$$

(9.7)

Where the (x_{limit}, y_{limit}) coordinates of the limiting position of the visibility are given in equation (5.11). Using (9.7) along with the definition of m_{limit} given in (5.10), one obtains:

$$\angle AOD = \mathrm{Cos}^{-1}\left[\left(\frac{r_p}{a}\right)\left(\frac{r_p}{c}\right) - \sqrt{\left[1-\left(\frac{r_p}{a}\right)^2\right]\left[1-\left(\frac{r_p}{c}\right)^2\right]}\right] \tag{9.8}$$

Where,

$$\frac{r_p}{c} = \frac{1}{1+\frac{h_b}{r_p}} \quad \text{and} \quad \frac{r_p}{a} = \frac{1}{1+\frac{h}{r_p}} \tag{9.9}$$

With the help of equations (9.8) and (9.9), the position of the satellite at any time 't' in the visibility region, $S(x_s(t), y_s(t))$, may be found using the following formula:

$$0 \le t \le \left(\frac{\angle AOD}{2\pi}\right)T_p$$

$$S(x_s(t), y_s(t)) = S\left(\underbrace{a \cdot \mathrm{Cos}\left[\angle AOD - \frac{\pi}{2} + \left(\frac{2\pi}{T_p}\right)t\right]}_{x_s(t)},\ \underbrace{a \cdot \mathrm{Sin}\left[\angle AOD - \frac{\pi}{2} + \left(\frac{2\pi}{T_p}\right)t\right]}_{y_s(t)}\right) \tag{9.10}$$

Where $\angle$ AOD is defined in equation (9.7) and T_p, the orbital period of the satellite is given in equation (5.16) above. Note that at t = 0, the equation (9.10) reduces to the satellite position (x_{limit}, y_{limit}), as can be seen from Figure 2.

The visibility and establishment of the link now depends upon the satellite being seen by the gondola of the parachute or balloon in this dynamic situation. The visibility can be determined using the following condition:

$$\left(x_{1}(t) \le x_{s}(t) \le x_{2}(t)\right) \bigcap \left(x_{1}(t) \le x_{s}(t) \le x_{2}(t)\right) \begin{cases} \text{TRUE} & \Rightarrow \quad \text{Orbiter is Visible} \\ \text{FALSE} & \Rightarrow \quad \text{Orbiter is not Visible} \end{cases} \tag{9.11}$$

It should be noted that the visibility of the orbiter from the balloon/parachute gondola now depends upon the initial condition of the balloon/parachute oscillatory motion as well as the position of the orbiter at that instant of time. A more detailed analysis may be performed, as was done for the non-oscillatory case; however, if the oscillations are small, the theory developed above will still be applicable.

This investigation into the balloon/parachute communications with an already existing orbiter spacecraft has the conclusions that follow.

10.0 Conclusions

A unified theory of telecommunications between a balloon/parachute to an orbiter using a dipole antenna is presented. The theory assumes that the transmitter and the receiver are both in the orbital plane of the orbiter. Presentation shows methods to calculate the following quantities:

- Relative positions of the balloon/parachute and the orbiter.

- Visibility time of the orbiter at the balloon/parachute for a defined gain angle of the dipole transmitting antenna.

- Useful dipole gain value directed towards the orbiter and the balloon/parachute to orbiter range value at any point of the orbiter's orbit.

- Computation of the maximum sustainable bit rate between the balloon/parachute to orbiter link at any relative positions of the two for different antenna diameters of the orbiter parabolic reflector antenna.

- Maximum attainable total received data bits at the orbiter.

The whole theory is presented in a manner such that the results produced may be applied to the balloon/parachute being at any planet of the solar system or the moons considered. Clear examples are given for the users to obtain the desired results quickly and efficiently.