INTEGRATED INS/GPS NAVIGATION FROM A POPULAR PERSPECTIVE

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ABSTRACT

Inertial navigation, blended with other navigation aids Global Positioning System (GPS) in particular, has gained significance due to enhanced navigation and inertial reference performance and dissimilarity for fault tolerance and anti-jamming. Relatively new concepts based upon using Differential GPS (DGPS) blended with Inertial (and visual) Navigation Sensors (INS) offer the possibility of low cost, autonomous aircraft landing. The FAA has decided to implement the system in a sophisticated form as a new standard navigation tool during this decade. There have been a number of new inertial sensor concepts in the recent past that emphasize increased accuracy of INS/GPS versus INS and reliability of navigation, as well as lower size and weight, and higher power, fault tolerance and long life.

The principles of GPS are not discussed; rather the attention is directed towards general concepts and comparative advantages. A short introduction to the problems faced in kinematics is presented. The intention is to relate the basic principles of kinematics to probably the most used navigation method in the future-INS/GPS. An example of the airborne INS is presented, with emphasis on how it works. The discussion of the error types and sources in navigation, and of the role of filters in optimal estimation of the errors then follows. The main question this paper is trying to answer is "What are the benefits of the integration of INS and GPS and how is this, navigation concept of the future achieved in reality?" The main goal is to communicate the idea about what stands behind a modern navigation method.

INTRODUCTION—CAPTURING A MOTION

In navigation the systems in which position parameters (e.g., the coordinates[1]) change with time, are referred to as kinematic processes or systems. Here we refer to a kinematic rather than a dynamic process—studies made under the heading of kinematics differ from dynamical investigation in that the concept of mass is not considered. Thus kinematics is sometimes referred to as the geometry of motion. A kinematic model

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describes a moving system (e.g., a vehicle or vessel) by putting parameter estimates at successive epochs in a mathematical relation.

A particle that is moving in space, and of position given by a system of curvilinear coordinates **x**, as the time *t* varies describes a certain spatial curve called trajectory. It is given simply by:

$$\mathbf{x} = \mathbf{x}(t). \tag{1}$$

Here **x** corresponds to position vector **r** of a vehicle or vessel. In navigation it is functionally related to position finding, thus obviously representing the main object of interest for avionics personnel working with navigation applications (see Figure 1). This figure shows the navigation process, differentiation feedback loop (Wells, 1996 based on Anderson, 1966). The first derivative of Equation 1 with respect to time is named velocity vector (**v**) of the particle. In navigation it comes into the center of the navigator's attention during the route following. The second derivative with respect to time of Equation 1 is called acceleration vector (**a**) of the particle. The aspect of navigation that deals with acceleration is guidance.

In dynamics particles are still idealized as points and their paths are represented by space curves, but because mass (*m*) is taken into account, the study of velocity and acceleration fields along the curves is replaced by a study of momentum (*m***v**) and force (*m***a**) fields. (Wrede, 1963).
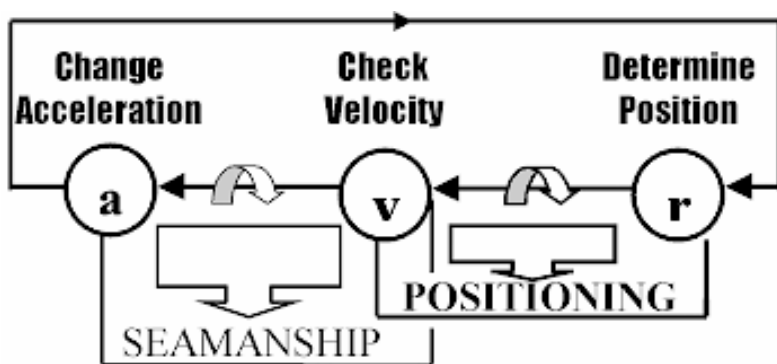

Figure 1. The navigational process and differentiation feedback loop

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Basically, a kinematic model is represented by the mathematical expression for the predictability of the motion. This predictability is

another way of saying that the position parameters are not entirely random, but have values related to their values at an earlier epoch. To illustrate this, we discuss the error/time frame. (see Figure 2)
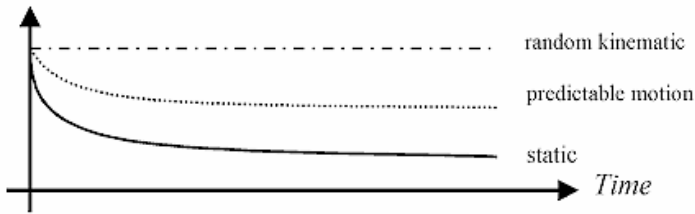
Mean error

Figure 2. Precision of positioning under various conditions

Note: From *The Kalman Filters in GPS Navigation*, by B. Merminod, 1989 NSW Australia: University of New South Wales. Copyrighted 1989 by University of New South Wales. Reprinted with permission.

Considering the purely random kinematic mode used to describe a motion, each determination of the position is independent of the others. Assuming that all other factors influencing the positioning system (e.g., the GPS) are constant, the precision of a fix is also constant in time (see Figure 2). If, on the other hand, the static mode is used for determining the position (say, a ground mark) each new measurement will contribute to the determination of the same position parameters. Extending an observation session makes more data available for the estimation of the same number of parameters, hence improving their precision. The predictability issue is raised once we accept that the difference in behavior of static versus kinematic solutions is not due to the movement itself, but to the a priori knowledge on the movement. If we could predict the movement of a vehicle as well as that of a ground mark (which, of course, in that case is not moving), the positions at different epochs could be perfectly related to one another and the precision obtainable would approach that of the static case. This applies, e.g., to an object moving along a known trajectory. Therefore, it is the predictability of the object's motion that permits the link between static mode (surveying) and well-behaved kinematic mode (most navigation) to be established. The dotted line shown on Figure 2 then represents the resulting improvement in the quality of the position determination.

INS/GPS INTEGRATION

Among today's trends in navigation are the integrated navigation systems, where the components (sensors) that are usually being integrated are the Inertial Navigation Systems (INS) and the Global Positioning System (GPS). Other sensors used in integration are, for instance, the Doppler Velocity Sensors (DVS). The purpose of combining navigation subsystems into an integrated system is to take advantage of complementary strengths of the subsystems. Thus navigation that is more reliable than that of individual subsystems is provided.

The INS computes the position by a sophisticated form of dead reckoning. In the simplest form of dead reckoning, as practiced by navigators in the past, the navigator multiplies the indicated (or estimated) speed by the estimated time enroute to obtain the estimated distance traveled from his starting location in a fixed direction usually provided by a magnetic compass. The assumptions made here are that velocity and heading are constant. The INS relies upon basically the same principle except it contains accelerometers that sense all specific forces including gravity. Further, it contains gyros that sense all angular rates experienced by the INS with respect to the inertial frame of reference.[2] With knowledge of angular rate, the INS computes the vehicle's orientation with respect to a geographic reference in the form of attitude (roll, pitch angles) and heading (see Figure 3). With the knowledge of vehicle acceleration, the INS computes the vehicle's velocity and change in position on the Earth, in form of latitude, longitude and altitude.
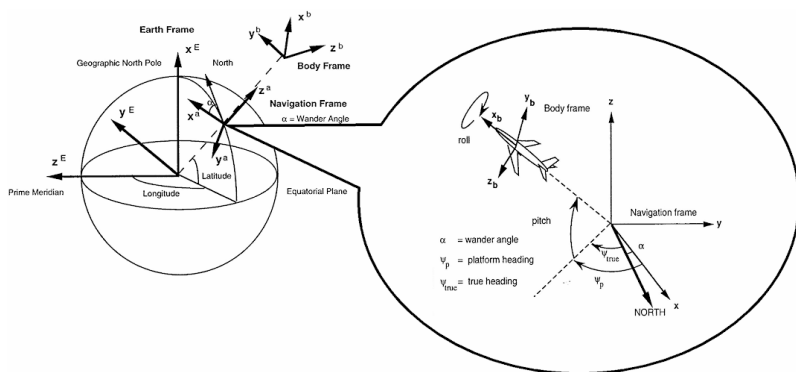

Figure 3. The body frame with respect to the reference frames used in navigation

Note: From "A Report on Navigation Systems Integration and on the Benefits and Costs of Integration," by B.M. Scherzinger, 1993, unpublished report. Reprinted with permission.

Three main forces that an INS has to take into account are: (a) Gravitational acting down; (b) Centrifugal due to Earth's rotation and sensed by gyros—a radial force acting outward from the object, unlike centripetal that acts toward the object; and (c) Coriolis[3] force in the direction of the movement, coming from compound acceleration of coriolis (in navigation: Coriolis correction of the sensed acceleration).

$$\mathbf{a}_c = 2\omega \times \mathbf{v}^1, \tag{2}$$

where ω (assumed to be constant—variations in earth's rotation can be neglected in navigation) represents the angular velocity of the rotating body (the Earth), and $\mathbf{v}^1$ relative velocity (with respect to the Earth). $\mathbf{a}_C$ correction is applied with respect to a frame relative to inertial space.

HOW IT WORKS

An INS designed to navigate on the Earth and in its atmosphere must first subtract the gravitational, centrifugal and the Coriolis effects in form of the corresponding accelerations, from the sensed specific accelerations in order to obtain the INS's acceleration with respect to the Earth. Then it has to subtract the Earth's rotation rate (15° per hour) from the sensed angular rates to obtain the INS's angular rate with respect to the Earth. The INS finally integrates the corrected accelerations and angular rates to obtain changes in velocity, position, attitude and heading, with respect to the Earth.

On Figure 4 is an example (Scherzinger, 1993) of the simplistic view of inertial space-stabilized navigation. The accelerometers are mounted on a platform that can rotate in the azimuth plane on the aircraft and are initially oriented North, East and Down, sensing North and East accelerations and the Down force. A single gyro mounted on the same platform senses the angular rate along the azimuth axis. The aircraft is initially at rest facing East, and accelerates to a velocity of 1 m/s over 1 second. The East accelerometer senses this as acceleration a_{East} that accelerates the airplane. The INS computes the changes in velocity v_{East} and position $\mathbf{r}_{East}$ as:

$$\left. \begin{aligned} v_{East}(t) &= v_{East}(t_0) + \int_{t_0}^{t} a_{East}(\tau)d\tau \\ r_{East}(t) &= r_{East}(t_0) + \int_{t_0}^{t} v_{East}(\tau)d\tau \end{aligned} \right\}. \tag{3}$$

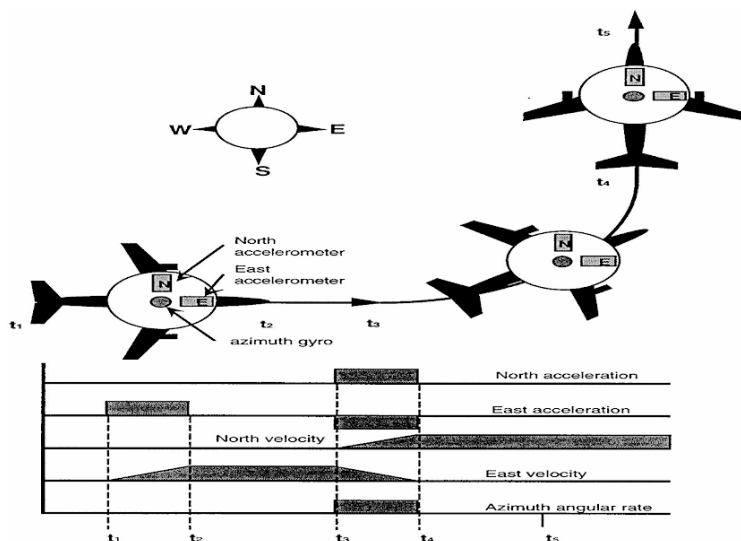Then the aircraft enters a turn and thereby changes its heading.

Figure 4. An example of the INS applied in air navigation

The azimuth-angular rate is sensed by the gyro and is used as a signal to a torque motor that rotates the platform about the azimuth axis in direction opposite to the sensed angular rate (clockwise in our case). The platform rotates with respect to the aircraft's centerline, and not with respect to the Earth's surface below the flying aircraft. Thus the North-East orientation of the accelerometers is being maintained. Worth noting here is that the aircraft's heading is the azimuth angle between the aircraft centerline and the accelerometer North axis. Finally, during the maneuver (a turn) as the aircraft changes its velocity from an East velocity to a North velocity this change is being described by a North acceleration and an East acceleration. The North and East accelerometers, naturally, sense these. In this way, the INS computes an increasing North velocity and decreasing East velocity during the turn. Once the turn is completed, the INS computes a constant North velocity since both the North and East accelerometers sense no aircraft accelerations (see Figure 4). Thus the stabilized platform has maintained its North-East orientation before, during, and after the turn.

In cases when the Down accelerometer senses a specific acceleration as the aircraft begins, for example, to climb, this (equation 4) is the resultant

$$a_{\text{Down}}^{\text{Sensed}} = a_{\text{Down}} + g\,, \tag{4}$$

(the sum) of the aircraft's vertical acceleration and the gravitational acceleration. Then the INS computes the aircraft altitude change from initial altitude h_0 and sensed acceleration, as:

$$h = h_0 + \iint (a_{\text{Down}}^{\text{Sensed}} - g)\, dt.$$ (5)

In a procedure separate from but analogue to the above-described one (maintaining the North-East orientation of the platform), the level orientation of the platform is being preserved as well. Here the platform ideally defines a navigation frame that is both locally stable and locally level. The accelerometer triad (North, East, and Down), in this case mounted on the platform, outputs a sensed acceleration vector given by:

$$a^{\text{Sensed}} = \begin{bmatrix} -a_{\text{North}}^{\text{Sensed}} \\ -a_{\text{East}}^{\text{Sensed}} \\ -a_{\text{Down}}^{\text{Sensed}} \end{bmatrix},$$ (6)

which is resolved in the navigation frame with axes oriented North, East and Down. Similarly, the gyro triad (North, East, and Down) outputs a vector of North, East and Down angular rates used for maintaining the platform's stability. All subsequent changes in position and velocity are then computed in the navigation frame defined by the platform.

Let it finally be mentioned how, especially for military applications, besides the above space stabilized (stable platform and gimbals) and local level, also in usage nowadays are so-called strap-down inertial equipment mechanization.

INS Errors

Most INS errors can be attributed to inertial sensors (instrumental errors). These are the residual errors exhibited by the installed gyros and accelerometers following calibration of the INS. The dominant error sources in the INS are shown in Table 1.

As mentioned above, error-characteristics of an INS are the behavior of the free-inertial INS following a nominal alignment (determination of position on take-off). For example, aircraft navigators are familiar with the two dominant error-characteristics. The first is Schüler[4] Oscillation, coming from the fact that the INS is behaving like a pendulum with the center of rotation in the center of the Earth. The second is Position Error Growth (PEG) is the most significant performance figure that characterizes

Table 1. Errors in the INS

Type/source	Description	Typical magnitude
Alignment *errors*	roll, pitch and heading errors	
Accelerometer bias *or offset*	a constant offset in the accelerometer output that changes randomly after each turn-on.	50-100µg $(1µg=9.81 \times 10^{-6} m/s^2)$
Accelerometer scale factor error	results in an acceleration error proportional to sensed acceleration	75-200 ppm
Accelerometer and gyro nonorthogonality	the axes of accelerometer and gyro uncertainty and misalignment	5"-25"
Gyro bias or drift due to **temperature changes** (and gas circulation*)	a constant gyro output without angular rate presence	0.002-0.01 °/hr
Gyro scale factor error from **temperature changes**	error in the assumed degrees per second per pulse	< 10 ppm
Random noise		spectral density of $(0.002°)^2/hr$

* when the gas gyro is used

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an INS. Dominant source of the PEG rate is the gyro bias, contributing 60 nmi/hr. Number of nautical miles per hour position error-rate, that it typically exhibits, classically characterizes an INS.

Lastly we speak of INS state errors (including initial state errors, and inaccuracies in the gravity field modeling) (Linkwitz & Hangleiter, 1988). Equation 4 from our example (Down accelerometer) applies to any sensed acceleration (East or North). Also, we can express the measured (sensed) vector of acceleration (**a**) by a physical law (Linkwitz Hangleiter, 1988):

$$\mathbf{a}(t) = \frac{d^2}{dt^2}\mathbf{x}(t) + \mathbf{b}(t) - \mathbf{g}(\mathbf{x}(t)) \tag{7}$$

$$\mathbf{a}(t)^{Sensed} = \frac{d^2}{dt^2}\mathbf{x}(t) + \mathbf{b}(t), \tag{8}$$

where **b** expresses rotational acceleration of the accelerometer frame with respect to inertial space, and **g** is the gravity vector. The above-mentioned errors are then exhibited as relative position-errors as the integration takes place:

$$\mathbf{v}(t) = \frac{d}{dt}\mathbf{x}(t) = \frac{d}{dt}\mathbf{x}(t_0) + \int_{t_0}^{t}[\mathbf{a}(\tau) - \mathbf{b}(\tau) + \mathbf{g}(\mathbf{x}(\tau))]d\tau \tag{9}$$

$$\mathbf{x}(t) = \mathbf{x}(t_0) + \int_{t_0}^{t}\left[\frac{d}{d\tau}\mathbf{x}(\tau)\right]d\tau. \tag{10}$$

For integration intervals of several minutes these position-errors are of the order of several centimetres, and grow to hundreds of metres for integration intervals of several hours (Linkwitz & Hangletier, 1988). Thus high (decimetre accuracy) short-term stability and poor (hectometre accuracy) long-term stability can characterize the error behavior of inertial positioning (Schwarz, 1986). The way to handle these errors consists in bringing the inertial platform to a complete rest (zero velocity update) after which velocity is reset to 0 value. However this is not feasible during the flight, for which an In-flight Update is performed. For further references on INS errors, see, for example Scherzinger, 1993.

To get a better understanding of the effect of the most significant INS errors, let us consider an aircraft on a 5,000 km long route. Assuming the average speed of v = 800 km/hr, we apply this information on the typical magnitudes column from Table 1:

1. *accelerometer bias;* 50-100μg
   - best-case scenario: 0.0004905 m/s^2
   - worst-case scenario: 0.000981 m/s^2

2. *accelerometer scale factor error*
   - best-case scenario: 75 ppm on 5,000 km trajectory: 375 m
   - worst-case scenario: 200 ppm on 5,000 km trajectory: 1000 m

3. *gyro bias*
   - best-case scenario: 0.002°/hr on 800 km: 28 m
   - worst-case scenario: 0.01°/hr on 800 km: 140 m

4. *gyro scale factor*
   - best-case scenario: 0 m
   - worst-case scenario: 10 ppm on 5,000 km: 0.05 km = 50 m

Summarizing the above errors we conclude that total INS error would be, in

- best-case scenario: 0.375 km over 5,000 km + 0.028 km over 800 km
          + 0.01 km over 5,000 km = 560 m ≈ **1/2 km**

- worst-case scenario:1 km over 5,000 km + 0.140 km over 800 km +
          0.05 km over 5,000 km = 1925 m ≈ **2km**.

It is indicative that the accuracy of a stand-alone INS decreases significantly: the error reaches the order of kilometres. The well-known accuracy of even absolute GPS positioning (stand-alone GPS) is about 100 m horizontally and 150 m vertically. Thus these two systems definitely represent complementary subsystems for the navigation integration, error-wise. In the following we will discuss some characteristics that make them complementary to one another for reasons other than dissimilarity for fault tolerance.

Of course, this very rough investigation did not include all the errors listed in Table 1, or errors from the discussion of equations 7 through 10. The results above should therefore be addressed with great caution.

GLOBAL POSITIONING SYSTEM IN NAVIGATION

Global Positioning System (GPS) can be regarded as a new navigation sensor. GPS provides range and range-rate measurements. The primary role of GPS is to provide highly accurate position and velocity worldwide, based on range and range-rate measurements. The acceleration vector is then determined from positions at different time epochs, by differentiation of these positions with respect to time. Worth noting is that by accuracy we mean how close the average measurement is the actual true value (accuracy measures systematic error), distinguishing it thus from precision that describes how close the measurements are to one another (precision measures random error).

Integration with one or more external systems capable of sensing forces, for example, with an INS, has the goal of achieving reliability in navigation, as the GPS signals may not be available at all times. In that sense, the basic idea behind the integration of GPS and INS is to estimate the inertial sensor errors online using GPS. This means an in-flight calibration (update) or identification of the INS's state and instrument errors in order to provide a precise inertial navigation solution—even during the loss of GPS signal (a few seconds to over a minute)—based on previous knowledge of INS errors. At the same time, the INS can be used to bridge cycle slips and times of loss of lock, but most important it can be

used to bridge the time between two GPS position computations. (Liang, 1992).

Usually a filter code (a mathematical algorithm) that will optimally process such (by an integrated system) collected information is written, burned into a chip and integrated into the integration structure itself. Of course, a filter implementation is not a necessity, but rather an option—an especially desirable one in military aircraft navigation—when losses of GPS signal due to sudden maneuvers are often. Due to a filter implementation the final solution becomes consistently reliable. For example, short-term highly reliable three-axis attitude and heading (from, e.g. INS) may accompany long-term highly accurate position and velocity (from, e.g. DGPS and say, a Kalman filter). The benefit of such (INS/GPS/filter) integration is then obvious. The INS error estimates may be used to improve INS/GPS navigation should GPS become unavailable.

Further, if there would be only INS, then the final solution performance would degrade consistently to the level prescribed by the available aiding sensors. Also, GPS can be subject to jamming and spoofing (Leick, 1995). The INS on the other hand is an autonomous navigator and therefore unaffected by external influences of this sort. Integration also proved to improve estimates of acceleration, attitude, and body rates that can be used for guidance and control. During testing at the NASA Ames Research Center in 1992, Precision (P) Code DGPS/INS positioning root mean square (RMS) achieved was 1 m horizontal and 3 m vertical (Liang, 1992).

GPS can be implemented in navigation in a few basic ways. It can be a stand-alone receiver, as a part of recreational application of GPS (but also a military application, e.g., in Gulf and Kosovo on Tornado aircraft). The operator reads the output on the receiver's display and manually inserts the GPS position into the main computer (if there is one) or applies it in manual navigation. Second, it can be input to a main computer via, for example, digital interfacing. Third, it can be a fixing aid as a part of an integrated navigation system, for example INS/GPS, and input with ability of the main computer to use the GPS data in the navigation solution together with other sensors and to aid GPS data with navigation data. Fourth, it can be input to mission computer, as a form of a military application of GPS: enabling fast adjusting (satellites reacquisition) to sudden maneuvers when the loss of the signal is highly possible using velocity (course and speed) data from INS.

Another benefit of integration is when DGPS is used for precision approach (autonomous all-weather landing)—in a (civil) aviation application of GPS. But such a task is not solved by DGPS itself. The standard integrity requirements for air navigation precision approach require GPS to be integrated with a complementary sensor system like an

INS (Liang, 1992). Only this way (by the strength of the integration), can the Instrument Landing System (ILS) or Microwave Landing System (MLS) (the expensive systems in use at the airports) be replaced by a worldwide (in a unique coordinate frame), all-weather system, that is not expensive and not dependent on the closeness of an airport flight-control.

GPS ERRORS; INS/GPS APPLICATION CLASSIFICATION BASED ON GPS TYPES

Position accuracy of GPS pseudo-range absolute positioning is affected by measurement noise (few metres) and unmodelled short- and long-term systematic effects of the order of a few tens of metres. The propagation of these errors into the position solution can be characterized by a Dilution of Precision (DOP) factor DOP is greater than 1, expressing the geometry between the satellite and the receiver. Therefore, GPS pseudo-range absolute positioning can be said to have medium (tens of metres) short- and long-term stability.

In pseudo-range GPS relative positioning, the position of a receiver is determined relative to another receiver at a known location, from simultaneous pseudo range observations to at least four GPS satellites. Although most of the systematic effects are eliminated in this differencing process, the errors in these position differences remain dominated by metre-level measurement noise and can be characterized by medium (tens of metres) short- and long term stability.

If one of the two stations involved in the relative positioning remains static and the other starts moving, simultaneous GPS carrier phase observations at two locations can be used to determine the change in relative position. As carrier phase measurement noise is much lower than pseudo-range noise, and most systematic errors are removed in the observation differencing, the relative position errors are of the order of a few centimetres over distances of a few tens of kilometres. Therefore, changes in relative positions from GPS carrier phases exhibit high short- and long-term stability.

A GPS receiver becomes an orientation and position sensor if pseudo-ranges and carrier phases are measured simultaneously through three different antennae mounted on a common antenna platform. The position is determined using the pseudo-range observations of one of the three antennae. The carrier-phase observations then determine two linearly independent relative position-vectors between the three antennae and, therefore, yield the platform orientation. (Linkwitz & Hangleiter, 1988).

After acquiring the above-described division for GPS positioning and orientating, to summarize our discussion on INS/GPS integration by

classifying the integration into four types. First integration in absolute positioning requires continuous reception of GPS signals in reasonable geometry (say DOP < 6). Due to the possible GPS signal loss, as well as in the high dynamic applications, INS derived position changes can serve as an interpolator. Typical applications are in precise air and marine navigation. Here the integration has to take place in real time. INS is considered as primary navigation system, GPS pseudo-ranges as position-updates provider, and a filter (e.g., Kalman) as the adequate formulation for the system integration. Second, integration in relative positioning provides an improved long-term stability resulting from the elimination of systematic errors in the GPS pseudo-ranges. Typical applications are in (hydrographic) surveying, inshore and river navigation and airborne photogrammetry. INS can be used as interpolator in case of GPS signal loss. Integration in orientation determination is where fiber-gyros are part of the INS. Fourth, is integration for gravity field determination. For more details about this type see, for example, Schwarz, 1986.

ESTIMATION, FILTERING, AND BLENDING

A final (blended) navigation solution is the correct navigation solution (either from a stand-alone or integrated sensors). The corrections to the, for example, INS solution can be computed within an integration (usually by the Kalman filter) which then compares the INS data with the aiding (from the integration) sensor data.
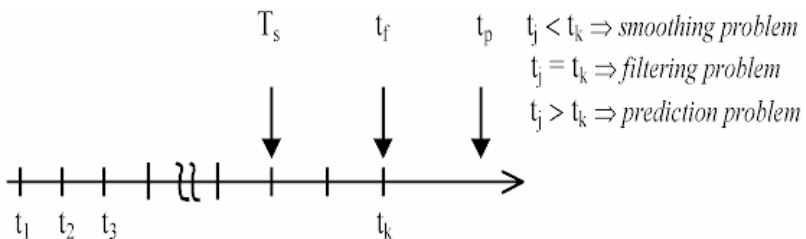

Figure 5. Three types of estimation problems

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Figure 5 considers the relations between prediction and filtering, as the two very first steps in an estimation procedure. To summarize and relate them to navigation, let us state how prediction understands the computation of expected position (and its precision) of the vehicle at some subsequent time t_k, based upon the latest measurement at t_{k-1}. In this way we now term

filtering to be a process of computing the vehicle's position at t_k (i.e., in real-time: while observations are taken also at t_k). Similarly, we define the third step in the estimation process as smoothing—the estimation of where the vehicle was (say, at time t_k) once all the measurements are post-processed to t_{k+1} . Here we actually talk about the reprocessing (of all the measurements) after the last measurement has been made and the filtering step has been completed.

Previously discussed predictability of the motion makes it an ideal candidate for filter estimation methods. If we imagine a vehicle traveling along a straight line, we soon realize how, after a couple of positions have been determined, future positions can be predicted by extrapolation. Here the uncertainty associated with the prediction grows with time—while a position predicted a few seconds ahead may be more accurate than new measurements-based determination, this will certainly not be the case if positions are predicted over some hours. Therefore, a kinematic model comprises two components: (a) the functional part—the prediction of a position based upon previous results, and (b) the stochastic part—the estimation of the precision associated with the predicted position (see Figure 5).

Similarly, we speak of two classes or approaches for modeling a trajectory. These depend on whether emphasis is on functional modeling or on stochastic modeling. This, however, is not a finite division. Functional modeling contains predicted positions whose stochastic properties were derived from the precision of the estimated parameters. On the other hand, in stochastic modeling contains a position for which the assumed uncertainty applies must be computed as a function of previous position estimates.

The reasoning behind the functional approach to kinematic modeling is to replace the time-varying parameters again (e.g., coordinates) by auxiliary constant parameters (e.g., coefficients of a polynomial). Thus, the outcome of the kinematic model takes the form of a direct measurement of (some or all of) the position parameters, with associated precision estimates. Now the role of a filter is to optimally combine these pseudo-measurements with new actual measurements, that is, to combine the position predicted via the kinematic model with a new position determination derived from subsequent measurements. Thus the kinematic system becomes the static one.

In practical terms, this means that a filter becomes an optimal estimator. By definition, an optimal estimator is a computational algorithm that processes measurements to deduce a minimum error estimate (in accordance with some stated criterion of optimality) of the state of a system. For this purpose the algorithm uses knowledge of system and

measurement dynamics, assumed statistics of system noises and measurement errors, and initial condition information (Gelb, 1994). Thus, for example, the Kalman filter encompasses both filtering (trying to cut off the noise from the signal) and estimating (optimality in providing state[5] of the system).

Functional approach (e.g., the polynomial filter) is suitable only for very well behaved motions. The stochastic approach possesses an important property. The noise added in each extrapolation progressively reduces the weight attributed to previous position determinations (thus enabling selective weighting of the information). This, so-called fading memory, together with the absence of predefined signature for the trajectory, increase the ability to adapt to a new system behavior, which with this being highly desirable in navigation, makes the stochastic approach in modeling the favorite one. The stochastic approach (e.g., the Kalman filter) to kinematic modeling thus makes the basis of filtering techniques. (Merminod, 1989).

In order for the final navigation solution to be computed, an integrated navigation system combines the navigation data supplied by complementary sensors. Navigation sensors are complementary if they meet the following conditions: (a) the set of sensors generates all the information required to compute a complete navigation solution, and (b) the sensors have complementary error dynamics—all of their error dynamics are observable. Two thus complementary sensors can calibrate each other's errors because their errors are separately observable in any linear combination of their outputs.

Applying this on, for example, an INS/GPS navigation system, we could compare the position obtained from, for example, INS with the one from GPS. The difference in, for example, North component of the position from these two methods equals the difference in the errors these quantities carry. This is obvious since they are actually sums of the true value of position and the specific error(s) (true-position components cancel out). Next we introduce the a priori knowledge on errors: the INS position error grows, say, on the order of 1 nmi/hr in the long-term, and is smooth in the short-term, with a strongly recognizable Schüler oscillation (full cycle every 84 minutes). Also, the GPS North position error is noisy in the short-term but of a constant offset (few metres) in the long-term. Clearly, the INS and GPS position errors exhibit complementary error dynamics. Applying a simple low-pass filter that smoothes out the random noise and passes the INS error plus small GPS offset, we simply subtract this estimate from the INS North position to obtain a blended North error, containing only the constant position offset from GPS that the filter passed. The blended solution now has the best characteristics of both the INS and the GPS: the position

solution is smooth and has a bounded error less than or equal to the GPS position error.

ENDNOTES

1. There are ten basic navigation parameters: three components of each of the vectors (1), **v** and **a**, plus a time tag (Xu, 1996). There can be more parameters, e.g. attitude components and their derivatives.

2. The *inertial frame of reference* is one which experiences no acceleration or angular rates (Newton's laws of motion apply without corrections for accelerations or rotations of the reference frame).

3. Gustave Gaspard de Coriolis (1792–1843).

4. Maximillian Schüler

5. For more details on observables and their properties, see, e.g., Vaníček & Krakiwsky, 1986.

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