A Hydrodynamic Theory for Spatially Inhomogeneous Semiconductor Lasers: I. Microscopic Approach

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Starting from the microscopic semiconductor Bloch equations (SBEs) including the Boltzmann transport terms in the distribution function equations for electrons and holes, we derived a closed set of diffusion equations for carrier densities and temperatures with self-consistent coupling to Maxwell's equation and to an effective optical polarization equation. The coherent many-body effects are included within the screened Hartree-Fock approximation, while scatterings are treated within the second Born approximation including both the in- and out-scatterings. Microscopic expressions for electron-hole (e-h) and carrier-LO (c-LO) phonon scatterings are directly used to derive the momentum and energy relaxation rates. These rates expressed as functions of temperatures and densities lead to microscopic expressions for self- and mutual-diffusion coefficients in the coupled density-temperature diffusion equations. Approximations for reducing the general two-component description of the electron-hole plasma (EHP) to a single-component one are discussed. In particular, we show that a special single-component reduction is possible when e-h scattering dominates over c-LO phonon scattering. The ambipolar diffusion approximation is also discussed and we show that the ambipolar diffusion coefficients are independent of e-h scattering, even though the diffusion coefficients of individual components depend sensitively on the e-h scattering rates. Our discussions lead to new perspectives into the roles played in the single-component reduction by the electron-hole correlation in momentum space induced by scatterings and the electron-hole correlation in real space via internal static electrical field. Finally, the theory is completed by coupling the diffusion equations to the lattice temperature equation and to the effective optical polarization which in turn couples to the laser field.

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I. INTRODUCTION

A typical semiconductor-based optoelectronic device, such as a diode laser, can be considered as consisting of three subsystems: an optical field, an electron-hole plasma (EHP), and a host crystal lattice. Light generation, propagation, amplification, and diffraction determine the behavior of the optical field, while electrical conduction, plasma diffusion, and carrier generation and recombination determine that of the plasma. The host crystal is very often represented by various phonon modes, with the longitudinal optical (LO) phonon mode being the most important one for optics and carrier transport of III-V semiconductors at room temperature. Obviously the whole system involves the interplay of optical, electrical and thermal processes. From the perspective of conversion and conservation, energy is stored in the form of photon energy, kinetic and thermal energy of the EHP, and thermal energy of the host crystal represented by the phonons excited and annihilated. Though various processes occur in different space and time scales, their couplings should be treated in a self-consistent fashion to arrive at correct coupled equations, or to appropriately decouple the equations in various limiting cases. While modeling and simulation of semiconductor lasers and other optoelectronic devices typically use the rate equations [1] or the semiconductor Bloch equations [2] that treat only the coupled electrical-optical subsystems, other approaches have focused on the thermal aspect [3]. In Refs. [4, 5], a self-consistent approach was attempted to combine all three subsystems and the relevant processes based on earlier work on plasma heating [6–9] in semiconductors. The approach is, however, valid only for a single-mode laser, or a spatially homogeneous laser.

There is plenty of manifestation of spatial inhomogeneities in a semiconductor device. Laser beam filamentation, dynamic beam steering, and multiple transverse mode formation and competition are some of the important examples where inclusion of spatial, or spatio-temporal variation is necessary. Additionally, with the inclusion of temperature variables, more spatial phenomena can be described, such as thermal lensing and formation of hot spots, and catastrophic optical damage (COD) in diode lasers. For advanced semiconductor lasers such as vertical-cavity surface-emitting lasers (VCSELs), or Master-Oscillator coupled with Power-Amplifier (MOPA), spatial inhomogeneity is a more prominent issue when efficient coupling of multimode VCSELs to multimode fibers are desired, or when careful transverse mode engineering is required for certain

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applications. In terms of theoretical description, spatial inhomogeneity of a semiconductor laser is usually dealt with using Maxwell's equation with diffraction terms, coupled with carrier diffusion equation [10]. This set of equations, which is a direct generalization of the rate equations to the spatially inhomogeneous case, can be further augmented by adding the material polarization equation to account for gain and refractive index dispersion [11]. This approach has recently been used to simulate MOPAs [12, 13] and VCSELs [14]. An alternative approach is to start from the Boltzmann-type of transport equation for the carrier distribution functions as described in Ref. [15]. This approach is a very general one, but it involves quite extensive computation, since k-resolved interband polarization equations have to be solved self-consistently for all space and time grid points. For nearly all practical purposes, lasers and other optoelectronic devices operating on time scales longer than a picosecond can be more efficiently dealt with by the moment-equation type of approach. To account for both carrier density and carrier energy (thus carrier temperature) inhomogeneities within the framework of the moment equations, one needs to derive a set of partial differential equations for up to the second order moments from the corresponding Boltzmann transport equations. Such an approach, which is alternatively called hydrodynamic approach, will lead to a generalization of the single-mode laser model in Ref. [4] to spatially inhomogeneous cases including thermal and heating effects. In the past, carrier diffusion equation has been derived from the Boltzmann transport equation in combination with Maxwell's equation and the optical polarization equations by several authors [15, 16]. Since only the zeroth-order moment equation is derived [15], plasma heating cannot be described. This lack of systematic derivation of the temperature equation creates problem in correctly choosing the energy or temperature diffusion coefficient [17]. Furthermore, whenever two or more types of diffusion processes exist, mutual diffusions, or cross diffusions, occur. The quantitative significance of these processes needs to be examined, which is only possible with a systematic derivation of the coupled diffusion equations and all the corresponding diffusion coefficients (DCs).

This work sets out to derive such a set of coupled macroscopic equations for carrier densities and carrier energies from the coupled Boltzmann-Bloch transport equations using moment-equation approach. We pay special attention to the treatment of carrier-carrier (c-c) scattering and carrier-LO phonon (c-LO) scattering. Different from previous approaches in this regard, microscopic electron-hole (e-h) and c-LO scattering terms in the Boltzmann transport equations are used directly to obtain the corresponding energy and momentum relaxation rates. Various DCs are then expressed in terms of these momentum relaxation rates. The energy relaxation rates are used to describe energy exchange between different subsystems, which translate to temperature variations consequently. Such an approach allows a detailed study of the DCs for a given quantum well structure. Detailed numerical results and analysis of these coefficients will be presented in a subsequent article.

The article is organized as follows. In Section II, we introduce the basic physical considerations and starting equations. This is followed by a general derivation of the moment equations and the treatment of the cutoff issue, which is central to any moment-equation based approach. The general form of the hydrodynamic equations for the general two-component situation is derived in the fourth section. The fifth section deals with the specialization to the cases of single-component approximation where we will discuss the consequence of the ultrafast e-h scattering for the drift momenta of electrons and holes. We will also discuss the well-known ambipolar diffusion approximation (ADA). In the sixth section, the very important issue of optical polarization is treated. We use two different approaches to close the hierarchy related to the k-resolved polarization equations. In Section VII the corresponding Maxwell's equation and the lattice temperature equation are introduced to form the complete set of equations for a more complete description of semiconductor lasers. Though we use laser as an exemplary device in this work, the treatment and the resulting equations can be used for other optoelectronic devices, such as photoconductors [18] and photodetectors [19], with little or no modification. In Section VIII we discuss and comment on some general aspects of our theory and sum up the major results of our article before we present detailed considerations in the appendices for scattering rates and DCs. Appendix A deals with momentum and energy relaxation rates due to e-h scattering, while the corresponding rates due to c-LO phonon scattering are treated in Appendix B. In Appendix C, we list DCs for the two-component case. Finally, expressions for all the DCs under the one-component approximation are given in the last Appendix.

II. MODEL AND BASIC EQUATIONS

We begin this section with some general remarks about the spirit of our approach. As is often assumed, laser-matter interaction in a semiconductor laser is described with an EHP model. The standard argument to support such a plasma model is that the typical room-temperature lasing density is around 10^{12} cm^{-2}, well above the Mott density. While we use this model in this article, its adequacy in a spatially inhomogeneous system needs special scrutiny. We comment on this in the final section of this article. Within the plasma model, Coulomb interaction is usually characterized by an excitonic enhancement of the optical transition and carrier density-dependent bandgap renormalization [2, 16] in the coherent part of the semiconductor Bloch equations (SBEs). In addition, this relatively high density leads to ultrafast carrier-carrier scattering within 100 femtoseconds, which dominates carrier dynamics on the short time scale and affect the interaction of the EHP with

a laser field in several important ways: First, these ultrafast collisions thermalize carriers in properly populated subbands [20], which justifies the standard assumption of quasiequilibrium for carrier distributions when longer time scale dynamics is of interest. Second, as will also be shown later in this article, ultrafast e-h scattering correlates populated subbands such that the whole EHP can be characterized with a single temperature and drift velocity, but different chemical potentials for different subbands under normal conditions. This understanding will be elaborated in the subsequent sections and in appendix A by examining in detail the effects of the scattering on momentum and energy relaxations. Due to these ultrafast collision events, local quasiequilibrium is established in less than a picosecond. This allows carrier distribution functions to be described locally by Fermi-Dirac distributions with finite drift momenta, while spatial inhomogeneity is taken into account by the space-dependent macroscopic "parameters" (densities and temperatures) of such distribution functions. Such a treatment further allows other slower processes be incorporated in a hydrodynamic fashion. Reasoning and understanding of this type underpin the moment-equation approach adopted in this work.

Specifically, we consider an intrinsic semiconductor quantum well of width w in the z direction and of area S in the x-y plane. The inhomogeneity occurs in the plane of the quantum well layer, while fixed profiles for carrier distribution and optical modes are assumed in the vertical direction to the quantum well due to tight quantum confinement and optical wave-guiding. The extension to include the vertical inhomogeneity can be naturally made when we include a vertical transport model, such as the one used in Ref. [21]. This is beyond the scope of the present article. In a typical edge emitting laser (EEL) , the inhomogeneity appears in the plane of the light propagation. For VCSELs, this plane is transverse to the light propagation. Our starting point for the semiconductor medium is the semiconductor Bloch equations [16, 22], generalized for the spatially inhomogeneous case in Ref. [15, 23], which may now be called Boltzmann-Bloch equations. The possible local charge unbalance requires that the Poisson equation be included. The complete set of equations required for such a system is therefore the Maxwell-Boltzmann-Bloch-Poisson equations (MBBP) [2, 15, 16, 22–24] for the non-equilibrium distribution functions $n^\alpha(\boldsymbol{k},\boldsymbol{r})$ ($\alpha = e, h$ for electrons and holes, respectively), interband polarization $p(\boldsymbol{k},\boldsymbol{r})$, electric potential $\Phi(\boldsymbol{r})$, and the laser field $\mathcal{E}(\boldsymbol{r},t)$, with $\boldsymbol{k}$ and $\boldsymbol{r}$ being the 2D vectors in k space and real space, respectively. The MBBP equations are collected as follows:

$$\tfrac{1}{c^2}\partial_t^2\mathcal{E}(\boldsymbol{r},t) - \nabla^2\mathcal{E}(\boldsymbol{r},t) = -\tfrac{1}{\varepsilon_0 c^2}\partial_t^2\left(\mathcal{P}_a + \mathcal{P}_b\right) , \tag{1}$$

$$\begin{aligned}\partial_t n^\alpha(\boldsymbol{k},\boldsymbol{r}) + \tfrac{1}{\hbar}\partial_{\boldsymbol{k}}\varepsilon^\alpha(\boldsymbol{k},\boldsymbol{r})\cdot\partial_{\boldsymbol{r}} n^\alpha(\boldsymbol{k},\boldsymbol{r}) - \tfrac{1}{\hbar}\partial_{\boldsymbol{r}}\left[\delta\epsilon^\alpha(\boldsymbol{k},\boldsymbol{r}) + q^\alpha\Phi(\boldsymbol{r})\right]\cdot\partial_{\boldsymbol{k}} n^\alpha(\boldsymbol{k},\boldsymbol{r}) \\ = R^\alpha(\boldsymbol{k},\boldsymbol{r}) + \partial_t n^\alpha(\boldsymbol{k},\boldsymbol{r})|_{col} ,\end{aligned} \tag{2}$$

$$\partial_t p(\boldsymbol{k},\boldsymbol{r}) = -\tfrac{i}{\hbar}\left[\varepsilon^e(\boldsymbol{k},\boldsymbol{r}) + \varepsilon^h(-\boldsymbol{k},\boldsymbol{r})\right]p(\boldsymbol{k},\boldsymbol{r}) - i\Omega(\boldsymbol{k},\boldsymbol{r})\left[n^e(\boldsymbol{k},\boldsymbol{r}) + n^h(-\boldsymbol{k},\boldsymbol{r}) - 1\right] + \partial_t p(\boldsymbol{k},\boldsymbol{r})|_{col} , \tag{3}$$

$$\partial_{\boldsymbol{r}}^2\Phi(\boldsymbol{r}) = -\tfrac{1}{\varepsilon_0\varepsilon_b}\tfrac{2}{V}\textstyle\sum_{\alpha,\boldsymbol{k}} q^\alpha n^\alpha(\boldsymbol{k},\boldsymbol{r}) , \tag{4}$$

with various terms given below:

$$R^\alpha(\boldsymbol{k},\boldsymbol{r}) = \Lambda^\alpha(\boldsymbol{k},\boldsymbol{r}) - \gamma_{nr} n^\alpha(\boldsymbol{k},\boldsymbol{r}) - B_{sp} n^e(\boldsymbol{k},\boldsymbol{r}) n^h(\boldsymbol{k},\boldsymbol{r}) + i\left[\Omega(\boldsymbol{k},\boldsymbol{r})p^*(\boldsymbol{k},\boldsymbol{r}) - \Omega^*(\boldsymbol{k},\boldsymbol{r})p(\boldsymbol{k},\boldsymbol{r})\right] , \tag{5}$$

$$\varepsilon^\alpha(\boldsymbol{k},\boldsymbol{r}) = \epsilon^\alpha(\boldsymbol{k}) + q^\alpha\Phi(\boldsymbol{r}) + \delta\epsilon^\alpha(\boldsymbol{k},\boldsymbol{r}) , \tag{6}$$

$$\delta\epsilon^\alpha(\boldsymbol{k},\boldsymbol{r}) = -\textstyle\sum_{\boldsymbol{k}'} n^\alpha(\boldsymbol{k}',\boldsymbol{r})V_{s,\boldsymbol{k}-\boldsymbol{k}'} + \delta_{\alpha,v}\sum_{\boldsymbol{k}'}(V_{s,\boldsymbol{k}-\boldsymbol{k}'} - V_{\boldsymbol{k}-\boldsymbol{k}'}) , \tag{7}$$

$$\hbar\Omega(\boldsymbol{k},\boldsymbol{r}) = \mu(\boldsymbol{k})\mathcal{E}(\boldsymbol{r},t) + \textstyle\sum_{\boldsymbol{k}'} p(\boldsymbol{k}',\boldsymbol{r})V_{s,\boldsymbol{k}-\boldsymbol{k}'} , \tag{8}$$

$$\mathcal{P}_a = \tfrac{2}{V}\textstyle\sum_{\boldsymbol{k}}\left[\mu^*(\boldsymbol{k})p(\boldsymbol{k},\boldsymbol{r}) + c.c.\right] , \tag{9}$$

$$\partial_t n^\alpha(\boldsymbol{k},\boldsymbol{r})|_{col} = \partial_t n^\alpha(\boldsymbol{k},\boldsymbol{r})|_{\alpha\alpha} + \partial_t n^\alpha(\boldsymbol{k},\boldsymbol{r})|_{eh} + \partial_t n^\alpha(\boldsymbol{k},\boldsymbol{r})|_{LO} , \tag{10}$$

$$\partial_t p(\boldsymbol{k},\boldsymbol{r})|_{col} = -\Gamma_o(\boldsymbol{k})p(\boldsymbol{k},\boldsymbol{r}) + \textstyle\sum_{\boldsymbol{k}'}\Gamma_i(\boldsymbol{k},\boldsymbol{k}')p(\boldsymbol{k}',\boldsymbol{r}) , \tag{11}$$

where $\partial_t \equiv \partial/\partial t$, $\partial_{\boldsymbol{r}} \equiv \partial/\partial\boldsymbol{r}$, $\partial_{\boldsymbol{k}} \equiv \partial/\partial\boldsymbol{k}$, $\partial_t^2 \equiv \partial^2/\partial t^2$, $\nabla^2 \equiv \partial_{\boldsymbol{r}}^2 + \partial^2/\partial z^2$, $\partial_{\boldsymbol{r}}^2 \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$. Also, $\varepsilon^\alpha(\boldsymbol{k},\boldsymbol{r})$ is the renormalized carrier energy, $\delta\epsilon^\alpha(\boldsymbol{k},\boldsymbol{r})$ is the correction to the single-particle carrier energy $\epsilon^\alpha(\boldsymbol{k})$ due to

exchange interaction and due to the Coulomb-hole self-energy. $\Omega(\boldsymbol{k},\boldsymbol{r})$ is the renormalized Rabi frequency. $V_{s,\boldsymbol{k}}$ is the screened Coulomb potential, for which we use the single plasmon pole model in this work, and $V_{\boldsymbol{k}}$ is the bare one. In addition, $\mu(\boldsymbol{k})$ is the interband optical dipole matrix element between electron state $|c\boldsymbol{k}\rangle$ and hole state $|v-\boldsymbol{k}\rangle$. $\mathcal{P}_b$ is the optical polarization of the unexcited semiconductor, while $\mathcal{P}_a$ accounts for the electronic contribution from photoexcitation, so the total material polarization $\mathcal{P}=\mathcal{P}_a+\mathcal{P}_b$. Their treatment together with Maxwell's equation is detailed in Section VI and Section VII. Furthermore, $\hbar$ is the Planck constant, $q^\alpha=\mp e$ is the carrier charge for electrons and holes, ε_0 is the electric constant, ε_0 is the relative permittivity of the unexcited semiconductor, $V=wS$ is the volume of the active region, $\delta_{\alpha,v}$ is the Kronecker delta, and $2\sum_{\boldsymbol{k}}$ means summation over all allowable momentum ($\boldsymbol{k}$) states, including spin (account for the 2), for a single subband. Additionally, the lumped generation-recombination (g-r) contributions $R^\alpha(\boldsymbol{k},\boldsymbol{r})$ consists of current injection term $\Lambda^\alpha(\boldsymbol{k},\boldsymbol{r})$, non-radiative recombination term with coefficient γ_{nr}, spontaneous emissions term with coefficient B_{sp}, and stimulated interaction term involving the renormalized Rabi frequency $\Omega(\boldsymbol{k},\boldsymbol{r})$. Detailed treatment of g-r term is available in Section VI. Finally, collisional contributions denoted by subscript *col* lead to decay in the interband polarization $\partial_t p(\boldsymbol{k},\boldsymbol{r})|_{col}$ and relaxation in the carrier distributions $\partial_t n^\alpha(\boldsymbol{k},\boldsymbol{r})|_{col}$. We include explicitly both the so-called out-scattering term $\Gamma_o(\boldsymbol{k})$ and the (non-diagonal) in-scattering term $\Gamma_i(\boldsymbol{k},\boldsymbol{k}')$. The dominant scatterings considered in this work are c-LO, α-α, and e-h scatterings, as separately denoted in Eq. (10).

Before we start the formal derivation of the moment equations, a few remarks are in order: First, parabolic band is assumed for electrons and holes with effective mass m_α, respectively. This approximation is valid when the well width (w) is small enough and plasma density is not too high, such that higher subbands are not populated and thus ignored. This means that carrier kinetic energy, given as $\epsilon^\alpha(\boldsymbol{k})=\hbar^2k^2/2m_\alpha$, is proportional to the second order moment. As we will see later, this simplification allows direct association of the second order moment with the total carrier kinetic energy and the first order current with this energy partially. As such, the extension of the following derivations to the case of non-parabolic bands is not straightforward. Second, all the scattering terms will be treated explicitly in the moment equations, without resorting to relaxation rate approximations or leaving them at the formal level. This is where our approach differs from those earlier approaches [15, 16] in deriving the moment equations; this will become more evident as we proceed with the derivation. Finally, short-hand notation will be adopted for convenience and brevity, such that $n^\alpha_{\boldsymbol{k}}$ stands for $n^\alpha(\boldsymbol{k},\boldsymbol{r})$, $\delta\epsilon^\alpha$ for $\delta\epsilon^\alpha(\boldsymbol{k},\boldsymbol{r})$, Φ for $\Phi(\boldsymbol{r})$, and so on, in the remaining part of this article unless indicated otherwise.

III. MOMENT EQUATIONS AND CUTOFF

We begin with introduction of the moment and current of the n-th order associated with the non-equilibrium distribution function $n^\alpha_{\boldsymbol{k}}$ as follows:

$$\psi^\alpha_n(\boldsymbol{r}) \equiv \frac{2}{S}\sum_{\boldsymbol{k}} F^\alpha_n n^\alpha_{\boldsymbol{k}}\,, \tag{12a}$$

$$\boldsymbol{J}^\alpha_n(\boldsymbol{r}) \equiv \frac{2}{S}\sum_{\boldsymbol{k}} \boldsymbol{v}^\alpha_{\boldsymbol{k}} F^\alpha_n n^\alpha_{\boldsymbol{k}}\,, \tag{12b}$$

where F^α_n, denoting the n-th order weight function, are 1, $\hbar\boldsymbol{k}$, and $\hbar^2k^2/2m_\alpha$ for $n=0,1,2$, respectively, and $\boldsymbol{v}^\alpha_{\boldsymbol{k}}\equiv\partial_{\boldsymbol{k}}\epsilon^\alpha_{\boldsymbol{k}}/\hbar=\hbar\boldsymbol{k}/m_\alpha$ for parabolic bands. We consider up to the second order for moments. Note that only the trace of the second order moment tensor needs consideration here, and it relates to energy. For clarity, we represent the first three considered moments and currents with conventional symbols: density $N^\alpha\equiv\psi^\alpha_0$, momentum $\boldsymbol{P}^\alpha\equiv\psi^\alpha_1$, energy $E^\alpha\equiv\psi^\alpha_2$; density current $\boldsymbol{J}^\alpha_N\equiv\boldsymbol{J}^\alpha_0$, momentum current $\boldsymbol{J}^\alpha_{\boldsymbol{P}}\equiv\boldsymbol{J}^\alpha_1$, energy current $\boldsymbol{J}^\alpha_E\equiv\boldsymbol{J}^\alpha_2$. $\boldsymbol{J}^\alpha_{\boldsymbol{P}}$ is a tensor despite its misleading vectorial notation. As is customary (see, e.g. Ref. [25]), we derive the moment equations by summing over all degrees of freedom, *i.e.*, applying $(2/S)\sum_{\boldsymbol{k}}$ on the Boltzmann transport equation [Eq. (2)] with the corresponding weight function F^α_n. It is straightforward to show that the first three moment equations for electrons and holes can be written as below:

$$\partial_t N^\alpha + \partial_{\boldsymbol{r}}\cdot\boldsymbol{J}^\alpha_N = R^\alpha_N\,, \tag{13}$$

$$\partial_t \boldsymbol{P}^\alpha + \partial_{\boldsymbol{r}}\cdot\boldsymbol{J}^\alpha_{\boldsymbol{P}} + N^\alpha\partial_{\boldsymbol{r}}\left(\delta\epsilon^\alpha+q^\alpha\Phi\right) = \boldsymbol{R}^\alpha_{\boldsymbol{P}} + \partial_t\boldsymbol{P}^\alpha|_{eh} + \partial_t\boldsymbol{P}^\alpha|_{LO}\,, \tag{14}$$

$$\partial_t E^\alpha + \partial_{\boldsymbol{r}}\cdot\boldsymbol{J}^\alpha_E + \partial_{\boldsymbol{r}}\left(\delta\epsilon^\alpha+q^\alpha\Phi\right)\cdot\boldsymbol{J}^\alpha_N = R^\alpha_E + \partial_t E^\alpha|_{eh} + \partial_t E^\alpha|_{LO}\,, \tag{15}$$

where we have neglected the weak k-dispersion in the many-body correction $\delta\epsilon^\alpha$. Terms on the right-hand side above are the result of summing the corresponding terms in Eq. (2) over all degrees of freedom after multiplied by the corresponding weight functions. For example,

$$R^{\alpha}_{N,P,E} = \frac{2}{S}\sum_{\boldsymbol{k}} F^{\alpha}_{N,P,E} R^{\alpha}(\boldsymbol{k},\boldsymbol{r}) \,. \tag{16}$$

Intuitively, α-α scattering does not change the total carrier number, momentum, and energy for each band, so they vanish from the above equations. Furthermore, e-h and c-LO scatterings do not alter the total carrier number within each band, but they do survive the summation and remain in Eqs. (14–15).

While the formal derivation of the above moment equations [Eqs. (13–15)] is exact and straightforward, these equations are not in a closed form yet. There are several reasons for this: (1) the first order current $\boldsymbol{J}^{\alpha}_{\boldsymbol{P}}$ is not completely given by the first and second order moments ($\boldsymbol{P}^{\alpha}$ and E^{α}) in the presence of anisotropy, where equations for the other (trace-less diagonal and off-diagonal) elements of the second order moment are required; (2) the second order current $\boldsymbol{J}^{\alpha}_{E}$ is connected to higher order moments; (3) the terms on the right-hand side depend on summation over the carrier distribution functions, thus are not fully known in terms of the first three moments and currents. These are well known reasons that exist in general for the moment-equation approach, no matter whether it is applied for transport problems in microelectronics or in fluid dynamics, which lead to the so-called *hierarchy problem*. In addition, there is an extra complication in optical problems as we are considering here: The carrier distribution functions are coupled to the interband polarization $p(\boldsymbol{k},\boldsymbol{r})$, another space and carrier momentum-dependent distribution function. As a result, we are essentially dealing with coupled Boltzmann transport equations for three distribution functions: $n^{e}(\boldsymbol{k},\boldsymbol{r})$, $n^{h}(\boldsymbol{k},\boldsymbol{r})$, and $p(\boldsymbol{k},\boldsymbol{r})$, though all transport terms involving explicit spatial variation of $p(\boldsymbol{k},\boldsymbol{r})$ are ignored [23]. In general, moment equations for all these distribution functions should be sought. To obtain a closed form of equations for these macroscopic quantities, two approaches are typically used: One is to derive the lower order of the moment equations and cut off the hierarchy by setting the higher order moments to zero. As it is evident, the coupling to the $p(\boldsymbol{k},\boldsymbol{r})$'s makes such moment cutoff impractical. Another approach is to assume that the distribution functions are well approximated by known distributions characterized by some macroscopic parameters. These macroscopic parameters can be linked to those moment variables. In fluid dynamics or in microelectronics, for example, one assumes that the system is locally described by the drifted Maxwell distribution [26]. Similarly, in semiconductor laser theory, the quasiequilibrium condition is well established [22, 24, 27]. Quasi-equilibrium here means that electrons and holes, driven out of mutual equilibrium by laser field and external pumping, are separately characterized by the equilibrium distributions of each subsystem in their inertia frame of reference. The physical mechanism responsible for the establishment of this quasiequilibrium is the ultrafast α-α scattering on the femtosecond time scale [28]. Using this quasiequilibrium assumption and neglecting the other elements in the second order moment tensor except its trace (valid for isotropic physical systems), we can readily truncate the *hierarchy* associated with $n^{\alpha}(\boldsymbol{k},\boldsymbol{r})$. We will come back to the hierarchy problem associated with $p(\boldsymbol{k},\boldsymbol{r})$ later.

First of all, we assume that the quasiequilibrium distributions of the EHP are given by the drifted Fermi-Dirac (DFD) distribution functions,

$$n^{\alpha}_{\boldsymbol{k}} = f^{\alpha}_{\boldsymbol{k}-\boldsymbol{k}^{\alpha}_{D}} \equiv \left\{1+\exp\left[\beta_{\alpha}(\epsilon^{\alpha}_{\boldsymbol{k}-\boldsymbol{k}^{\alpha}_{D}} - \mu^{\alpha}_{F})\right]\right\}^{-1} \,, \tag{17}$$

where $\boldsymbol{k}^{\alpha}_{D}$ is the drift wavevector and μ^{α}_{F} is the chemical potential. Moreover, $\beta_{\alpha} = 1/k_{B}T^{\alpha}$, where k_{B} is the Boltzmann constant and T^{α} is the temperature for electrons or holes. The drift wavevector is related to the first order moment and μ^{α}_{F} is given by $\beta_{\alpha}\mu^{\alpha}_{F} = \ln\left[\exp\left(\pi\beta_{\alpha}\hbar^{2}N^{\alpha}/m_{\alpha}\right)-1\right]$ in 2D case. We note that three parameters are needed to characterize a DFD function given by Eq. (17). A total of six parameters for the electron and hole distribution functions can be uniquely associated with the six moment variables. With the aid of the known functional form of the DFDs, the right-hand-side terms in Eqs. (13–15) are calculated as functions of carrier densities, drift wavevectors, and temperatures. Finally, using the definition of moments and currents, Eqs. (12), with $n^{\alpha}_{\boldsymbol{k}}$ replaced by the DFD function, we can show that currents of the first three orders depend on the moments as follows:

$$\boldsymbol{J}^{\alpha}_{N} = N^{\alpha}\boldsymbol{u}^{\alpha} \,, \tag{18}$$

$$\boldsymbol{J}^{\alpha}_{P} = \boldsymbol{u}^{\alpha}\boldsymbol{P}^{\alpha} + W^{\alpha}\boldsymbol{I} \,, \tag{19}$$

$$\boldsymbol{J}^{\alpha}_{E} = 2\boldsymbol{u}^{\alpha}W^{\alpha} + \tfrac{1}{2}\boldsymbol{u}^{\alpha}\boldsymbol{u}^{\alpha}\cdot\boldsymbol{P}^{\alpha} \,, \tag{20}$$

where $\boldsymbol{I}$ is the unit tensor,

$$W^{\alpha} \equiv \frac{2}{V}\sum_{\boldsymbol{k}} \frac{\hbar^{2}k^{2}}{2m_{\alpha}} f^{\alpha}_{\boldsymbol{k}} \tag{21}$$

is the thermal part of the carrier energy E^{α}, and

$$E^{\alpha} = W^{\alpha} + \frac{1}{2}\boldsymbol{u}^{\alpha}\cdot\boldsymbol{P}^{\alpha} \,. \tag{22}$$

Above, $\boldsymbol{P}^{\alpha} = N^{\alpha}\hbar\boldsymbol{k}^{\alpha}_{D}$ and a drift velocity is introduced:

$$\boldsymbol{u}^{\alpha} = \boldsymbol{P}^{\alpha}/m_{\alpha}N^{\alpha} = \hbar\boldsymbol{k}^{\alpha}_{D}/m_{\alpha} \,.$$

Obviously the carrier energy E^{α} is related to temperature via its thermal energy part given by Eq. (21). Therefore, these relations between currents and moments connect all dynamical variables in a closed form, leading to a closed set of equations for $\{N^{\alpha}, \boldsymbol{P}^{\alpha}, W^{\alpha}\}$.

IV. GENERAL TWO-COMPONENT MOMENT EQUATIONS

The moment equations given by Eqs. (13–15) are now closed for $\{N^\alpha,\ \boldsymbol{P}^\alpha,\ W^\alpha\}$ after applying the relations between the moments and currents given in Eqs. (18–22). The resultant equations are written more specifically as follows:

$$\partial_t N^\alpha + \partial_{\boldsymbol{r}} \cdot (\boldsymbol{u}^\alpha N^\alpha) = R_N^\alpha \ , \tag{23}$$

$$\partial_t \boldsymbol{P}^\alpha + \partial_{\boldsymbol{r}} \cdot (\boldsymbol{u}^\alpha \boldsymbol{P}^\alpha) + \partial_{\boldsymbol{r}} W^\alpha + N^\alpha \partial_{\boldsymbol{r}} \left(\delta\epsilon^\alpha + q^\alpha \Phi\right) = \boldsymbol{R}_P^\alpha + \partial_t \boldsymbol{P}^\alpha|_{eh} + \partial_t \boldsymbol{P}^\alpha|_{LO} \ , \tag{24}$$

$$\partial_t W^\alpha + \partial_{\boldsymbol{r}} \cdot (2\boldsymbol{u}^\alpha W^\alpha) - \boldsymbol{u}^\alpha \cdot \partial_{\boldsymbol{r}} W^\alpha = R_W^\alpha + \partial_t W^\alpha|_{eh} + \partial_t W^\alpha|_{LO} \ , \tag{25}$$

where $2\boldsymbol{u}^\alpha W^\alpha = \boldsymbol{J}_W^\alpha$ is the thermal energy current density. In addition,

$$R_W^\alpha = R_E^\alpha + \tfrac{1}{2} m_\alpha \boldsymbol{u}^\alpha \cdot \boldsymbol{u}^\alpha R_N^\alpha - \boldsymbol{u}^\alpha \cdot \boldsymbol{R}_P^\alpha \ , \tag{26}$$

$$\partial_t W^\alpha|_{eh} = \partial_t E^\alpha|_{eh} - \boldsymbol{u}^\alpha \cdot \partial_t \boldsymbol{P}^\alpha|_{eh} \ , \tag{27}$$

$$\partial_t W^\alpha|_{LO} = \partial_t E^\alpha|_{LO} - \boldsymbol{u}^\alpha \cdot \partial_t \boldsymbol{P}^\alpha|_{LO} \ . \tag{28}$$

Equations (23–25) are the general form of the moment equations which describe the lateral spatial-temporal carrier dynamics in a semiconductor laser. These equations should be solved together with the Poisson equation, Eq. (4), the still k-resolved polarization equations, and Maxwell's equation. The scattering terms in the above equations are specified in their general form or in the linearized form in Appendices A and B. From Eqs. (26–28), we see that terms $\{R_W^\alpha,\ \partial_t W^\alpha|_{eh},\ \partial_t W^\alpha|_{LO}\}$ differ from $\{R_E^\alpha,\ \partial_t E^\alpha|_{eh},\ \partial_t E^\alpha|_{LO}\}$ correspondingly by nonlinear terms in P^α. As it turns out (see Appendices A and B), the nonlinear terms drop out in the thermal energy relaxation expressions. Finally, it can be shown that these terms vanish for R_W^α as well if translational invariance in k space is assumed for the lumped g-r term $R^\alpha(\boldsymbol{k}, \boldsymbol{r})$ in Eq. (2), which intuitively makes sense as internal energy generation and recombination should be independent of translational momenta.

To simplify the above set of equation further, we intend to eliminate the momentum equation (24). First, we point out that the g-r term $\boldsymbol{R}_P^\alpha$ is negligible as compared to the two scattering terms since momentum relaxation is dominated by ultrafast scattering events, given that no appreciable momentum transfer occurs accompanying the generation and recombination processes. Furthermore, as shown in Appendices A and B, the two scattering terms can be linearized, as a consequence of the assumption of quasiequilibrium for the electron-hole plasma. Possible violation of this assumption comes from the presence of a strong electric field in the quantum well plane that tends to drive the system out of quasiequilibrium and into a nonlinear regime. Substituting the scattering terms by their linearized forms as given in Eqs. (A8) and (B6), the momentum equation (24) is now rewritten as follows:

$$\begin{aligned} \partial_t \boldsymbol{P}^\alpha \ &+ \ \partial_{\boldsymbol{r}} \cdot (\boldsymbol{u}^\alpha \boldsymbol{P}^\alpha) + \partial_{\boldsymbol{r}} W^\alpha + N^\alpha \partial_{\boldsymbol{r}} \left(\delta\epsilon^\alpha + q^\alpha \Phi\right) \\ &= \ -m_r \bar{\gamma}_{eh} \left(\boldsymbol{u}^\alpha - \boldsymbol{u}^\beta\right) - \gamma_{LO}^\alpha \boldsymbol{P}^\alpha \ , \end{aligned} \tag{29}$$

where $\alpha \neq \beta$, $m_r \bar{\gamma}_{eh}/m_\alpha N^\alpha$ corresponds to the momentum relaxation rate due to e-h scattering, $m_r = m_e m_h/(m_e + m_h)$ is the reduced mass, and γ_{LO}^α is the momentum relaxation rate due to c-LO scattering. To obtain an explicit expression for the density current for each carrier type, we adopt the adiabatic elimination approximation for the above equation [16]. For a weakly inhomogeneous system, we can further ignore the nonlinear (second on the left side) term in $\boldsymbol{P}^\alpha$. The resulting solution for the momentum is given as

$$\begin{aligned} \boldsymbol{P}^\alpha \ &= \ -m_\alpha \mu_\alpha \sum_{\nu=e,h} \left[\partial_{\boldsymbol{r}} W^\nu + N^\nu \partial_{\boldsymbol{r}} \left(\delta\epsilon^\nu + q^\nu \Phi\right)\right] \\ &\quad -m_\alpha \mu_\alpha \eta_\alpha \left[\partial_{\boldsymbol{r}} W^\alpha + N^\alpha \partial_{\boldsymbol{r}} \left(\delta\epsilon^\alpha + q^\alpha \Phi\right)\right] \ , \end{aligned} \tag{30}$$

where the two factors are defined by

$$\mu_\alpha \ = \ \frac{\gamma_{eh}^\beta}{\gamma_{LO}^e \gamma_{LO}^h (m_e + m_h) + \gamma_{eh}^\beta \sum_{\nu=e,h} m_\nu \gamma_{LO}^\nu} \ , \tag{31}$$

$$\eta_\alpha \ = \ \frac{\gamma_{LO}^\beta}{\gamma_{eh}^\beta} \frac{m_e + m_h}{m_\alpha} \ , \tag{32}$$

where $\alpha, \beta \in \{e,\ h\} \mid \alpha \neq \beta$ and $\gamma_{eh}^\beta = \bar{\gamma}_{eh}/N^\beta$. The two terms in the above equation have distinct physical meanings: The first term relates to the e-h scattering that tends to equilibrate the two carrier types, while the second term describes the equilibration process between each carrier type and the LO phonon subsystem. We mention that similar formulation and results for the e-h scattering have been derived previously in the study of the negative mobility of minority carriers in semiconductor quantum wells [29–34].

Finally, equations for temperatures are very often preferred over those for thermal energies. Fortunately, a

unique transformation exists under the DFD approximation, since $W^\alpha = \bar{W}^\alpha[\mu_F^\alpha(N^\alpha, T^\alpha), T^\alpha] = W^\alpha(N^\alpha, T^\alpha)$. It is given below:

$$\partial_t T^\alpha = j_W^\alpha \partial_t W^\alpha - j_N^\alpha \partial_t N^\alpha \,, \tag{33}$$

and the resultant temperature equation goes as:

$$\begin{aligned}\partial_t T^\alpha \;+\; & \partial_r \cdot \boldsymbol{J}_T^\alpha - j_W^\alpha \boldsymbol{u}^\alpha \cdot \partial_r W^\alpha + \partial_r j_N^\alpha \cdot \boldsymbol{J}_N^\alpha - \partial_r j_W^\alpha \cdot \boldsymbol{J}_W^\alpha \\ & = j_W^\alpha \left(R_W^\alpha + \partial_t W^\alpha|_{LO} + \partial_t W^\alpha|_{eh}\right) - j_N^\alpha R_N^\alpha \,, \end{aligned} \tag{34}$$

where the temperature current $\boldsymbol{J}_T^\alpha = j_W^\alpha \boldsymbol{J}_W^\alpha - j_N^\alpha \boldsymbol{J}_N^\alpha$ and

$$j_W^\alpha = \left(\partial_{T^\alpha} W^\alpha|_{N^\alpha}\right)^{-1} \,, \tag{35a}$$

$$j_N^\alpha = \partial_{N^\alpha} W^\alpha|_{T^\alpha} \left(\partial_{T^\alpha} W^\alpha|_{N^\alpha}\right)^{-1} \,. \tag{35b}$$

To summarize this section, Eqs. (23,34) form the closed set of diffusion equations in terms of carrier densities and temperatures for each component after we replace momentum $\boldsymbol{P}^\alpha$ or $\boldsymbol{u}^\alpha$ in Eqs. (23,34) with its adiabatic solution of Eq. (30). The corresponding density and temperature currents, $\boldsymbol{J}_N^\alpha$ and $\boldsymbol{J}_T^\alpha$, are given by

$$\boldsymbol{J}_N^\alpha = \boldsymbol{P}^\alpha / m_\alpha \,, \tag{36}$$

$$\boldsymbol{J}_T^\alpha = \left[2 j_W^\alpha (W^\alpha / N^\alpha) - j_N^\alpha\right] \boldsymbol{J}_N^\alpha \,. \tag{37}$$

The above currents contain the gradients of four macroscopic variables: N^α and T^α for $\alpha = e, h$. In general, we can introduce a 4×4 diffusion matrix, D_{XY}, with $X, Y \in \{N^e, N^h, T^e, T^h\}$. While the diagonal elements of this matrix represent the self-diffusion coefficients, the off-diagonal elements denote various mutual- or cross-diffusion coefficients. A complete list of all the coefficients is given in Appendix C.

V. SINGLE-COMPONENT APPROXIMATION

In this section, we consider two approximations that allow reduction of the general two-component description of the electron-hole plasma to a single-component one: in the limit of strong electron-hole scattering and the often used *ambipolar diffusion approximation*. Detailed analysis is provided for the comparison of the two approaches and they are found to produce consistent results in the linear regime of description.

A. The Limiting Case of Strong Electron-Hole Scattering

While we used the quasiequilibrium assumption in the above derivation, which is due to ultrafast carrier-carrier (α-α) scattering within each band, e-h scattering, which is on the same time scale [29, 35], has been retained in Eqs. (13-15), together with c-LO scattering (in subpicosecond range) and carrier diffusion and energy transport (in nanosecond range). Thus self-consistency demands that we further consider the dynamical correlation between electrons and holes imposed by e-h scattering. In this subsection, we take on the issue of how e-h scattering reduces the general two-component description to a single-component one for the EHP near quasiequilibrium. As shown in Appendix A, detailed balance (DB) requirement for quasiequilibrium in the sole presence of e-h scattering leads to these conditions:

$$T^e = T^h \,, \tag{38}$$

$$\boldsymbol{u}^e = \boldsymbol{u}^h \,, \tag{39}$$

which is intuitively apparent as electron-hole scattering is meant for equilibration between the two carrier types. The above conditions are the same as the *a posteriori* requirements needed for the ambipolar diffusion approximation [36], and they now permit us to settle the issue of reducing the original two-component problem to a single-component one if the EHP is initially neutral in real space. Further discussions along the line of standard ADA reduction will be presented in the next subsection.

To examine this issue in greater detail, we consider the dynamics around the DB state by looking at the equations for momenta and energies with scattering terms linearized around the DB state. The linearized scatterings terms are derived in Appendix A and the corresponding momentum equations are given by Eq. (29). As we see from the adiabatic solution to these equations [see Eq. (30)], in general, Eq. (39) is not valid. This means that the DB is not sustainable and the corresponding single-component reduction does not hold. However, when $\gamma_{LO}^\alpha \ll \gamma_{eh}^\alpha$, the second term in Eq. (30) can be neglected. The omission of the second term conveniently leads to the conclusion of $\boldsymbol{u}^e = \boldsymbol{u}^h = \boldsymbol{u}$, if $N^e = N^h = N$. Thus

$$\boldsymbol{u} = -\frac{\mu}{N} \sum_{\nu=e,h} \left[\partial_r W^\nu + N \partial_r \left(\delta\epsilon^\nu + q^\nu \Phi\right)\right] \,, \tag{40}$$

where $\mu = \mu_e = \mu_h$ as defined in Eq. (31). Take a note of the difference between this line of single-component reduction and the standard ADA line, as presented in next subsection. Here the drift velocities are equal, irrespective of the internal electric field ($-\partial_r \Phi$). In other words, internal field is not required to maintain equal velocities. The ultrafast e-h scattering alone maintains the charge neutrality if the system is neutral initially. It is clear from the above discussion that the validity of equal drift velocities for the two components of unequal masses requires that e-h scattering dominate carrier-LO phonon scattering. This is intuitively easy to understand from the physical point of view. The role of e-h scattering is to correlate electrons and holes dynamically and equilibrate their drift velocities. By contrast, c-LO phonon

scattering is to generate different individual drift momenta because of unequal masses and therefore unequal scattering rates with LO phonons. When the e-h scattering strength is comparable to that of c-LO scattering, the unequal c-LO phonon scattering rates for electrons and holes will be enough to counteract the homogenizing role played by e-h scattering, thus resulting in different drift velocities. At this point, one may argue if the DB is still a valid concept in this situation. We point out that the DB condition as expressed for e-h scattering alone in Appendix A is no longer true. Rather, the DB between in- and out-scatterings for a given k-state must include c-LO phonon scattering as well, as the latter becomes non-negligible. Since c-LO phonon scattering is sensitively dependent on temperature with e-h scattering being on density, it is clear that conditions (38–39) will no longer be valid for high temperature and relatively low carrier density.

For now, let us continue discussion of the limiting case when c-LO phonon scattering is much weaker than e-h scattering. The hydrodynamic equality of drift velocities of electrons and holes means that $N^e = N^h = N$ will be maintained if the EHP is neutral initially, according to Eq. (23), the continuity equation. Major g-r contributions in R_N^α are the same for electrons and holes, which will be labeled as R_N under the single-component approximation and examined further in the next section. As charge neutrality under strong e-h scattering can be maintained dynamically, therefore, the Poisson equation is automatically satisfied. The density equations for electrons and holes are reduced to a single one for the plasma density N:

$$\partial_t N + \partial_r \cdot \boldsymbol{J}_N = R_N \ . \tag{41}$$

To obtain the corresponding energy equation in the single-component case, we notice that the temperature equality indicated in Eq. (38) signifies interdependence of the energy equations for electrons and holes. Because e-h scattering conserves the total energy of the EHP subsystem, it is natural to take the total carrier energy as the second dynamic variable. Applying Eq. (39) for the drift velocities when summing up Eq. (25) for electrons and holes, it is found that the total thermal energy obeys the following equation of dynamics:

$$\partial_t W + \partial_r \cdot \boldsymbol{J}_W = R_W + \partial_t W|_{LO} \ , \tag{42}$$

where $W = W^e + W^h$, $\boldsymbol{J}_W = 2u\boldsymbol{W}$, $R_W = R_W^e + R_W^h$, $\partial_t W|_{LO} = \partial_t W^e|_{LO} + \partial_t W^h|_{LO}$. In deriving the above equation, we have assumed weak inhomogeneity in the system so that nonlinear terms have been dropped. Up to this point, the set of moment equations for the EHP has been reduced to only two: one for plasma density N and one for its total thermal energy W. Similar to Section IV, an equation for plasma temperature $T = T^e = T^h$ is derived as follows:

$$\begin{aligned} &\partial_t T + \partial_r \cdot \boldsymbol{J}_T + \partial_r j_N \cdot \boldsymbol{J}_N - \partial_r j_W \cdot \boldsymbol{J}_W \\ &= j_W \left(R_W + \partial_t W|_{LO}\right) - j_N R_N \ , \end{aligned} \tag{43}$$

where $\boldsymbol{J}_T = j_W \boldsymbol{J}_W - j_N \boldsymbol{J}_N$.

Bearing great resemblance to ordinary diffusion equations, the equations derived here, Eqs. (41) and (43), include many-body corrections and apply to a neutral EHP. To define the related diffusion coefficients, the currents in the equations need to be expressed in terms of the gradients of plasma density and temperature:

$$\boldsymbol{J}_N = -D_{NN}\,\partial_r N - D_{NT}\,\partial_r T \ , \tag{44}$$

$$\boldsymbol{J}_T = -D_{TN}\,\partial_r N - D_{TT}\,\partial_r T \ . \tag{45}$$

Explicit expressions for the DCs in terms of material parameters and thermodynamic variables, N and T, are given in Appendix D.

B. The Ambipolar Diffusion Approximation

The single-component reduction discussed in the last subsection is valid only when e-h scattering is much stronger than any other scatterings. In a typical III-V semiconductor device, this is true only for the case of high carrier density and low temperature where e-h scattering is predominant over c-LO scattering. At room temperature, these two scatterings are about the same order of magnitude. Thus the reduction procedure above becomes questionable. Another single-component reduction procedure is the so-called ambipolar diffusion approximation. Even though it seems to us quite difficult to justify purely from scattering analysis the validity of the ADA, numerical simulation by Held et al. [37, 38] has indeed shown that the ADA is a quite good approximation at nanosecond time scale and at high density. The difference between standard ADA and the reduction procedure described in the last subsection can be seen in the following way. From Eq. (30), we notice that the total current consists of diffusive and conductive parts: one proportional to the gradient of the thermal energy and the other one to the field ($-\partial_r \Phi$), respectively. When other scatterings are negligibly small compared to e-h scattering, the diffusive currents for electrons and holes are equal when the densities are the same, so that there is no need for the conductive currents. In the more general cases, however, non-zero conductive currents are needed to maintain the total currents for electrons and holes to be the same. By requiring the electron and hole currents to be the same, an expression for the non-zero internal field is obtained. Substitution of the internal field expression into either one of the density current expressions leads to the ambipolar density current:

$$\boldsymbol{J}_N^{am} = -D_{NN}^{am}\partial_r N - D_{NT}^{am}\partial_r T \ , \tag{46}$$

where the ambipolar diffusion coefficients are given below:

$$D_{NX}^{am} = \frac{S_X^e + S_X^h}{m_e\gamma_{LO}^e + m_h\gamma_{LO}^h} \tag{47}$$

and

$$S_X^\alpha = \partial_X W^\alpha + N\partial_X \delta\epsilon^\alpha \ , \ (X = N, T) \ . \tag{48}$$

As seen from the above expressions for the ambipolar DCs, they are independent of the e-h scattering rates. This is somewhat surprising at first, since it seems to us no previous work has explicitly noticed this point [39]. A plausible explanation is as follows: While e-h scattering is important for electrons and holes to effectively move together and thus maintain the validity of the ambipolar diffusion approximation, e-h scattering itself should not affect the diffusivity of the e-h ensemble since such scattering drives only internal dynamics. Rather, scatterings of the e-h ensemble with the ambient determines its diffusive capability in the ambient, which is the LO phonon subsystem in our model.

To close this section we point out that the results derived for the limiting case of a predominant e-h scattering in Subsection V A and with standard ADA procedure in this subsection agree with each other. First of all, the derived diffusion equations for plasma density and temperature are identical. At the same time two approaches produce the same expressions for the DCs. The proof is easily seen by comparing Eq. (47) with its counterpart Eqs. (D4–D5) in Appendix D where explicit expressions for all the DCs are available for the single-component case. The results for the ambipolar temperature current can be derived in exactly the same way as in Section IV and thus will not be shown. Finally, we mention that we refer to the DCs for the single-component case simply as the ambipolar diffusion coefficients without distinction afterwards.

VI. CARRIER GENERATION AND RECOMBINATION: CLOSURE OF k-RESOLVED POLARIZATION HIERARCHY

As mentioned, the moment equations still depend on k-resolved polarization $p(\boldsymbol{k}, r)$ through the stimulated interaction contributions $R_N|_{stim}$ to the density equation (41) and $R_W|_{stim}$ to the energy equation (43). The complete source contributions are specified as follows:

$$R_N = \tfrac{\eta_N J\Theta}{ew} - \gamma_{nr}N - B_{sp}N^2 + R_N|_{stim} \ , \tag{49}$$

$$R_W = \tfrac{\eta_E J\Theta}{ew}\Delta E_g - \gamma_{nr}W - B_{sp}NW + R_W|_{stim} \ , \tag{50}$$

where a couple of approximations are made to obtain the analytical form of expressions for both the injection term and the spontaneous recombination term. First, we neglect the detailed carrier capture kinetics that carriers undergo when entering the active region from the electrodes. Instead, an empirical model is adopted by assuming instantaneous carrier capture process for the injection current density J with quantum efficiency η_N and η_E. The factor Θ represents the spatial profile of the pumping current and ΔE_g is the bandgap offset between the quantum well and the barrier material. Second, the spontaneous recombination term assumes a bilinear form as we ignore the correlation between electrons and holes. The stimulated interaction terms in the density and energy equations are given by

$$R_N|_{stim} = \frac{2}{S}\sum_{\boldsymbol{k}} R_{stim}(\boldsymbol{k}, \boldsymbol{r}) \ , \tag{51}$$

$$R_W|_{stim} = \frac{2}{S}\sum_{\boldsymbol{k}} \frac{\hbar^2 k^2}{2m_r} R_{stim}(\boldsymbol{k}, \boldsymbol{r}) \ , \tag{52}$$

where $R_{stim}(\boldsymbol{k}, \boldsymbol{r})$ is given by the last term in Eq. (5), which contains $p(\boldsymbol{k}, \boldsymbol{r})$ explicitly.

To close this hierarchy, approximation to the $p(\boldsymbol{k}, \boldsymbol{r})$ is necessary. This issue has been addressed in the past for different special cases in two ways. The first is to eliminate the polarization equation adiabatically as done in Ref. [16] for the total density equation and in Ref. [40] for the kinetic energy equation. An alternative approach is to replace the sum of the k-resolved polarization with an effective polarization [11]. We will outline both approaches with slight generalization beyond what has been published in either case.

We begin with the adiabatic elimination of the polarization. As was done in Ref. [41], the same adiabatic elimination can be performed in the presence of non-diagonal scattering terms in the $p(\boldsymbol{k}, r)$ equation. To proceed, we introduce the slowly varying temporal amplitudes for the laser field and polarization:

$$\mathcal{E}(\boldsymbol{r}, t) = \tfrac{1}{2}\left[E(\boldsymbol{r}, t)\ e^{-i\omega_0 t} + c.c.\right] \ , \tag{53}$$

$$p(\boldsymbol{k}, \boldsymbol{r}) = \tfrac{1}{2}\left[\bar{p}(\boldsymbol{k}, \boldsymbol{r})\ e^{-i\omega_0 t} + c.c.\right] \ . \tag{54}$$

Then the k-resolved polarization equation is rewritten under the rotating-wave approximations as

$$\begin{aligned} \partial_t \bar{p}(\boldsymbol{k}, \boldsymbol{r}) \ = \ & -\left[\Gamma_o(\boldsymbol{k}) + i\delta_{\boldsymbol{k}}\right]\bar{p}(\boldsymbol{k}, \boldsymbol{r}) \\ & -i\frac{\mu_{\boldsymbol{k}}}{\hbar}\mathcal{D}(\boldsymbol{k})E \\ & +\sum_{\boldsymbol{k}'}\left[\Gamma_i(\boldsymbol{k}, \boldsymbol{k}') - \frac{i}{\hbar}\mathcal{D}(\boldsymbol{k})V_{s,\boldsymbol{k}-\boldsymbol{k}'}\right]\bar{p}(\boldsymbol{k}', \boldsymbol{r}) \ , \end{aligned} \tag{55}$$

where

$$\hbar\delta_{\boldsymbol{k}} = E_g(T_L) + \hbar^2 k^2/(2m_r) + \delta\epsilon^e + \delta\epsilon^h - \hbar\omega_0 \ , \tag{56}$$

$$\mathcal{D}(\boldsymbol{k}) = f_{\boldsymbol{k}}^e + f_{\boldsymbol{k}}^h - 1 \ . \tag{57}$$

After introducing the k-resolved susceptibility χ_k by definition:

$$\bar{p}(\boldsymbol{k},\boldsymbol{r}) = \varepsilon_0\varepsilon_b V\frac{\chi_k}{\mu_k^*}E\,, \tag{58}$$

we can write the polarization equation under the adiabatic elimination approximation in the form:

$$\chi_k = \chi_k^0 + \sum_{k'}\mathcal{K}(\boldsymbol{k},\boldsymbol{k}')\chi_{k'}\,, \tag{59}$$

where the kernel $\mathcal{K}$ and the zeroth-order solution are given by

$$\chi_k^0 = -i\frac{|\mu_k|^2\mathcal{D}(\boldsymbol{k})}{\varepsilon_0\varepsilon_b V\hbar\left[\Gamma_o(\boldsymbol{k})+i\delta_k\right]}\,, \tag{60}$$

$$\mathcal{K}(\boldsymbol{k},\boldsymbol{k}') = \frac{\hbar\Gamma_i(\boldsymbol{k},\boldsymbol{k}') - iV_{s,\boldsymbol{k}-\boldsymbol{k}'}\mathcal{D}(\boldsymbol{k})}{\hbar\left[\Gamma_o(\boldsymbol{k})+i\delta_k\right]}\,\frac{\mu_k^*}{\mu_{k'}^*}\,. \tag{61}$$

This equation can be solved either by matrix inversion or by an approximation [41] similar to the Páde method as in the case of relaxation rate approximation [16]. Then the final expressions for the stimulated interaction contributions to density and energy changes can be written as

$$R_N|_{stim} = i\frac{\varepsilon_0\varepsilon_b}{2\hbar}\sum_k(\chi_k^*-\chi_k)|E|^2\,, \tag{62}$$

$$R_W|_{stim} = \left[iV\frac{\varepsilon_0^2\varepsilon_b^2}{2\hbar}\sum_{k,k'}\frac{\hbar^2(k^2-k'^2)}{2m_r}\frac{\chi_k^*}{\mu_k}V_{s,k-k'}\frac{\chi_k'^*}{\mu_{k'}}\right.$$
$$\left.+i\,\frac{\varepsilon_0\varepsilon_b}{2\hbar}\sum_k\frac{\hbar^2k^2}{2m_r}(\chi_k^*-\chi_k)\right]|E|^2\,. \tag{63}$$

While adiabatic elimination of the polarization leads to a simple closure of the total set of equations, the resultant equations have a severe deficiency, especially in the presence of the spatial inhomogeneity. As was discussed in detail in Ref. [11], the reason is that the adiabatic elimination completely neglects gain dispersion. This the reason that some alternative time-dependent polarization equation was sought. One remedy is the so-called Effective Bloch equation (EBE) approach as discussed in detail in Ref. [11]. The issue of how to combine the EBE approach with the energy equation was partly addressed in Ref. [17] for the free-carrier case with a phenomenological polarization decay constant. In the following, we will follow the EBE approach to construct the kinetic energy equation.

The EBE approach retains an effective (total) polarization equation by microscopically computing the total susceptibility and reconstructing the total polarization $\bar{P}(\boldsymbol{r},t)$. As a result, an equation for $\bar{P}(\boldsymbol{r},t)$ is obtained which is similar in form to the standard non-diagonal Bloch equation for a two-level system. The procedure has been described in detail in Ref. [11] and will not be repeated here. As can be easily seen, the total density equation depends only on the total polarization after summing over $\boldsymbol{k}$ [11]. Therefore, $R_N|_{stim}$ will no longer depend on $p(\boldsymbol{k},\boldsymbol{r})$. The remaining difficulty is to deal with the total kinetic energy equations which still depend on k-resolved polarization.

Using the definition $P_a = (2/V)\sum_k \mu_k^*\bar{p}(\boldsymbol{k},\boldsymbol{r})$, we obtain from Eq. (55):

$$\partial_t P_a = -i\delta_0 P_a - i\frac{2}{\hbar V}\sum_k\mu_k^*\frac{\hbar^2k^2}{2m_r}\bar{p}(\boldsymbol{k},\boldsymbol{r})$$
$$- i\frac{2E}{\hbar V}\sum_k|\mu_k|^2\mathcal{D}(\boldsymbol{k})\,, \tag{64}$$

where $\delta_0 = \delta_k|_{k=0} - \sum_{k'}V_{s,k-k'}$. From Eq. (64), we can construct the following relationship:

$$E^*\partial_t P_a + E\partial_t P_a^* = i\delta_0\left[EP_a{}^* - E^*P_a\right]$$
$$- i\frac{2}{\hbar V}\sum_k\frac{\hbar^2k^2}{2m_r}\left[\mu_k^*E^*\bar{p}(\boldsymbol{k},\boldsymbol{r}) - \mu_kE\bar{p}^*(\boldsymbol{k},\boldsymbol{r})\right]\,. \tag{65}$$

Using this relation, we have

$$R_W|_{stim} = \frac{1}{2}Re\{E^*(i\delta_0P_a + \partial_tP_a\} + R_{W,CA}\,. \tag{66}$$

While the first term now depends only on total polarization and its derivative, the last term, standing for the Coulomb-assisted (CA) energy change, unfortunately still depends on $p(\boldsymbol{k},\boldsymbol{r})$:

$$R_{W,CA} = \frac{i}{2\hbar V}\sum_{k,k'}V_{s,k-k'}\frac{\hbar^2k^2}{2m_r}\left[\bar{p}(\boldsymbol{k}',\boldsymbol{r})p^*(\boldsymbol{k},\boldsymbol{r}) - p^*(\boldsymbol{k}',\boldsymbol{r})\bar{p}(\boldsymbol{k},\boldsymbol{r})\right]$$
$$= \frac{i}{2\hbar V}\sum_{k,k'}\frac{\hbar^2(k^2-k'^2)}{2m_r}\bar{p}(\boldsymbol{k}',\boldsymbol{r})V_{s,k-k'}p^*(\boldsymbol{k},\boldsymbol{r})\,.$$

At this stage, we use the adiabatic elimination results for $\bar{p}(\boldsymbol{k}, r)$ to close the set of equations, since gain dispersion has now been included with the effective polarization equations.

Expressing the right hand side of the total energy equation in terms of $\partial_t \bar{P}$ was first done in Ref. [17]. Comparing the first term of Eq. (66) with the corresponding equation in Ref. [17], we find that the term proportional to the polarization decay rate is absent in our expression. This is because there the relaxation rate is a k-independent constant introduced phenomenologically. Here we have taken into account all (both diagonal and non-diagonal) scattering terms in the polarization equations. The sum rule for the total scattering terms leads to the disappearance of the linear decay term that survived the k-summation in Ref. [17] by contrast.

Finally, plasma heating due to stimulated interactions results in a corresponding temperature change which is given by

$$\partial_t T|_{stim} = j_W R_W|_{stim} - j_N R_N|_{stim} \ . \tag{67}$$

To summarize this subsection, we have outlined two approaches to close the set of equations for the total kinetic (thermal) energy and carrier density by approximating the polarization equations in two different ways.

VII. LASER FIELD EQUATION AND LATTICE TEMPERATURE EQUATION

In this section, we specify Maxwell's equation in a more concrete form. Our derivation will include frequency dependence of the background refractive index and the so-called thermal lensing effect, where the background index depends on lattice temperature. After making the slowly-varying envelop approximation to Eq. (1), it is written as

$$-\nabla^2 E - \frac{\omega_0^2}{c^2}E - \frac{2i\omega_0}{c^2})\partial_t E = \frac{\omega_0^2}{\varepsilon_0 c^2}(P_a + P_b) + \frac{2i\omega_0}{\varepsilon_0 c^2}\partial_t P_b \ , \tag{68}$$

where the envelope functions P_a and P_b are defined through

$$\mathcal{P}_{a,b} = \frac{1}{2}\left[P_{a,b}e^{-i\omega_0 t} + c.c.\right] \ . \tag{69}$$

We assume that the background polarization satisfies the following "constitutive relation" in frequency space (with tilde added to the top of a variable):

$$\tilde{P}_b(\omega) = \varepsilon_0 \chi_b(\omega, T_L)\tilde{E}(\omega) \ , \tag{70}$$

where T_L stands for lattice temperature and χ_b is in general complex with real and imaginary parts defined as usual: $\chi_b = \chi_b' + i\chi_b''$. We Taylor-expand χ_b around a given lattice temperature T_L^0 and the reference frequency ω_0:

$$\chi_b(\omega, T_L) = \chi_b^0 + (T - T_L^0)\frac{\partial \chi_b'}{\partial T_L} + (\omega - \omega_0)\frac{\partial \chi_b'}{\partial \omega} + \dots \ , \tag{71}$$

where $\chi_b^0 = \chi_b(\omega_0, T_L^0) = [(n_b^0)^2 - 1] + i\chi_b^{0\prime\prime}$ and we assume that there is no temperature and frequency dependences of the imaginary part of χ_b (absorption). Substituting the Taylor-expansion into Eq. (70) and Fourier-transform the resultant expression to time domain, we will obtain the time-domain relation between $P_b(t)$ and $E(t)$. After straightforward algebraic manipulation, the equation for the slowly varying envelope is written as follows:

$$\begin{aligned} \frac{-i}{2K}(\nabla^2 + K^2)E \ + \ & \frac{n_b^0}{c}(\beta_\omega + \beta_T + \frac{i\alpha_b}{K})\partial_t E \\ = \ & \frac{i\omega_0^2}{2\varepsilon_0 c^2}P_a - \frac{\alpha_b}{2}E + \frac{iK}{2}\beta_T E \ , \end{aligned} \tag{72}$$

where we have adopted the following shorthand notation:

$$K = \frac{\omega_0 n_b^0}{c} \ , \tag{73a}$$

$$\beta_\omega = 1 + \frac{\omega_0}{n_b^0}\frac{\partial n_b}{\partial \omega} \ , \tag{73b}$$

$$\beta_T = \frac{2(T_L - T_L^0)}{n_b^0}\frac{\partial n_b}{\partial T_L} \ , \tag{73c}$$

$$\alpha_b = K\frac{\chi_b^{0\prime\prime}}{(n_b^0)^2} \ . \tag{73d}$$

The physical meanings of these parameters are obvious: β_T describes the index change with temperature and accounts for the thermal lensing effects observed in high power laser operation. β_ω describes the background index dispersion and thus $\beta_\omega n_b^0$ gives the group velocity index. Lastly, α_b describes the background absorption.

To complete our hydrodynamic model, we include the lattice temperature equation, which is modified from Ref. [4]:

$$\begin{aligned} \partial_t T_L \ - \ & \partial_r(K_{ph}C_{ph}^{-1}\partial_r T_L) = -\gamma_a(T_L - T_a) \\ + \ & C_{ph}^{-1}\left(\partial_t W|_{LO} + \hbar\omega_0\gamma_{nr}N + \frac{J^2 R}{wS}\right) \ , \end{aligned} \tag{74}$$

This equation takes into account equilibration to the ambient temperature T_a with a phenomenological rate γ_a, heat transfer from the EHP to the lattice due to c-LO scattering, temperature rise due to energy transfer from nonradiative recombination of e-h pairs, and certainly Joule heating by current injection into an active device with a congregate resistance R. Only phonon part of the specific heat C_{ph} of the material is responsible for

lattice temperature change, and the negligibly small difference between constant-pressure and constant-volume specific heat of the semiconductor material is ignored. In addition, a heat conduction term due to phonon diffusion is added with a thermal conductivity K_{ph}. Finally, Eq. (74), together with Eqs. (41), (43), (72), the effective Bloch equations (not listed here, but see [11]), and appropriate boundary conditions forms a complete hydrodynamic laser theory. This set of equations describes couplings between lasing and heating self-consistently for a spatially inhomogeneous semiconductor laser.

VIII. CONCLUDING REMARKS

In conclusion, we have derived a *coupled diffusion model* (CDM) for the hydrodynamic variables—carrier densities and temperatures—for the electron-hole plasma in a semiconductor quantum well optical device. The derivation is self-consistent in the sense that the CDM is coupled to the optical polarization and the laser field, such that all relevant processes and variables are treated on the same footing. The major difference of this article from the earlier work is that carrier-carrier and carrier-LO phonon scatterings are treated explicitly in the moment equations when deriving the energy and momentum decay rates. As such, all diffusion coefficients are given as functions of the densities and temperatures via microscopical contributions. There are a few consequences with such a derivation. First, the resultant CDM not only consists of self-diffusion terms as most phenomenological models do, but also contains mutual-diffusion terms between electron variables and hole variables and between temperatures and densities. Second, many-body effects of Hartree-Fock type appear in all of the diffusion coefficients very naturally, leading to modification of the coefficients, which will be discussed in connection with the numerical results for the coefficients in a subsequent article. Furthermore, our explicit treatment leads to some new perspectives about the single-component reduction and the well-known ambipolar diffusion approximation. Finally, such a detailed treatment leads to the conclusion that the ambipolar DCs do not dependent on e-h scattering. This work enables us to obtain and analyze all the DCs for an optical device in the general two-component case and under the ambipolar diffusion approximation from a microscopic point of view. Such detailed numerical results will be presented in a subsequent article.

Another insight we gain from this study is the role of electron-hole correlation. The degree of such correlation determines whether the plasma subsystem can be adequately described as an ensemble of correlated pairs or a gaseous state of individual species, which in turn requires either an effective single-component or two-component description. Within the scope of this work there are two distinct ways to establish such correlation: one is by e-h scattering in k space, and the other is by static Coulomb interaction in real space. These two types of correlation are responsible for the two types of single-component reduction, respectively. The first type of reduction described in Subsec. V A relies solely on k-space correlation (*i.e.*, e-h scattering), and therefore the internal field that leads to real-space e-h correlation is not needed. For the second type of reduction shown in Subsec. V B, real-space correlation is sufficient, and no assumption for e-h scattering and c-LO scattering is necessary. In reality, both real-space and k-space correlations exist and are responsible for the establishment of an effective single-component behavior in most cases. We note that the ambipolar diffusion approximation has not been derived *self-consistently* in the literature so far. Theoretical treatments thus far are mere adaptation to experimental observations, rather than rigorous derivation. The combined usage of these two types of correlation may eventually lead to a systematic derivation of the ambipolar diffusion approximation. This will help elucidate when the ambipolar diffusion approximation is out of question.

Finally, a few words about the plasma model (ignoring the excitons) are in order. As mentioned at the beginning of Section II, a plausible argument that supports the plasma model is the relatively high density in a typical laser. While this certainly holds in the phenomenological rate equation model which lacks of spatial dependence, this becomes a much weaker argument in a spatially inhomogeneous laser. It is well known that carrier density is not uniform in the lateral direction of an edge-emitting laser and in the transverse direction of a surface-emitting laser due to pumping and carrier diffusion process and, to a lesser degree, due to the interaction with laser light field. While at the center of such a nonuniform distribution, the density is certainly above the Mott density, but the density decreases to zero as we move towards the device edges. Typically at about the half-value position of the distribution, the density drops below the Mott density. Such low density regions cause two problems: First, bandgap renormalization at high density level leads to the spectral overlap of the gain peak, where laser is regularly designed to operate, with the exciton peak of the low density region. Thus the laser field incurs strong excitonic absorption. This is especially true for gain guided devices where the field tends to spread more into low density regions. For a very short-cavity device where mode spacing is quite large, we may be able to design a laser to avoid such overlap of the laser cavity modes with the excitonic features. This becomes difficult for regular edge- or surface-emitting lasers where the longitudinal or transverse modes become closely spaced such that some modes will always fall near the excitonic features. The effect of such absorption is expected to be weak in a strongly index guided device, nevertheless. The second problem is related to the treatment of momentum and energy decay processes which affect carrier diffusion. The plasma model is no longer valid in the low density regions, as noted. Since the diffusion processes in such regions play a role for the overall carrier distribution and in particular for the density at the center, an appropriate treatment

of the e-h subsystem is thus necessary. Such a treatment would require a theory that takes into account all the intermediate situations from an excitonic solid, or condensate, to a pure plasma phase. This is obviously an issue beyond the scope of this work, but one needs to be aware of this issue when dealing with a strongly inhomogeneous lasing system.

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APPENDIX A: MOMENTUM AND ENERGY RELAXATION RATES DUE TO e-h SCATTERING

It is well known that ultrafast e-e and h-h scatterings lead to carrier thermalization with a rate around 50 femtosecond at typical lasing density [20, 22, 24, 27, 43]. However, the role and consequences of e-h scattering is rather unclear. In this appendix, we will study such a consequence of the e-h scattering within the second Born approximation. Specifically, we will show that the DB requirement of the microscopic e-h scattering processes lead to equal temperatures and drift velocities for electrons and holes. Linearization around the DB state allows us to obtain the momentum and energy relaxation rates. Similar rates for the Maxwell distributions have been obtained in Ref. [37].

We assume that each band is in quasiequilibrium and described by a drifted Fermi-Dirac distribution function, $n_{\boldsymbol{k}}^{\alpha} \equiv f_{\boldsymbol{k}-\boldsymbol{k}_D^{\alpha}}^{\alpha}$, as given in Eq. (17), where $\alpha = e, h$. Using the second Born approximation [44], the change rate in the distributions due to e-h scattering can be written as,

$$\partial_t n_{\boldsymbol{k}}^{\alpha}|_{eh} = \left(\frac{2}{S}\right)^2 \sum_{\boldsymbol{k}',\boldsymbol{q}\neq 0} \frac{2\pi}{\hbar} V_{s,\boldsymbol{q}}^2 \delta\left(\Delta\epsilon_{eh}\right) \mathcal{T}\,, \qquad \text{(A1)}$$

where $V_{s,\boldsymbol{q}}$ is the Fourier component of the screened Coulomb potential, $\Delta\epsilon_{eh}$ in the delta function, which stands for the difference in the total kinetic energy between in- and out-scattering, is given by $\epsilon_{\boldsymbol{k}}^{\alpha} + \epsilon_{\boldsymbol{k}'+\boldsymbol{q}}^{\beta} - \epsilon_{\boldsymbol{k}'}^{\beta} - \epsilon_{\boldsymbol{k}+\boldsymbol{q}}^{\alpha}$ where $\alpha \neq \beta$, and $\mathcal{T}$ is defined as

$$\begin{aligned}\mathcal{T} \;\equiv\; & n_{\boldsymbol{k}+\boldsymbol{q}}^{\alpha} n_{\boldsymbol{k}'}^{\beta} (1 - n_{\boldsymbol{k}}^{\alpha})(1 - n_{\boldsymbol{k}'+\boldsymbol{q}}^{\beta}) \\ & - n_{\boldsymbol{k}}^{\alpha} n_{\boldsymbol{k}'+\boldsymbol{q}}^{\beta} (1 - n_{\boldsymbol{k}'}^{\beta})(1 - n_{\boldsymbol{k}+\boldsymbol{q}}^{\alpha})\,. \qquad \text{(A2)}\end{aligned}$$

If the EHP is in quasiequilibrium, the DB condition requires that $\mathcal{T} = 0$. Using $1 - n_{\boldsymbol{k}}^{\alpha} = n_{\boldsymbol{k}}^{\alpha} \exp[(\epsilon_{\boldsymbol{k}-\boldsymbol{k}_D^{\alpha}}^{\alpha} - \mu_F^{\alpha})/k_B T^{\alpha}]$, we can factorize $\mathcal{T}$ into the following form

$$\mathcal{G}\left[1 - \mathrm{e}^{\frac{\epsilon_{\boldsymbol{k}+\boldsymbol{q}}^{\alpha} - \epsilon_{\boldsymbol{k}}^{\alpha} - \hbar\boldsymbol{q}\cdot\boldsymbol{u}^{\alpha}}{k_B T^{\alpha}} - \frac{\epsilon_{\boldsymbol{k}'+\boldsymbol{q}}^{\beta} - \epsilon_{\boldsymbol{k}'}^{\beta} - \hbar\boldsymbol{q}\cdot\boldsymbol{u}^{\beta}}{k_B T^{\beta}}}}\right]\,,$$

where $\mathcal{G} = n_{\boldsymbol{k}+\boldsymbol{q}}^{\alpha} n_{\boldsymbol{k}'}^{\beta} (1 - n_{\boldsymbol{k}}^{\alpha})(1 - n_{\boldsymbol{k}'+\boldsymbol{q}}^{\beta})$ and $\boldsymbol{u}^{\alpha} \equiv \hbar\boldsymbol{k}_D^{\alpha}/m_{\alpha}$ is the drift velocity. Using energy conservation, the above expression can be rearranged as

$$\mathcal{T} = \mathcal{G}\left\{1 - \exp\left[\frac{\epsilon_{\boldsymbol{k}+\boldsymbol{q}}^{\alpha} - \epsilon_{\boldsymbol{k}}^{\alpha} - \hbar\boldsymbol{q}\cdot\boldsymbol{u}^{\alpha}}{k_B}\left(\frac{1}{T^{\alpha}} - \frac{1}{T^{\beta}}\right) + \frac{\hbar\boldsymbol{q}}{k_B T^{\beta}}\cdot\left(\boldsymbol{u}^{\beta} - \boldsymbol{u}^{\alpha}\right)\right]\right\}\,, \qquad \text{(A3)}$$

by adding and subtracting the term $\hbar\boldsymbol{q}\cdot\boldsymbol{u}^{\alpha}/k_B T^{\beta}$. Since Equation (A2) is valid for arbitrary $\boldsymbol{k}$ and $\boldsymbol{q}$, and $\mathcal{G}$ contains no algebraic roots, it is then straightforward to conclude that detailed balance leads to

$$T^e = T^h\,, \qquad \text{(A4)}$$

$$\boldsymbol{u}^e = \boldsymbol{u}^h\,. \qquad \text{(A5)}$$

These equalities are the result of the DB requirement under the condition of strong e-h scattering and have been used for the single-component reduction in that limiting case. They are duplicated in Subsection V A as Eqs. (38–39).

Now we consider the deviation of the EHP from the DB state. We make a linear Taylor expansion for the exponential term inside the curly bracket in $\mathcal{T}$, and the resultant expression is given below:

$$\mathcal{T} \approx -\mathcal{G}_1\left(T^{\alpha} - T^{\beta}\right) - \mathcal{G}_2\cdot\left(\boldsymbol{u}^{\alpha} - \boldsymbol{u}^{\beta}\right)\,.$$

Plugging this linearized expression into Eq. (A1), we thus obtain the momentum and energy relaxation rates.

The momentum relaxation rate due to e-h scattering, as appeared in Eq. (14), is defined as

$$\partial_t \boldsymbol{P}^\alpha|_{eh} \equiv \frac{2}{S}\sum_{\boldsymbol{k}} \hbar\boldsymbol{k}\ \partial_t n^\alpha_{\boldsymbol{k}}|_{eh} \tag{A6}$$
$$= \left(\frac{2}{S}\right)^3 \sum_{\boldsymbol{k},\boldsymbol{k}',q\neq 0} \frac{2\pi}{\hbar} V^2_{s,q}\delta\left(\Delta\epsilon_{eh}\right) \mathcal{T}\hbar\boldsymbol{k}$$
$$= -\left(\frac{2}{S}\right)^3 \sum_{\boldsymbol{k},\boldsymbol{k}',q\neq 0} \frac{2\pi}{\hbar} V^2_{s,q}\delta\left(\Delta\epsilon_{eh}\right) \hbar\boldsymbol{k}$$
$$\times \left[\mathcal{G}_1\left(T^\alpha - T^\beta\right) + \boldsymbol{\mathcal{G}}_2\cdot\left(\boldsymbol{u}^\alpha - \boldsymbol{u}^\beta\right)\right] .$$

We notice that there are two terms in the momentum relaxation, but ensuing proof shows that the coefficient of the temperature difference term $(T^\alpha - T^\beta)$ vanishes under the DB condition. This leads to the equation for momentum relaxation due to e-h scattering given in Eq. (29). To prove this, we notice that the prefactor of the term with $(T^\alpha - T^\beta)$ is proportional to

$$\sum_{\boldsymbol{k},\boldsymbol{k}',q\neq 0} V^2_{s,q}\delta\left(\Delta\epsilon_{eh}\right) n^\alpha_{\boldsymbol{k}+\boldsymbol{q}} n^\beta_{\boldsymbol{k}'}(1-n^\alpha_{\boldsymbol{k}})(1-n^\beta_{\boldsymbol{k}'+\boldsymbol{q}})$$
$$\times \frac{\epsilon^\alpha_{\boldsymbol{k}+\boldsymbol{q}} - \epsilon^\alpha_{\boldsymbol{k}} - \hbar\boldsymbol{q}\cdot\boldsymbol{u}^\alpha}{k_B}\ \boldsymbol{k} =$$
$$\sum_{\boldsymbol{k},\boldsymbol{k}',q\neq 0} V^2_{s,q}\delta\left(\Delta\epsilon_{eh}\right) f^\alpha_{\boldsymbol{k}+\boldsymbol{q}} f^\beta_{\boldsymbol{k}'}(1-f^\alpha_{\boldsymbol{k}})(1-f^\beta_{\boldsymbol{k}'+\boldsymbol{q}})$$
$$\times \frac{\epsilon^\alpha_{\boldsymbol{k}+\boldsymbol{q}} - \epsilon^\alpha_{\boldsymbol{k}}}{k_B}\left(\boldsymbol{k}+\boldsymbol{k}^\alpha_D\right) , \tag{A7}$$

which are obtained by using Eq. (A5) inside the delta function after making the following translational transformations:

$$\boldsymbol{k} - \boldsymbol{k}^\alpha_D = \boldsymbol{k}_1 ,$$
$$\boldsymbol{k}' - \boldsymbol{k}^\beta_D = \boldsymbol{k}_2 ,$$

then change the notation back, *i.e.*, $\boldsymbol{k}_1 \to \boldsymbol{k}$ and $\boldsymbol{k}_2 \to \boldsymbol{k}'$. Due to inversion symmetry in the prefactor of $(\boldsymbol{k}+\boldsymbol{k}^\alpha_D)$, only the term proportional to $\boldsymbol{k}^\alpha_D$ in Eq. (A7) survives. We rewrite its prefactor $(\times k_B)$ below:

$$\sum_{\boldsymbol{k},\boldsymbol{k}',q\neq 0} V^2_{s,q}\delta\left(\Delta\epsilon_{eh}\right) f^\alpha_{\boldsymbol{k}+\boldsymbol{q}} f^\beta_{\boldsymbol{k}'}(1-f^\alpha_{\boldsymbol{k}})(1-f^\beta_{\boldsymbol{k}'+\boldsymbol{q}})\left(\epsilon^\alpha_{\boldsymbol{k}+\boldsymbol{q}} - \epsilon^\alpha_{\boldsymbol{k}}\right) .$$

We recognize that the expression denotes the integrated energy difference between in-scattering and out-scattering with respect to states $|\alpha\boldsymbol{k}\rangle$ and $|\beta\boldsymbol{k}'+\boldsymbol{q}\rangle$ under detailed balance, and thus vanishes. Therefore, the momentum relaxation term is reduced to a simple rate equation form:

$$\partial_t \boldsymbol{P}^\alpha|_{eh} = -m_r\bar{\gamma}_{eh}\left(\boldsymbol{u}^\alpha - \boldsymbol{u}^\beta\right) . \tag{A8}$$

where the momentum relaxation rate is given by

$$\bar{\gamma}_{eh} = -\left(\frac{2}{S}\right)^3 \sum_{\boldsymbol{k},\boldsymbol{k}',q\neq 0} \frac{2\pi}{\hbar} V^2_{s,q}\delta\left(\Delta\epsilon_{eh}\right) \tag{A9}$$
$$\times f^\alpha_{\boldsymbol{k}+\boldsymbol{q}} f^\beta_{\boldsymbol{k}'}(1-f^\alpha_{\boldsymbol{k}})(1-f^\beta_{\boldsymbol{k}'+\boldsymbol{q}})\frac{\hbar q_x}{k_B T^\beta}\frac{\hbar k_x}{m_r} .$$

This can be obtained by making the same replacements and using the inversion symmetry argument. We also assumed that the direction of $\boldsymbol{u}^\alpha - \boldsymbol{u}^\beta$ is taken as the x direction. The positive-definiteness of the above relaxation rate can be verified easily after accounting for the isotropy of the EHP. Then we may replace $q_x k_x$ in the summation by $\boldsymbol{k}\cdot\boldsymbol{q}/2$, which is related to $(\epsilon^\alpha_{\boldsymbol{k}+\boldsymbol{q}} - \epsilon^\alpha_{\boldsymbol{k}}) - \epsilon^\alpha_{\boldsymbol{q}}$. Since the energy difference term vanishes after integration, the momentum relaxation rate takes the neat form:

$$\bar{\gamma}_{eh} = \frac{m_\alpha}{2m_r k_B T^\beta}\left(\frac{2}{S}\right)^3 \sum_{\boldsymbol{k},\boldsymbol{k}',q\neq 0} \frac{2\pi}{\hbar} V^2_{s,q}\delta\left(\Delta\epsilon_{eh}\right)$$
$$\times f^\alpha_{\boldsymbol{k}+\boldsymbol{q}} f^\beta_{\boldsymbol{k}'}(1-f^\alpha_{\boldsymbol{k}})(1-f^\beta_{\boldsymbol{k}'+\boldsymbol{q}})\epsilon^\alpha_{\boldsymbol{q}} . \tag{A10}$$

Next, we consider the energy relaxation term appeared in Eq. (15) which is specifically given as

$$\partial_t E^\alpha|_{eh} \equiv \frac{2}{S}\sum_{\boldsymbol{k}} \frac{\hbar^2 k^2}{2m_\alpha}\partial_t n^\alpha_{\boldsymbol{k}}|_{eh} \tag{A11}$$
$$= \left(\frac{2}{S}\right)^3 \sum_{\boldsymbol{k},\boldsymbol{k}',q\neq 0} \frac{2\pi}{\hbar} V^2_{s,q}\delta\left(\Delta\epsilon_{eh}\right) \mathcal{T}\ \frac{\hbar^2 k^2}{2m_\alpha}$$
$$= -\left(\frac{2}{S}\right)^3 \sum_{\boldsymbol{k},\boldsymbol{k}',q\neq 0} \frac{2\pi}{\hbar} V^2_{s,q}\delta\left(\Delta\epsilon_{eh}\right) \frac{\hbar^2 k^2}{2m_\alpha}$$
$$\times \left[\mathcal{G}_1\left(T^\alpha - T^\beta\right) + \boldsymbol{\mathcal{G}}_2\cdot\left(\boldsymbol{u}^\alpha - \boldsymbol{u}^\beta\right)\right] .$$

Through exactly the same manipulation and arguments as we did for the momentum relaxation, the energy relaxation term cab be written in the following form,

$$\partial_t E^\alpha|_{eh} = -\Gamma^\alpha_{eh}\left(T^\alpha - T^\beta\right)$$
$$\cdot \quad -m_r\bar{\gamma}_{eh}\boldsymbol{u}^\alpha\cdot\left(\boldsymbol{u}^\alpha - \boldsymbol{u}^\beta\right) , \tag{A12}$$

where the energy relaxation rate due to temperature difference is given by

$$\Gamma^\alpha_{eh} = \left(\frac{2}{S}\right)^3 \sum_{\boldsymbol{k},\boldsymbol{k}',q\neq 0} \frac{2\pi}{\hbar} V^2_{s,q}\delta\left(\Delta\epsilon_{eh}\right) \tag{A13}$$
$$\times f^\alpha_{\boldsymbol{k}+\boldsymbol{q}} f^\beta_{\boldsymbol{k}'}(1-f^\alpha_{\boldsymbol{k}})(1-f^\beta_{\boldsymbol{k}'+\boldsymbol{q}})\epsilon^\alpha_{\boldsymbol{k}}\frac{\epsilon^\alpha_{\boldsymbol{k}} - \epsilon^\alpha_{\boldsymbol{k}+\boldsymbol{q}}}{k_B T^\alpha T^\beta} .$$

Before closing this appendix, it is worth making two observations: (1) the relaxation rate equation for energy, Eq. (A12), is consistent with our previous results, Eq.

(27), and (A8). Thus, we prove, within linear expansion treatment, that

$$\partial_t W^\alpha|_{eh} = -\Gamma^\alpha_{eh}\left(T^\alpha - T^\beta\right) . \qquad \text{(A14)}$$

The corresponding temperature change rate due to e-h scattering can be written, according to Eq. (34), as follows:

$$\partial_t T^\alpha|_{eh} = j^\alpha_W \partial_t W^\alpha|_{eh} = -\gamma^{eh}_{T^\alpha}\left(T^\alpha - T^\beta\right) , \qquad \text{(A15)}$$

where the relaxation rate is given by $\gamma^{eh}_{T^\alpha} = j^\alpha_W \Gamma^\alpha_{eh}$. Furthermore, we point out that the relaxation rates are different for electrons and holes. The reason is two fold: first and foremost, the Γ^α_{eh} factor is inversely proportional to the carrier mass. As compared to Eq. (A10), we immediately see a factor m_r/m_α is recovered for Eq. (A13), given that $\bar{\gamma}_{eh}$ is carrier mass independent. Second, the j^α_W factor depends on the carrier mass in the quantum regime. Therefore, the temperature relaxation rate is inversely proportional to the carrier mass at low density. (2) the second equality, Eq. (A5), is the same as what is required in the *ambipolar diffusion approximation* [15, 16]. The requirement is consistent with the intuitive understanding of incoherent collisions between different species in the EHP. These interactions mean additional frictional force between the oppositely charged species, in addition to the force relative to the ambient, caused by interacting with mainly LO phonons. Such frictional drag may lead to a reversal in the drift direction of carriers, as corroborated in the study of the negative mobility of minority carriers in semiconductor quantum wells [29–34]. By contrast, the ambipolar diffusion approximation is deduced by use of charge neutrality, which is a static condition. Interestingly, given a neutral initial condition for a system, it will evolve and preserve charge neutrality since there is no charge separation as the oppositely charged components move at the same velocity, as governed by the equation of continuity, that is Eq. (41), and Eq. (A5). Therefore, having ultrafast carrier scattering in the EHP, the ambipolar diffusion approximation is redundant for an initially neutral system.

APPENDIX B: MOMENTUM AND ENERGY RELAXATION RATES DUE TO c-LO SCATTERING

Interaction of the EHP with the host semiconductor crystal is dominated by the inelastic carrier-LO phonon scattering at room temperature, while collisions with other phonon branches are relatively weak and neglected. In semiconductor quantum well structures, as the well width is reduced, phonon modes could become confined in the growth direction and interface modes are introduced for small enough width [45]. On the other hand, if the quantum well is not too narrow, calculation of the scattering rates using bulk modes produces similar results to that obtained by incorporating both the confined and the interface modes for a semiconductor quantum well, if no mode-specific physics is concerned [46, 47]. In this work, we therefore use bulk LO phonon modes of the Einstein model for the 8 nm quantum well structure. Under the assumption of quasiequilibrium for the EHP and equilibrium for phonons, the momentum and energy relaxation rates due to LO phonon scattering, $\partial_t \boldsymbol{P}^\alpha|_{LO}$ and $\partial_t W^\alpha|_{LO}$ are worked out in this appendix.

First, let us put forth the necessary microscopic ingredients [45]. The rate of change in carrier distribution due to c-LO phonon scattering, according to Fermi's golden rule, is given by

$$\begin{aligned}\partial_t n^\alpha_{\boldsymbol{k}}|_{LO} \;=\; & \frac{2\pi}{\hbar}\sum_{\boldsymbol{q},q_z}|\hat{H}_e|^2\left[\delta\left(\Delta\epsilon^+_{LO}\right) n^\alpha_{\boldsymbol{k}+\boldsymbol{q}}(1-n^\alpha_{\boldsymbol{k}})\right. \\ & \left. - \delta\left(\Delta\epsilon^-_{LO}\right) n^\alpha_{\boldsymbol{k}}(1-n^\alpha_{\boldsymbol{k}-\boldsymbol{q}})\right] \\ & + \frac{2\pi}{\hbar}\sum_{\boldsymbol{q},q_z}|\hat{H}_a|^2\left[\delta\left(\Delta\epsilon^-_{LO}\right) n^\alpha_{\boldsymbol{k}-\boldsymbol{q}}(1-n^\alpha_{\boldsymbol{k}})\right. \\ & \left. - \delta\left(\Delta\epsilon^+_{LO}\right) n^\alpha_{\boldsymbol{k}}(1-n^\alpha_{\boldsymbol{k}+\boldsymbol{q}})\right] , \qquad \text{(B1)}\end{aligned}$$

where phonon wavevector $\boldsymbol{Q} \equiv (\boldsymbol{q}, q_z)$ is expressed in its in-plane component $\boldsymbol{q}$ and vertical component q_z, ω_{LO} is the phonon circular frequency, $\hat{H}_e$ and $\hat{H}_a$ are the carrier-LO phonon scattering matrix elements for phonon emission and absorption, $\Delta\epsilon^+_{LO} = \epsilon^\alpha_{\boldsymbol{k}} - \epsilon^\alpha_{\boldsymbol{k}+\boldsymbol{q}} + \hbar\omega_{LO}$, and $\Delta\epsilon^-_{LO} = \epsilon^\alpha_{\boldsymbol{k}} - \epsilon^\alpha_{\boldsymbol{k}-\boldsymbol{q}} - \hbar\omega_{LO}$. Using the Fröhlich Hamiltonian, the matrix elements are given as

$$|\hat{H}_e|^2 \;=\; \frac{e^2\hbar\omega_{LO}}{2\varepsilon_0\varepsilon_p V}|G(q_z)|^2\frac{N_{\boldsymbol{Q}}+1}{Q^2} , \qquad \text{(B2a)}$$

$$|\hat{H}_a|^2 \;=\; \frac{e^2\hbar\omega_{LO}}{2\varepsilon_0\varepsilon_p V}|G(q_z)|^2\frac{N_{\boldsymbol{Q}}}{Q^2} , \qquad \text{(B2b)}$$

where $1/\varepsilon_p = 1/\varepsilon_\infty - 1/\varepsilon_s$ with ε_s (ε_∞) being the static (high-frequency) relative permittivity of the unexcited semiconductor. The overlapping integral, $G(q_z)$, is defined as $\int_{-\infty}^{\infty} dz \chi_i(z)\chi_j(z)\exp(iq_z z)$, where $\chi_i(z)$ is the real and normalized carrier envelop wavefunction in the i-th subband which has been always taken as the ground state in this work. The phonon density is governed by Bose-Einstein distribution, $N_{\boldsymbol{Q}} = 1/\left[\exp(\beta_{LO}\hbar\omega_{LO}) - 1\right]$, with $\beta_{LO} = 1/k_B T_{LO}$ and T_{LO} being the LO phonon temperature. Conservation of momentum has already been explicitly considered in Eq. (B1).

Before calculating the momentum and energy relaxation rates, we point out that we will still work in the linear regime, as in the preceding appendix, but now in the drift velocities themselves. (Given the femtosecond scattering time scale at typical lasing density [43, 48], it is reasonable to expect the drifted carrier distributions are small perturbations from the non-drift Fermi-Dirac distributions.) The linear expansion is well known as

$$n_{\boldsymbol{k}}^{\alpha} \approx f(\epsilon_{\boldsymbol{k}}^{\alpha}) - (\boldsymbol{k}_D^{\alpha} \cdot \partial_{\boldsymbol{k}} \epsilon_{\boldsymbol{k}}^{\alpha})\, f'(\epsilon_{\boldsymbol{k}}^{\alpha}) \,, \tag{B3}$$

where $f'(\epsilon_{\boldsymbol{k}}^{\alpha})$ is the derivative of the Fermi-Dirac distribution function $f(\epsilon_{\boldsymbol{k}}^{\alpha})$ with respect to the carrier energy.

Now we are prepared to compute the rates. The momentum relaxation term is defined in the same manner as in the carrier-carrier scattering case, that is

$$\partial_t \boldsymbol{P}^{\alpha}|_{LO} \equiv \frac{2}{S} \sum_{\boldsymbol{k}} \hbar \boldsymbol{k}\; \partial_t n_{\boldsymbol{k}}^{\alpha}|_{LO} \,.$$

Inversion symmetry leads to zero contribution from the non-drift part of the distribution, so we only need to focus on the linear term in $\boldsymbol{k}_D^{\alpha}$ in Eq. (B3). Lengthy but straightforward integration over the phonon wavevector $\boldsymbol{Q}$ produces a concise integrated solution. We only give a few intermediate steps in the deduction and the main result below:

$$\begin{aligned}
\partial_t \boldsymbol{P}^{\alpha}|_{LO} = {} & \frac{\pi e^2 \omega_{LO}}{(2\pi)^4 \varepsilon_0 \varepsilon_p} \int d\boldsymbol{k}\; \hbar \boldsymbol{k} \int dq\; \frac{F_{11}(q)}{q} \\
\times \Bigg(& \delta(\Delta\epsilon_{LO}^{-}) \left\{ \frac{\hbar(\boldsymbol{k}-\boldsymbol{q}) \cdot \boldsymbol{u}^{\alpha}}{k_B T^{\alpha}} (N_{LO} + f_{\boldsymbol{k}}^{\alpha}) f_{\boldsymbol{k}-\boldsymbol{q}}^{\alpha} (1 - f_{\boldsymbol{k}-\boldsymbol{q}}^{\alpha}) + \frac{\hbar \boldsymbol{k} \cdot \boldsymbol{u}^{\alpha}}{k_B T^{\alpha}} \left[f_{\boldsymbol{k}-\boldsymbol{q}}^{\alpha} - (N_{LO}+1) \right] f_{\boldsymbol{k}}^{\alpha} (1 - f_{\boldsymbol{k}}^{\alpha}) \right\} \\
& - \delta(\Delta\epsilon_{LO}^{+}) \left\{ \frac{\hbar \boldsymbol{k} \cdot \boldsymbol{u}^{\alpha}}{k_B T^{\alpha}} (N_{LO} + f_{\boldsymbol{k}+\boldsymbol{q}}^{\alpha}) f_{\boldsymbol{k}}^{\alpha} (1 - f_{\boldsymbol{k}}^{\alpha}) + \frac{\hbar(\boldsymbol{k}+\boldsymbol{q}) \cdot \boldsymbol{u}^{\alpha}}{k_B T^{\alpha}} \left[f_{\boldsymbol{k}}^{\alpha} - (N_{LO}+1) \right] f_{\boldsymbol{k}+\boldsymbol{q}}^{\alpha} (1 - f_{\boldsymbol{k}+\boldsymbol{q}}^{\alpha}) \right\} \Bigg) \,.
\end{aligned}$$

where $N_{LO} = N_{\boldsymbol{Q}}$ since the LO phonons are dispersionless in the Einstein model and

$$\begin{aligned}
F_{11}(q) \;\equiv\; & \frac{q}{\pi} \int_{-\infty}^{\infty} dq_z\; |G(q_z)|^2 / (q^2 + q_z^2) \\
= \; & \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{\infty} dz_2\; \chi_1(z_1) \chi_1(z_2) \\
& \times \exp(-q|z_1 - z_2|) \chi_1(z_2) \chi_1(z_1) \,.
\end{aligned} \tag{B5}$$

Then we evaluate the integrals in the momentum relaxation term individually by using polar coordinates for both $\boldsymbol{k}$ and $\boldsymbol{q}$. During the integration the direction of the drift wavevector is taken as in the x direction. After term-collecting and -canceling, the final momentum relaxation term is found to be

$$\partial_t \boldsymbol{P}^{\alpha}|_{LO} = -\gamma_{LO}^{\alpha} \boldsymbol{P}^{\alpha} \,, \tag{B6}$$

with the momentum relaxation rate given by

$$\begin{aligned}
\gamma_{LO}^{\alpha} \;=\; & c_0 \int_0^{\infty} I_0(k) \left\{ \left[N_{LO} + f(\epsilon_+) \right] f(\epsilon_+) \left[1 - f(\epsilon_+) \right] \right. \\
& \left. + \left[N_{LO} + 1 - f(\epsilon) \right] f(\epsilon_+) \left[1 - f(\epsilon_+) \right] \right\} d\epsilon \,,
\end{aligned} \tag{B7}$$

where $c_0 = (m_{\alpha} e^2 \omega_{LO}) / (8\pi^2 \hbar^2 \varepsilon_0 \varepsilon_p k_B T)$ and $\epsilon_+ = \epsilon + \hbar\omega_{LO}$. We define $k_+ = \sqrt{k^2 + 2m_{\alpha}\omega_{LO}/\hbar}$, which is used in the integral:

$$I_0(k) = \int_{k_+ - k}^{k_+ + k} \frac{q^2 F_{11}(q)\; dq}{\sqrt{(2kq)^2 - (q^2 - 2m_{\alpha}\omega_{LO}/\hbar)^2}} \,. \tag{B8}$$

Similar to the carrier scattering, it is found, after careful evaluation of the carrier energy relaxation due to LO phonon scattering, that the higher-order drift term cancels out according to Eq. (28) as far as the carrier thermal energy is concerned. Thus we will only show the partial contribution from the non-drift first term in Eq. (B3) below. To assist computing the energy relaxation term, summation over the phonon wavevector in Eq. (B1) can be carried out directly so that the procedure becomes more transparent. After tedious algebraic manipulations, the expression becomes

$$\begin{aligned}
\partial_t n_{\boldsymbol{k}}^{\alpha}|_{LO} \;=\; & c_1 \left(I_1(k) \left\{ N_{LO} f(\epsilon_{\boldsymbol{k}}^{\alpha} - \hbar\omega_{LO}) \left[1 - f(\epsilon_{\boldsymbol{k}}^{\alpha}) \right] \right. \right. \\
& \left. - (N_{LO}+1) f(\epsilon_{\boldsymbol{k}}^{\alpha}) \left[1 - f(\epsilon_{\boldsymbol{k}}^{\alpha} - \hbar\omega_{LO}) \right] \right\} \\
& + I_2(k) \left\{ (N_{LO}+1) f(\epsilon_{\boldsymbol{k}}^{\alpha} + \hbar\omega_{LO}) \left[1 - f(\epsilon_{\boldsymbol{k}}^{\alpha}) \right] \right. \\
& \left. \left. - N_{LO} f(\epsilon_{\boldsymbol{k}}^{\alpha}) \left[1 - f(\epsilon_{\boldsymbol{k}}^{\alpha} + \hbar\omega_{LO}) \right] \right\} \right) \,,
\end{aligned} \tag{B9}$$

where $c_1 = (m_{\alpha} e^2 \omega_{LO}) / (2\pi\hbar^2 \varepsilon_0 \varepsilon_p)$, while the two k-dependent integrals are given as

$$I_1(k) = \int_{k - k_-}^{k + k_-} \frac{F_{11}(q)\; dq}{\sqrt{(2kq)^2 - (q^2 + 2m_{\alpha}\omega_{LO}/\hbar)^2}} \,, \tag{B10}$$

$$I_2(k) = \int_{k_+ - k}^{k_+ + k} \frac{F_{11}(q)\; dq}{\sqrt{(2kq)^2 - (q^2 - 2m_{\alpha}\omega_{LO}/\hbar)^2}} \,, \tag{B11}$$

where $k_- = \sqrt{k^2 - 2m_{\alpha}\omega_{LO}/\hbar}$. The integrals are related by the transformation: $I_1(k_+) = I_2(k)$. Using the above equations for $I_{1,2}(k)$ and integrating over the polar angle

of wavevector $\boldsymbol{k}$, the carrier thermal energy relaxation is determined to be

$$\begin{aligned}\partial_t W^\alpha|_{LO} &= -c_2 \int_0^\infty d\epsilon\, I_2(k) \\ &\quad \times \{(N_{LO}+1) f(\epsilon_+)\,[1-f(\epsilon)] \\ &\quad - N_{LO} f(\epsilon)\,[1-f(\epsilon_+)]\}\ , \end{aligned} \tag{B12}$$

where $c_2 = (m_\alpha^2 e^2 \omega_{LO}^2)/(2\pi^2\hbar^3\varepsilon_0\varepsilon_p)$. Plugging the Fermi-Dirac distributions in, it can be shown that a factor proportional to the temperature difference between the carriers and the host lattice exists. From equation (34), we can write the temperature change due to c-LO scattering in the form

$$\begin{aligned}\partial_t T^\alpha|_{LO} &= j_W^\alpha \partial_t W^\alpha|_{LO} \\ &= -\Gamma_{LO}^\alpha (T^\alpha - T_{LO})\ , \end{aligned} \tag{B13}$$

where Γ_{LO}^α is defined as follows: We note that the expression inside the braces of integrand in (B12) can be rewritten as follows:

$$\begin{aligned}&(N_{LO}+1) f(\epsilon_+)\,[1-f(\epsilon)] - N_{LO} f(\epsilon)\,[1-f(\epsilon_+)] \\ &= (N_{LO}+1) f(\epsilon_+)\,[1-f(\epsilon)] \\ &\quad \times \{1 - \exp\,[(\beta_\alpha - \beta_{LO})\hbar\omega_{LO})]\}\ . \end{aligned} \tag{B14}$$

Obviously the term is proportional to the temperature difference between the carriers and the phonons. Therefore, the constant can be explicitly written as

$$\begin{aligned}\Gamma_{LO}^\alpha &= c_2 j_W^\alpha (N_{LO}+1) \int_0^\infty d\epsilon\, I_2(k) f(\epsilon_+)\,[1-f(\epsilon)] \\ &\quad \times \frac{1-\exp\,[(\beta_\alpha - \beta_{LO})\hbar\omega_{LO})]}{T_\alpha - T_{LO}}\ . \end{aligned} \tag{B15}$$

The constant is positively definitive for $T^\alpha > T_{LO}$. By taking the limit of $T^\alpha \to T_{LO}$ in the above expression, we then obtain Newton's cooling rate between electrons (or holes) and LO phonons:

$$\begin{aligned}\Gamma_{LO}^\alpha &= c_2 j_W^\alpha (N_{LO}+1) \frac{\hbar\omega_{LO}}{k_B T_{LO}^2} \\ &\quad \times \int_0^\infty d\epsilon\, I_2(k) f(\epsilon_+)\,[1-f(\epsilon)]\ . \end{aligned} \tag{B16}$$

Here, the results are independent of the assumption that the lattice and LO phonons are in thermal equilibrium: $T_L = T_{LO}$, despite that it is assumed so in this work.

APPENDIX C: CARRIER DIFFUSION COEFFICIENTS IN GENERAL TWO-COMPONENT CASE

In this appendix, we only give the expressions for density-related diffusion coefficients of a general two-component plasma in semiconductor quantum wells. Using the solutions for electron ($\alpha = e$) and hole ($\alpha = h$) momenta $\boldsymbol{P}^\alpha$ given in Eq. (30), we can write the density current for the α-component in the form:

$$\boldsymbol{J}_{N^\alpha} = -\sum_X D_{N^\alpha X} \partial_r X - \frac{\sigma^\alpha}{q^\alpha} \partial_r \Phi\ , \tag{C1}$$

where $X \in \{N^e, N^h, T^e, T^h\}$. We have introduced various DCs and conductivities which are listed as follows:

$$D_{N^\alpha N^\alpha} = \mu_\alpha \left[(1+\eta_\alpha) S_{N^\alpha}^\alpha + H_{N^\alpha}^\beta\right]\ , \tag{C2}$$

$$D_{N^\alpha N^\beta} = \mu_\alpha \left[(1+\eta_\alpha) H_{N^\beta}^\alpha + S_{N^\beta}^\beta\right]\ , \tag{C3}$$

$$D_{N^\alpha T^\alpha} = \mu_\alpha \left[(1+\eta_\alpha) S_{T^\alpha}^\alpha + H_{T^\alpha}^\beta\right]\ , \tag{C4}$$

$$D_{N^\alpha T^\beta} = \mu_\alpha \left[(1+\eta_\alpha) H_{T^\beta}^\alpha + S_{T^\beta}^\beta\right]\ , \tag{C5}$$

$$\sigma^\alpha = \mu_\alpha e^2 \left[N^\alpha(1+\eta_\alpha) - N^\beta\right]\ , \tag{C6}$$

where $\alpha \neq \beta$ and

$$\begin{aligned} S_X^\alpha &= \partial_X W^\alpha + N^\alpha \partial_X \delta\epsilon^\alpha\ ,\ (X = N^\alpha, T^\alpha)\ , \\ H_X^\alpha &= N^\alpha \partial_X \delta\epsilon^\alpha\ ,\ (X = N^\beta, T^\beta)\ . \end{aligned}$$

The factor μ_α is defined in Eq. (31). The various rates used above have been given in the first two appendices. The corresponding temperature currents and associated DCs can be readily written down according to the relation in Eq. (37) and will not be listed here.

APPENDIX D: SELF- AND MUTUAL-DIFFUSION COEFFICIENTS UNDER THE SINGLE-COMPONENT APPROXIMATION

In this appendix, all diffusion coefficients for the single component case are explicitly given in terms of material parameters, scattering rates, and distribution functions of the EHP in a semiconductor quantum well structure. We consider the limiting case of strong e-h scattering. This is true when the plasma density is high enough such that e-h scattering dominates over other scatterings for momentum relaxation. At the end, we introduce terminology for the diffusion coefficients, which shall be extensively used in the subsequent article.

From the relations obtained in Subsection V A, it is easy to show that the currents for density and temperature are given, respectively, by

$$\boldsymbol{J}_N = N\boldsymbol{u}\ , \tag{D1}$$

$$\boldsymbol{J}_T = [2j_W(W/N) - j_N]\,\boldsymbol{J}_N\ , \tag{D2}$$

where we have used $\boldsymbol{J}_T = j_W \boldsymbol{J}_W - j_N \boldsymbol{J}_N$ and $\boldsymbol{J}_W = 2\boldsymbol{u}W$ with W being the total plasma thermal energy.

Recall that the drift velocity of plasma $\boldsymbol{u}$ is given by Eq. (40) and now we rewrite it in terms of the gradients of density and temperature as follows:

$$\begin{aligned} \boldsymbol{u} &= -\frac{\mu}{N}\left(\partial_N W + N\partial_N \delta\epsilon_g\right)\partial_r N \\ &\quad -\frac{\mu}{N}\left(\partial_T W + N\partial_T \delta\epsilon_g\right)\partial_r T\,, \end{aligned} \tag{D3}$$

where $\delta\epsilon_g = \delta\epsilon^e + \delta\epsilon^h$ is the total Hartree-Fock bandgap renormalization. As defined in Eqs. (44) and (45), the DCs are finally written as

$$D_{NN} = \mu\left(\partial_N W + N\partial_N \delta\epsilon_g\right)\,, \tag{D4}$$
$$D_{NT} = \mu\left(\partial_T W + N\partial_T \delta\epsilon_g\right)\,, \tag{D5}$$
$$D_{TN} = \left[2j_W(W/N) - j_N\right] D_{NN}\,, \tag{D6}$$
$$D_{TT} = \left[2j_W(W/N) - j_N\right] D_{NT}\,. \tag{D7}$$

In the two-dimensional case, analytical expressions for the energy and carrier density are given by

$$W^\alpha = \frac{m_\alpha \left(k_B T^\alpha\right)^2}{\pi\hbar^2}\int_{-\beta_\alpha\mu_F^\alpha}^{\infty} \ln\left(1+\mathrm{e}^{-x}\right) dx\,, \tag{D8}$$

$$N^\alpha = \frac{m_\alpha k_B T^\alpha}{\pi\hbar^2}\ln\left[1+\exp(\beta_\alpha\mu_F^\alpha)\right]\,, \tag{D9}$$

The derivatives of W_α with respect to N^α and T^α can also be written explicitly:

$$\partial_{N^\alpha} W^\alpha|_{T^\alpha} = k_B T^\alpha \theta_\alpha\,, \tag{D10}$$
$$\partial_{T^\alpha} W^\alpha|_{N^\alpha} = 2k_B\beta_\alpha W^\alpha - k_B N^\alpha\theta_\alpha\,, \tag{D11}$$

where θ_α, defined as

$$\theta_\alpha = \left[1+\exp\{-\beta_\alpha\mu_F^\alpha\}\right]\ln\left[1+\exp\{\beta_\alpha\mu_F^\alpha\}\right]\,, \tag{D12}$$

can be considered as a degeneracy factor, which goes to one in the nondegenerate limit. Both (D10) and (D11) take their classical values in the nondegenerate limit.

We also note from the definition of μ [Eq. (31)] that

$$\mu = 1/[m_e\gamma_{LO}^e + m_h\gamma_{LO}^h] \tag{D13}$$

when the c-LO scattering can be ignored compared with e-h scattering. In this case, all the diffusion coefficients defined in this appendix are the same as in Subsection V B. This means that the two approaches lead to the same definition of ambipolar DCs.

Before closing this appendix and for the benefit of discussions in the text, we introduce the following terminology for the DCs. The coefficient is dubbed *self-diffusion* when it relates the gradient of a variable to its current, and *mutual-diffusion* otherwise. In addition, variable name is used to label the coefficient that relates to the variable gradient. For example, D_{TN} is called *mutual-diffusion density coefficient* accordingly.

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