2001 NASA/ASEE SUMMER FACULTY FELLOWSHIP PROGRAM

JOHN F. KENNEDY SPACE CENTER
UNIVERSITY OF CENTRAL FLORIDA

FLUID DYNAMICS OF A PRESSURE REDUCING INLET

JOHN M. RUSSELL
Associate Professor, Aerospace Engineering
Florida Institute of Technology
NASA colleague: FREDERICK W. ADAMS, YA-D2

ABSTRACT

Instruments for the monitoring of hazardous gases in and near the space shuttle collect sample gas at pressures on the order of one atmosphere and analyze their properties in an ultra-high vacuum by means of a quadrupole-mass-spectrometer partial pressure tranducer. Sampling systems for such devices normally produce the required pressure reduction through combinations of vacuum pumps, fluid Tees and flow restrictors (*e.g.* orifices, sintered metal frits or capillaries). The present work presents an analytical model of the fluid dynamics of such a pressure reduction system which enables the calculation of the pressure in the receiver vessal in terms of system parameters known from the specifications for a given system (*e.g.* rated pumping speeds of the pumping hardware and the diameters of two orifices situated in two branches of a fluid Tee). The resulting formulas will expedite the fine tuning of instruments now under development and the design of later generations of such devices.

FLUID DYNAMICS OF A PRESSURE REDUCING INLET

JOHN M. RUSSELL

1. FLOW GEOMETRY; OBJECTIVE

Figure 1.1 is a schematic diagram of a fluid Tee.

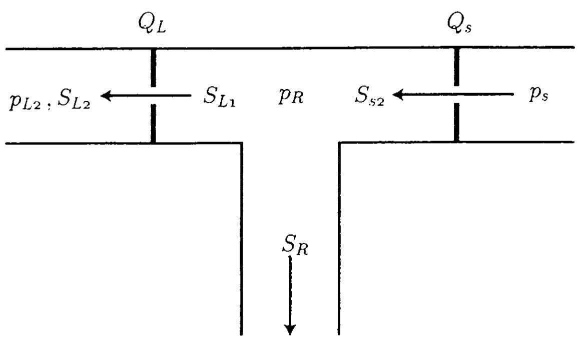

Fig. 1.1

The top left branch of the Tee discharges into an ultra high vacuum envelope containing a partial pressure transducer (*e.g.* a Quadrupole Mass Spectrometer gas analyzer). The volumetric throughput through the partial pressure transducer is presumed to be given by the rated pumping speed, S_{L2}, of a high-vacuum pump of turbomolecular type. The bottom branch of the Tee discharges into a low-vacuum foreline pump (*e.g.* of scroller type). The top left and top right branches of the Tee each contain an orifice of prescribed diameter.

As the arrows indicate, the Tee has one inlet and two outlets. In keeping with the notation conventions in vacuum literature, I have denoted *volumetric rate of transport of gas across a cross section* by the symbol S. The symbol p denotes local *gas pressure* and the symbol $Q := pS$ denotes the *pressure volume throughput* of gas at a given cross section. The numerical subscripts 1 and 2 denote conditions on the upstream and the downstream sides of an orifice, respectively. The letter subscripts S, L and R denote *supply*, *leak*, and *roughing* (pump) respectively.

The objective of the present discussion will be *to show how one may calculate the two outlet pressures, namely p_R and p_{L2} from various constants of the system.* The be specific, the constants of the system consist of the members of the following list:

$$S_{L2}\ ,\ S_R\ ,\ T_s\ ,\ T_L\ ,\ d_s\ ,\ d_L\ ,,\ k\ ,\ m\ ,\ \gamma\ ,\ p_s \tag{1.1}$$

in which (for example):

S_{L2}=pumping speed at the inlet to a high-vacuum turbopump (Alcatel model ATH30+):
$\begin{cases} 20\ \text{L/s} & \text{for } He, \\ 30\ \text{L/s} & \text{for } N_2, \end{cases}$

S_R=pumping speed at the inlet of a Dry Scroll type roughing pump (Varian model TriScroll 300):
8.8 ft^3/min (or 4.15314 L/s),

T_s=temperature of the supply gas upstream of the top right orifice in Fig. 1.1:
295.15 °K (22 °C),

T_L=temperature of the gas in the Tee between the two orifices in Fig. 1.1:
295.15 °K (22 °C),

d_s=diameter of the top right orifice in Fig. 1.1:
0.009 in,

d_L=diameter of the top left orifice in Fig. 1.1:
0.002 in,

k=BOLTZMANN constant:
1.38066×10^{-23} J/°K,

m=mass of an individual gas particle:
$\begin{cases} 4.0026\ \text{amu} & \text{for } He, \\ 28.0134\ \text{amu} & \text{for } N_2, \end{cases}$
in which the notation 'amu' stands for *atomic mass unit* (1 amu=1.66054×10^{-27} kg),

γ=ratio, c_p/c_v, of specific-heat-at-constant-pressure to specific-heat-at-constant-volume of the gas:
$\begin{cases} 1.667 & \text{for } He, \\ 1.4 & \text{for } N_2, \end{cases}$

p_s=pressure of the supply line (upstream of the top right orifice in Fig. 1.1):
400 Torr.

2. APPLICATIONS OF THE LAW OF CONSERVATION OF MASS

In order make the following analytical solution self contained I will preface it with a discussion of the consequences of the law of conservation of mass and the equation of state of an ideal gas as they apply to the present problem. Recall, first, that one may write the equation of state of an ideal gas in the form (*cf.* CHAPMAN AND COWLING, 1970, p38)[1]

$$p = nkT\ , \tag{2.1}$$

in which n is the *number density of particles*, k (as stated above) is the BOLTZMANN constant and T is the absolute temperature. Let V denote the volume of a given sample of gas. If one multiplies both sides of (2.1) by V and writes $nV := N$ for the *number of particles in the sample*, one gets

$$pV = NkT\ . \tag{2.2}$$

If (as above) m is the mass of a single particle in the sample then $Nm := M$ is the *total mass of the gas in the sample.* If one writes $N = M/m$ in (2.2) one gets

$$pV = M\frac{k}{m}T\ . \tag{2.3}$$

Consider a steady-state process. Then a special case of (2.3) is

$$p\Delta V = \Delta M\frac{k}{m}T\ , \tag{2.4}$$

in which ΔV and ΔM are the values of of volume and mass of a small sample of gas transported across a fixed cross section during the time interval Δt. If one divides (2.4) by Δt one gets

$$p\frac{\Delta V}{\Delta t} = \frac{\Delta M}{\Delta t}\frac{k}{m}T\ . \tag{2.5}$$

If one passes to the limit $\Delta t \to 0$ one may write, in turn

$$\lim_{\Delta t\to 0}\left(\frac{\Delta V}{\Delta t}\right) := S\ ,$$

in which S, as above, is the volumetric throughput across the generic cross section and

$$\lim_{\Delta t\to 0}\left(\frac{\Delta M}{\Delta t}\right) := \dot{M}\ ,$$

in which $\dot{M}$ is the corresponding *mass throughput* across the same cross section. Equation (2.5) thus becomes, in the limit $\Delta t \to 0$,

$$pS = \dot{M}\frac{k}{m}T\ , \tag{2.6}$$

Let the subscripts $(\ )_1$ and $(\ )_2$ refer to any two distinct cross sections of conduit between which there are no brances (or leaks). I have already introduced the assumption that the flow is steady in time. Then the law of conservation of mass requires that

$$\dot{M}_1 = \dot{M}_2\ . \tag{2.7}$$

If one eliminates $\dot{M}$ from the left and right members by means of (2.6) one gets

$$\left(\frac{pS}{(k/m)T}\right)_1 = \left(\frac{pS}{(k/m)T}\right)_2\ . \tag{2.8}$$

In many vacuum applications, the changes in absolute temperature, T, along the conduit between two cross sections is small compared to, say, the average value over that interval. In that case, one may take

$$T_1 \approx T_2 \tag{2.9}$$

in (2.8). In the mean time, the factor k is a universal physical constant and m is constant for a given gas mixture. One may thus simplify (2.8) by cancelling the common factor $(k/m)T$ in the right and left members to obtain

$$(pS)_1 = (pS)_2 := Q\ . \tag{2.10}$$

The symbol Q thus denotes a parameter that is constant between any two distinct cross sections of conduit provided there are no branches or leaks in between. This model (which is subject to the uniform temperature assumption) may be quite accurate even though the individual factors, p and S, contributing to the product $Q = pS$ undergo large changes (*e.g.* p may decrease by a factor 10^{-3} while S increases by the factor 10^3).

Now suppose that there is a branch in a conduit. Then in steady flow the law of conservation of mass requires that the net flow of mass into the branch be balanced by the net outflow from it, *i.e.*

$$\sum \dot{M}_{\text{in}} = \sum \dot{M}_{\text{out}}\ .$$

One may, as before, eliminate $\dot{M}$ by means of (2.6) to get

$$\sum\left(\frac{pS}{(k/m)T}\right)_{\text{in}} = \sum\left(\frac{pS}{(k/m)T}\right)_{\text{out}}\ . \tag{2.11}$$

The uniform temperature assumption, (2.9), enables one to cancel T from the left and right members and k/m cancels for the same reasons as before. Thus,

$$\sum (pS)_{\text{in}} = \sum (pS)_{\text{out}}\ . \tag{2.12}$$

In the particular example of the flow in the fluid Tee shown in Fig. 1.1 equation (2.12) implies that

$$p_R S_{s2} = p_R S_R + p_R S_{L1}\ .$$

If one cancels the common factor p_R one gets

$$S_{s2} = S_R + S_{L1}\ . \tag{2.13}$$

3. ON THE FLOW THROUGH THE ORIFICES

I will restrict attention in the present problem to the case in which the pressure, p_R, is less than half of the pressure on the upstream side of the supply orifice. In the case when the flow through the supply orifice in the regime of *continuum* gas flow the conditions are such that the gas speed at the throat of the orifice just attains the speed of sound. According the equations of Gasdynamics (*cf.* LIEPMANN & ROSHKO, 1957, §§2.10 and 5.3)[2] one may derive the following equation for Q_s

$$Q_s = p_s \pi \frac{d_s^2}{4} \sqrt{\frac{kT_s}{m}} \left(\frac{2\gamma}{\gamma + 1} \right)^{1/2} \left(\frac{2}{\gamma + 1} \right)^{1/(\gamma - 1)} . \tag{3.1}$$

Here, d_s is the diameter of the supply orifice, γ is the ratio of specific heats c_p/c_v of the gas (namely 1.6667 for a monotomic gas or 1.4 for a diatomic one). Equation (3.1) is compatible with the equations in LIEPMANN & ROSHKO[2] (though none of the equations they write is equivalent to an explicit solution for Q). One may find equation (3.1) as written above in O'HANLON, 1989, p29[3].

I will further restrict attention to the case when flow through the left orifice in Fig. 1.1 is in the regime of *free molecule flow* (as opposed to contiuum flow). One may then relate the pressure-volume throughput, Q_L, though that orifice to the difference between the pressures on the two sides and to the *conductivity*, C_L defined by

$$C_L = \frac{Q_L}{p_R - p_{L2}} . \tag{3.2}$$

The kinetic theory of gases (*cf.* JEANS, 1940, pp 58–60)[4] furnishes an estimate of the rate of effusion of a gas through a circular hole in a plate subject to the assumptions that the fluid is in the regime of free-molecular flow and the remote pressure in the receiver vessal is a perfect vacuum. If one adjusts this formula to allow for nonzero (but unequal) pressures on the two sides of the hole one can arrange the result into a formula having the structure of (3.2) with the conductivity, C_L, given by

$$C_L = \frac{d_L^2}{4} \sqrt{\frac{\pi k T_L}{2m}} , \tag{3.3}$$

(see also O'HANLON, 1989, equations 2.2 and 3.19) [3].

4. ALGEBRAIC SOLUTION FOR THE OUTLET PRESSURES

As stated in §**1** above suppose that the parameters listed in (1.1) are given and that the two pressures p_R and p_{L2} are sought. One begins by noting that all of the terms in the right member of equation (3.3) are in the list (1.1) of given data. Thus, *the conductivity, C_L, of the orifice in the top left of Fig. 1.1 is known.*

Note that the reciprocal of (3.2) is

$$\frac{1}{C_L} = \frac{p_R - p_{L2}}{Q_L} = \frac{p_R}{Q_L} - \frac{p_{L2}}{Q_L} \tag{4.1}$$

But the general identity $Q := pS$ implies that $p = Q/S$. In particular, $p_R = Q_L/S_{L1}$ and $p_{L2} = Q_L/S_{L2}$. The outermost equality in (4.1) thus becomes

$$\frac{1}{C_L} = \frac{Q_L}{S_{L1}} \frac{1}{Q_L} - \frac{Q_L}{S_{L2}} \frac{1}{Q_L}$$

or

$$\frac{1}{C_L} = \frac{1}{S_{L1}} - \frac{1}{S_{L2}} \tag{4.2}$$

after simplification. One may arrange this result in the equivalent form

$$\frac{1}{S_{L1}} = \frac{1}{C_L} + \frac{1}{S_{L2}} .$$

The reciprocal of this equation is

$$S_{L1} = \frac{1}{\dfrac{1}{C_L} + \dfrac{1}{S_{L2}}} .$$

If one multiplies the right member by $1 = C_L/C_L$ one gets

$$S_{L1} = \frac{C_L}{1 + \dfrac{C_L}{S_{L2}}} . \tag{4.3}$$

Now (2.13) asserts that $S_{s2} = S_R + S_{L1}$. One concludes from (4.3) that

$$\begin{aligned} S_{s2} &= S_R + \frac{C_L}{1 + \dfrac{C_L}{S_{L2}}} \\ &= S_R \left[1 + \frac{\dfrac{C_L}{S_R}}{1 + \dfrac{C_L}{S_{L2}}} \right] , \end{aligned}$$

or

$$S_{s2} = S_R \left[\frac{1 + \dfrac{C_L}{S_{L2}} + \dfrac{C_L}{S_R}}{1 + \dfrac{C_L}{S_{L2}}} \right] . \qquad (4.4)$$

Recall that the general identity $Q := pS$ implies that $p = Q/S$. In particular, $p_R = Q_s/S_{s2}$. If one eliminates S_{s2} from the identity $p_R = Q_s/S_{s2}$ by means of (4.4) one gets

$$p_R = \frac{Q_s}{S_R} \left[\frac{1 + \dfrac{C_L}{S_{L2}}}{1 + \dfrac{C_L}{S_{L2}} + \dfrac{C_L}{S_R}} \right] \qquad (4.5)$$

To calculate the pressure, p_{L2}, downstream of the top left orifice in Fig. 1.1 one applies equation (2.10) at two stations of the top left branch of the Tee, namely stations upstream and downstream of the top left orifice. The result is

$$p_{L2} S_{L2} = Q_L = p_R S_{L1} . \qquad (4.6)$$

If one divides through by S_{L2} the outermost equality becomes

$$p_{L2} = \frac{S_{L1}}{S_{L2}} p_R .$$

If one eliminates S_{L1} from this identity by means of (4.3) one gets

$$p_{L2} = \frac{1}{S_{L2}} \left(\frac{C_L}{1 + \dfrac{C_L}{S_{L2}}} \right) p_R$$

or

$$p_{L2} = \left(\frac{\dfrac{C_L}{S_{L2}}}{1 + \dfrac{C_L}{S_{L2}}} \right) p_R .$$

If one eliminates p_R by means of (4.5) one gets

$$p_{L2} = \left(\frac{\dfrac{C_L}{S_{L2}}}{1 + \dfrac{C_L}{S_{L2}}} \right) \frac{Q_s}{S_R} \left[\frac{1 + \dfrac{C_L}{S_{L2}}}{1 + \dfrac{C_L}{S_{L2}} + \dfrac{C_L}{S_R}} \right] .$$

or

$$p_{L2} = \frac{C_L}{S_{L2}} \frac{Q_s}{S_R} \frac{1}{\left(1 + \dfrac{C_L}{S_{L2}} + \dfrac{C_L}{S_R}\right)} . \qquad (4.7)$$

In summary, equations (4.5) and (4.7) [with Q_s and C_L determined by (3.1) and (3.3), respectively] fulfill the stated purpose of finding the two outlet pressures in terms of the given data listed in (1.1) above.

Having taken the trouble to calculate p_R, one may as well write down the corresponding formula for the leak rate into the analyzer cell, namely the pressure-volume throughput, Q_L, though the top left orifice in Fig. 1.1. Thus, from (4.6), we have $Q_L = p_{L2} S_{L2}$ so (4.7) implies that

$$Q_L = \frac{C_L}{S_R} \frac{Q_s}{\left(1 + \dfrac{C_L}{S_{L2}} + \dfrac{C_L}{S_R}\right)} , \qquad (4.8)$$

5. NUMERICAL EXAMPLE

If one carries out an appropriate set of unit conversions, then equation (3.1) yields, under the foregoing assumptions, the following values for the pressure-volume throughput, Q_s, into the Tee from the supply line:

$$Q_s = \begin{cases} 12.2744 \ \text{atm(cm)}^3/\text{s} & \text{for } He, \\ 4.37454 \ \text{atm(cm)}^3/\text{s} & \text{for } N_2. \end{cases} \qquad (5.1)$$

Similarly, equation (3.3) yields, under the foregoing assumptions, the following values for the conductivity C_L, of the top left orifice in Fig 1:

$$C_L = \begin{cases} 0.633135 \ (\text{cm})^3/\text{s} & \text{for } He, \\ 0.239323 \ (\text{cm})^3/\text{s} & \text{for } N_2. \end{cases} \qquad (5.2)$$

Continuing in the same vein equation (4.7) yields, under the foregoing assumptions, the following values for the pressure, p_{L2}, downstream of the top left orifice in Fig. 1.1 (*i.e.* into the analyzer cell):

$$p_{L2} = \begin{cases} 7.11469 \times 10^{-5} \ \text{Torr} & \text{for } He, \\ 6.39054 \times 10^{-6} \ \text{Torr} & \text{for } N_2. \end{cases} \qquad (5.3)$$

Likewise equation (4.5) yields, under the foregoing assumptions, the following values for the pressure, p_R, in the interior of the Tee in Fig. 1.1 (*i.e.* into the input port of the roughing pump):

$$p_R = \begin{cases} 2.24752 \ \text{Torr} & \text{for } He, \\ 0.801084 \ \text{Torr} & \text{for } N_2. \end{cases} \qquad (5.4)$$

Finally, equation (4.8) yields, under the foregoing assumptions, the following values for the pressure-volume throughput, Q_L, through the top left orifice of the Tee in Fig. 1.1 (this quantity represents the leak rate into the analyzer cell) :

$$Q_L = \begin{cases} 1.87085 \times 10^{-3}\ \mathrm{atm(cm)^3/s} & \text{for } He, \\ 2.52064 \times 10^{-4}\ \mathrm{atm(cm)^3/s} & \text{for } N_2. \end{cases} \tag{5.5}$$

6. DESIGN CHARTS

I have presented several of the more useful results in §4 so as to isolate, wherever possible, expressions such as

$$1 + \frac{C_L}{S_{L_2}} \tag{6.1}$$

or

$$1 + \frac{C_L}{S_R} . \tag{6.2}$$

Note from the numerical examples of §**5** that C_L is typically of the order of one $(\mathrm{cm})^3$/sec or smaller while S_{L_2} is of the order of tens of L/s. Thus, C_L/S_{L_2} is typically of the order of 10^{-4} and thus small compared to one in the present examples. Furthermore, S_R is on the order of a few L/s, so C_L/S_R is again small compared to one in the present examples. To an engineering approximation one may thus approximate expressions such as (6.1) and (6.2) by one.

When one makes an approximation of the sort described in the last paragraph, one finds that the pressure p_R in the intermediate space between the two orifices in Fig. 1.1 is effectively independent of the diameter d_{L_2} of the orifice in the to left branch of the Tee and of the pumping speed S_{L_2} of the pump downstream of it. In other words, *to an engineering approximation* (with an error on the order of one tenth of one percent) *the pressure p_R depends upon only the pumping speed S_R of the foreline pump and the diameter d_s of the supply orifice.*

I have applied the algorithm described in the last section to a set of input parameters significantly larger than the one summarized in §**1** above. Expecting that the results for p_R to be effectively independent of d_{L_2} and S_{L_2} I arbitrarily took the values of those parameters to be their nominal ones for the present version of the HGS2000 system (*cf.* §**1** above) while retaining the accurate versions of the equations of §**4** (*i.e. not* approximating expressions such as (6.1) and (6.2) by one). The result of such a set of calculations is shown in Fig. 6.1 nearby.

In order not to limit the results needlessly, I carried out a calculation of the *ratio* p_{L_2}/p_R as a function of the parameters S_{L_2} and d_L. Figures 6.1 and 6.2 thus enable one to estimate the value of, say, p_{L_2} as a function of the *four* variables

$$S_R \quad , \quad d_s \quad , \quad S_{L_2} \quad , \quad d_L$$

in terms of data presented in just two sets of log-log plots.

7. RECOMMENDATIONS

It may happen that a mass-spectrometer gas analyzer that features a two-orifice pressure-reduction inlet may perform well for a given set of orifice diameter when Nitrogen is the background gas but less well when Helium is in the background. Adjustment of the orifice sizes may improve the performance of the system when Helium is the background gas but have the unintended consequence of degrading the performance when Nitrogen is again in the background.

One engineering approach that may enable one to get the best of both worlds is to incorporate two distinct pairs of orifices of which only one is active for a given background gas (the inactive pair being isolated by electronically controlled valves). I would thus recommend the consideration of such an option should it happen that there is no combination of orifice sizes that leads to satisfactory performance of the HGS2000 system for both a Nitrogen and a Helium background.

I have not discussed the application of the design charts 6.1 and 6.2 to the estimation of the partial pressures of trace gases in a background. I will simply assert that the charts in Fig. 6.2 were constructed under the assumption that the flow through the second orifice (in the top left branch of Fig. 1.1) is in the free-molecule flow regime. They should thus apply with equal validity when p_R represents the *partial* pressure of a tracer gas. The present model of the flow through first, orifice, by contrast, presumes that the flow there is in the continuum flow regime. The value of the effective molecular weight (and the ratio γ of specific heats) of fluid through it must, therefore be that of the background gas.

8. ACKNOWLEDGEMENTS

This summer is the eighth in which I have worked as a NASA/ASEE Summer Faculty Research Fellow and the sixth in which Ric Adams has been my

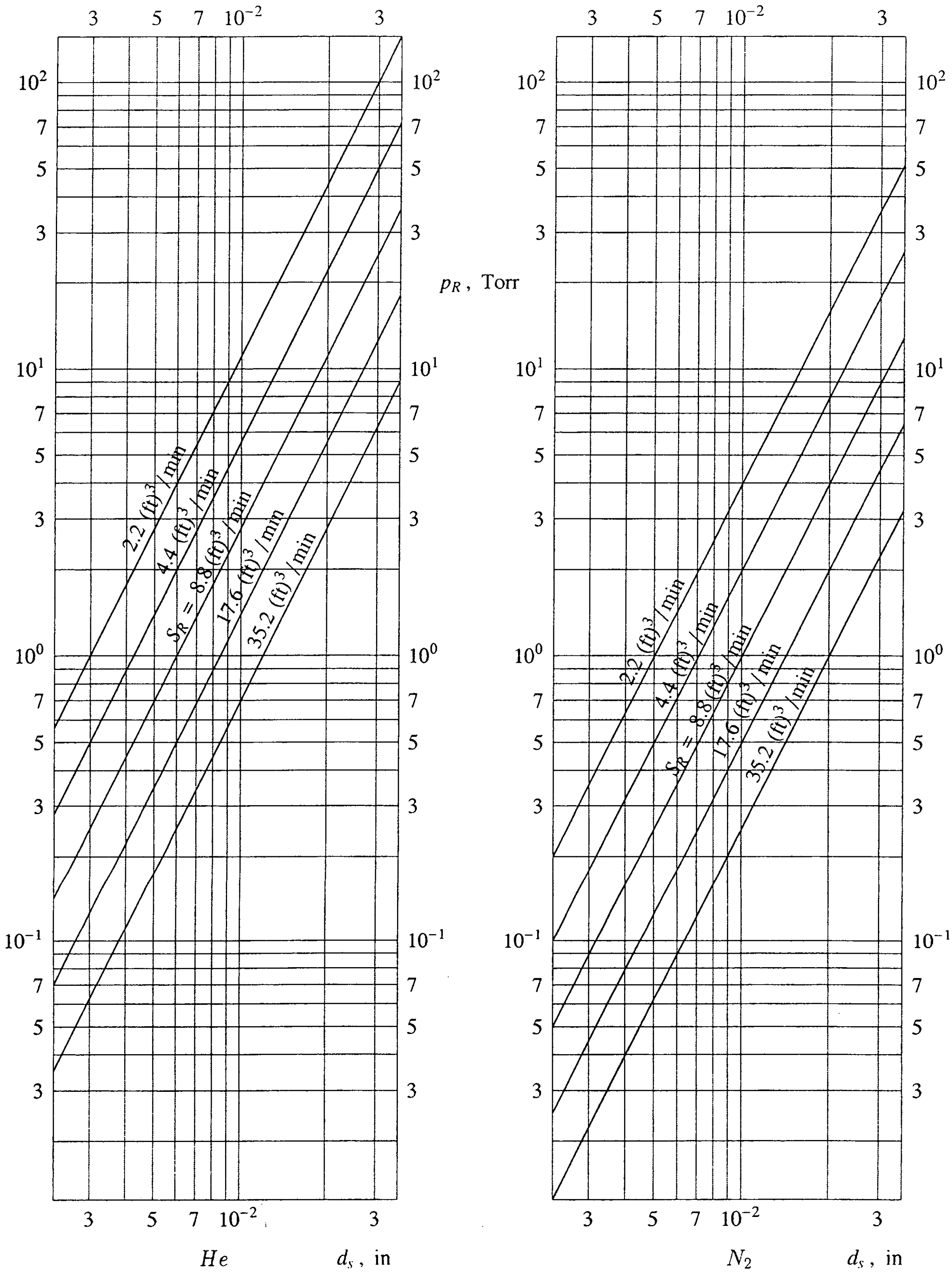

Fig. 6.1 Pressure in the intermediate portion of the Tee in Fig. 1.1.

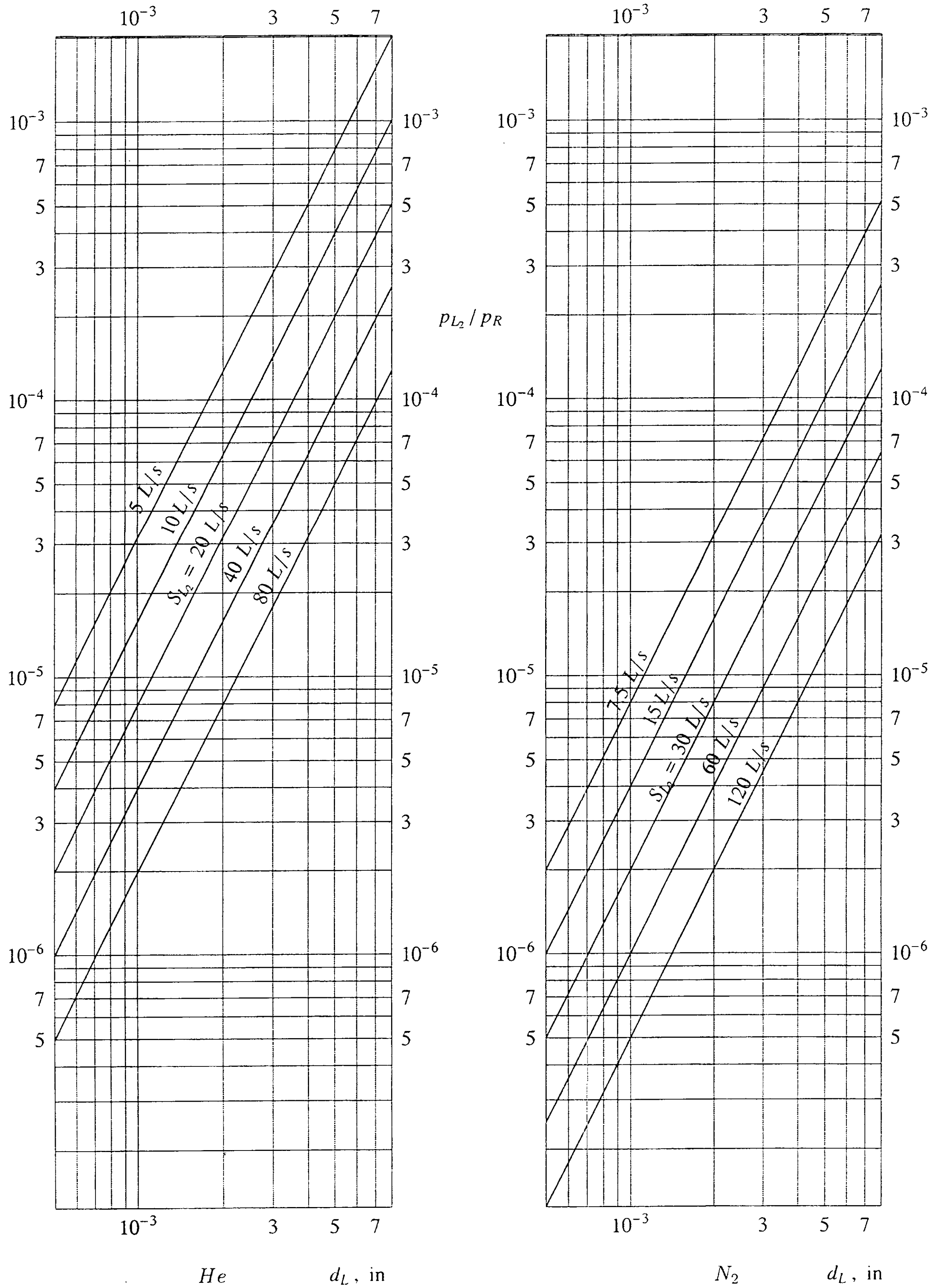

Fig. 6.2 Pressure downstream of the second (*i.e.* top left) orifice in the Tee in Fig. 1.1.

NASA colleague. I am, as so many times before, indebted to him for suggesting the present problem and providing much material and moral support. I am also indebted to his NASA supervisor, BILL HELMS, to his NASA colleague DUKE FOLLESTEIN, to Drs. TIM GRIFFEN and RICHARD ARKIN and Mr. GUY NAYLOR of DYNACS and, of course, to Dr. RAY HOSLER and CASSIE SPEARS of the Unviersity of Central Florida for their able administration of the Summer Faculty Fellow Program.

REFERENCES

[1] CHAPMAN, S. & COWLING, T.G. *The mathematical theory of non-uniform gases.* Third edition, Cambridge University Press, 1970

[2] LIEPMANN, H.W. & ROSHKO, A. *Elements of gasdynamics.* John Wiles & Sons, 1957.

[3] O'HANLON, JOHN F. *A user's guide to vacuum technology.* Second edition, Wiley-Interscience, 1989.

[4] JEANS, SIR JAMES *An introduction to the kinetic theory of gases.* Cambridge, 1940.