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Prepared for  
Photonics West  
sponsored by the International Society for Optical Engineering  
San Jose, California, January 19–25, 2002

National Aeronautics and  
Space Administration

Glenn Research Center

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# Polarimetric Glucose Sensing Using Brewster Reflection off of Eye Lens: Theoretical Analysis

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## ABSTRACT

An important task of in vivo polarimetric glucose sensing is to find an appropriate way to optically access the aqueous humor of the human eye. In this paper two different approaches are analyzed theoretically and applied to the eye model of Le Grand. First approach is the tangential path of Coté *et al.*, and the second is a new scheme of this paper of applying Brewster reflection off the eye lens.

## 1. INTRODUCTION

The non-invasive detection of blood glucose levels in humans is an ambitious goal for managing diabetes. Diabetes is the fourth leading cause of mortality in the United States. Diabetes can lead to severe complications over time. These can include blindness, renal and cardiovascular diseases, peripheral neuropathy associated with limb. The poor blood circulation in lower extremities of the body can lead to gangrene and subsequent amputation. Over 100 million diabetics world wide could greatly benefit by better managing their diabetes if their glucose levels can be monitored frequently and non-invasively without the pain and inconvenience of presently used fingerstick blood tests or implants.

Several non-invasive optical techniques have the potential of avoiding the disadvantages of the standard fingerstick method, and thus enable a more frequent determination of glucose levels. These methods are primarily based upon the measurement of the glucose concentration in the aqueous humor using polarimetry [2-3], dermal, epidermal, interstitial fluid, and sweat using absorption spectroscopy. Test sites being explored include eye, fingertips, cuticle, finger web, forearm and ear lobe. However, many hurdles remain before these products reach the commercial marketplace.

Glucose concentration in the aqueous humor closely mimics glucose levels in the blood [4-5]. Polarimetric techniques use the property of glucose as an optically active analyte, that rotates the plane of polarization of incident linearly polarized light in accordance with the law [2-5]:

$$\alpha = [\alpha]_{\lambda, pH}^T \cdot l \cdot c, \quad (1)$$

where  $[\alpha]_{\lambda, pH}^T$  is the specific rotation of glucose,  $l$  the optical pathlength through the sample and  $c$  the glucose concentration in the sample. Thus, knowing the pathlength  $l$  inside the sample, the glucose concentration  $c$  can be calculated by observing the rotation  $\alpha$ , that the glucose molecules induce to linearly polarized light. Previously, such a measurement was performed by passing a beam of linearly polarized light crosswise (tangentially) through the aqueous and by observing  $\alpha$  on the other end [1-2]. To best of our knowledge, to this date, no commercial device exploiting this simple optical working principle has been realized. We believe the major reason for this is that the proposed optical path plays a critical role due to the limits imposed by the shape of the cornea.

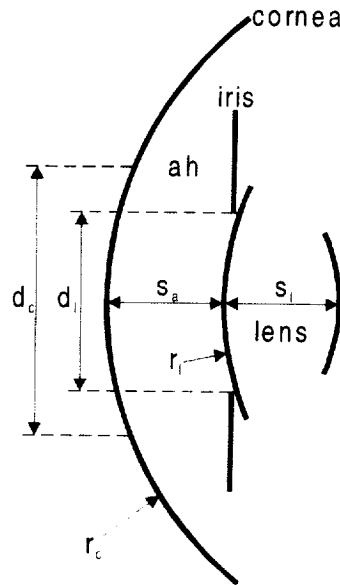
In this paper two different approaches for optically accessing the aqueous humor are analyzed theoretically with respect to their geometrical properties of light propagation and applying them to the eye model of Le Grand [6]. First, the tangential optical path proposed by Coté *et al.* [1-2] is investigated. Second, the new optical scheme using Brewster reflection off the lens will be introduced.

## 2. THEORETICAL ANALYSIS

The beam path of light propagation inside the eye is calculated applying the Snell's law of refraction for transitions between the media of different refractive indices. As the ocular media are assumed to be homogeneous and isotropic the calculation of the beam path is reduced to simple geometric considerations.

### 2.1. Eye-model

In the present investigation the eye model of Le Grand is applied [6]. The essential properties of this model are given in Table 1 and Fig. 1.



**Figure 1:** Eye-model of Le Grand; ah=aqueous humor.

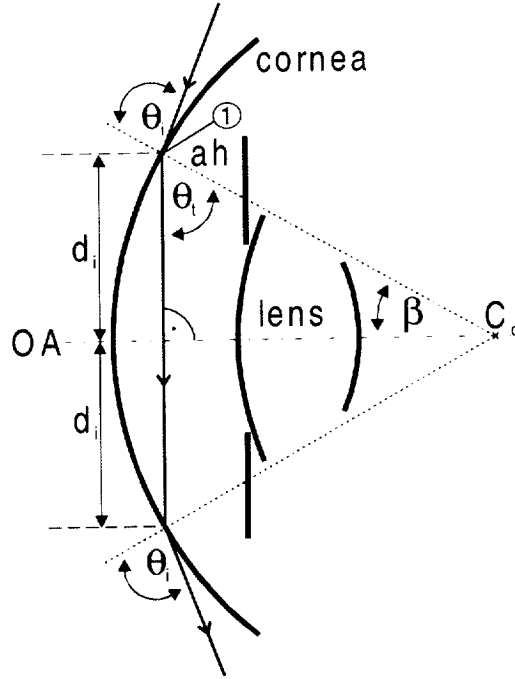
**Table 1:** Parameters of Le Grand eye-model.

Medium	Front diameter [mm]	Radius [mm]	Spacing [mm]	Refractive index
Cornea	$d_c=10$	$r_c=8$	—	$n_c=1.336$
Aqueous	—	—	$s_a=5.7763-6.3740$	$n_h=1.336$
Lens (ant.)	$d_l=2-8$	$r_l=10.2$	$s_l=4$	$n_l=1.4208$

As shown in Fig. 1, in this model the optical system of the eye consists of three centered, spherical refracting surfaces. Thus, the optical system shows rotational symmetry. Cornea and aqueous humor are assumed to have equal refractive indices with an effective value that accounts for this simplification. Therefore, there is only one effective refracting surface between air and aqueous humor. All ocular media are assumed to be homogeneous and isotropic. Thus, the linear birefringence of the cornea will be left out of consideration. This means no limitation for the validity of the present study as this effect can be discriminated from the circular birefringence of the glucose through an accurate analysis of the polarization state of the output beam [7]. Moreover, the shell structure of the lens is neglected and an average refractive index is considered. These assumptions provide us with a relatively simple optical system for the calculations.

## 2.2. Tangential path

In this approach, incident linearly polarized light is passed tangentially through the aqueous humor, as is shown in Fig. 2.



**Figure 2:** Tangential optical path: The incident light (1) is linearly polarized. Abbreviations: OA=optical axis, ah=aqueous humor,  $C_c$ =center point of sphere of cornea.

We calculate the minimum distance of the incidence point (1) from the optical axis (OA),  $d_i$ , that is necessary to achieve the tangential path. The incident beam at the air-aqueous humor interface has to meet the condition to be smaller than  $90^\circ$  with respect to the normal to the surface at the intersection point:

$$\theta_i < 90^\circ \quad (2)$$

Although the condition  $\theta_i=90^\circ$  is not practicable because the beam would not enter the eye we use this condition as the limit for the applicability of the present approach.

Applying the Snell's law for this boundary condition we get the refraction angle of the transmitted beam:

$$\theta_t = \arcsin\left(\frac{1}{n_h}\right) \quad (3)$$

with:  $\theta_i = 90^\circ$ ,

where the refractive index of air is set to unity.

The minimum distance  $d_i$  of the incidence point from the optical axis is then given by:

$$d_i = r_c \cdot \sin(\beta) , \quad (4)$$

where  $\beta$  is defined as:

$$\beta = 90^\circ - \theta_t . \quad (5)$$

Thus, using Eq. (3) and the values given in Table 1,  $d_i$  can be calculated to:

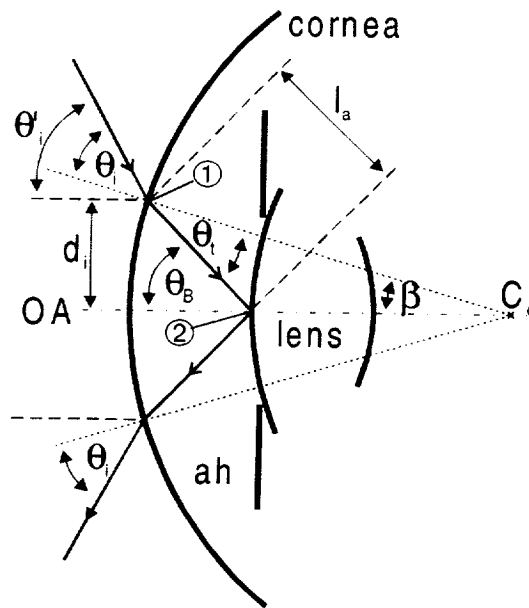
$$d_i = r_c \cdot \sin(90^\circ - \theta_t) = 5.305 \text{ mm} . \quad (6)$$

As  $\theta_i$  has to meet  $\theta_i < 90^\circ$ , it follows due to the Snell's law that  $\theta_i$  amounts to  $\theta_i < \arcsin(1/n_h)$  and thus, the value for the distance of the incidence point from the optical axis yields  $d_i > 5.3\text{mm}$ .

However, as reported in Table 1, the average human eye exposes only a diameter of  $d_c=10\text{mm}$  of its surface for optical access [6]. Thus, for this eye-model, the tangential path is theoretically not achievable.

### 2.3. Brewster reflection off the lens

As shown in Fig. 3, we propose a novel approach to access the aqueous humor of the eye. Part of the incident beam is reflected off the lens at the Brewster's angle yielding a purely linearly polarized reflected beam (2), orientated perpendicularly to the plane of refraction (paper plane). On its way out of the eye the polarization state of this beam is rotated by the glucose in the aqueous humor and thus carries the concentration information. Notice, that the propagation path shows symmetry with respect to OA.



**Figure 3:** Brewster reflection off the lens. Abbreviations: OA=optical axis, ah=aqueous humor,  $C_c$ =center point of sphere of cornea.

Referring to Figs. 1 and 3 we consider the triangle given by the corner points at position (1), (2) and  $C_c$ . The cosine rule, applied to this triangle, delivers a second order equation for  $l_a$ , which can be solved as:

$$l_a = -(r_c - s_a) \cos(\theta_B) + \sqrt{(r_c - s_a)^2 \cos^2(\theta_B) - s_a^2 + 2s_a r_c} = 6.496 \text{ mm} . \quad (7)$$

This leads directly to:

$$d_i = l_a \cdot \sin(\theta_B) = 4.73 \text{ mm} . \quad (8)$$

Applying simple geometrical relations, the entrance angle  $\theta_i'$  can be calculated by the following steps:

$$\begin{aligned} \beta &= \arcsin\left(\frac{d_i}{r_c}\right) = 36.27^\circ \\ \theta_i &= \theta_B - \beta = 10.49^\circ \\ \theta_i &= \arcsin(n_h \cdot \sin(\theta_i)) = 14.08^\circ \\ \theta_i' &= \theta_i + \beta = 50.35^\circ . \end{aligned} \quad (9)$$

In contrast to the approach discussed in 2.2, within the limits of this eye model, the anterior diameter  $d_c$  of the cornea is sufficient to accept a beam entering at distance  $d_i$  from the ocular axis. The entrance angle  $\theta_i'$  is, as well, not critical to be achieved.

### 3. CONCLUSIONS

A detailed geometrical analysis of two different schemes for optically accessing the aqueous humor of the human eye for in vivo polarimetric glucose measurements is presented. The newly proposed optical path using Brewster reflection off the lens is demonstrated to be theoretically applicable. In conclusion, Brewster reflection off the lens has the potential to be the future instrumental arrangement for in vivo polarimetric glucose sensing. This work is in progress.

### 4. REFERENCES

1. G.L. Coté, M.D. Fox, R.B. Northrop. "Noninvasive Optical Polarimetric Glucose Sensing using a True Phase Measurement Technique," IEEE Transactions on Biomedical Engineering, Vol. 39, No. 7, pp. 752–756, 1992.
2. B.D. Cameron, G.L. Coté, "Noninvasive Glucose Sensing Utilizing a Digital Closed-Loop Polarimetric Approach," IEEE Transactions on Biomedical Engineering, Vol. 44, No. 12, pp. 1221–1227, 1997.
3. D.P. Hutchinson, "Personal glucose monitor," United States Patent 4 901 728, 1990.
4. R.J. McNichols and G.L. Coté, "Optical Glucose Sensing in Biological Fluids: An Overview," J. Biomedical Optics, 5(1):5–16. Review, Jan. 2000.
5. C.A. Browne and F.W. Zerban. "Physical and Chemical Methods of Sugar Analysis," New York, London: Chapman & Hill, 1941.
6. Y. Le Grand and S.G. El Hage, Physiological Optics (Springer-Verlag, Berlin, 1980).
7. L. Rovati and R. Ansari, Optics letter, submitted 2001.

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE March 2002		3. REPORT TYPE AND DATES COVERED Technical Memorandum
4. TITLE AND SUBTITLE  Polarimetric Glucose Sensing Using Brewster Reflection off of Eye Lens: Theoretical Analysis			5. FUNDING NUMBERS  WU-101-51-00-00	
6. AUTHOR(S)  Stefan Böckle, Luigi Rovati, and Rafat R. Ansari				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  National Aeronautics and Space Administration John H. Glenn Research Center at Lewis Field Cleveland, Ohio 44135-3191			8. PERFORMING ORGANIZATION REPORT NUMBER  E-13172	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)  National Aeronautics and Space Administration Washington, DC 20546-0001			10. SPONSORING/MONITORING AGENCY REPORT NUMBER  NASA TM-2002-211354 SPIE 4624-24	
11. SUPPLEMENTARY NOTES  Prepared for Photonics West sponsored by the International Society for Optical Engineering, San Jose, California, January 19-25, 2002. Stefan Böckle and Luigi Rovati, University of Brescia, Department of Electronics for the Automation, via Branze 38, I-25123, Brescia, Italy; and Rafat R. Ansari, NASA Glenn Research Center. Responsible person, Rafat R. Ansari, organization code 6712, 216-433-5008.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT  Unclassified - Unlimited Subject Category: 51  Available electronically at <a href="http://gltrs.grc.nasa.gov/GLTRS">http://gltrs.grc.nasa.gov/GLTRS</a> This publication is available from the NASA Center for AeroSpace Information, 301-621-0390.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  An important task of in vivo polarimetric glucose sensing is to find an appropriate way to optically access the aqueous humor of the human eye. In this paper two different approaches are analyzed theoretically and applied to the eye model of Le Grand. First approach is the tangential path of Coté, et al. (G.L. Coté, M.D. Fox, and R.B. Northrop: Noninvasive Optical Polarimetric Glucose Sensing Using a True Phase Measurement Technique. IEEE Transactions on Biomedical Engineering, vol. 39, no. 7, pp. 752-756, 1992.) and the second is a new scheme of this paper of applying Brewster reflection off the eye lens.				
14. SUBJECT TERMS  Non-invasive glucose sensor; Polarimetry; Diabetes			15. NUMBER OF PAGES  11	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT  Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE  Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT  Unclassified	20. LIMITATION OF ABSTRACT	