A NEW APPROXIMATE
CHIMERA DONOR CELL SEARCH ALGORITHM

4TH SYMPOSIUM ON OVERSET COMPOSITE GRID
& SOLUTION TECHNOLOGY
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PRESENTATION OUTLINE

OBJECTIVE/MOTIVATION

APPROACH
- > NUMERICAL APPROACH
- > DONOR CELL SEARCH ALGORITHM

TWO-ZONE ISOLATED WING RESULTS

THREE-ZONE WING/BODY RESULTS

CONCLUDING REMARKS

OBJECTIVE/MOTIVATION

OBJECTIVE:

> TO DEVELOP CHIMERA-BASED FULL POTENTIAL METHODOLOGY WHICH IS COMPATIBLE WITH OVERFLOW (EULER/NAVIER-STOKES CHIMERA FLOW SOLVER)

> TO DEVELOP A FAST DONOR CELL SEARCH ALGORITHM THAT IS COMPATIBLE WITH THE CHIMERA FULL POTENTIAL APPROACH

MOTIVATION:

> DESIRE TO SIGNIFICANTLY SPEED TURNAROUND TIME FOR AERODYNAMIC ANALYSIS AND DESIGN

> FULL-POTENTIAL+BL MUCH FASTER THAN NAVIER-STOKES (UP TO X100 FASTER)

> CHIMERA FULL POTENTIAL COULD USE EXISTING OVERFLOW INFRASTRUCTURE (GEOMETRY SETUP AND POST PROCESSING SOFTWARE)

> USER WILL HAVE FLOW SOLVER OPTION:
- FULL POTENTIAL
- EULER
- NAVIER-STOKES

(UTILIZE NAVIER-STOKES ONLY WHEN REQUIRED AND MORE APPROXIMATE FP APPROACH WHEN APPROPRIATE)

NUMERICAL APPROACH

> **GOVERNING EQUATIONS: CONSERVATIVE FULL POTENTIAL EQUATION MAPPED FROM PHYSICAL DOMAIN (X, Y, Z) TO COMPUTATIONAL DOMAIN (ξ, η, ζ)**

$$\xi = \xi(X,Y,Z)$$
$$\eta = \eta(X,Y,Z)$$
$$\zeta = \zeta(X,Y,Z)$$

> **SPATIAL DIFFERENCING SCHEME:**
- **SUBSONIC FLOW: CENTRAL DIFFERENCING (SECOND-ORDER ACCURATE)**
- **SUPERSONIC FLOW (TWO OPTIONS):**
  - **FIRST-ORDER UPWIND**
  - **SECOND-ORDER UPWIND USING A SOLUTION LIMITER**

> **ITERATION SCHEME: ZONE-BY-ZONE FULLY IMPLICIT AF2 SCHEME**

> **ZONAL CHARACTERISTICS:**
- **IBLANK ARRAY MODIFICATIONS MADE IN SPATIAL AND ITERATION SCHEMES AS APPROPRIATE**
- **STANDARD CHIMERA UPDATES AT ALL ZONAL INTERFACES (TRI-LINEAR INTERPOLATION ON THE VELOCITY POTENTIAL)**
- **DENSITY EXTRAPOLATED AT ZONAL INTERFACES**

GOVERNING EQUATIONS

CARTESIAN COORDINATES

$$(\rho\phi_x)_x + (\rho\phi_y)_y + (\rho\phi_z)_z = 0$$

$$\rho = \left[1 - \frac{\gamma - 1}{\gamma + 1}(\phi_x^2 + \phi_y^2 + \phi_z^2)\right]^{\frac{1}{\gamma - 1}}$$

TRANSFORMED COORDINATES $[\xi = \xi(X,Y,Z),\ \eta = \eta(X,Y,Z),\ \zeta = \zeta(X,Y,Z)]$

$$\left(\frac{\rho U}{J}\right)_\xi + \left(\frac{\rho V}{J}\right)_\eta + \left(\frac{\rho W}{J}\right)_\zeta = 0$$

$$\rho = \left[1 - \frac{\gamma - 1}{\gamma + 1}(U\phi_\xi + V\phi_\eta + W\phi_\zeta)\right]^{\frac{1}{\gamma - 1}}$$

CONTRAVARIANT VELOCITY COMPONENTS

$$U = A_1\phi_\xi + A_4\phi_\eta + A_5\phi_\zeta,\ V = A_4\phi_\xi + A_2\phi_\eta + A_6\phi_\zeta,\ W = A_5\phi_\xi + A_6\phi_\eta + A_3\phi_\zeta$$

METRIC QUANTITIES

$$A_1 = \nabla\xi \bullet \nabla\xi, \quad A_2 = \nabla\eta \bullet \nabla\eta, \quad A_3 = \nabla\zeta \bullet \nabla\zeta$$

$$A_4 = \nabla\xi \bullet \nabla\eta, \quad A_5 = \nabla\xi \bullet \nabla\zeta, \quad A_6 = \nabla\eta \bullet \nabla\zeta$$

$$J = \xi_x\eta_y\zeta_z + \xi_y\eta_z\zeta_x + \xi_z\eta_x\zeta_y - \xi_z\eta_y\zeta_x - \xi_y\eta_x\zeta_z - \xi_x\eta_z\zeta_y$$

SPATIAL DISCRETIZATION SCHEME

$$L\phi^n_{i,j,k} = \overleftarrow{\delta}_\xi\left(\frac{\tilde{\rho}U}{J}\right)^n_{i+1/2,j,k} + \overleftarrow{\delta}_\eta\left(\frac{\rho V}{J}\right)^n_{i,j+1/2,k} + \overleftarrow{\delta}_\zeta\left(\frac{\rho W}{J}\right)^n_{i,j,k+1/2} = 0$$

$$\begin{aligned}\tilde{\rho}_{i+1/2,j,k} = \rho_{i+1/2,j,k} - \nu_{i+1/2,j,k}[\rho_{i+1/2,j,k} - \rho_{i-1/2,j,k} \\ -\Psi_{i+1/2,j,k}(\rho_{i-1/2,j,k} - \rho_{i-3/2,j,k})]\end{aligned}$$

$$\nu_{i+1/2,j,k} = \begin{cases} 2.46625(2\rho^* - \rho_{i+1/2,j,k} - \rho_{i-1/2,j,k})C & \text{if} \quad \rho_{i,j,k} \le \rho^* \\ 0 & \text{if} \quad \rho_{i,j,k} > \rho^* \end{cases}$$

$$\Psi_{i+1/2,j,k} = \begin{cases} 1 - C_2\Delta & \text{if} \quad r_{i+1/2,j,k} \ge 0 \\ 0 & \text{if} \quad r_{i+1/2,j,k} < 0 \end{cases}$$

$$r_{i+1/2,j,k} = \frac{\rho_{i+1/2,j,k} - \rho_{i-1/2,j,k}}{\rho_{i-1/2,j,k} - \rho_{i-3/2,j,k}}$$

$$\Psi_{i+1/2,j,k} = \Psi_{i+1/2,j,1}(C_3)^{-(k-1)}$$

AF2 CHIMERA ITERATION SCHEME
(C-GRID TOPOLOGIES)

UPPER SURFACE (FLOW ALIGNED WITH ξ DIRECTION):

SWEEP 1: $$\boxed{(\alpha - IB_{i,j,k}\overrightarrow{\delta}_{\xi})f^n_{i,j,k} = IB_{i,j,k}\alpha\omega L\phi^n_{i,j,k}}$$

SWEEP 2: $$\boxed{(\alpha - IB_{i,j,k}\delta_{\eta\eta})g^n_{j,k} = f^n_{i,j,k} + IB_{i,j,k}\alpha^2 C^n_{i-1,j,k}}$$

SWEEP 3: $$\boxed{(\alpha - IB_{i,j,k}\delta_{\zeta\zeta})C^n_{i,j,k} = g^n_{j,k}}$$

LOWER SURFACE (FLOW ALIGNED WITH - ξ DIRECTION):

SWEEP 1 $$\boxed{(\alpha + IB_{i,j,k}\overleftarrow{\delta}_{\xi})f^n_{i,j,k} = IB_{i,j,k}\alpha\omega L\phi^n_{i,j,k}}$$

SWEEP 2: $$\boxed{(\alpha - IB_{i,j,k}\delta_{\eta\eta})g^n_{j,k} = f^n_{i,j,k} + IB_{i,j,k}\alpha^2 C^n_{i+1,j,k}}$$

SWEEP 3: $$\boxed{(\alpha - IB_{i,j,k}\delta_{\zeta\zeta})C^n_{i,j,k} = g^n_{j,k}}$$

CHIMERA INTERPOLATION SCHEME

SCHEME CHARACTERISTICS:

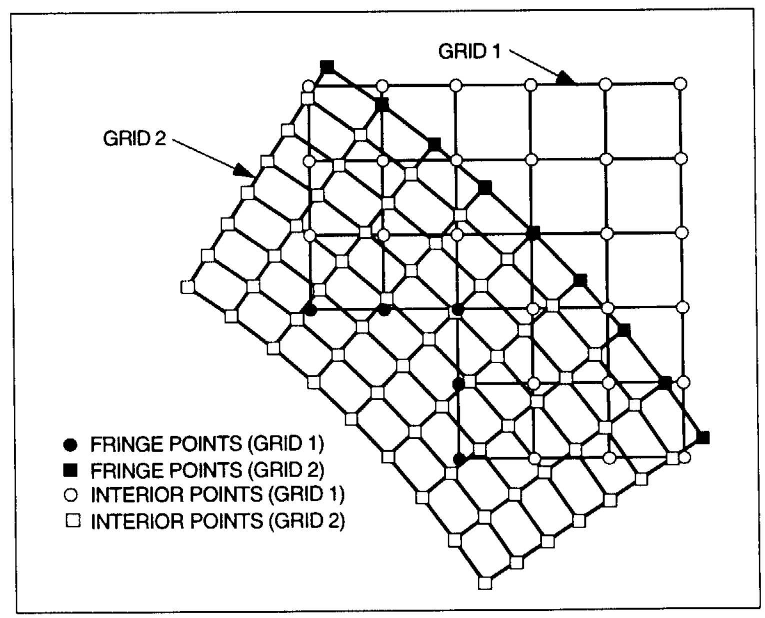
> TRI-LINEAR INTERPOLATION OF VELOCITY POTENTIAL USED TO OBTAIN FRINGE POINT VALUES

> IBLANK ARRAY USE IDENTICAL TO TRADITIONAL CHIMERA APPROACH

> SPECIAL DENSITY COMPUTATION REQUIRED AT FRINGE POINTS

> SPECIAL LOGIC REQUIRED WHEN PERFORMING DONOR CELL SEARCHES FOR FRINGE POINTS ON VORTEX SHEET

SCHEMATIC OF TYPICAL GRID-TO-GRID INTERFACE



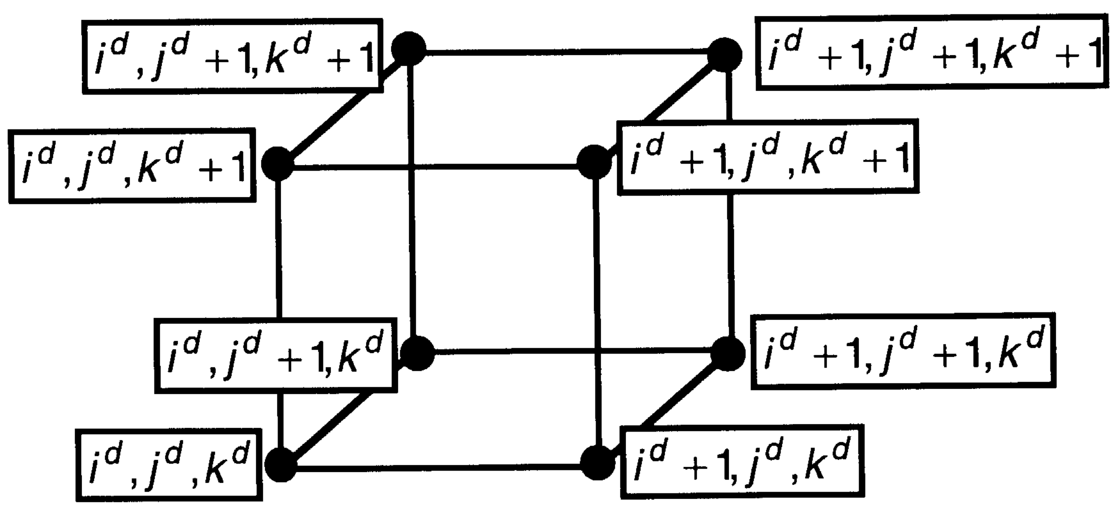
DONOR CELL SEARCH ALGORITHM
PROBLEM STATEMENT

GIVEN LOCATION OF ARBITRARY IGBP (INTERGRID BOUNDARY POINT)

$$x^r,\ y^r,\ z^r$$

FIND THE THREE INDICES i^d, j^d, k^d THAT DEFINE THE CELL SURROUNDING x^r, y^r, z^r

THIS "DONOR CELL" IS DEFINED BY THE x, y, z VALUES AT THE FOLLOWING 8 POINTS:



DONOR CELL SEARCH ALGORITHM
BASIC IDEA

RELATIONAL INFORMATION BETWEEN COMPUTATIONAL DOMAIN (ξ, η, ζ) **AND PHYSICAL DOMAIN** (x, y, z)

$$d\xi = \xi_x dx + \xi_y dy + \xi_z dz$$
$$d\eta = \eta_x dx + \eta_y dy + \eta_z dz$$
$$d\zeta = \zeta_x dx + \zeta_y dy + \zeta_z dz$$

USE NUMERICAL APPROXIMATION TO COMPUTE DONOR CELL LOCATION

$$i^d - i^n = \xi_x^n(x^r - x^n) + \xi_y^n(y^r - y^n) + \xi_z^n(z^r - z^n)$$
$$j^d - j^n = \eta_x^n(x^r - x^n) + \eta_y^n(y^r - y^n) + \eta_z^n(z^r - z^n)$$
$$k^d - k^n = \zeta_x^n(x^r - x^n) + \zeta_y^n(y^r - y^n) + \zeta_z^n(z^r - z^n)$$

WHERE ξ_x^n, ξ_y^n, **ETC. ARE STANDARD NUMERICALLY EVALUATED METRICS AND** x^n, y^n, z^n **IS AN ARBITRARY STARTING POINT IN THE DONOR GRID. TO IMPROVE ACCURACY THESE QUANTITIES ARE EVALUATED AT CELL CENTERS, I.E., AT** $i^n + 1/2, j^n + 1/2, k^n + 1/2$

DONOR CELL SEARCH ALGORITHM

ALTERNATE NOTATION

$$M^d = M^n + H^n(R^r - R^n)$$

WHERE

$$M = \begin{pmatrix} i \\ j \\ k \end{pmatrix}, \qquad H = \begin{pmatrix} \xi_x & \xi_y & \xi_z \\ \eta_x & \eta_y & \eta_z \\ \zeta_x & \zeta_y & \zeta_z \end{pmatrix}, \qquad R = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

MODIFICATION FOR INTEGER CHOPPING

$$M^d = M^n + H^n(R^r - R^n) + I_s^n$$

WHERE I_s^n **IS A COLUMN MATRIX DEFINED BY** $I_s^n = 0.5\,\mathrm{sgn}[H^n(R^r - R^n)]$

WHEN STARTING CELL x^n, y^n, z^n **IS FAR REMOVED FROM DESIRED DONOR CELL , MUST ITERATE**

$$M^{n+1} = M^n + H^n(R^r - R^n) + I_s^n, \quad n = 1, 2, \cdots, MAXIT$$

WHERE $MAXIT \sim 10$

DONOR CELL SEARCH ALGORITHM

GRID INDEX LIMITS:

if $M^{n+1} < 1$ then $M^{n+1} = 1$

if $M^{n+1} > M_{\max} - 1$ then $M^{n+1} = M_{\max} - 1$

DONOR CELL CONVERGENCE CRITERIA:

$$| H^n(R^r - R^n) + I_s^n | \leq 1.0 + \mathrm{TOL} \qquad \text{WHERE TOL~0.05}$$

NEAREST NEIGHBOR CONVERGENCE CRITERIA:

IF DONOR CELL CONVERGENCE CRITERIA IS NOT SATISFIED AND

$$n = \mathrm{MAXIT}$$

OR

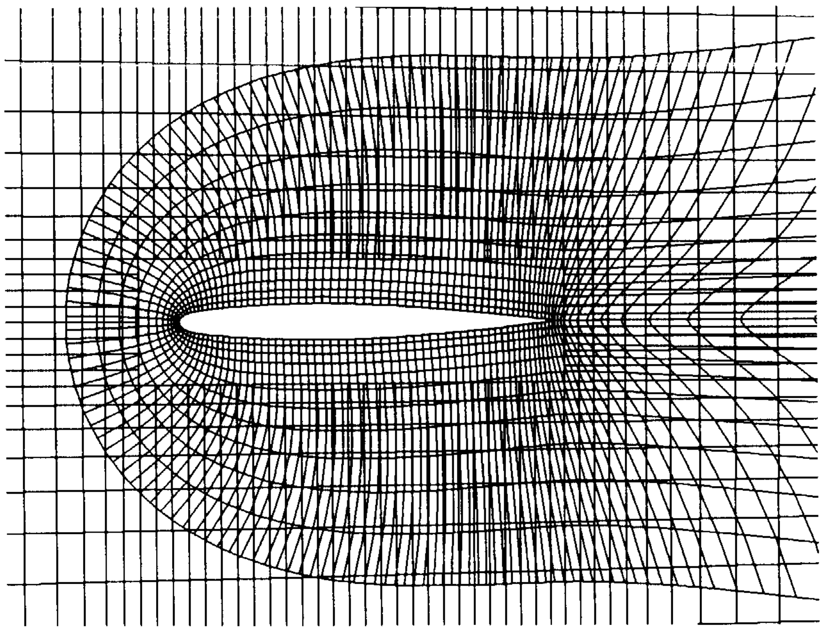
$$i^{n+1} = i^n, \quad j^{n+1} = j^n, \quad k^{n+1} = k^n$$

THEN AN ALTERNATE CRITERIA IS USED

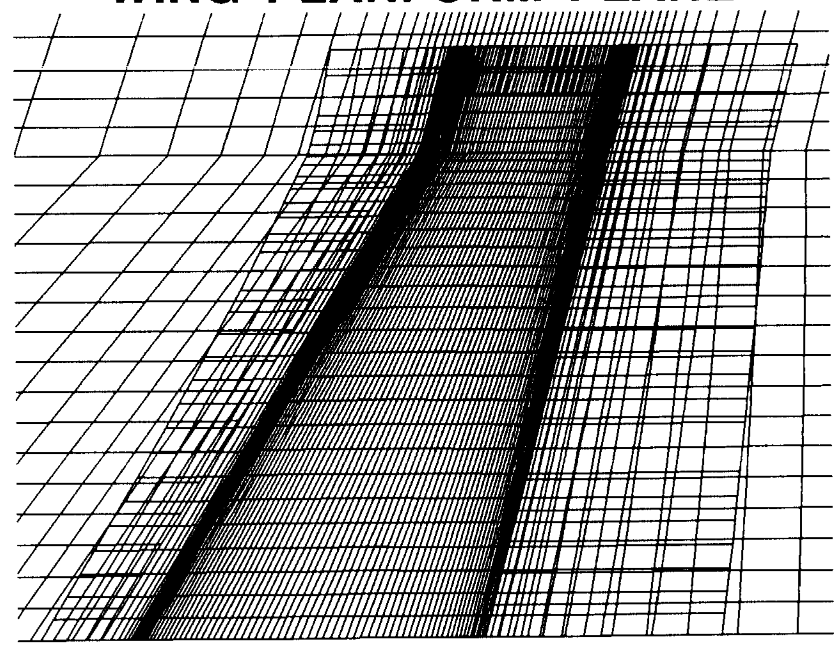
$$| H^n(R^r - R^n) + I_s^n | \leq 2.0$$

TWO-ZONE CHIMERA GRID ARRANGEMENT AROUND THE ONERA M6 WING

WING ROOT PLANE



WING PLANFORM PLANE



DONOR CELL SEARCH STATISTICS

ONERA M6 WING, $M_\infty = 0.84, \alpha = 3.06°$

FIRST LINE IN EACH TABLE FROM EXACT SEARCH ALGORITHM

FINE GRID: INNER GRID = 201X41X17, OUTER GRID = 109X49X74, IGBP=15299

TOL	NMISS	NNN	NAVG	CPU (sec)	MFLOPS	C_L	C_D
----	----	----	----	5.980	120	.2913	.01081
0.00	427	633	2.17	0.265	234	.2912	.01081
0.02	1131	18	1.83	0.253	244	.2912	.01081
0.10	3426	0	1.81	0.234	263	.2912	.01081
0.30	7991	0	1.77	0.243	254	.2912	.01080
1.00	10710	0	1.71	0.228	271	.2911	.01079

COARSE GRID: INNER GRID = 101X23X9, OUTER GRID = 55X25X38, IGBP=4259

TOL	NMISS	NNN	NAVG	CPU (sec)	MFLOPS	C_L	C_D
----	----	----	----	0.726	85	.2850	.01031
0.00	185	338	2.59	0.066	125	.2846	.01030
0.02	601	12	1.95	0.060	134	.2846	.01030
0.10	1795	0	1.90	0.056	143	.2845	.01029
0.30	2878	0	1.84	0.055	147	.2841	.01026
1.00	3448	0	1.77	0.052	154	.2844	.01026

NMISS - NUMBER OF IGBPs OUTSIDE THEIR DONOR CELLS AS DETERMINED BY EXACT SEARCH ALGORITHM

NNN - NUMBER OF NEAREST NEIGHBOR CELLS USED AS DONOR CELLS AS DETERMINED BY NEAREST NEIGHBOR TEST

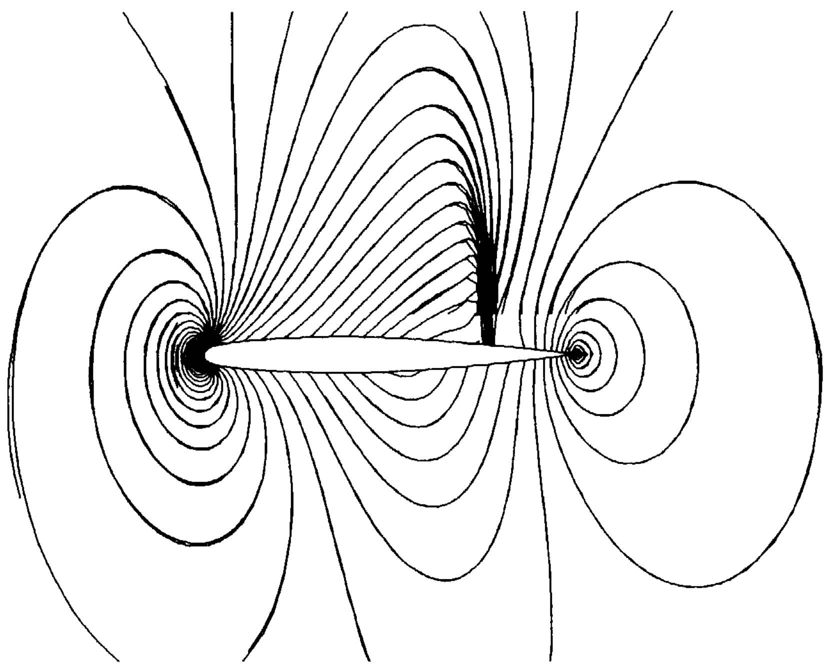
NAVG - NUMBER OF DONOR CELL SEARCH ITERATIONS PER IGBP

MACH NUMBER CONTOUR COMPARISONS

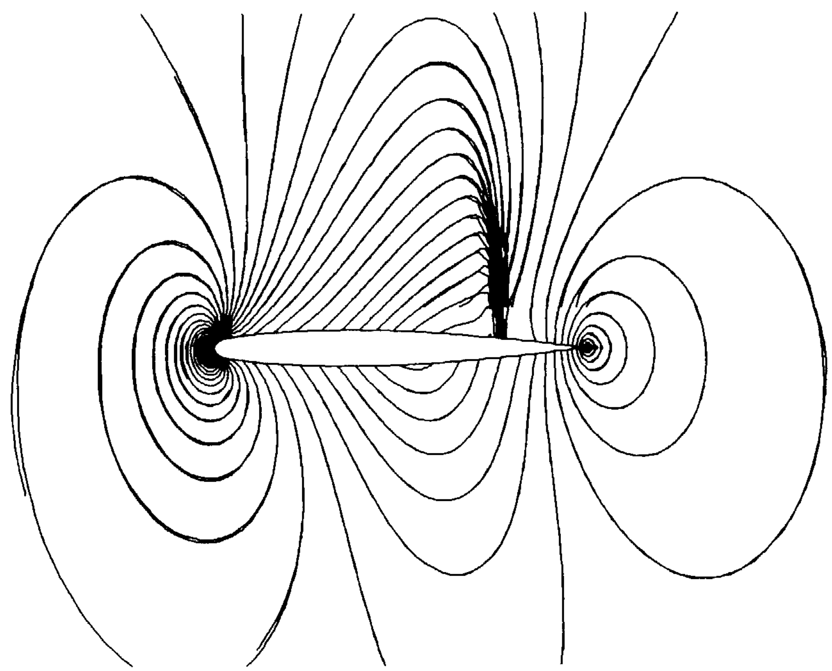
WING ROOT SYMMETRY PLANE

ONERA M6 WING, $M_\infty = 0.84, \alpha = 3.06°$

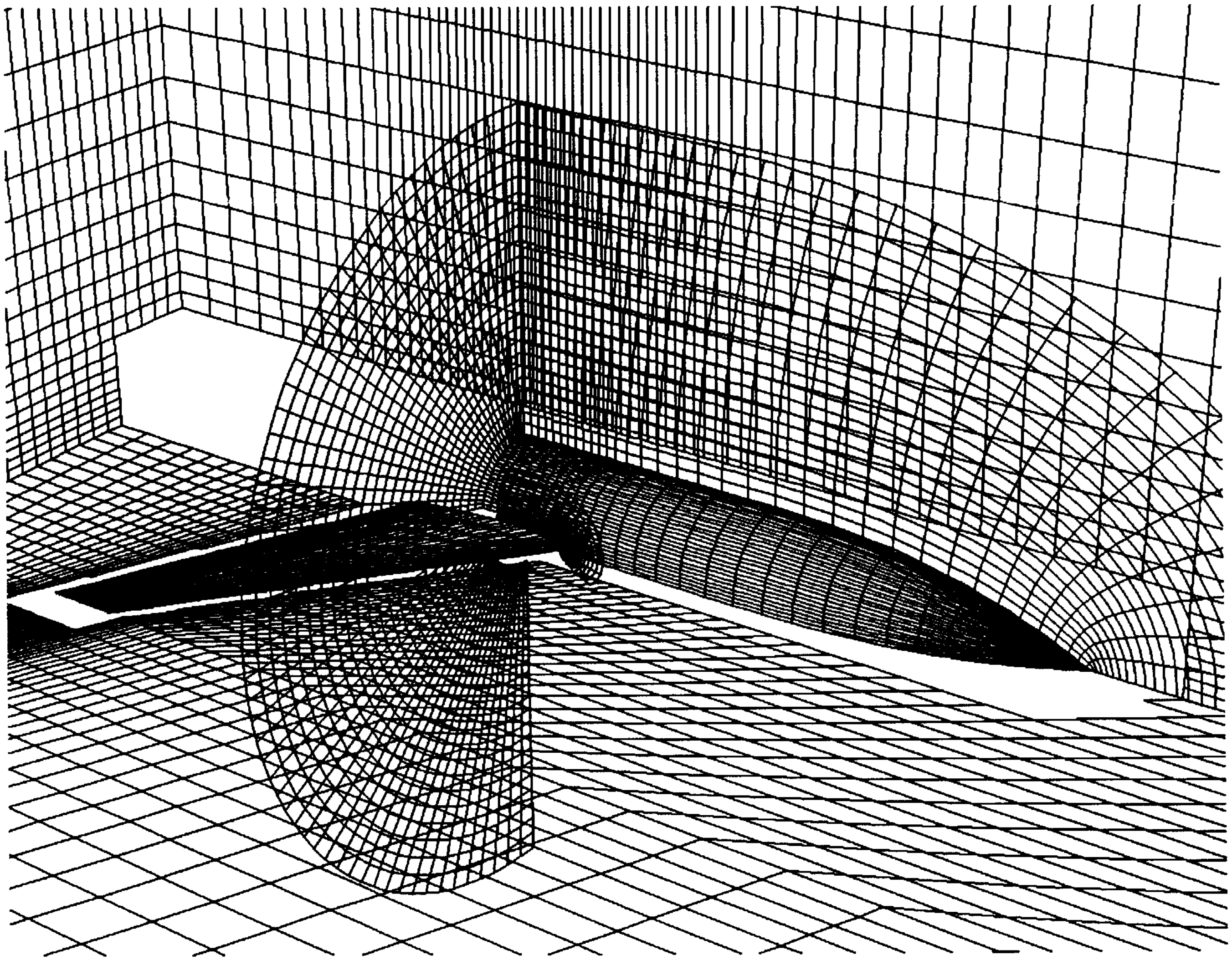
TOL =0.0



TOL=1.0



THREE-ZONE GRID ABOUT WING/BODY GEOMETRY
RAE WING A + CYLINDRICAL BODY B2



DONOR CELL SEARCH STATISTICS

GRID REFINEMENT STUDY
THREE GRID-ZONE CASE
RAE WING + B2 BODY
TOL=0.1

GRIDS			TOTAL	IGBP	NNN	NAVG	CPU	MFLOPS
GRID1	GRID2	GRID3	POINTS					
101X23X9	73X13X30	61X25X38	107390	7631	6	5.72	0.220	60
151X32X13	109X19X44	81X33X50	287590	15480	2	5.35	0.433	81
201X41X17	145X25X58	101X41X62	607089	26423	6	4.99	0.714	102

IGBP - INTERGRID BOUNDARY POINTS
NNN - NUMBER OF NEAREST NEIGHBOR CELLS USED AS DONORS
NAVG - NUMBER OF DONOR CELL SEARCH ITERATIONS PER IGBP
CPU - CPU TIME (SEC) ON A SINGLE-PROCESSOR CRAY C-90
MFLOPS - PROCESSING RATE (MILLION FLOATING POINT OPERATIONS PER SEC)

DONOR CELL SEARCH STATISTICS

RAE WING A WITH B2 FUSELAGE

THREE-ZONE GRID

WING GRID = 201X41X17, FUSELAGE GRID = 145X25X58, OUTER GRID = 101X41X62

IGBP = 26423

TOL	NNN	NAVG	CPU (sec)	MFLOPS
0.00	903	5.11	0.738	100
0.03	34	5.01	0.721	102
0.10	6	4.99	0.714	102
0.30	4	4.95	0.714	103
1.00	0	4.89	0.692	106

NNN - NUMBER OF NEAREST NEIGHBOR CELLS USED AS DONORS

NAVG - NUMBER OF DONOR CELL SEARCH ITERATIONS PER IGBP

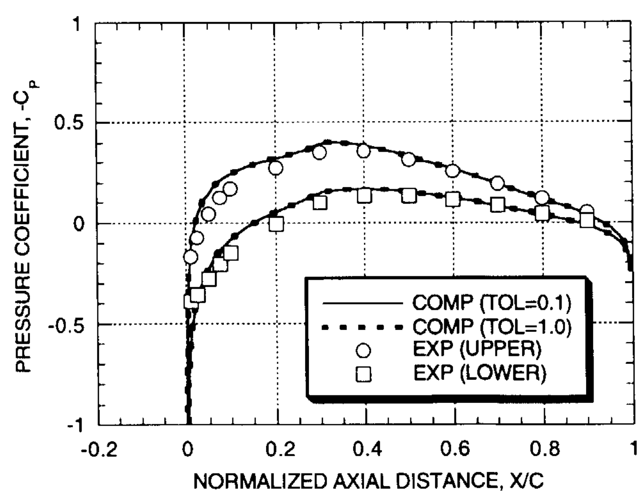
WING SURFACE PRESSURE COMPARISONS

RAE WING A + BODY B2

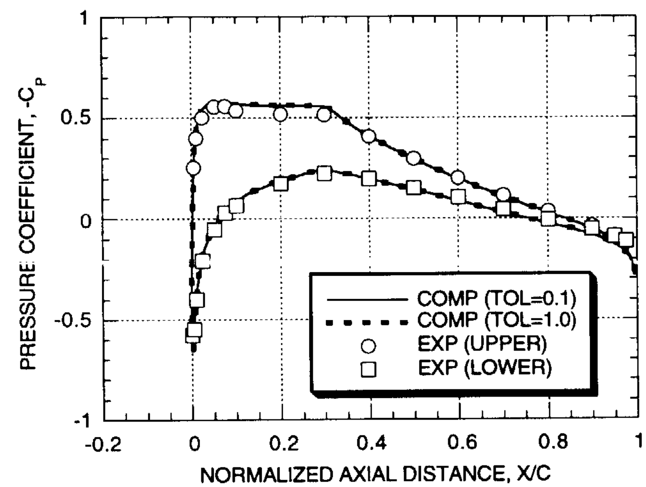
$M_\infty = 0.82, \alpha = 2°$

THREE-ZONE GRID
WING GRID = 201X41X17
FUSELAGE GRID = 145X25X58
OUTER GRID = 101X41X62
t_{RAVG} = 73 sec
t_{OH} = 17 sec

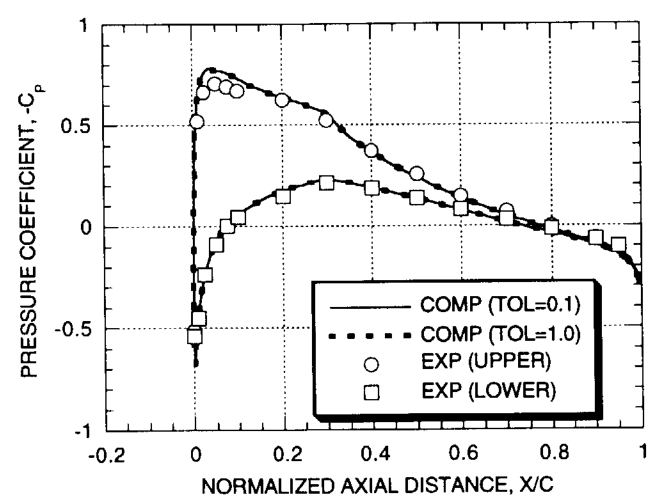
2Y/B=0.17



2Y/B=0.40



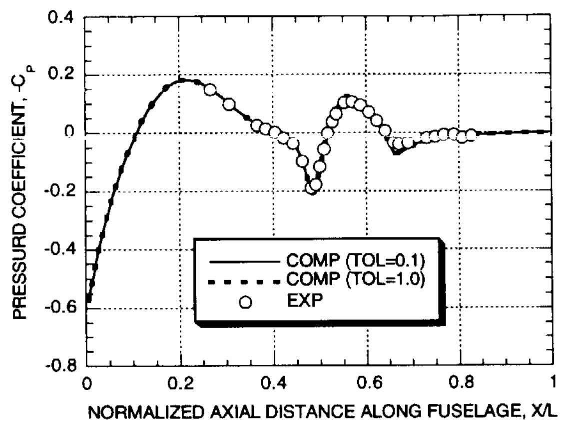
2Y/B=0.85



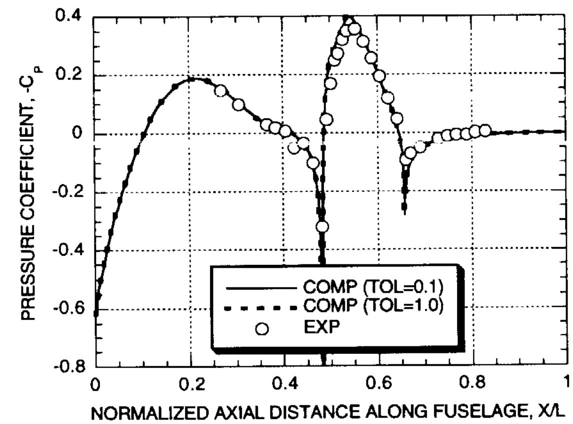
FUSELAGE SURFACE PRESSURE COMPARISONS

RAE WING A + BODY B2, $M_\infty = 0.82, \alpha = 2°$

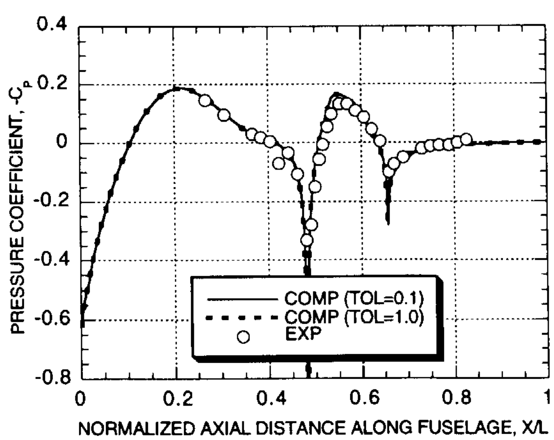
$\varphi = -15°$



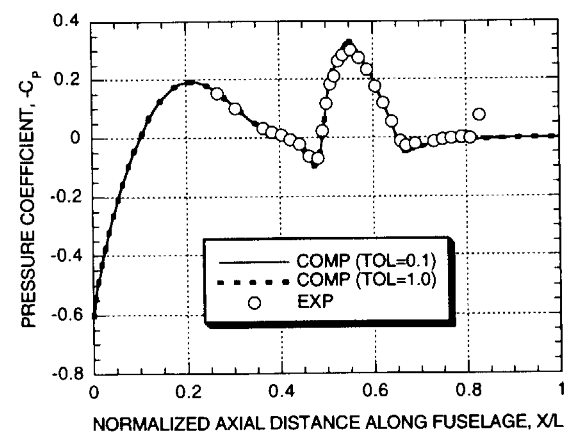
$\varphi = +0°$



$\varphi = -0°$

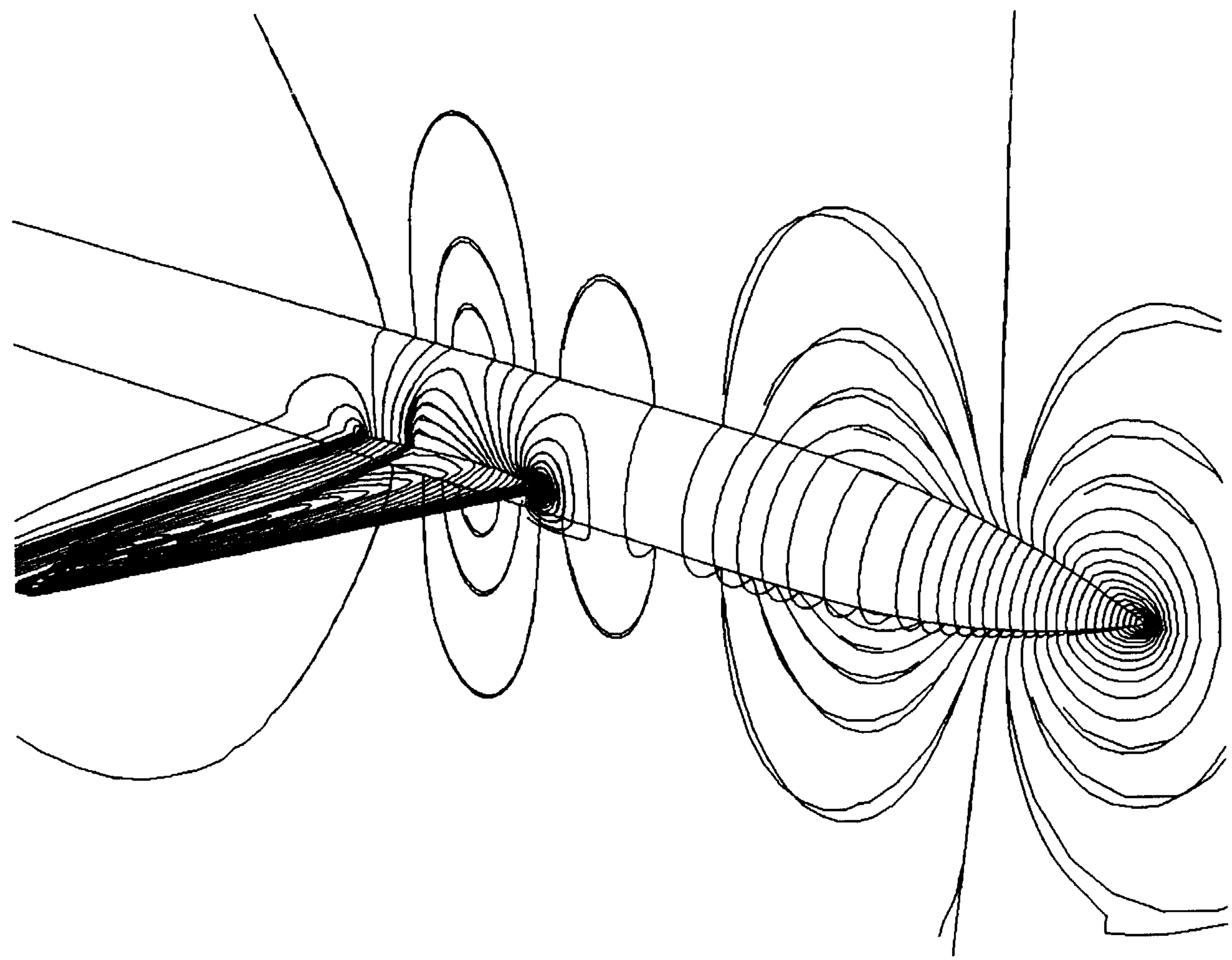


$\varphi = +15°$



MACH NUMBER CONTOURS

RAE WING A WITH B2 FUSELAGE, $M_\infty = 0.9, \alpha = 0°$
WING GRID = 201X41X17, FUSELAGE GRID = 145X25X58, OUTER GRID = 101X41X62



CONCLUDING REMARKS

> A NEW DONOR CELL SEARCH ALGORITHM SUITABLE FOR USE WITH A CHIMERA-BASED FULL POTENTIAL SOLVER HAS BEEN PRESENTED AND EVALUATED

> NEW SEARCH ALGORITHM IS EXTREMELY FAST AND SIMPLE PRODUCING DONOR CELLS AS FAST AS 60,000 PER SEC (CRAY C-90 SINGLE PROCESSOR)

> NEW SEARCH ALGORITHM IS APPROXIMATE, IE, DONOR CELLS MAY BE APPROXIMATED BY A NEAREST NEIGHBOR CELL

- ON A FINE GRID THIS ONLY HAPPENS 3-4% OF THE TIME WITH NO IMPACT ON SOLUTION ACCURACY OR CONVERGENCE EFFICIENCY
- EVEN WHEN THE NUMBER OF NEAREST NEIGHBOR CELLS USED TO APPROXIMATE ACTUAL DONOR CELLS IS FORCED UP TO 70%, THERE IS NO IMPACT ON ACCURACY OR CONVERGENCE EFFICIENCY

FUTURE DIRECTIONS

> MULTI-ZONE COMPLEX GEOMETRY APPLICATIONS

> GLOBALLY IMPLICIT SCHEMES/MULTI-GRID SCHEMES

> BOUNDARY LAYER APPLICATIONS

> WIND TUNNEL WALL AND MODEL SUPPORT COMPUTATIONS

> PARALLELIZATION STUDIES

> STRUCTURES COUPLING

> DESIGN/OPTIMIZATION APPLICATIONS