DESIGN AND STRESS ANALYSIS OF LOW-NOISE ADJUSTED BEARING CONTACT SPIRAL BEVEL GEARS

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Abstract

An integrated computerized approach for design and stress analysis of low-noise spiral bevel gear drives with adjusted bearing contact is proposed. The procedure of computations is an iterative process that requires four separate procedures and provide: (a) a parabolic function of transmission errors that is able to reduce the effect of errors of alignment on noise and vibration, and (b) reduction of the shift of bearing contact caused by misalignment. Application of finite element analysis enables us to determine the contact and bending stresses and investigate the formation of the bearing contact. The design of finite element models and boundary conditions is automated and does not require intermediate CAD computer programs for application of general purpose computer program for finite element analysis.

1 Introduction

Design and stress analysis of spiral bevel gears is an important topic of research that has been performed by many scientists [1 - 6]. Reduction of noise and stabilization of bearing contact of misaligned spiral bevel gear drives is still a very challenging topic of research although the manufacturing companies (The Gleason Works (USA), Klingelnberg-Oerlikon (Germany-Switzerland)) have developed analysis tools and outstanding equipment for manufacture of such gear drives.

The problem of design of spiral bevel gears is that machine tool settings for manufacturing of spiral bevel gears are not standardized and have to be specially determined for each set of parameters of design to guarantee the required quality of the gear drives. A new approach is proposed for the solution of this problem that is based on the following considerations:

(1) The gear machine-tool settings are considered as given (adapted, for instance, from the Gleason summary). The to-be-determined pinion machine-tool settings have to provide the observation of assigned conditions of meshing and contact of the gear drive.

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(2) Low noise of the gear drive is achieved by application of a predesigned parabolic function of transmission errors of a limited maximum value of 6-8 arcsec. A predesigned parabolic function of transmission errors is able to absorb almost linear discontinuous functions of transmission errors caused by errors of alignment. Such transmission errors are the source of high noise and vibration.

(3) The provided orientation of the bearing contact has to reduce its shift caused by the errors of alignment of the gear drive.

(4) The design procedure developed is an iterative process based on simultaneous application of local synthesis and tooth contact analysis (TCA). The local synthesis provides assigned conditions of meshing and contact at the mean contact point of tangency of pinion and gear tooth surfaces. The TCA computer program is applied for simulation of the conditions of meshing and contact for the entire meshing process.

Finite element method (FEM) is used for stress analysis and investigation of formation of the bearing contact. A model of three pairs of teeth complemented with the boundary conditions is applied for the finite element analysis (FEA). The development of contact models is automated and does not require application of CAD computer programs.

2 Basic Ideas of Developed Approach

Local Synthesis. The mean contact point is chosen on the gear tooth surface (Fig. 1). The length of the major axis of the instantaneous contact ellipse at the mean contact point, $2a$, the direction of the tangent to the contact path on gear tooth surface, η_2, and the derivative of the gear ratio function m'_{12} are chosen at point M. The gear ratio function is given by $m_{12} = \omega^{(1)}/\omega^{(2)}$ where $\omega^{(1)}$ and $\omega^{(2)}$ are the angular velocities of the pinion and gear rotations. The local synthesis program determines the pinion machine-tool settings considering as known the gear machine-tool settings and parameters a, η_2, and m'_{12} [7]. The program requires solution of ten equations for ten unknowns but six of the ten equations are represented in echelon form. The algorithm of local synthesis includes relations between principal curvatures and directions proposed in [7, 8, 9].

Tooth Contact Analysis (TCA). The algorithm of tooth contact analysis is based on conditions of continuous tangency of pinion and gear tooth surfaces [7]. The TCA computer program determines the function of transmission errors and the bearing contact obtained for each iteration when the input variable parameters a, η_2, m'_{12} of the respective iteration are applied.

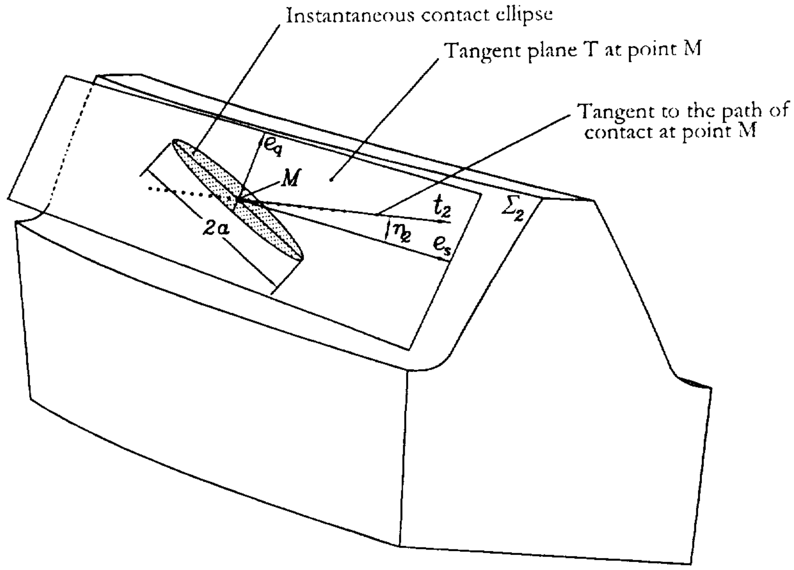


Figure 1: Illustration of parameters η_2 and a applied for local synthesis.

The computational procedure is divided into four separately applied procedures that are performed as follows:

Procedure 1: This procedure is directed at obtaining the assigned orientation of the bearing contact and is performed as follows:

(a) The local synthesis and TCA are applied simultaneously. The varied parameter is m'_{12} and parameters a and η_2 are taken as constant. The orientation of η_2 is initially chosen for a longitudinally oriented bearing contact. The errors of alignment are taken equal to zero.

(b) Using the output of TCA it becomes possible to obtain numerically the path of contact on gear tooth surface Σ_2 and determine its projection L_T on plane T that is tangent to Σ_2 at point M (Fig. 1).

(c) The goal of the iterative process is to obtain $L_T^{(n)}$ as a straight line for the process of meshing that is investigated for one cycle of meshing given by $-\pi/N_1 \leq \phi_1 \leq \pi/N_1$. This goal is achieved by variation of m'_{12} and the sought-for solution is obtained analytically as follows:

(i) The numerically obtained projection is represented by a polynomial function

$$y_t(x_t, m'^{(i)}_{12}) = \beta_0(m'_{12})^{(i)} + \beta_1(m'_{12})^{(i)}x_t + \beta_2(m'_{12})^{(i)}x_t^2 \qquad (1)$$

(ii) Variation of $m_{12}^{\prime(i)}$ $(i = 1, 2, 3, \ldots, n)$ in the iterative process based on simultaneous application of local synthesis and TCA, enables us to obtain a path of contact when $\beta_2 = 0$ and $L_T^{(n)}$ becomes a straight line. Figure 2 shows various lines $L_T^{(1)}$, $L_T^{(2)}$, and the desired shape $L_T^{(n)}$.

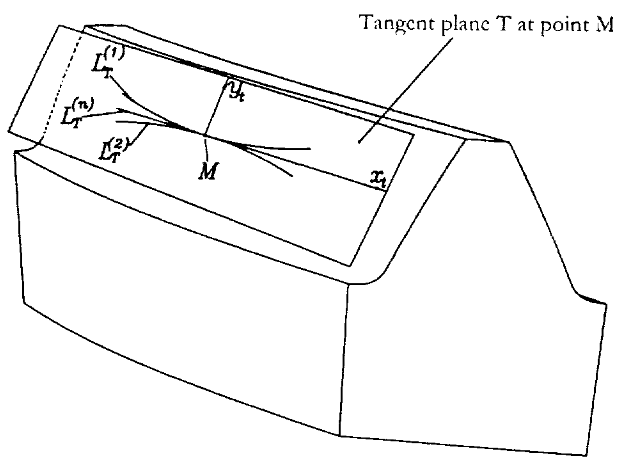


Figure 2: Projections of various paths of contact L_T on tangent plane T.

(iii) Solution of Eq. (1) for $\beta_2(m'_{12})^{(i)} = 0$, where m'_{12} is the varied parameter, is based on the secant method [10].

Procedure 2: As a result of Procedure 1, the path of contact $L_T^{(n)}$ is represented by a straight line. However, the output of the TCA for the obtained function of transmission errors, function $\Delta\phi_2^{(n)}(\phi_1)$ where $-\pi/N_1 \leq \phi_1 \leq \pi/N_1$, is of unfavorable shape and magnitude. The goal of Procedure 2 is transformation of $\Delta\phi_2^{(n)}(\phi_1)$ into a parabolic function and limitation of the magnitude of maximum transmission errors. This goal is achieved by application of *modified roll* for pinion generation.

Modified roll means that during the pinion generation the angles of rotation of the pinion and the cradle of the generating machine, designated as ψ_1 and ψ_{c1}, respectively, are related as follows

$$\psi_1^{(j)}(\psi_{c1}) = m_{1c}\psi_{c1} - b_2^{(j)}\psi_{c1}^2 - b_3^{(j)}\psi_{c1}^3 \tag{2}$$

where m_{1c} is the first derivative of function $\psi_1(\psi_{c1})$ at $\psi_{c1} = 0$ that is obtained by application of local synthesis [7]. The superscript in Eq. (2) indicates that the j-th iteration is considered.

Transformation of the function of transmission errors $\Delta\phi_2^{(n)}(\phi_1)$ is achieved as follows:

(i) The function of transmission errors $\Delta\phi_2^{(n)}(\phi_1)$ is the output of TCA obtained at the n-th iteration of Procedure 1 and is represented numerically. We represent $\Delta\phi_2^{(n)}(\phi_1)$ as a polynomial function of the third order designated as

$$\Delta\phi_2^{(j)}(\phi_1) = a_0^{(j)} + a_1^{(j)}\phi_1 + a_2^{(j)}\phi_1^2 + a_3^{(j)}\phi_1^3, \qquad \frac{-\pi}{N_1} \leq \phi_1 \leq \frac{\pi}{N_1} \qquad (3)$$

The designation $j = 1, 2, \ldots, k$ means that an iterative process for modification of $\Delta\phi_2^{(n)}(\phi_1)$ is considered. Function $\Delta\phi_2^{(j)}(\phi_1) \equiv \Delta\phi_2^{(n)}$ is obtained at the final iteration of Procedure 1.

(ii) The goal of Procedure 2 is to transform the function of transmission errors and obtain

$$\Delta\phi_2^{(k)}(\phi_1) = -a_2^{(k)}\phi_1^2, \qquad \frac{-\pi}{N_1} \leq \phi_1 \leq \frac{\pi}{N_1} \qquad (4)$$

$$\left|\Delta\phi_2^{(k)}(\phi_1)\right|_{max} = a_2^{(k)}\left(\frac{\pi}{N_1}\right)^2 = \Delta\Phi \qquad (5)$$

where $\Delta\Phi$ is the chosen maximum level of transmission errors.

(iii) The goals mentioned above are obtained by variation of coefficients $b_2^{(j)}$ and $b_3^{(j)}$ of the function of modified roll. The secant method is applied for this purpose wherein variations of $b_2^{(j)}$ and $b_3^{(j)}$ are performed separately.

Procedure 3: The goal of Procedure 3 is to reduce the shift of the bearing contact caused by errors of alignment and this is achieved by the proper change of orientation of L_T assigned initially in Procedure 1. Procedure 3 is performed as follows:

(i) Computer programs developed for local synthesis and TCA are again applied simultaneously, but the expected errors of alignment are simulated.

(ii) The effect of all errors of alignment on the shift of the bearing contact is investigated separately. The sensitivity of the shift of L_T is reduced by the proper choice of parameter η_2 (Fig. 1), that is the varied parameter of local synthesis in Procedure 3.

(iii) Application of procedure 3 enables us to obtain the resultant orientation of L_T that differs from the longitudinally one. The optimal orientation of L_T depends on the design parameters of the gear drive, particularly on the gear ratio and on the required range of errors of alignment.

After completion of Procedures 1, 2, and 3, the obtained pinion machine-tool settings guarantee that the designed gear drive is indeed a low-noise gear drive with reduced sensitivity to errors of alignment.

Procedure 4: The goal of Procedure 4 is the stress analysis and investigation of formation of the bearing contact (see details in Section 5).

3 Derivation of Equations of Gear Tooth Surfaces

We recall that the machine-tool settings for the gear are considered as given and the to-be-derived equations allow the gear tooth surfaces to be determined. The head-cutter is provided with blades that are rotated about the Z_g axis of the head-cutter (Fig. 3) during the process of generation. Both sides of the gear tooth are generated simultaneously. The profiles of the blade consist of two parts (Fig. 3): (i) of a straight lines, and (ii) of a rounded edge formed by circular arcs connected to the previously mentioned straight lines. The blades by rotation about the Z_g axis form the head-cutter generating surfaces.

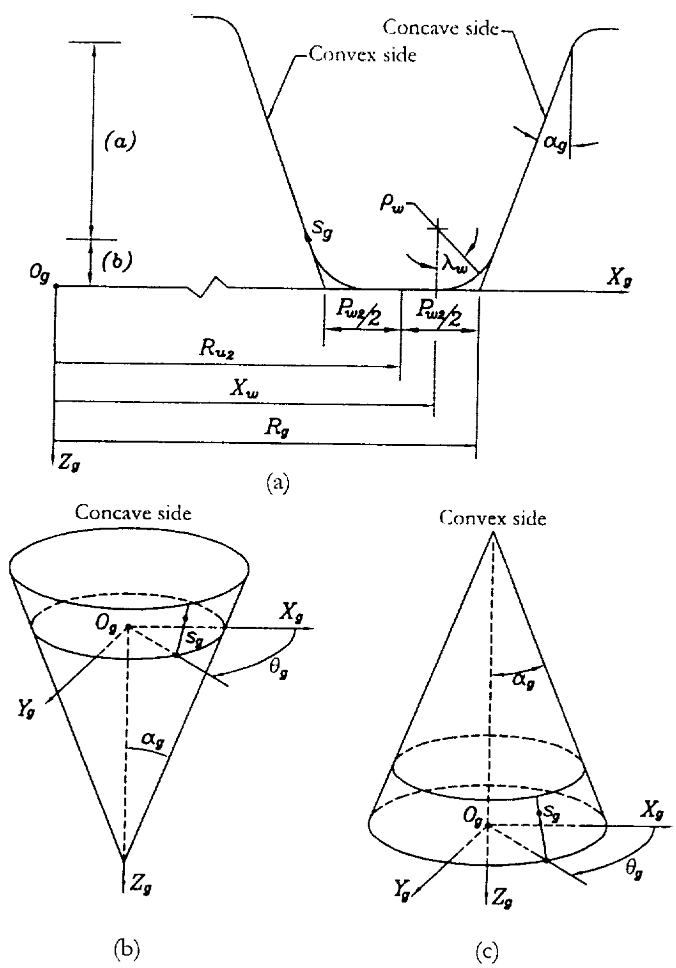


Figure 3: Blade and generating cones for gear generating tool: (a) illustration of head-cutter blade; (b) and (c) generating tool cones for concave and convex sides.

Applied Coordinate Systems. Coordinate systems S_{m_2}, S_{a_2}, and S_{b_2} are the fixed ones and they are rigidly connected to the cutting machine (Fig. 4). The movable coordinate systems S_2 and S_{c_2} are rigidly connected to the gear and the cradle, respectively. Coordinate system S_g is rigidly connected to the gear head-cutter. It is considered that the head-cutter is a cone, and the rotation of the head cutter about the Z_g axis does not affect the process of generation. The head-cutter is mounted on the cradle and coordinate system S_g is rigidly connected to the cradle coordinate system S_{c_2}. The cradle and the gear perform related rotations about the S_{m_2} axis and the S_{b_2} axis, respectively. Angles ψ_{c_2} and ψ_2 are related and represent the current angles of rotation of the cradle and the gear. The ratio of gear roll is designated as m_{2c_2} and is determined as

$$m_{2c_2} = \frac{\omega^{(2)}}{\omega^{(c_2)}} = \frac{d\psi_2}{dt} \div \frac{d\psi_{c2}}{dt} \qquad (6)$$

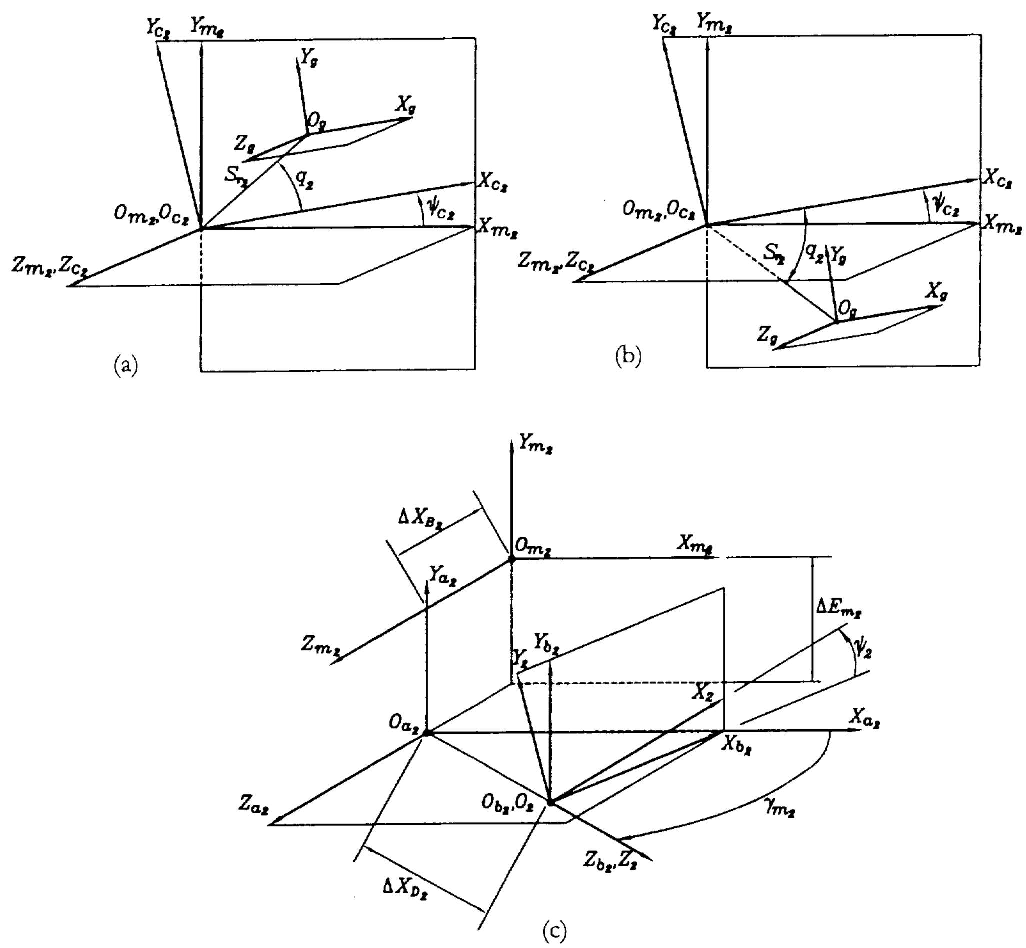


Figure 4: Coordinate systems applied for the pinion generation: (a) and (b) illustration of tool installment for generation of righ- and left-hand pinions; (c) illustration of corrections of machine-tool settings.

The installment of the tool on the cradle is determined by parameters S_{r_2} and q_2, that are called radial distance and basic cradle angle. Parameters ΔX_{B_2}, ΔE_{m_2}, ΔX_{D_2}, and γ_{m_2} represent the settings of the gear. Figures 4(a) and (b) show the installment of the head-cutter for right-hand and left-hand gears, respectively.

Derivation of Gear Tooth Surfaces. The head-cutter generating surface is represented in coordinate system S_g by vector function $\mathbf{r}_g(s_g, \theta_g)$ where s_g and θ_g are the surface parameters.

The family of generating surfaces is represented in coordinate system S_2 rigidly connected to the gear by the matrix equation

$$\mathbf{r}_2(s_g, \theta_g, \psi_2) = \mathbf{M}_{2g}(\psi_2)\ \mathbf{r}_g(s_g, \theta_g) \tag{7}$$

where ψ_2 is the generalized parameter of motion.

The equation of meshing is represented as

$$f_{2g}(s_g, \theta_g, \psi_2) = 0 \tag{8}$$

and determined as [7, 9]

$$\left(\frac{\partial \mathbf{r}_2}{\partial \theta_g} \times \frac{\partial \mathbf{r}_2}{\partial s_g}\right) \cdot \frac{\partial \mathbf{r}_2}{\partial \psi_2} = 0 \tag{9}$$

or as [7, 8, 9]

$$\mathbf{N}_g \cdot \mathbf{v}_g^{(g2)} = 0 \tag{10}$$

Here $\mathbf{N}_g(s_g, \theta_g)$ is the normal to the head-cutter surface represented in coordinate system S_g and $\mathbf{v}_g^{(g2)}$ is the relative velocity represented in S_g.

Equations (7) and (8) determine the gear tooth surface by three related parameters.

4 Derivation of Equations of Pinion Tooth Surfaces

The two sides of the pinion tooth surfaces are generated separately. The machine-tool settings applied for generation of each tooth side are determined separately by application of Procedures 1, 2, and 3 previously mentioned. Profile blades of pinion head-cutters are represented in Fig. 5.

Applied Coordinate Systems. Coordinate systems applied for generation of pinion are similar to those represented in Fig. 4. Coordinate systems S_1 and S_2 are rotated about the Z_{m_1} axis and Z_{b_1} axis, respectively (see Fig. 4 for equivalent coordinate systems Z_{m_2} and Z_{b_2} for gear generation), and their rotations are related by a polynomial function $\psi_1(\psi_{c_1})$ wherein modified roll is applied (see below). The ratio of instantaneous angular

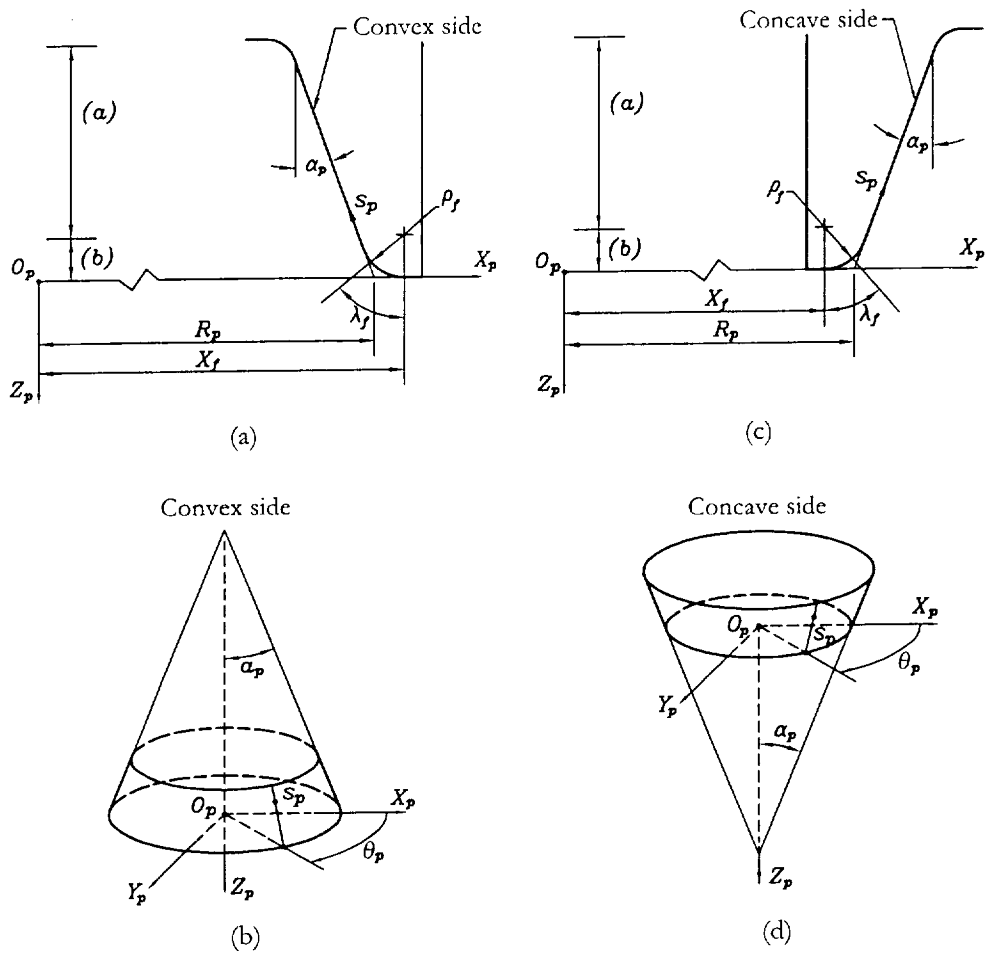


Figure 5: Blades and generating cones for pinion generating tool with straight blades: (a) convex side blade; (b) convex side generating cone; (c) concave side blade; (d) concave side generating cone.

velocities of the pinion and the cradle is defined as $m_{1c}(\psi_1(\psi_{c1})) = \omega^{(1)}(\psi_{c1})/\omega^{(c)}$. The magnitude of m_{1c} is called ratio of roll or velocity ratio. Parameters ΔX_{D_1}, ΔX_{B_1}, ΔE_{m_1}, and γ_{m_1} are the basic machine tool settings for pinion generation.

Derivation of Pinion Tooth Surfaces. The pinion head-cutter surface is represented by vector function $\mathbf{r}_p$ (s_p, θ_p) (Fig. 5) where (s_p, θ_p) are the surface parameters. The family of head-cutters is represented in coordinate system S_1 rigidly connected to the pinion by the matrix equation

$$\mathbf{r}_1(s_p, \theta_p, \psi_{c1}) = \mathbf{M}_{1p}(\psi_{c_1})\mathbf{r}_p(s_p, \theta_p) \tag{11}$$

Unlike the case of gear generation, modified roll is applied for generation of the pinion, and function $\psi_1(\psi_{c_1})$ relates the angles of rotation of the pinion and the cradle of the pinion generating machine by a polynomial but not linear function [see Eq. (2)]. The equation of meshing is represented as

$$f_{p1}(s_p, \theta_p, \psi_{c1}) = 0 \tag{12}$$

and is determined by application of approaches similar to those represented by Eqs. (9) or (10). Equations (11) and (12) determine the pinion tooth surfaces by three related parameters.

5 Application of Finite Element Analysis

Application of finite element analysis permits: (i) investigation of the bearing contact wherein multiple sets of teeth may be in contact under load simultaneously, and (ii) determination of contact and bending stresses.

Application of finite element method [11] requires the development of the finite element model mesh, the definition of possible contacting surfaces, and the establishment of boundary conditions for loading the gear drive with the given torque. Finite element analysis is performed by application of general purpose computer program [12].

The development of the solid models and finite element meshes using CAD computer programs is expensive, requires skilled users of those computer programs, and has to be done for every case of gear geometry and position of meshing to be investigated.

The authors have developed a modified approach to perform the finite element analysis that has the following advantages:

(a) Computational procedures for synthesis, analysis, and generation of finite element models are integrated in developed computer programs. Graphical interpretation of the output is obtained by using commercially available graphical subroutines.

(b) The generation of the finite element mesh required for FEA is performed automatically using the equations of the surfaces of the tooth and its portion of the rim. Nodes of the finite element mesh that belong to the tooth surfaces of pinion (gear) model guarantee to simulate points of the real tooth surfaces of the pinion (gear). Loss of accuracy due to the development of solid models using CAD computer programs is avoided. The boundary conditions for the pinion and the gear are set automatically as well.

(c) The formation of the bearing contact for a cycle of meshing can be investigated and edge contact can be discovered and avoided.

(d) Setting of boundary conditions for gear and pinion is obtained automatically. Nodes on the sides and bottom part of the rim portion of the gear are considered as fixed.

Nodes on the two sides and bottom part of the rim portion of the pinion build a rigid surface. The rigid body reference node is located on the pinion axis of rotation with all degrees of freedom except the rotation around the axis of rotation of the pinion that is fixed to zero. The torque is applied directly to the remaining degree of freedom of the rigid body reference node.

(e) Definition of contacting surfaces for the contact algorithm of the finite element computer program [12] is automatic as well and requires definition of the master and slave surfaces. Generally, the master surface is chosen as the surface of the stiffer body or as the surface with the coarser mesh if the two surfaces are located on structures with comparable stiffness. Finite element mesh for the pinion is more refined than the one for the gear due to larger curvatures. The gear and pinion tooth surfaces are considered as the master and slave surfaces, respectively, for the contact algorithm.

6 Numerical Examples

Three examples of design of face-milled spiral bevel gears with various design parameters have been investigated. The output of computations for the mentioned examples are represented in Fig. 6 and shows:

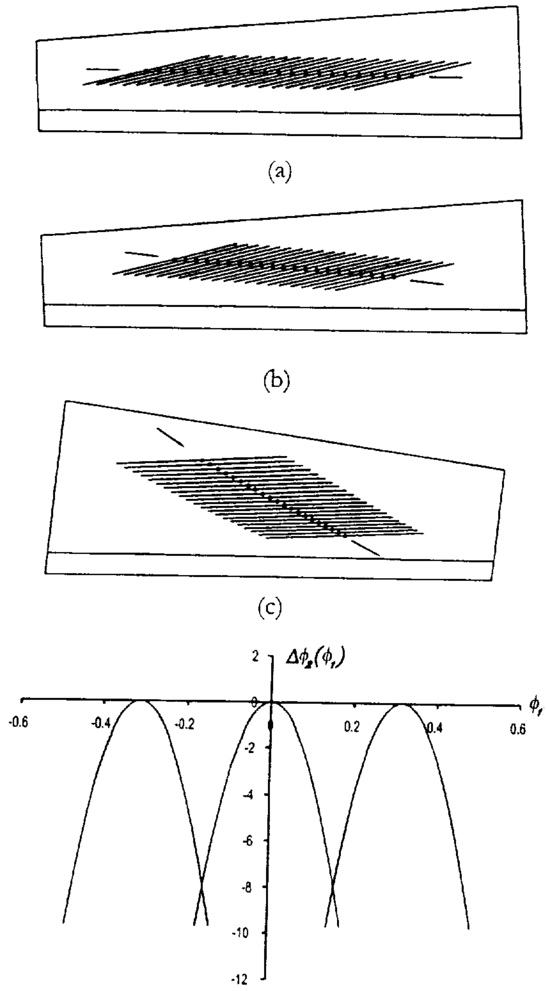
(i) The bearing contact has a longitudinal direction in Example 1 for an aligned gear drive [Fig. 6(a)].

(ii) The bearing contact in Example 2 [Fig. 6(b)] is deviated from the longitudinal direction for reduction of the shift of bearing contact caused by errors of alignment.

(iii) Example 3 of design [Fig. 6(c)] shows that the gear drive with a gear ratio close to 1 ($N_2/N_1 = 26/17$) is very sensitive to errors of alignment. The shift of bearing contact due to misalignment is reduced since larger deviation of the bearing contact from the longitudinal direction has been provided.

(iv) The function of transmission errors [Fig. 6(d)] for all three examples is indeed a parabolic function up to 8 arcsec of maximum amount.

Finite element analysis has been performed for all three examples. Figure 7 shows the formation of the bearing contact at the heel position of the gear for one of the studied gear drives.



(d)

Figure 6: Bearing contact on gear tooth surfaces and predesigned function of transmission errors for: (a) longitudinally oriented bearing contact for gear drive of example 1; (b) adjusted bearing contact for gear drive of example 2; (c) adjusted bearing contact for gear drive of example 3; (d) function of transmission errors for examples 1, 2, and 3.

7 Conclusions

Based on the study conducted the following conclusions can be made:

(1) An integrated computerized approach has been developed for design of low-noise spiral bevel gears with an adjusted bearing contact based on the following ideas:

(i) A parabolic function of transmission errors of limited value of maximum transmission errors is applied. Such a function is able to absorb linear discontinuous functions of transmission errors caused by misalignment.

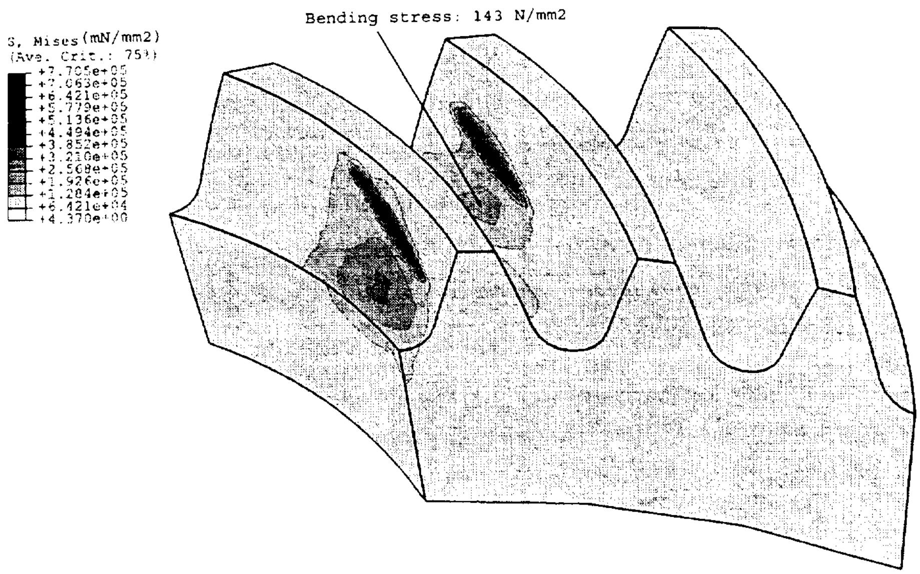


Figure 7: Bearing contact at heel contact point on gear tooth surfaces for spiral bevel gear drive.

(ii) The orientation of the bearing contact is adjusted for reduction of the shift of the bearing contact caused by errors of alignment.

(iii) The approach developed is an iterative procedure based on simultaneous application of local synthesis and tooth contact analysis (TCA) using modified roll for pinion generation.

(iv) The contacting model for finite element analysis is formed by three teeth and the boundary conditions, it is automatically designed, and does not need intermediate CAD computer programs for application of finite element analysis.

(v) The same computer language is applied for numerical computations performed for all stages of design, and automatic generation of finite element models.

(2) The approach developed may be also applied for design of formate cut spiral bevel gears, hypoid gear drives and other types of gear drives.

Acknowledgments

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References

[1] Bibel, G. D., Kumar, A., Reddy, S., and Handschuh, R. F., *Contact Stress Analysis of Spiral Bevel Gears Using Finite Element Analysis,* ASME Journal of Mechanical Design, Vol. 117, No. 2, pp. 235–240, 1995.

[2] Gosselin, C., Nonaka, T., Shiono, Y., Kubo, A., and Tatsuno, T., *Identification of the Machine Setting of Real Hypoid Gear Tooth Surfaces,* Journal of Mechanical Design, Vol. 120, pp. 429–440, 1998.

[3] Lewicki, D. G., Handschuh, R. F., Henry, Z. S., and Litvin, F. L., *Low-Noise, High Strength Spiral Bevel Gears for Helicopter Transmission,* J. Propul. Power, Vol. 10, No. 3, 1994.

[4] Handschuh, R. F. and Litvin, F. L., *A method for Determining Spiral-Bevel Gear Tooth Geometry for Finite Element Analysis,* NASA Technical Paper 3096, AVS-COM Technical Report 91-C-020, 1991.

[5] Litvin, F. L., Wang, A. G., and Handschuh, R. F., *Computerized Design and Analysis of Face-Milled Uniform Tooth Height Spiral Bevel Gear Drives,* ASME Journal of Mechanical Design, Vol. 118, No. 4, pp. 573–579, 1996.

[6] Litvin, F. L., Wang, A. G., and Handschuh, R. F., *Computerized Generation and Simulation of Meshing and Contact of Spiral Bevel Gears with Improved Geometry,* Journal Computer Methods in Applied Mechanics and Engineering, Vol. 158, No. 1-2, pp. 35–64, 1998.

[7] Litvin, F. L., *Gear Geometry and Applied Theory,* Prentice Hall, Inc., Englewood Cliffs, New Jersey, 1994.

[8] Litvin, F. L., *Theory of Gearing,* Nauka (in Russian), 1968.

[9] Litvin, F. L., *Theory of Gearing,* NASA RP-1212 (AVSCOM 88-C-C035), Washington, D. C., 1989.

[10] Press, W. H., Teukolsky, S. A., Vetterling, W. T., and Flannery, B. P., *Numerical Recipes in Fortran 77: The Art of Scientific Computing,* Cambridge University Press, New York, 2nd Ed., 1992.

[11] Zienkiewicz, O. C. and Taylor, R. L., *The Finite Element Method,* John Wiley & Sons, 5th Ed., 2000.

[12] Hibbit, Karlsson & Sirensen, Inc., *ABAQUS/Standard 6.1 User's Manual,* 1800 Main Street, Pantucket, RI 20860-4847, 1998.