

Optimal Management of Redundant Control Authority for Fault Tolerance¹

N. Eva Wu and Jianhong Ju

Department of Electrical Engineering, Binghamton University, Binghamton, NY 13902-6000 USA
evawu,bg24615@binghamton.edu

Abstract

This paper is intended to demonstrate the feasibility of a solution to a fault tolerant control problem. It explains, through a numerical example, the design and the operation of a novel scheme for fault tolerant control. The fundamental principle of the scheme was formalized in [5] based on the notion of normalized nonspecificity. The novelty lies with the use of a reliability criterion for redundancy management, and therefore leads to a high overall system reliability.

1. Introduction

The relationship between the overall control system reliability and control/diagnostic/redundancy management was discussed in [5], which provides a guideline for high reliability designs. In the process of such a design however, calculations are required for control system performance, diagnostic resolution, and reconfiguration coverage. These calculations are shown to be feasible in this paper by an example.

The plant model to be used in our example is taken from μ -toolbox (See [1] and references therein) with some changes in the system description and design objectives. The model has two inputs: elevator command and canard command; two outputs: angle of attack and pitch angle; and four states: forward velocity, angle of attack, pitch rate and pitch angle. Control objectives were originally specified on vertical transition, pitch pointing and direct lift. The elevators are regarded as the primary control effectors, and the canards as the secondary. Therefore, some aerodynamically redundant control authority exists in the pitch axis. The same model has been used by the author in a number of other studies related to fault tolerant control[2, 3, 4].

The organization of the paper is as follows. The issue of control performance evaluation for a set of controllers is discussed in Section 2, and control performance is evaluated for the above described model which is subject to a certain adverse operating conditions. The result of on-line estimation of the adverse conditions is presented in Section 3, with a focus on how the uncertainty associated with the estimate is represented. Section 4 describes a redundancy management criterion, based on which a control selection process is simulated. The selection process combines the information obtained from a control performance evaluation with that obtained from an adverse condition estimation. Some unresolved issues for future study are briefly discussed in Section 5.

2. Control design and performance evaluation

In this section, the pitch axis controller of the above system

is considered. The controller is required to be capable of accommodating adverse conditions associated with loss of control effectiveness. This is accomplished by designing a set of controllers, each of which compensates a certain set of adverse conditions. The state-space linear model

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t), \quad x(0) = x_0$$

derived at a fixed operating point is used. The following modifications to the original model are made. The impairment representing the loss of actuator effectiveness is parameterized by actuator effectiveness factors. The factors enter the model in the form $B = B_0 + B_0\Delta$, where $\Delta = \text{diag}\{\delta_1, \delta_2\}$. Each δ_i ranges between 0, representing no loss of effectiveness, and -1 , representing complete loss of effectiveness in the i th effector. One of the frequency dependent weighting factors reflecting multiplicative plant uncertainty in the original design[1] is changed to a small constant value to ease the change detection, because this uncertainty enters the plant model at the same location as the actuator effectiveness factors.

Two controllers are designed for two different nominal values of (δ_1, δ_2) . For each controller, a closed-loop performance measure is calculated in terms of $1/\mu$, as a function of δ_1 and δ_2 , where μ is the structured singular value[1] of the closed-loop system. Both the controller design and the performance evaluation are carried out using the μ -toolbox[1]. The results of control performance evaluation are plotted in Fig.1. Also shown in Fig.1 is a plane indicating prescribed performance level m_T . When the control performance level falls below this threshold, a failure is declared. This threshold, together with the evaluated control performance surfaces, determine the performance recoverable region. The challenge of recovering the control performance lies with the fact that at any given time the value of (δ_1, δ_2) is not known.

3. Control effectiveness estimation and uncertainty representation

In this section issues related to the estimation of severity of loss of control effectiveness is discussed. The uncertainty description of the estimate is converted into possibilistic terms. This allows us, in the process of controller selection, to fuse the information obtained through estimation with that through control performance evaluation. The information fusion will be discussed in the next section.

The task of control effectiveness estimation is accomplished by using an adaptive Kalman filtering algorithm. The reader is referred to [4] for an estimation performed on the same plant model. An important feature of this estimation algorithm is that a set of covariance-dependent forgetting factors is introduced into the part of the filtering

¹This research was supported by grants from NSF (ECS-9615956), NASA (NCC-1-336), Xerox Corporation (HE 1321-98)

algorithm that estimates the control effectiveness. As a result, the change in the control effectiveness is accentuated to help achieve a more accurate estimate more rapidly.

For any estimate to be useful in decision making, it must be accompanied by a description of uncertainty. Fig.2(a) shows the probability density function of the estimate of control effectiveness obtained 2.5 seconds into the system operation, and 0.3 seconds after a simulated adverse condition at $(\delta_1, \delta_2) = (-0.85, -0.65)$ has occurred. This probability distribution is then transformed under the uncertainty invariant principle[5] into the possibility distribution shown in Fig.2(b).

4. Redundancy management

By combining Fig.1 and Fig.2(b), it is clear that a redundancy management policy is needed in order that the most suitable controller is selected at any given time. This section discusses the policy and calculation of the risk associated with each decision.

Define $F = ((\delta_1, \delta_2), m_F(\delta_1, \delta_2))$, as the fuzzy set associated with the possibility function of estimate $(\hat{\delta}_1, \hat{\delta}_2)$, where m denotes a membership function. Fig.2(b) gives an example of F . Define $C_k = ((\delta_1, \delta_2), m_{C_k}(\delta_1, \delta_2))$, $k = 1, 2$ as the fuzzy set representing the control performance resulted from using controller k . Fig.1(a) depicts both C_1 and C_2 . Now assume that C_k is being used. Express F as the union of two sets $F = F_k^u \cup F_k^l$ where

$$F_k^u = \{F : m_{C_k} \geq m_T\}, \quad F_k^l = \{F : m_{C_k} < m_T\}.$$

Let c_k , $k = 1, 2$, denote the likelihood of successful failure accommodation when controller k is selected. More specifically,

$$c_k = \frac{\int_0^1 HL(\alpha F_k^u) d\alpha}{\int_0^1 HL(\alpha F) d\alpha}, \quad \bar{c}_k = \frac{\int_0^1 HL(\alpha F_k^l) d\alpha}{\int_0^1 HL(\alpha F) d\alpha},$$

where αF is the α -cut of fuzzy set F . $HL(\cdot)$ denotes the Hartley-like measure[5]

$$HL(A) = \min_{t \in T} \{ \log_2 \left[\prod_{i=1}^2 (1 + |A_{it}|) + |A| - \prod_{i=1}^2 |A_{it}| \right] \},$$

where T is the set of all unitary transformations on the N -dimensional Euclidean space, $|A_{it}|$ is the Lebesgue measure of the projection of set A on to the i th axis of the unitary transformed coordinate system under transformation t . c_k and $1 - \bar{c}_k$ (which can be shown to be no greater than c_k under some conditions) are called the upper bound and the lower bound of fault tolerance coverage for controller k , respectively. The redundancy management policy

$$\arg \max_k \{c_k, k = 1, 2\}$$

is used, under which the controller that maximizes the likelihood of successful accommodation of the given uncertain adverse condition is selected. The reader is referred to [5] for a rigorous treatment and proofs of the properties of this redundancy management policy.

For the adverse condition simulated in the previous section, the upper and lower fault coverage bounds are calculated as functions of control performance threshold. The results are shown in Fig.3. The implications of these plots

are obvious. When the control performance threshold value is required to be no less than 0.65, for example, controller 1 ought to be selected. Fig.4 illustrates the result of a simulation for a switched control system. The system begins by using controller 2. The result of no switching is shown in Fig.4(a) after the simulated reduction control effectiveness occurs; while in Fig.4(b), controller 2 is switched to controller 1 three seconds after the effectiveness reduction occurs.

References

- [1] Balas, G.J., Doyle, J.C., Glover, K., Packard, A., and Smith, R., *μ -Analysis and Synthesis Toolbox* (for use with MATLAB), Chapter 7, The Math Works, 1995.
- [2] Wu, N.E., and Chen, T.J., Feedback design in control reconfigurable systems, *International Journal of Robust and Nonlinear Control*, vol.6, pp.560-570, 1996.
- [3] Wu, N.E., Zhou, K., and Salomon, G., On Reconfigurability, to appear in *Proc. IFAC Symposium on Safe-process*, 2000.
- [4] Wu, N.E., Zhang, Y., and Zhou, K., Detection, estimation, and accommodation of loss of control effectiveness, to appear in *International Journal of Adaptive Control and Signal Processing*, 2000.
- [5] Wu, N. E., and Klir, G., Optimal redundancy management in reconfigurable control systems based on normalized nonspecificity, to appear in *International Journal of Systems Science*, 2000.

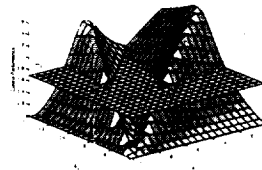


Fig.1 Control performance and threshold

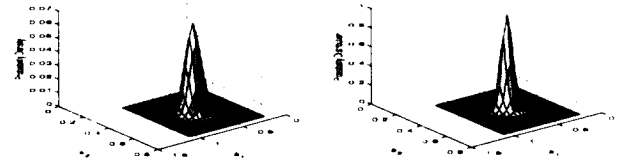


Fig.2 probability and possibility distributions of a control effectiveness estimate

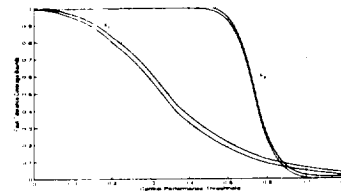


Fig.3 Coverage bounds versus performance threshold

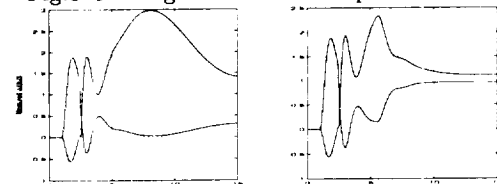


Fig.4 Step response without and with controller switch