A Pulse-Compression Method for Process Monitoring\textsuperscript{1}

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Abstract
This paper describes, from a deterministic viewpoint, an on-line process monitoring method for small signal behavior of time invariant stable systems at a given operating point. The monitoring is accomplished by using an m-sequence modulated low-pass wavelet as the probing signal, and a cyclic reference signal as the correlator. This method allows to achieve high resolution, high signal to noise ratio monitoring, and the acquired data can be used for on-line diagnosis.

1 Introduction
Health monitoring and diagnosis of industrial processes provide situational awareness and feedback information that help optimize operational effectiveness, prevent catastrophic failures, and decrease maintenance costs. Existing approaches to process monitoring include model-based, knowledge-based, and multivariable statistical approaches\textsuperscript{[8]}. This paper applies the concept of pulse-compression probing to process monitoring. In this case, a compressible finite duration probing signal stimulates a process. The response is processed by a correlator with a cyclic reference signal which compresses the response of the probing signal into a narrow source wavelet, called the compressed version of the probing signal. Functionally the compressed version of the probing signal equivalently excites the process to yield the probing output from which the impulse response that reflects the small signal characteristics of the process can be extracted. Deviation of the impulse response from its normality is an indication of abnormality of the process. The acquired information can then be used for further process diagnosis.

The pulse-compression method has been used in seismic and acoustic probing applications\textsuperscript{[2, 3]}. This paper considers the application of the method to on-line process monitoring. The principle, the properties of the method, the selection of parameters of the probing signal, and the implementation issues will be discussed. In addition, its relation to cross-correlation\textsuperscript{[5]} technique in least squares linear system identification will be delineated.

2 Pulse-compression probing method
Let $u(t) = u_1(t) + u_2(t)$ be the input signal at one of the input ports of a stable process, where $u_1(t)$ is a prescribed set point or a general reference profile, $u_2(t)$ is a finite duration probing signal, $y_1(t)$ be the response at one of the output ports of the process to $u(t)$ in the absence of $u_2(t)$, $y_2(t)$ be the process response corresponding to the perturbation by $u_2(t)$, $h(t)$ be the impulse response that reflects the small signal characteristics of response $y$ to $u$ at the current operating condition, $s(t)$ be a cyclic reference signal, and $z(t) = z_1(t) + z_2(t)$ be the probing output, where $z_1$ and $z_2$ correspond to $y_1(t)$ and $y_2(t)$, respectively. Let '$*$' denote the convolution operation, and '$\circledast$' denote the correlation operation. Suppose the probing signal $u_2(t)$ is sufficiently small. Then from Fig.1 the following can be seen to hold.

$$z_2(t) = y_2(t) \circledast s(t) = h(t) * u_2(t) \circledast s(t) = h(t) * e(t),$$ \hspace{1cm} (1)

where $e(t)$ is called an equivalent excitation and is given by

$$e(t) = u_2(t) \circledast s(t).$$ \hspace{1cm} (2)

If $e(t) = \delta(t)$, then $z_2(t) = h(t)$, and the small signal characteristics is extracted. In the following development, we attempt to synthesize $e(t)$ by designing signals $u_2(t)$ and $s(t)$. Note that $u_1(t)$ is fixed by the process operation requirement. The signal design problem amounts to designing $u_2(t)$ and $s(t)$ such that the following holds.

$$y(t) \approx y_1(t),$$ \hspace{1cm} (3)

or

$$\frac{\text{rms}(y_2)}{\text{rms}(y_1)}$$

is minimized, and

$$z(t) \approx z_2(t) = h(t).$$ \hspace{1cm} (4)

or

$$\frac{\text{rms}(z_2)}{\text{rms}(z_1)}$$

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is maximized. Condition (3) implies that \( u_2 \) should be constraint in both power and magnitude. The power constraint is dictated by the acceptable signal to noise ratio in response \( y(t) \), and the magnitude constraint is dictated by the requirement to exit the process only in its linear region around the current operating point. Condition (4) is dictated by the acceptable signal to noise ratio in response \( z(t) \). Note that the roles of signal and noise are reversed in (3) and in (4). The finite duration requirement on the probing signal \( u(t) \) is consistent with the requirement of a small time delay in the probing output that benefits the process monitoring. It is a hard requirement in some applications where the duration is limited, for example, by the interval between the transmitting time and the receiving time of the probing signal.

\( u_2 \) and \( s(t) \) satisfying (3) and (4) can be selected to be of the following forms, based on [2, 3].

(i) Reference signal \( s(t) \) of the correlator.
The reference signal is the cyclic repetition of an impulsive \( m \)-sequence of order \( n \) with bit duration \( t_0 \) and intensity 1. An example of an impulsive \( m \)-sequence \( m_s(t) \) is shown in Fig.2(b) where \( n = 5 \), resulting a maximum sequence duration of \((2^5 - 1) \times t_0 = 31t_0\).

(ii) Probing signal \( u_2(t) \).

\[
u_2(t) = p(t) \ast m_s(t)
\]

where \( p(t) \) is a narrow low pass wavelet of magnitude \( A \). Fig.2(a) shows two examples of unity magnitude wavelets. One is a rectangular pulse and the other is a raised cosine pulse. The use of a rectangular wavelet will be assumed from this point on for simplicity. Smoother probing signals such as the \( m \)-sequence modulated raised cosine is in general more desirable but more complex in signal generation.

In its original form, a discrete \( m \)-sequence \( m(k) \) of order \( n \) and magnitude \( a \) is a pseudo-random binary sequence (PRBS) of maximum length \( 2^n - 1 \) with the following properties [4, 7].

\[
\frac{1}{2^n - 1} \sum_{k=1}^{2^n-1} m(k) = \frac{a}{2^n - 1}
\]

\[
\frac{1}{2^n - 1} \sum_{k=1}^{2^n-1} m(k)m(k+l) = \begin{cases} 
  a^2, & k = 0, \pm(2^n - 1), \pm2(2^n - 1), \cdots \\
  -\frac{a^2}{2^n - 1}, & \text{elsewhere}
\end{cases}
\]

Note that the second property holds when at least one of the two \( m(k) \)'s involved is cyclically repeated [3].

The probing output is given by

\[
z(t) = y(t) \otimes s(t) = y_1(t) \otimes s(t) + y_2(t) \otimes s(t).
\]

The first term in (8) depends on the fixed input and the process. Therefore \( y_1(t) \) can be assumed to be weakly correlated with \( s(t) \). Applying (6), the following inequality can be considered valid

\[
|y_1(t) \otimes s(t)| \leq \frac{\max_t |y_1(t)| \cdot p(t - (2^n - 1)t_0)}{(2^n - 1)t_0}.
\]

Let \( T = (2^n - 1)t_0 \). The second term of (8) can be expressed in a more detailed manner as

\[
h(t) \ast m_s(t) \otimes s(t) = h(t) \ast \left\{ 2^n p(t) \sum_i \delta(t - iT) - p(t) \sum_j \delta(t - jT) \right\}.
\]

When \( n \) is sufficiently large, we have

\[
y_2(t) \otimes s(t) \approx h(t) \ast \frac{p(t - iT)}{t_0}.
\]

If in addition, wavelet \( p(t) \) is sufficiently narrow with respect to the quickest variations in \( h(t) \) the above term approximates \( Ah(t) \). For the above term to be sufficiently dominant in \( z(t) \), it must be sufficiently larger than \( \max_t |y_1(t)|/[(2^n - 1)t_0] \) in (9) which again requires sufficiently large \( n \).

### 3 Signal parameters and implementation issues

Two conditions must be met to arrive at \( z(t) \approx Ah(t) \). These are a sufficiently large \( n \) and a sufficiently small \( t_0 \). In this case \( z(t) \) is called an estimate of \( h(t) \), denoted by \( \hat{h}(t) \). This section will discuss in more rigorous terms the meaning of large \( n \) and small \( t_0 \).

Recall that \( t_0 \) is the bit duration of the impulsive \( m \)-sequence in use. It is also the width of the equivalent excitation \( e(t) \) and that of the low-pass wavelet \( p(t) \). Therefore \( t_0 \) determines the level of details of \( h(t) \) observable from the probing output \( z(t) \), or the resolution of \( \hat{h}(t) \). In order to observe the full details in \( h(t) \), \( t_0 \) should be determined by using Shannon's sampling theorem [1], which states that if the Fourier transform of \( h(t) \) is zero outside of \(-\omega_0, \omega_0\), then \( h(t) \) can be fully reconstructed based only on its sampled values at a sampling rate higher than \( \omega_s = 2\omega_0 \). In practice, \( h(t) \) is not strictly band limited, and one may settle with a sampling rate about 5 times the estimated bandwidth \( \omega_{BW}/(2\pi) \) of \( h(t) \). This rate translates to the condition

\[
t_0 \leq 1/\omega_{BW}.
\]

The probing output repeats itself every \( T \) seconds. It is obvious that \( T \) must be longer than the "memory" of \( h(t) \) in order to avoid the time domain aliasing. This would require that knowledge of how long it takes for
the slowest significant modes of \( h(t) \) to decay to approximately zero. A reasonable assessment is \( T = 5\tau_{\text{slow}} \)
where \( \tau_{\text{slow}} \) is the time constant of the slowest mode of
\( h(t) \). Given \( T \) and \( t_0 \), \( n \) can be determined through

\[ n \geq \log_2\left( \frac{T}{t_0} + 1 \right) \]  

(12)

Therefore, for a specified \( T \), a smaller \( t_0 \) (a higher resolution) results in a larger \( n \). In practice, however, the above requirement on \( n \) is often much less stringent than that imposed by the signal to noise ratio \( \text{rms}(z_2)/\text{rms}(z_1) \). For every unit increment in \( n \), this signal to noise ratio is doubled. Another direct benefit for using a larger \( n \) is the reduction in bias as shown in (7). Note however, that both a smaller \( t_0 \) and a larger \( n \) lead to an increased computational burden in carrying out the cross-correlation. In addition, from the time a probing signal is applied to the time the probing output appears in its entirety (one complete \( h(t) \), \( 2T \) must have gone by. Therefore, whenever the signal to noise ratio at \( z \) allows, \( T \) thus \( n \) should be minimized, and whenever the resolution on \( h \) allows, \( t_0 \) should be maximized.

The pulse-compression method for process monitoring can be easily implemented digitally, though all the above discussion was carried out in the continuous domain. The digital data storage solves the problem of long delay times normally encountered in using the cross-correlation technique on-line for processes with large time constants[5]. In this case, the impulses in the impulsive \( m \)-sequence are replaced by pulses, the lowpass wavelet can be replaced by its samples at a sampling rate as low as one sample per bit duration \( (t_0) \). The \( m \)-sequence can be generated by an \( n \)-stage shift register[4]. The digital signal acquisition and application from and to the process can be accomplished by using A/D and D/A converters.

4 Relation to the cross-correlation technique

The same pseudo-random excitation is used in both the pulse-compression method for process monitoring introduced in this paper and in the cross-correlation method for impulse response identification for linear systems casted in least squares formulation[6, 5]. Therefore the two methods are in fact equivalent, and enjoy the same advantages. Since the methods are nonparametric, little a priori knowledge is required of the process model. Processing and memory requirements are moderate and can be controlled through trading off with the monitoring/identification resolution/accuracy. As long as the process is stable, divergence is not possible. Though not considered in this paper, it has been shown[5, 6] that the cross-correlation method has a superb noise rejection capability. The pulse-compression method differs from the cross-correlation method in the following aspects. The pulse compression for process monitoring problem is formulated entirely in the continuous domain; the probing signal and the reference signal in our pulse-compression method are not the same; our method is applicable to nonlinear systems; the analysis is carried out from a deterministic view; the flexibility of using a general lowpass wavelet (as opposed to rectangular only) is introduced; the unnessess of the periodic repetition of the probing signal[7] for less noisy process is clarified and therefore reduce the time delay in cross-correlating. Use of the monitoring data for diagnosis or identification purpose is possible[6].

For a complete development of the cross-correlation method using the \( m \)-sequence for linear system identification, the reader is referred to Hill and McMurtry[5].

5 An example

In this section, the pulse-compression method for process monitoring is applied to a simulated electro-hydraulic system that is subject to leakage. The system exhibits a first order small signal characteristics with a bandwidth of approximately 5 rad/sec. In order to be able to recognize a small percentage change in leakage coefficient, bit duration \( t_0 \) is chosen to be 0.04sec. Using the setup of Fig.1, a continuous probing signal \( (u_2(t)) \) one tenth of the magnitude of a windowed sinusoidal signal \( (u_1(t)) \) is applied to the system. \( u_2(t) \) is generated through a 15th order impulsive \( m \)-sequence modulated rectangular wavelet. Response \( y(t) \) at the system output is shown in Fig.3. The magnitude ratio of \( y_1 \) to \( y_2 \) is about 7. Response \( z(t) \) is shown in Fig.4, the magnitude ratio of \( z_2 \) to \( z_1 \) is approximately 40, yielding a very accurate estimate of impulse response. This is achieved at the expense of a long delay of approximately 42 (= \( 2T \)) minutes before the entire impulse response is observed. In practice that the noise level would not necessitate the pursuit of this accuracy, and a shorter delay in acquiring monitoring data is desirable.

6 Conclusions

A pulse-compression method for on-line monitoring of the small signal behavior is described. The parameters in the probing signal can be chosen to balance between the speed of processing and the accuracy of the acquired monitoring data, which is the impulse response of the process at the current operating condition. The paper elucidates the concept of stimulating all significant modes of the small signal characteristics of a process through the compression of an elongated small magnitude poring signal of finite duration into
a near impulsive equivalent excitation signal. Besides the application in process monitoring as discussed in this paper, the pulse-compression method has found applications in seismic acquisition[2, 3], and in linear system identification[5, 6, 7] under a different formalization. Other possible applications include earth crust movement monitoring, near surface engineering applications, paleontologic study, mechanical malfunction in machineries, and biomedical imaging applications.

References


\[
\begin{align*}
\text{Fig.1 A process monitoring schematic} \\
\text{Fig.2 Lowpass wavelet examples and a probing signal} \\
\text{Fig.3 Hydraulic system response to sinusoidal with/without probing signal} \\
\text{Fig.4 Probing output to probing input with/without sinusoidal input}
\end{align*}
\]