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An automated method to compute orbital re-entry trajectories with heating constraints

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Determining how to properly manipulate the controls of a re-entering re-usable launch vehicle (RLV) so that it is able to safely return to Earth and land involves the solution of a two-point boundary value problem (TPBVP). This problem, which can be quite difficult, is traditionally solved on the ground prior to flight. If necessary, a nearly unlimited amount of time is available to find the "best" solution using a variety of trajectory design and optimization tools. The role of entry guidance during flight is to follow the pre-determined reference solution while correcting for any errors encountered along the way. This guidance method is both highly reliable and very efficient in terms of onboard computer resources. There is a growing interest in a style of entry guidance that places the responsibility of solving the TPBVP in the actual entry guidance flight software. Here there is very limited computer time. The powerful, but finicky, mathematical tools used by trajectory designers on the ground cannot in general be converted to do the job. Non-convergence or slow convergence can result in disaster. The challenges of designing such an algorithm are numerous and difficult. Yet the payoff (in the form of decreased operational costs and increased safety) can be substantial. This paper presents an algorithm that incorporates features of both types of guidance strategies. It takes an initial RLV orbital re-entry state and finds a trajectory that will safely transport the vehicle to Earth*. During actual flight, the computed trajectory is used as the reference to be flown by a more traditional guidance method.

Overview

In the design of entry trajectories, two control variables are normally available: alpha (angle of attack) and phi (bank angle)[1]. The entry guidance algorithm developed in this paper (referred to as EGuide) primarily attempts to find and adjust *bank angle* profiles to meet final state constraints while maintaining a constant or pre-determined angle of attack profile. EGuide contains a classic shooting method as its solver. When solving a specific entry problem, EGuide adjusts parameters to meet specified goals using a Newton method that has been configured to generate its Jacobian matrix by flying predictive simulations. For orbital entry, most of the heavy computational load of the Newton process can occur prior to the de-orbit burn. This is what is meant by an orbital entry planner. The solution derived on-board can be used to supply trajectory information to a more traditional profile-following guidance law such as Dukeman's LQR[2]. It is also a natural setup to run as a predictor/corrector guidance. EGuide contains a planning stage and functions both as a predictor/corrector and as a profile follower using LQR.

Development and testing of EGuide has been carried out using the MAVERIC vehicle simulation. MAVERIC is a full 6-DOF simulation developed to test GNC flight software for the X33.

Shooting Method

EGuide solves the TPBVP using mnewt[3]. Mnewt** generates its Jacobian matrix by measuring the effect of control changes on the final state of simulated flights. EGuide uses a self-contained 3-DOF-trajectory simulation (independent from MAVERIC) that models motion over a rotating oblate Earth with a US62 standard atmosphere. The equations of motion are integrated using a 4^{th} order Runge-Kutta algorithm with a fixed step size of 1 second.

Sub-orbital Re-entry Guidance

Part of EGuide is dedicated to solving the TPBVP of a sub-orbital entry trajectory. Specifically, it tries to figure out how to deliver a vehicle from a variety of widely dispersed sub-orbital entry conditions to a Terminal Area Energy Management (TAEM) interface box within an acceptable tolerance of altitude, range, and heading.

To solve the sub-orbital problem, a linear equation is used to define the bank angle. At each time point during an EGuide trajectory simulation the commanded bank angle is computed using the following formulation:

$$\varphi_{cmd} = B_1 + B_2\left(t_{current} - t_{init}\right) \qquad (1)$$

The sign of φ_{cmd} is assigned to maintain the heading angle within a specified corridor. Alpha angle is predefined as a function of mach number. Through shooting, EGuide identifies the individual values that parameters B_1 and B_2 must take so that the vehicle will fly to TAEM.

Solving a sub-orbital entry trajectory problem is essential in the process that EGuide uses to solve for an orbital re-entry trajectory. It is also a required function to participate in the Advanced Guidance and Control project[4]. This project uses the MAVERIC vehicle simulation loaded with an X33 vehicle model as its testing platform and provides a scored evaluation to the participating algorithms. Nominal and off-nominal X33 missions are included in the testing criteria. For a nominal X33 flight, the EGuide planning phase takes place shortly after main engine cutoff (MECO). Once parameters B_1 and B_2 from equation (1) have been found, the trajectory is recorded and LQR is activated to guide the vehicle to TAEM.

Sub-Orbital Results

Alternative bank angle formulations

Constant heat-rate tracking

Although equation (1) can be used to generate valid trajectories from orbital re-entry states, it does not contain enough flexibility to consistently return practical solutions. Heat and dynamic pressure constraints, which have previously been ignored, must somehow be addressed to assure safe flight conditions. Heating in particular can be effectively controlled using a bank angle formulation designed to maintain a constant heat rate during flight.

From the 3-DOF equations of motion for a vehicle flying over a spherical Earth we have[5]:

$$\dot{r} = V \sin\gamma$$

$$\dot{V} = -D - \frac{\mu}{r^2}\sin\gamma$$

$$\dot{\gamma} = \frac{1}{V}\left\{L\cos\sigma + \left(V^2 - \frac{\mu}{r}\right)\frac{\cos\gamma}{r} + 2\omega V\cos\varphi\cos\phi\right\} \qquad (2)$$

For a vehicle entering a planetary atmosphere, the time rate of average heat input per unit area can be estimated with the expression[6]

$$\dot{Q} = C\sqrt{\rho}V^n \qquad (3)$$

where n = 3.15, and C is a constant. Heat-rate tracking guidance begins with the definition of an error term

$$e = \dot{Q} - \dot{Q}_{ref} \qquad (4)$$

and the intent for this term to exhibit the behavior of a stable second order feedback system. To accomplish this, it is substituted into the following classical second order system

$$\ddot{e} + 2\zeta\omega_n\dot{e} + \omega_n^2 e = 0 \qquad (5)$$

The three preceding equations along with the equations of motion yield the following bank angle formulation:

$$\varphi_{cmd} = \cos^{-1}\left\{\frac{\left(\dot{\gamma}_{cmd} - \frac{v}{r} + \frac{\bar{g}}{v}\right)\frac{v}{D}}{\left(\frac{L}{D}\right)_{est}}\right\}$$

$$\dot{\gamma}_{cmd} = \frac{-a - 2\zeta\omega_n\ddot{Q} - \omega_n^2(\dot{Q} - \dot{Q}_{ref})}{b} \qquad (6)$$

where a and b are expressions that are determined in the second time derivative of equation (3). Thus, given a reference heat-rate $\dot{Q}_{ref}$ and using equation (6) to generate bank angle control commands, the vehicle is expected to track a constant heat rate.

Orbital re-entry planning

In an orbital re-entry, high heating starts near the beginning of the flight back into the atmosphere when speeds are still close to orbital velocity. Re-entry guidance can begin as soon as there is enough dynamic pressure to maintain adequate control of the vehicle. The EGuide planning simulation is configured to use heat rate tracking guidance at the onset of orbital re-entry. Once the vehicle has been safely transported through high heating, heat rate tracking guidance is deactivated and the sub-orbital guidance formulation is used for the remainder of the flight. The trajectory is fully characterized by four parameters. The first three $\dot{Q}_{ref}$, B1, and B2 are from the heat rate tracking and sub-orbital guidance formulations. The fourth parameter is the time chosen to terminate heat rate tracking and switch to sub-orbital guidance (Fig. 2).

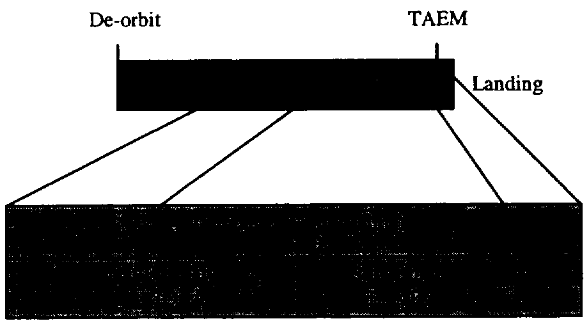

Fig. 2 The four parameters of an EGuide orbital re-entry trajectory

Selection of the guidance switch time is based on observed bank angle behavior during heat rate tracking. As illustrated in Figure 3, in the very first part of an orbital re-entry, heat rate tracking guidance may modulate the bank angle quickly as the control "latches on" to the specified reference heat rate. This initial transient behavior gives way to a slower varying bank angle which is decreasing in magnitude as the control tracks the reference. If heat rate tracking is allowed to remain active indefinitely, the magnitude of the bank angle eventually begins to increase and finally becomes excessive as the control struggles to track a reference heat rate it can no longer sustain. The main feature of interest in the bank angle vs. time plot of Figure 3 is the local minimum that occurs at approximately 800 seconds (Point A). Point A is designated as the guidance switch time. In actuality, good solutions to the orbital re-entry problem may exist by switching guidance formulations anywhere along the bank angle profile between points A and B. However, Point A offers the advantage of being an event that is easily detectable. EGuide is programmed to find Point A by simulating an orbital re-entry (using heat rate tracking guidance exclusively) and looking for the last occurrence of a bank angle minimum. Additionally, the fact that the bank angle magnitude must increase after Point A to maintain the reference

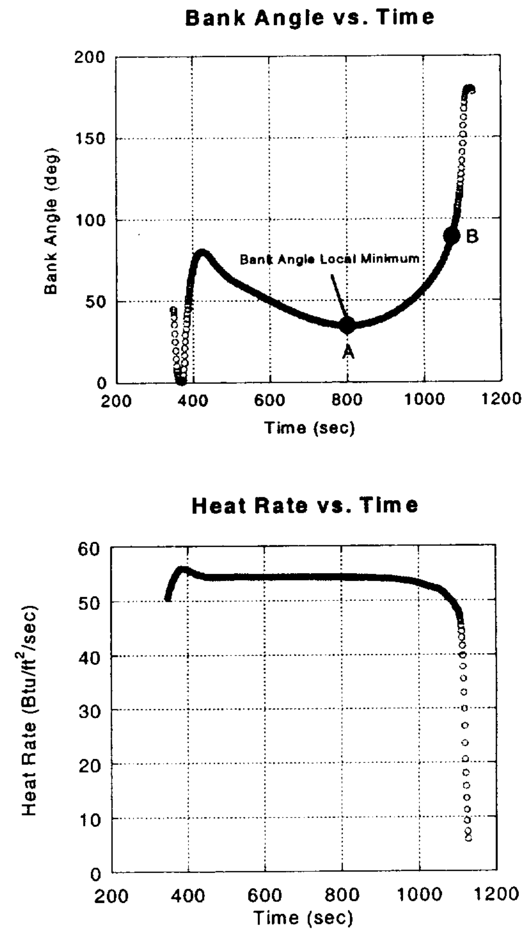

Fig. 3 Heat rate tracking guidance in action

heat rate indicates that the high heating portion of the flight has past.

Orbital re-entry results

References

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