STEADY STATE TRANSPIRATION COOLING IN POROUS MEDIA UNDER LOCAL, NON-THERMAL EQUILBRIUM FLUID FLOW

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Abstract

An analytical solution to the steady-state fluid temperature for 1-D transpiration cooling has been derived. Transpiration cooling has potential use in the aerospace industry for protection against high heating environments for re-entry vehicles. Literature for analytical treatments of transpiration cooling has been largely confined to the assumption of thermal equilibrium between the porous matrix and fluid. In the present analysis, the fundamental fluid and matrix equations are coupled through a volumetric heat transfer coefficient and investigated in non-thermal equilibrium. The effects of varying the thermal conductivity of the solid matrix and the heat transfer coefficient are investigated. The results are also compared to existing experimental data.

Introduction

Transpiration cooling is the process of injecting a fluid (generally serving as a coolant) into a porous matrix, which could serve as a protective barrier against high temperature environments for a re-entry vehicle. In order to utilize a transpiration cooling analysis approach to solving physical applications, a well-developed understanding of the heat transfer and fluid flow characteristics must be obtained. In the report by J.C.Y. Koh et. al. "Investigation of Fluid Flow and Heat Transfer in Porous Matrices for Transpiration Cooling", the fundamental equations for steady state transpiration cooling are stated [1]. However, the solution of the fluid temperature distribution generates results that are not consistent with the physical model requirements as a result of the lack of formal boundary conditions. The results of which can create conditions that violate energy conservation. Therefore, an investigation of the fluid temperature solution presented by Koh is conducted to understand the inconsistencies with the model, and derive an alternative fluid temperature solution. Once these steady state transpiration equations have been established, they can be used as a guide for understanding behavior of heat transfer in porous matrices and also for further transient studies of transpiration cooling.

Transpiration cooling has been treated in the literature by numerous authors. Heat conduction textbooks generally treat transpiration cooling with the assumption of thermal equilibrium between the matrix and fluid. The assumption leads to defining an effective conductivity for the fluid and solid matrix [2]. Curry and Cox conducted numerical studies of the transient effects of transpiration cooling [3]. Using a non-equilibrium solution, they determined that for a high conductivity of the solid matrix, the equilibrium solution is a valid assumption. However, the lower the conductivity of the solid, the more desperate the fluid solution diverges from that of the solid. Additionally, the volumetric

heat transfer coefficient does not affect the response significantly compared to the thermal conductivity of the solid matrix [3].

Transpiration Cooling Model

The physical model utilized in Koh's report is used in this analysis. A flat plate with finite thickness, L, shown in Figure 1, is used for the derivation of the steady state fluid and matrix temperature distributions. The analysis is based on a one-dimensional model with constant material properties.
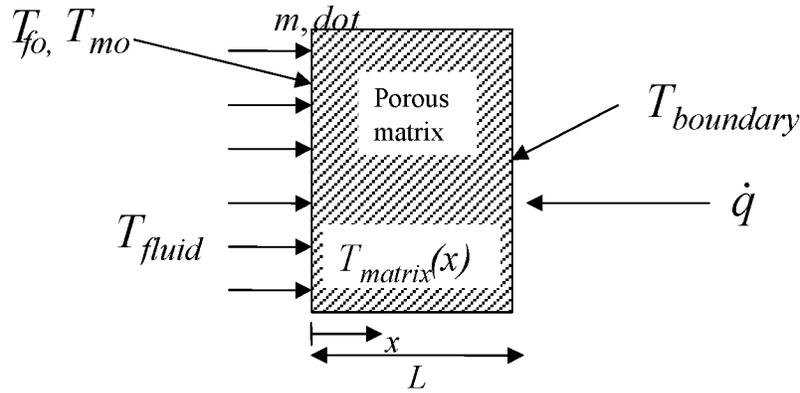

Figure 1: Transpiration Cooling Model

The boundary conditions for the transpiration cooling model are taken from Figure 1. A fluid is injected into porous matrix at a constant mass flow rate, $\dot{m}$, with a temperature T_{fo}. The temperature of the matrix at the entrance, $x=0$, is T_{mo}. A flux is imposed at $x=L$ from the environment which induces a constant temperature boundary condition at the exit, T_{mw}.

The energy balance equations for the fluid and the matrix in non-dimensional form are used to define the governing differential equations for the transpiration cooling model. The energy equation for the fluid stated by Koh is,

$$\frac{d\theta_f}{d\eta} = \frac{h'L}{\dot{m}c_p}\left(\theta_m - \theta_f\right) = A\left(\theta_m - \theta_f\right) \tag{1}$$

where

$$\theta_f = \frac{T_f - T_{f,i}}{T_{mw} - T_{f,i}} \tag{2}$$

and c_p is the specific heat of the fluid, k_m is the effective conductivity of the matrix, h' is the heat transfer coefficient for internal convection, and $\eta = x/L$. Equation (1) states that

heat is transferred from the matrix to the fluid via convection [1]. Conduction from the matrix to the fluid raises the enthalpy of the fluid and is given by,

$$\frac{d\theta_m}{d\eta} = \frac{m, dotc_p L}{k_m} \theta_f = B\theta_f \tag{3}$$

where

$$\theta_m = \frac{T_m - T_{f,i}}{T_{mw} - T_{f,i}} \tag{4}$$

Matrix Temperature Distribution

From the energy balance equations for the transpiration cooling model, the steady state equations for the fluid and matrix temperature distributions are derived [1]. Equation (3) can be solved in terms of the fluid temperature and substituted into equation (1). The resulting equation is a linear homogeneous second order ordinary differential equation with constant coefficients previously solved by Koh, which describes the non-dimensional steady-state matrix temperature:

$$\frac{d^2\theta_m}{d\eta^2} + A\frac{d\theta_m}{d\eta} - AB\theta_m = 0 \tag{5}$$

The boundary conditions imposed upon the matrix are used to solve equation (5) which are given by,

$$\theta_m(\eta = 0) = \frac{T_{m,o} - T_{f,i}}{T_{mw} - T_{f,i}} = \theta_{m,o} \tag{6}$$

$$\theta_m(\eta = 1) = \frac{T_{mw} - T_{f,i}}{T_{mw} - T_{f,i}} = 1 \tag{7}$$

Imposing the boundary conditions on the solution of equation (5) results in the non-dimensional matrix temperature,

$$\theta_m = \frac{1 - \theta_{mo} e^{r_2}}{e^{r_1} - e^{r_2}} e^{r_1 \eta} - \frac{1 - \theta_{mo} e^{r_1}}{e^{r_1} - e^{r_2}} e^{r_2 \eta} \tag{8}$$

where, r_1 and r_2 are the roots of the characteristic equation defined by equation (5) shown here,

$$r_1 = \frac{A}{2}\left(-1 + \sqrt{1 + 4B/A}\right)$$ (9)

$$r_2 = \frac{A}{2}\left(-1 - \sqrt{1 + 4B/A}\right)$$ (10)

Equation (8) is the steady state transpiration cooling equation for the matrix temperature along the η coordinate. This equation, first derived by Koh, will be necessary for determining an alternative steady state equation for the fluid temperature distribution, which is discussed in the following section.

Derivation of Alternative Fluid Temperature Distribution

The fluid temperature distribution is determined by substituting the matrix temperature solution into one of the energy balance equations, equation (1) or (3). However, the results from this substitution produce different results based upon which equation is chosen for the substitution. The fluid temperature presented by Koh utilized equation (3). However, no boundary condition for the fluid at the entrance is required. Since, the results from this method do not permit entrance conditions from being incorporated, an alternative solution which imposes a fluid entrance boundary condition is derived.

Equation (1) can be used to solve for the fluid temperature. The fluid entrance boundary condition can be incorporated into the solution since equation (1) is a non-homogeneous first order differential equation of the form,

$$\frac{d\theta_f}{d\eta} + A\theta_f = A\theta_m$$ (11)

where θ_m is defined by equation (8). The solution to equation (11) is approached by first multiplying equation (11) by the function, $e^{A\eta}$, which produces,

$$e^{A\eta}\frac{d\theta_f}{d\eta} + e^{A\eta}A\theta_f = e^{A\eta}A\theta_m$$ (12)

Noting that the left-hand side of equation (12) is equivalent to the derivative $D_x\left[e^{A\eta}\cdot\theta_f\right]$, the following expression can be obtained,

$$e^{A\eta}\theta_f = \int e^{A\eta}A\theta_m d\eta + C$$ (13)

where C is a constant of integration and defined by the boundary condition imposed upon this problem. Substituting equation (8) into equation (12) and solving for θ_f yields,

$$\theta_f = e^{-A\eta}\left[\int e^{A\eta} A\left(\frac{1-\theta_{mo}e^{r_2}}{e^{r_1}-e^{r_2}}e^{r_1\eta} - \frac{1-\theta_{mo}e^{r_1}}{e^{r_1}-e^{r_2}}e^{r_2\eta} \right)d\eta + C \right]$$ (14)

which can be simplified to the following form,

$$\theta_f = A\left[\frac{1-\theta_{mo}e^{r_2}}{\left(e^{r_1}-e^{r_2}\right)\left(r_1+A\right)}\left(e^{r_1\eta}\right) - \frac{1-\theta_{mo}e^{r_1}}{\left(e^{r_1}-e^{r_2}\right)\left(r_2+A\right)}\left(e^{r_2\eta}\right) + \frac{C}{A}e^{-A\eta} \right]$$ (15)

The constant of integration is solved by imposing the fluid entrance boundary condition, which is defined as,

$$\theta_f\left(\eta=0\right) = \frac{T_{f,o}-T_{f,i}}{T_{mw}-T_{f,i}} = \theta_{f,o}$$ (16)

The boundary condition is imposed on equation (15) and results in the following expression for the non-dimensional fluid temperature distribution,

$$\theta_f = A\left[\frac{1-\theta_{mo}e^{r_2}}{\left(e^{r_1}-e^{r_2}\right)\left(r_1+A\right)}\left(e^{r_1\eta}\right) - \frac{1-\theta_{mo}e^{r_1}}{\left(e^{r_1}-e^{r_2}\right)\left(r_2+A\right)}\left(e^{r_2\eta}\right) + \right.$$
$$\left. \left(\frac{\theta_{f,0}}{A} - \left(\frac{1-\theta_{mo}e^{r_2}}{\left(e^{r_1}-e^{r_2}\right)\left(r_1+A\right)} - \frac{1-\theta_{mo}e^{r_1}}{\left(e^{r_1}-e^{r_2}\right)\left(r_2+A\right)} \right) \right)e^{-A\eta} \right]$$ (17)

which simplifies to

$$\theta_f = A\left[\frac{1-\theta_{mo}e^{r_2}}{\left(e^{r_1}-e^{r_2}\right)\left(r_1+A\right)}\left(e^{r_1\eta}-e^{-A\eta}\right) - \frac{1-\theta_{mo}e^{r_1}}{\left(e^{r_1}-e^{r_2}\right)\left(r_2+A\right)}\left(e^{r_2\eta}-e^{-A\eta}\right) + \frac{\theta_{f,0}}{A}e^{-A\eta} \right]$$ (18)

Equation (18) states the steady state transpiration cooling equation for the fluid temperature. It is a function of primarily of the distance along the x direction, and based on the boundary condition of the matrix and fluid at the entrance of the porous matrix.

Results

The steady state transpiration cooling equations for the fluid and the matrix are functions of a single variable, η. In Figure 2, the alternative fluid temperature solution and steady state matrix temperature are plotted versus the non-dimensionalized coordinate, η. The entrance fluid boundary condition is assumed to be the temperature of the fluid reservoir with a temperature of $T_{fo}=T_{fi}=600$ °F and the matrix boundary condition at the entrance is $T_{mo} = 500$ °F. At the exit, T_{mw} is 1500 °F, which is due to the environmental heating.
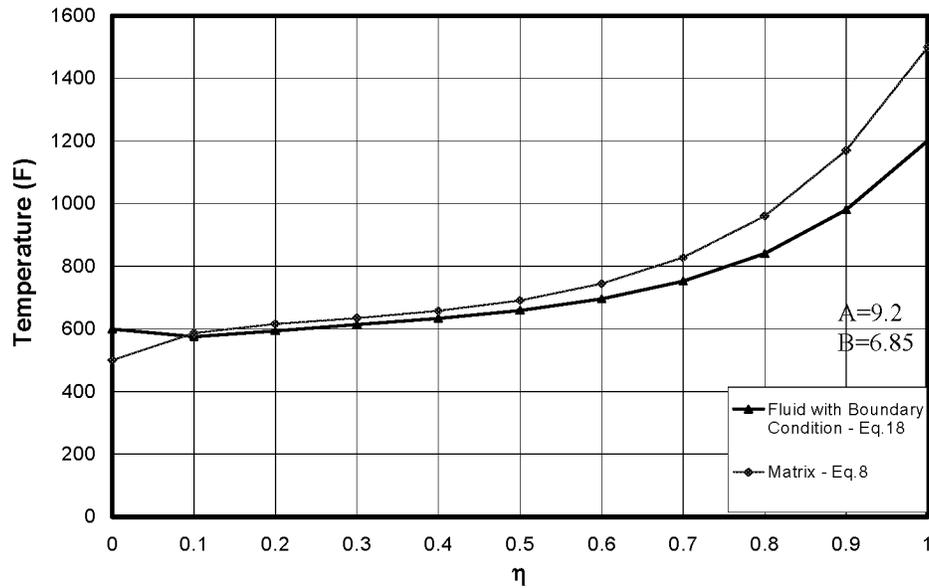
Figure 2: Fluid and Matrix Temperature Distributions along the Porous Matrix

Using equation (8) and (18), the alternative fluid temperature solution and the matrix solution are shown in Figure 2. The fluid and matrix temperature profiles along the η direction obey the conservation of energy and satisfy the physical boundary conditions imposed upon the model. A comparison of the results from the Koh study and the alternative method of calculating the fluid temperatures is shown in Figure 3.
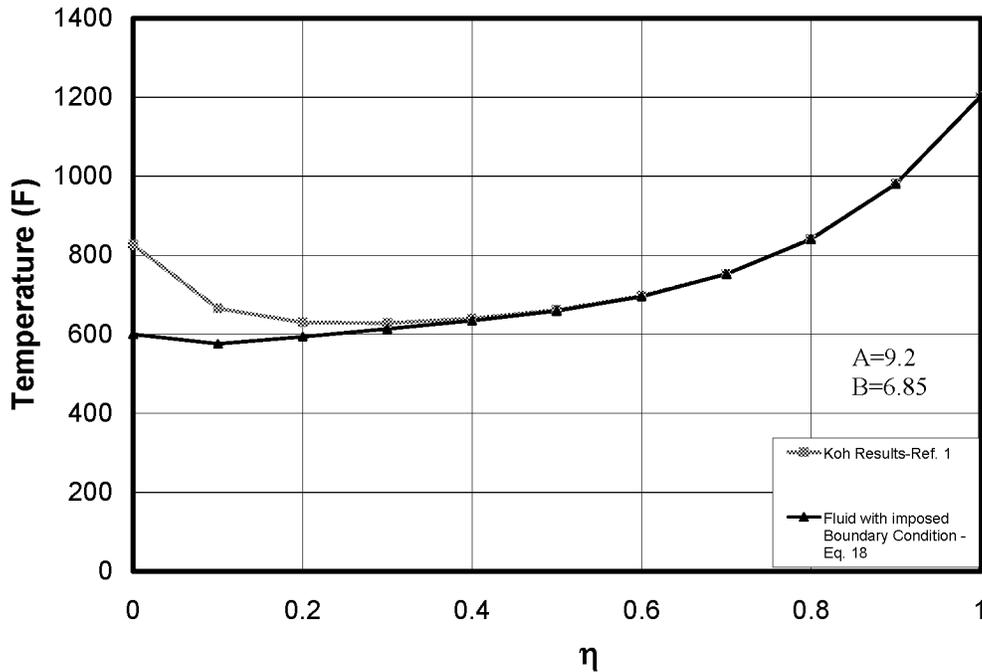
Figure 3: Fluid Temperature Solution Comparison

In Figure 3, the solution presented in [1] results in an entrance fluid temperature that does not adhere to the boundary conditions imposed upon the system. However, the alternative solution given by equation (18) has an imposed boundary condition, which satisfies the physical boundary conditions of the transpiration cooling model.

The parameters that govern the temperature distribution for the fluid and matrix are A and B defined in equations (1) and (3). The inverse of the thermal conductivity of the solid matrix is defined in parameter B. The effect of varying the thermal conductivity is shown in Figure 4.
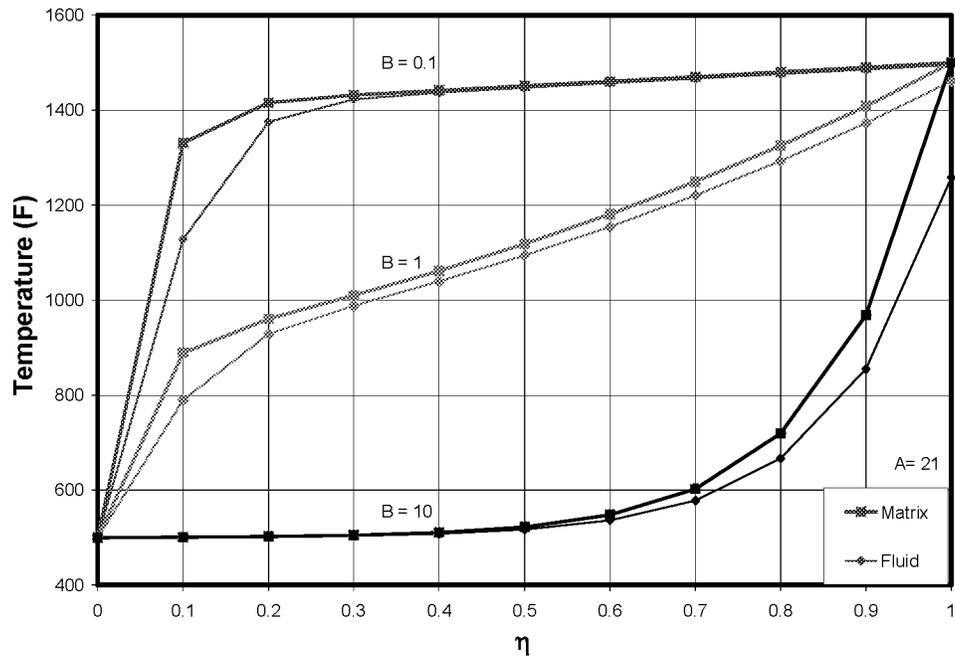
Figure 4: Predicted Temperature Distributions with Varying Thermal Conductivity

Since increasing the thermal conductivity results in a decreasing B, the temperature distribution changes dramatically with the amount of heat that is allowed to flow through the matrix. It is also clear that the temperature distribution is highly dependent on the solid matrix conductivity. The volumetric heat transfer coefficient is represented in parameter A. The effect of varying A is illustrated in Figure 5.
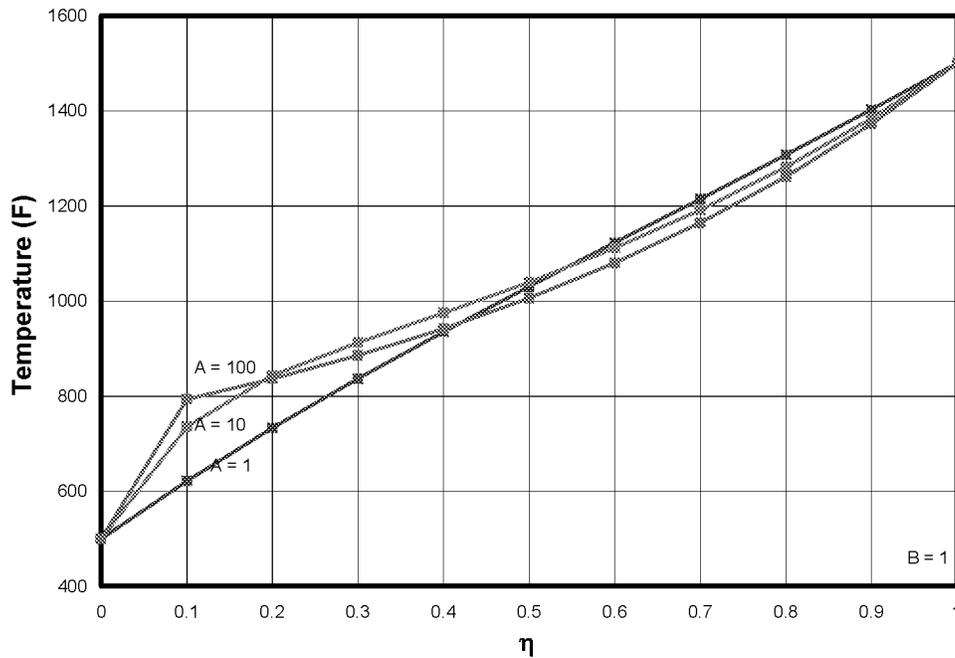
Figure 5: Predicted Temperature Distributions with Varying Heat Transfer Coefficient

The heat transfer coefficient does not significantly change the temperature distribution of the solid matrix. For increasing heat transfer coefficient, h', the fluid temperature (not shown in Figure 5) will converge on the matrix temperature solution, however it will not significantly affect the matrix temperature distribution. This phenomenon demonstrated by the present analysis therefore conforms to previous numerical studies of porous media heat transfer [3].

The results of the present analysis are also compared to previous experimental data [4]. The experimental data is based on air flowing through uniformly packed beds at various Reynolds numbers. Spheres were used in the experiment approximately .5 inch in diameter. However, for materials that would fit transpiration cooling applications, the porous matrix diameter would be reduced by several magnitudes of order. The reduction in the porous matrix diameter affects the mass flow rate, which is accounted for in the present analysis. Iron-constantan thermocouples were imbedded in the spheres to determine matrix temperatures. Gas temperatures were determined by an energy balance equation in finite difference form [1].
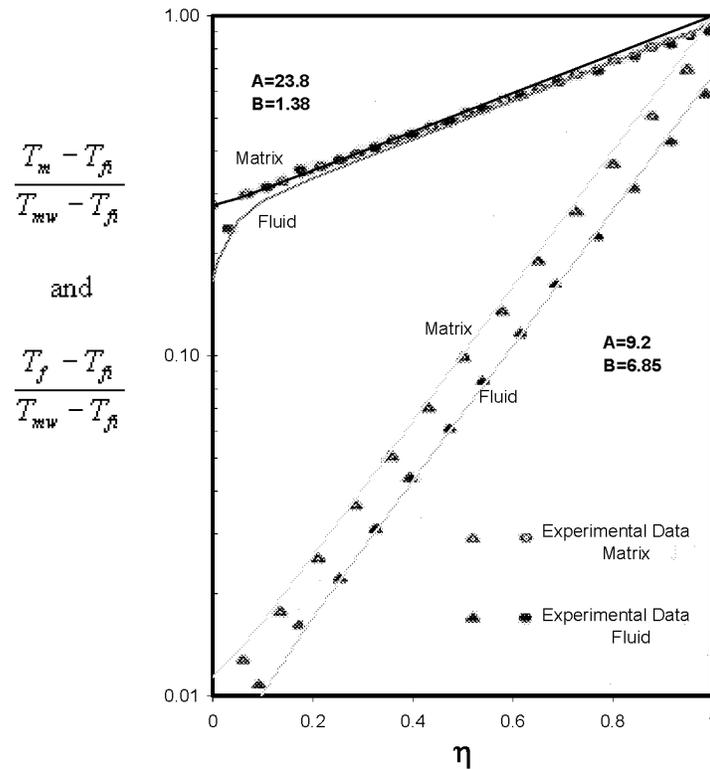
$$\frac{T_m - T_{fi}}{T_{mw} - T_{fi}}$$

and

$$\frac{T_f - T_{fi}}{T_{mw} - T_{fi}}$$

Figure 6: Comparison of Theoretical and Experimental Results

As illustrated in Figure 6, the analysis agrees well with the two sets of experimental data. For the large heat transfer coefficient condition, characterized by a higher value of A, it is seen that the difference between the matrix and fluid temperatures is small. For the small thermal conductivity condition, characterized by a higher value for B, the heat flow becomes significantly reduced agreeing with both the experimental data and previous numerical studies.

Conclusion

The fluid temperature solution presented by Koh produces results that are not physically realistic. An alternative method for deriving the steady state fluid temperature has been presented. This method allows for the inclusion of the entrance boundary condition for the fluid. Furthermore, the two solutions are compared to one another and the alternative fluid solution adheres to the physical system requirements. The effect of increased thermal conductivity of the solid matrix is significant. However, the effect of changing the volumetric heat transfer coefficient was small. The results also compared well to the existing experimental data. This analysis may further the understanding not only of steady state behavior, but also the transient responses in transpiration cooling.

References

1. Koh, J. C. Y, et al. "Investigation of Fluid Flow and Heat Transfer in Porous Matrices for Transpiration Cooling." Boeing Company Technical Report AFFDL-TR-65-228, 1968.

2. Gebhardt, B. Heat Conduction and Mass Transfer. New York: McGraw-Hill Inc., 1993.

3. Curry, D. M., and Cox, J. E. "The Effect of the Porous Material Characteristics on the Internal Heat and Mass Transfer." ASME, 73 HTD-49, 1973.

4. Turnacliff, R.D., "An Experimental Study of Local Convective Heat Transfer and Pressure Drop for Laminar and Turbulent Flow of Air within a Uniformly Packed Bed of Spheres," Ph.D. Thesis, University of Minnesota, 1957.

Acknowledgements

Nomenclature

A eq. (12), dimensionless
B eq. (13), dimensionless
c_p specific heat (Btu/lb.-F)
h' volumetric heat transfer coefficient (Btu/hr-ft^3-F)
k thermal conductivity (Btu/hr-ft-F)
l length (ft.)
m,dot mass flow rate, (lb./hr)
T Temperature, (F)
x distance measured from inlet of matrix (ft.)
η dimensionless variable, x/l
θ dimensionless variable, $\dfrac{T - T_{fi}}{T_{mw} - T_{fi}}$

Subscripts

m matrix solid
f fluid
o inlet (x=0)
fi fluid reservoir
fo fluid inlet (x=0)
w wall (x=L)