

## **Probabilistic Fatigue: Computational Simulation**

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### **ABSTRACT**

Fatigue is a primary consideration in the design of aerospace structures for long term durability and reliability. There are several types of fatigue that must be considered in the design. These include low cycle, high cycle, combined for different cyclic loading conditions – for example, mechanical, thermal, erosion, etc.

The traditional approach to evaluate fatigue has been to conduct many tests in the various service-environment conditions that the component will be subjected to in a specific design. This approach is reasonable and robust for that specific design. However, it is time consuming, costly and needs to be repeated for designs in different operating conditions in general.

Recent research has demonstrated that fatigue of structural components/structures can be evaluated by computational simulation based on a novel paradigm. Main features in this novel paradigm are progressive telescoping scale mechanics, progressive scale substructuring and progressive structural fracture, encompassed with probabilistic simulation. These generic features of this approach are to probabilistically telescope scale local material point damage all the way up to the structural component and to probabilistically scale decompose structural loads and boundary conditions all the way down to material point. Additional features include a multi-factor interaction model that probabilistically describes material properties evolution, any changes due to various cyclic load and other mutually interacting effects. The objective of the proposed paper is to describe this novel paradigm of computational simulation and present typical fatigue results for structural components. Additionally, advantages, versatility and inclusiveness of computational simulation versus testing are discussed. Guidelines for complementing simulated results with strategic testing are outlined. Typical results are shown for computational simulation of fatigue in metallic composite structures to demonstrate the versatility of this novel paradigm in predicting a priori fatigue life.

**PROBABILISTIC FATIGUE:  
COMPUTATIONAL SIMULATION**

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## **BACKGROUND:**

- Fatigue is a primary consideration in the design of aerospace structures for long-term durability and reliability.
- There are several types of fatigue that must be considered in the design, such as: low cycle, high cycle, combined for different cyclic loading conditions – for example, mechanical, thermal, erosion, etc.
- The traditional approach to evaluate fatigue has been to conduct many tests in the various service-environment conditions that the component will be subjected to in a specific design.
- This approach is reasonable and robust for that specific design.
- However, it is time consuming, costly and needs to be repeated for designs in different operating conditions in general.

# Hierarchical-Technology-Benefit-Estimator

by Using Telescoping Scale Concepts/Mechanics

Estimate order

1st Aircraft

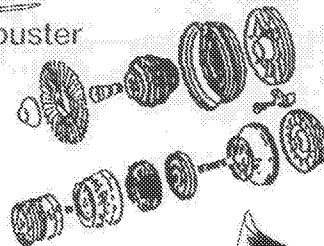
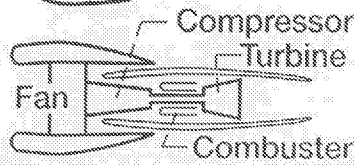
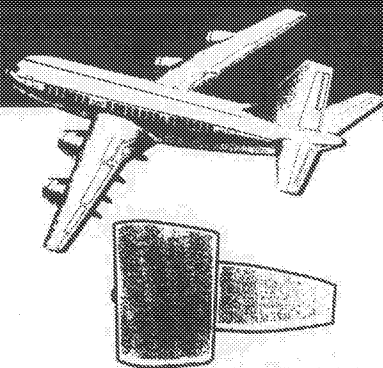
3rd Engine

4th Sections

5th Assemblies

6th Blades

12th Materials



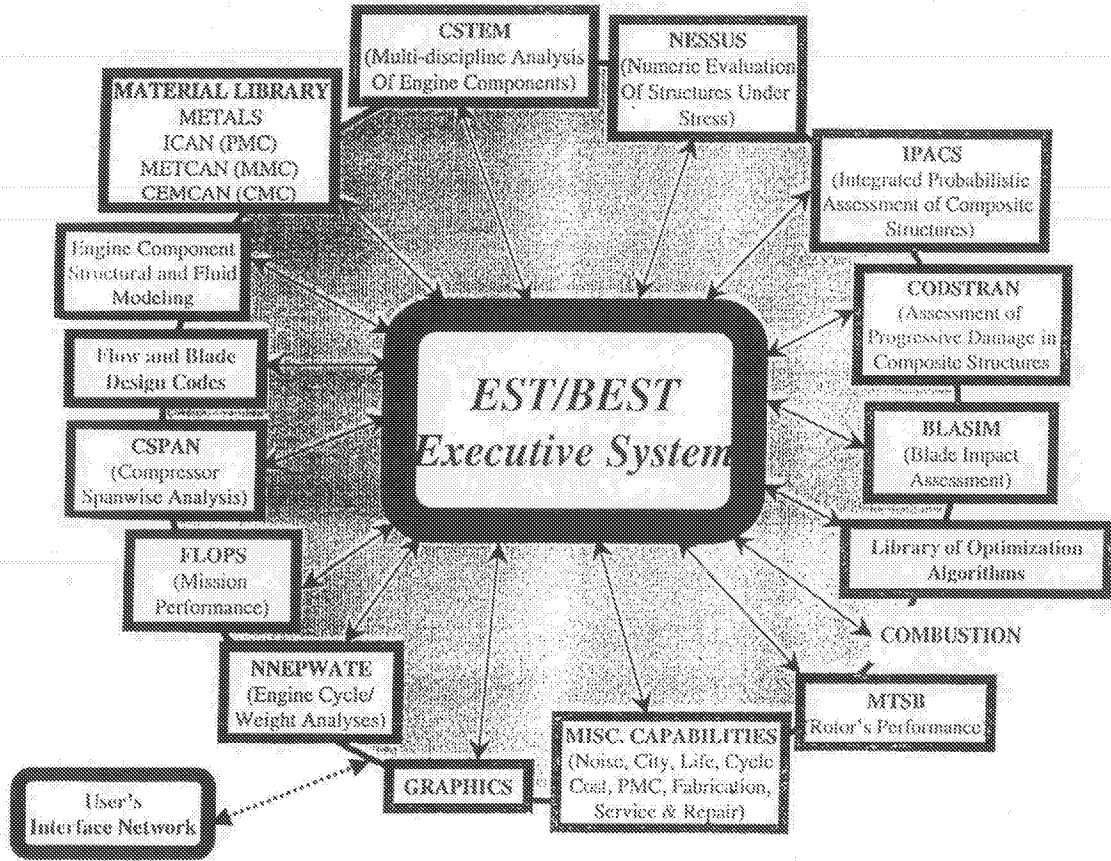
Increasing input detail

Increasing benefits estimate accuracy

## Quantified Engine and Aircraft Benefits

- |                           |                         |
|---------------------------|-------------------------|
| Emissions                 | Engine cost             |
| Noise                     | Direct operating cost   |
| Component weights         | Indirect operating cost |
| Specific fuel consumption | Life cycle cost         |
| Thrust                    | Return on investment    |
| Range                     | Risk                    |
| Capacity                  | Development time        |
| Speed                     | Development cost        |
| Life                      |                         |
| Pre-certification testing |                         |

# EST/BEST ENGINE STRUCTURES TECHNOLOGY BENEFIT ESTIMATOR



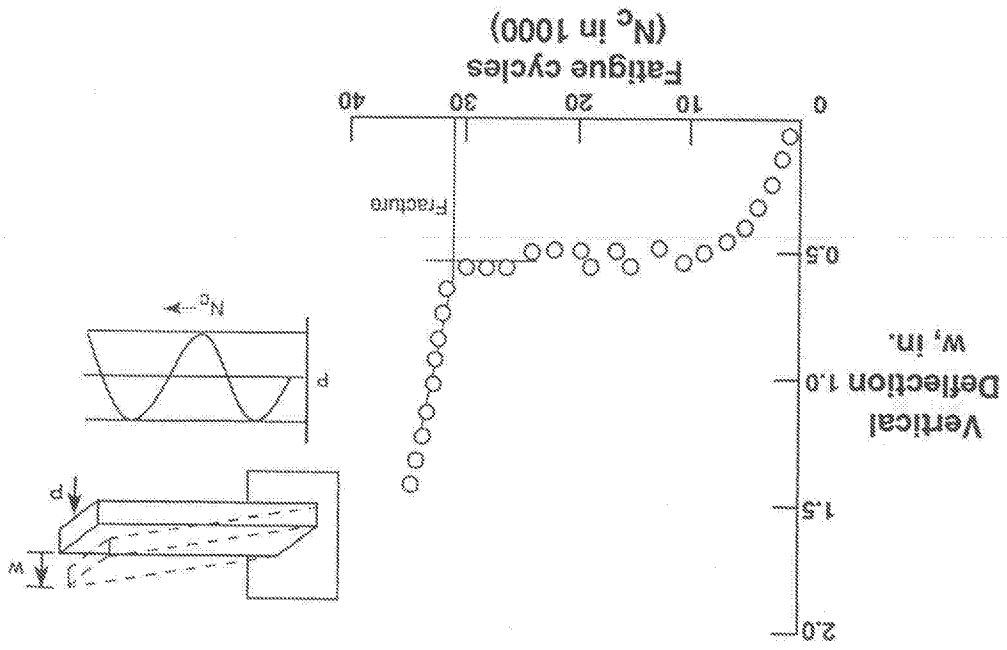
**NEED:**

- There is a continuing need to develop a method to reduce cost in long-life fatigue evaluations.
- Recent research has demonstrated that fatigue of structural components/structures can be evaluated by computational simulation based on a novel paradigm.

**OBJECTIVE:**

- Describe the novel paradigm and present structural component results that illustrate versatility, inclusiveness and that - YES IT CAN BE DONE!

**Progressive Fracture Under Cyclic Load**  
 (Experimental data: Mandel, et al)



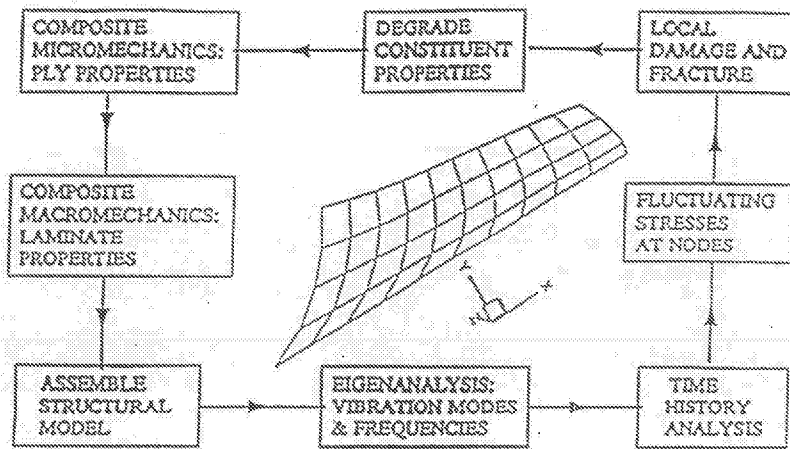


## **APPROACH:**

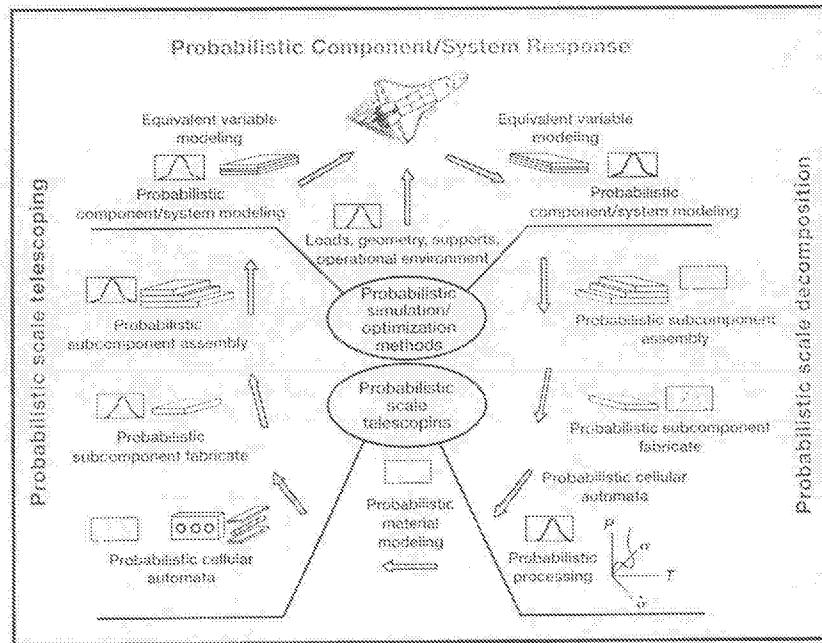
### **MAIN FEATURES IN THIS NEW PARADIGM ARE:**

- Progressive Structural Fracture
- MFIM for Material Behavior
- Telescopic Scale Mechanics

# COMPUTATIONAL SIMULATION CYCLE



# Non-Deterministic/Non-Traditional (ND/NT) Methods for Design to Cost in the Presence of Uncertainties (Simulation Iterative Cycle)



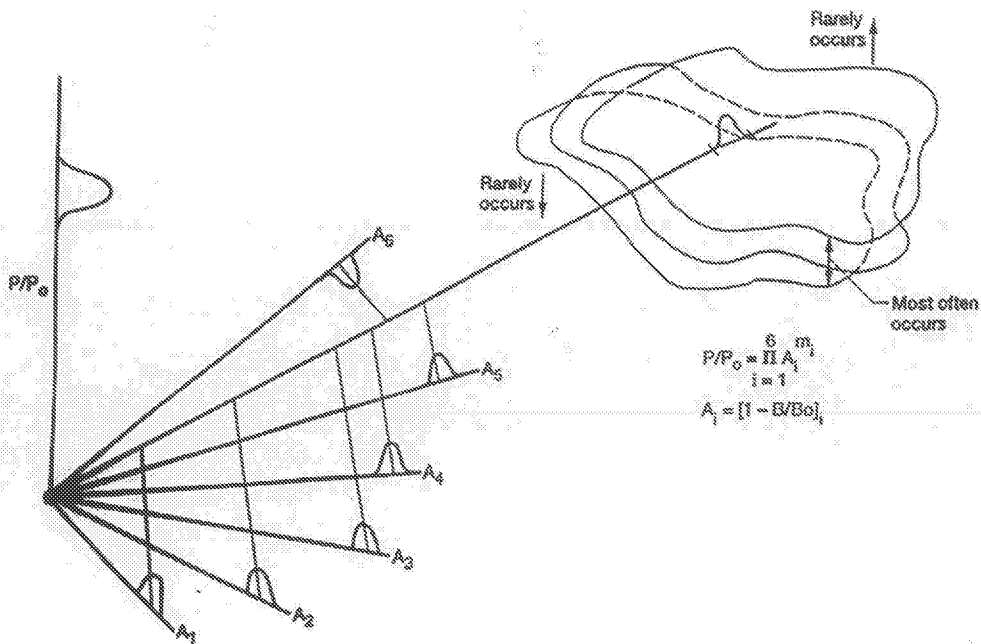
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C.C. Chenis  
NASA/GRC

## **TYPICAL RESULTS:**

- MFIM – Behavior Illustration
- Blade Thermomechanical Fatigue – GRC Simulation
- Two-Stage Rotor Fatigue – GRC Simulation
- Tank Fatigue – SWRI Simulation
- Large Shell Fatigue – GRC Simulation

# MULTI-FACTO-INTERACTION MODEL WITH SUBSTRUCTURING



### Time dependent Multi-Factor Interaction Equation (MFIE):

$$\frac{P}{P_o} = \left( \frac{T_{gw} - T}{T_{gw} - T_o} \right)^m \left( 1 - \frac{\sigma}{S_f} \right)^n \left( 1 - \frac{\sigma t}{S_f t_f} \right)^q \left( 1 - \frac{\sigma_M N_M}{S_f N_{fM}} \right)^r \left( 1 - \frac{\sigma_T N_T}{S_f N_{fT}} \right)^u$$

where:

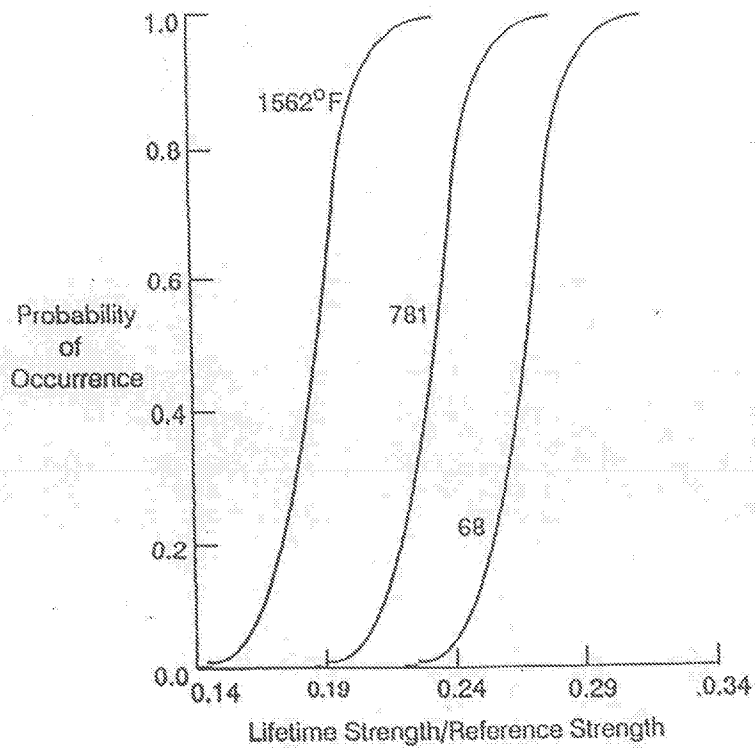
- P - Property
- T - temperature
- S - strength
- $\sigma$  - stress
- N - number of cycles
- t - time

subscripts:

- gw - wet glass temperature
- o - reference condition
- f - final condition
- M - mechanical load
- T - thermal cyclic load.

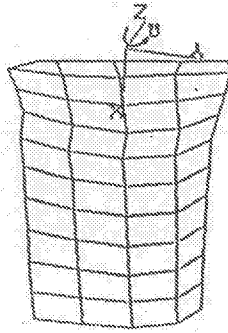
superscripts: m, n, q, r and u are exponents for the material that describe the behavior path from the reference to the final values

**PMBM-SIMULATED LIFETIME STRENGTH FOR A NICKEL-BASED  
SUPERALLOY SUBJECTED TO 3162 STRESS CYCLES  
AND 100 HOURS OF CREEP**



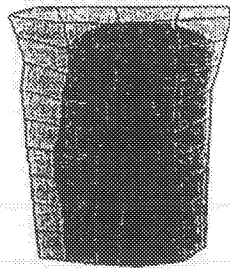
# THERMAL MECHANICAL LOADS ON SSME BLADE

CENTRIFUGAL LOADS

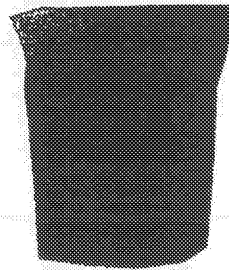
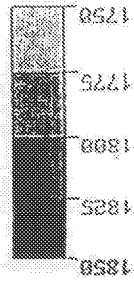


$\Omega$  = ROTATIONAL SPEED  
= 40,000 RPM

PRESSURE (psi)

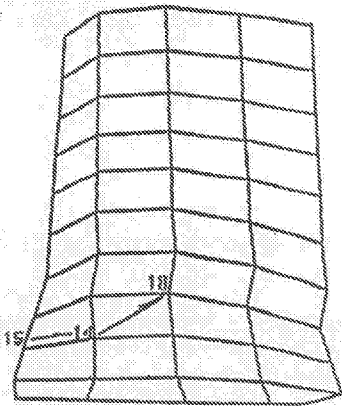


TEMPERATURE (°F)



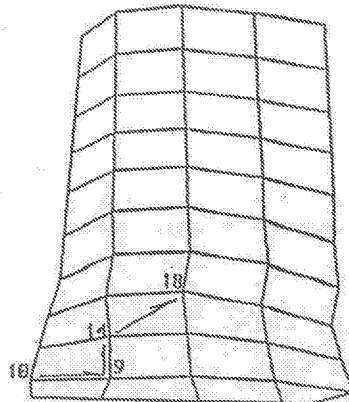


### PROBABILITY OF COMPONENT DAMAGE PROPAGATION PATH CAUSED BY 100,000 FATIGUE CYCLES



PATH A

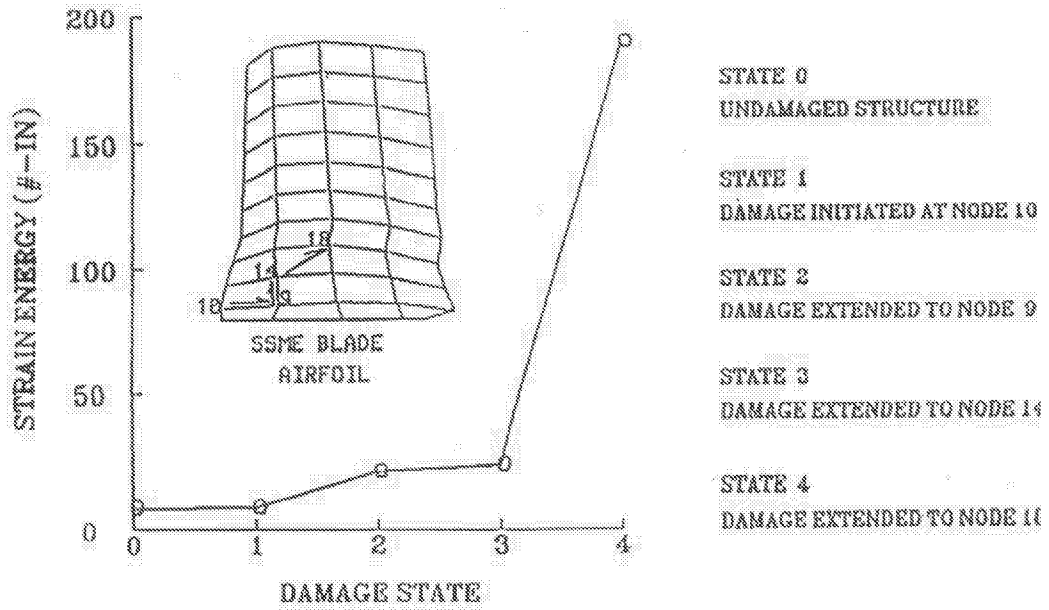
PROBABILITY OF  
PATH A OCCURS  
= 0.00001



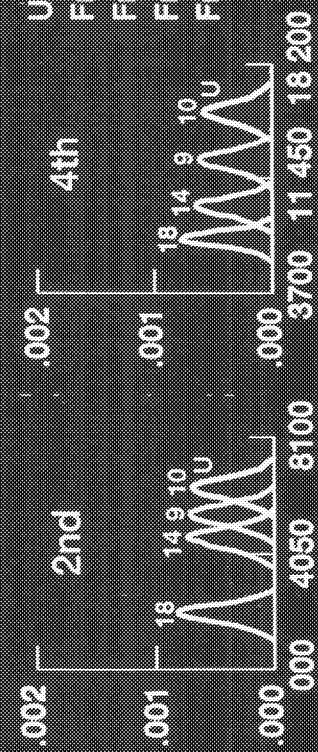
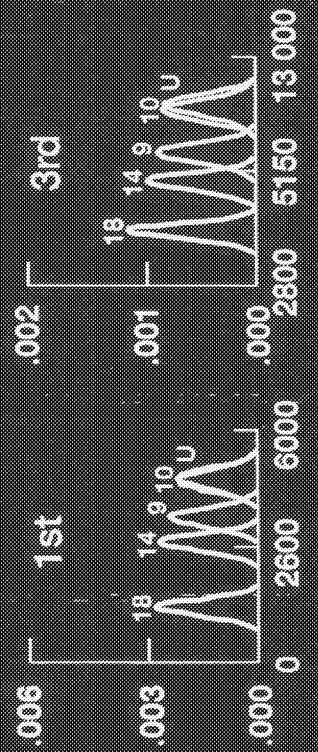
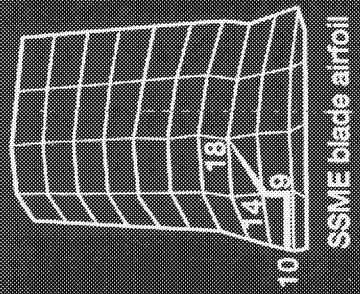
PATH B

PROBABILITY OF  
PATH B OCCURS  
= 0.0002

### STRAIN ENERGY INCREASES AS THE DAMAGE PROGRESSES

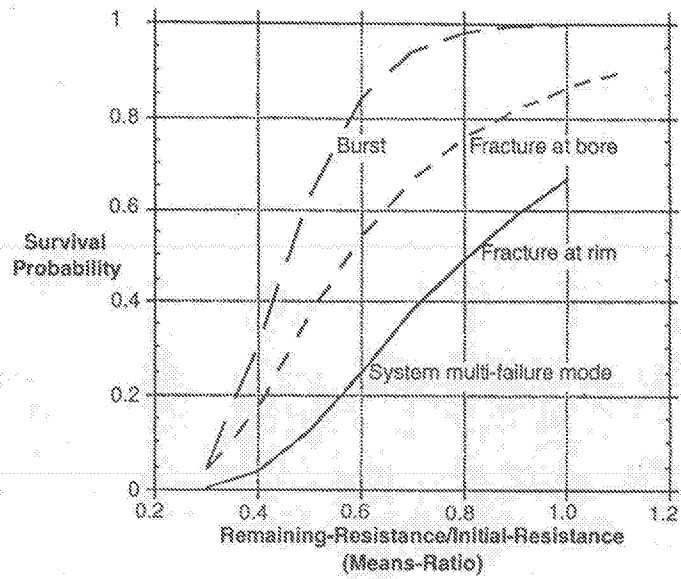
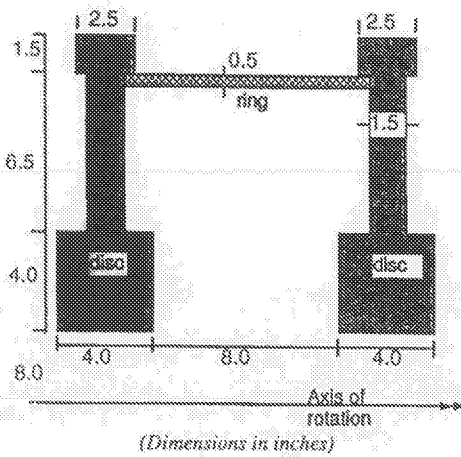


# Natural Frequencies Decreases as Fracture Progresses



Undamaged structure (i)  
 Fracture initiated at node 9  
 Fracture extended to node 14  
 Fracture extended to node 18

## Rotor System Survival Probability Under Multiple Failure Modes



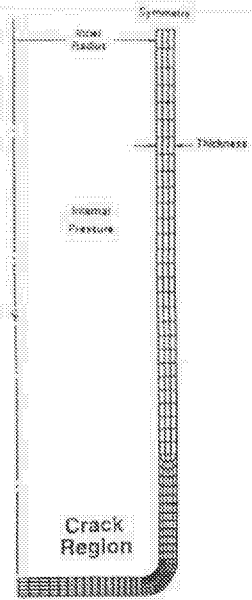
Failure Mode	S	R
1. Disc burst	Average stress	Burst strength
2. Fracture at bore	Fracture life	10,000 cycles
3. Fracture at rim	Fracture life	10,000 cycles
4. Progressive damage	Yielding of the ring	Yield strength

*Note: Yielding of ring affects all other modes through mutual interaction.*

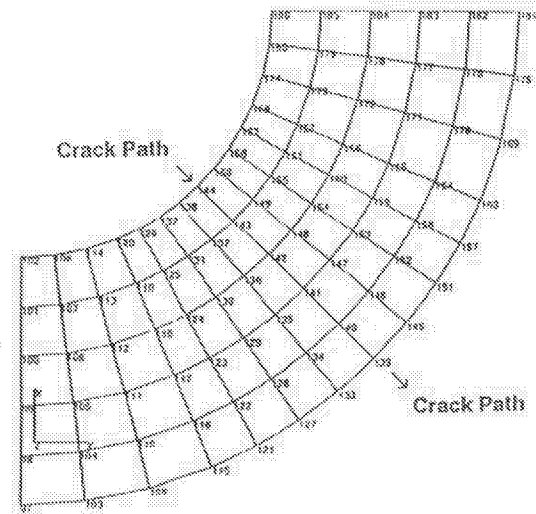
## Sensitivity Factors of Rotor System Failure Probability

Name	ln u(i) Space
E_ROT	0.016011
E_RIN	-0.002698
ROTOR DENS	0.438499
RING DENS	-0.000386
SPEED	0.850827
TEMPE	0.170793
BURST	-0.011983
RINGY	0.073086
RK1C	-0.061872
AO	0.057976
C	0.133702
NI	-0.000008
Kt	0.060917
A_LCF	-0.005132
TOLER	0.0

### STRUCTURAL SYSTEM RELIABILITY CONSIDERING PROGRESSIVE FRACTURE EXAMPLE



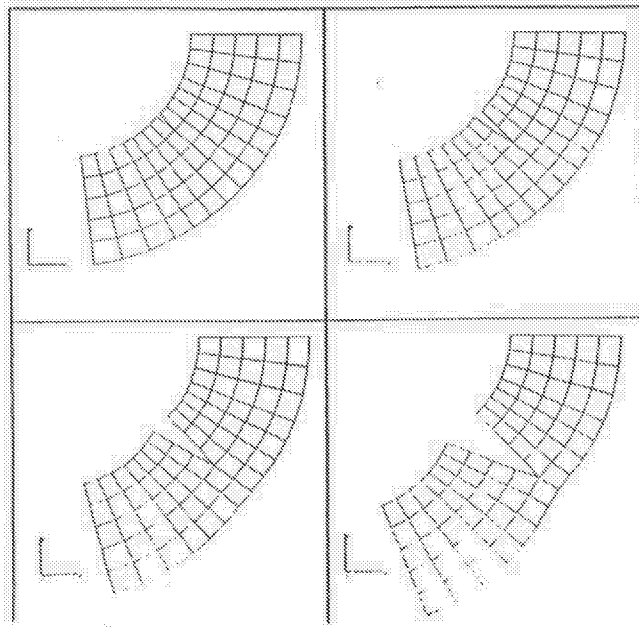
Finite Element Model of Axisymmetric Structure Under Internal Pressure



Crack Path Region

## STRUCTURAL SYSTEM RELIABILITY CONSIDERING PROGRESSIVE FRACTURE EXAMPLE

**Crack Growth: Bottom Events Modeled through Node Unzipping. Each Bottom Event Corresponds to Crack Initiation or a Crack Growth Increment**



## STRUCTURAL SYSTEM RELIABILITY CONSIDERING PROGRESSIVE FRACTURE EXAMPLE

### Fatigue Calculations

**Number of Cycles to Grow Crack  
Computed Using Crack Growth Law  
Given Crack Increment**

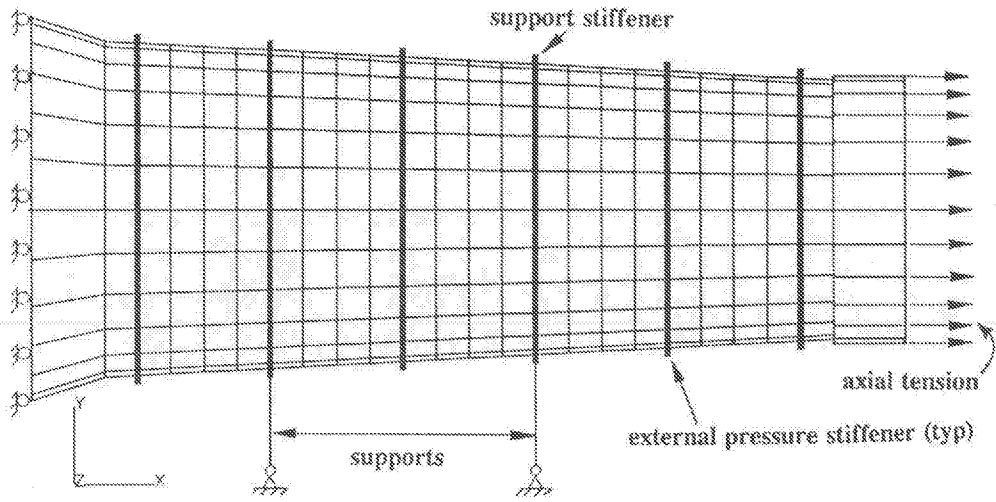
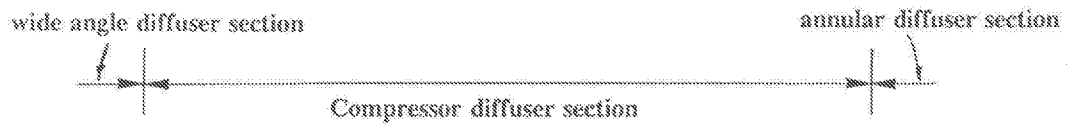
Cycles to Failure Results	
Event	# Cycles
Crack Initiation at node 144	$1.00 \times 10^{04}$
Fracture, N 144 → N 143	$1.80 \times 10^{04}$
Fracture, N 143 → N 142	$5.99 \times 10^{03}$
Fracture, N 142 → N 141	$2.99 \times 10^{03}$
<b>Total</b>	<b><math>3.70 \times 10^{04}</math></b>

### Paris Crack Growth Law

$$\frac{da}{dn} = C (\Delta k)^m$$

$$N_f = \frac{2[a_f^{(1-n/2)} - a_i^{(1-n/2)}]}{C (2-n) (Y \sigma_{max} \sqrt{\pi})^n} \quad n \neq 2$$

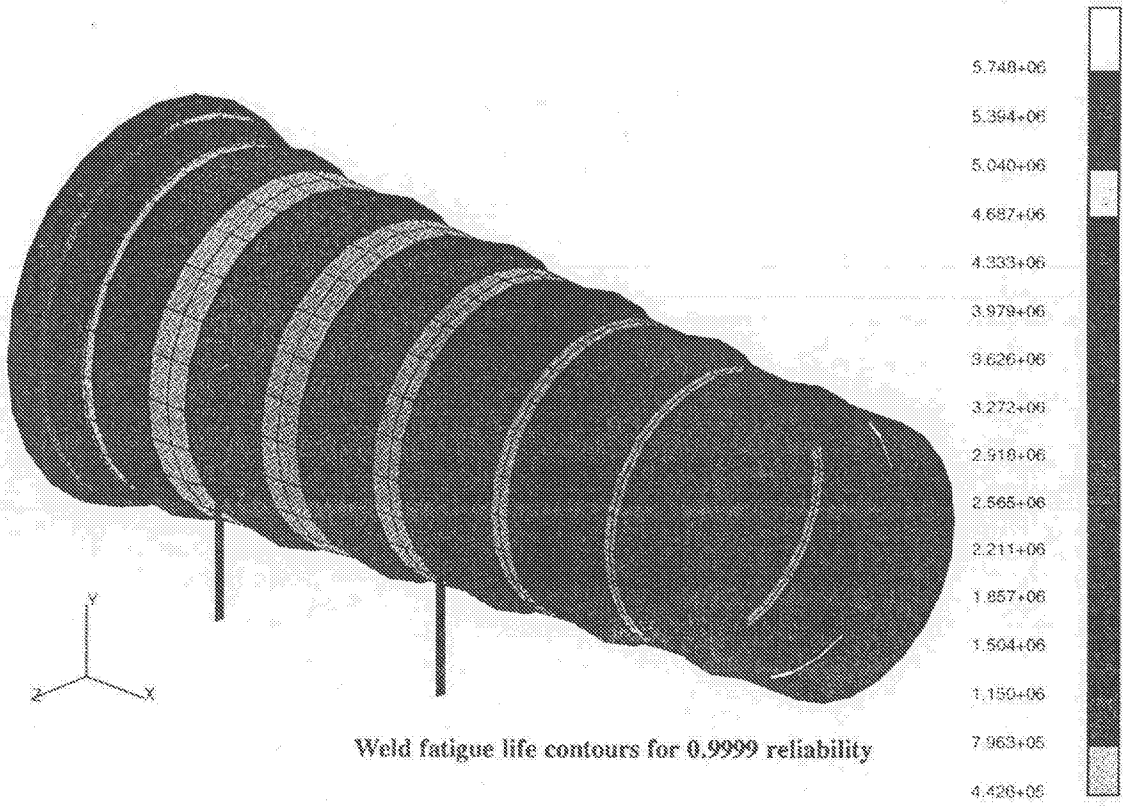




National Wind Tunnel - structure and components

## National Wind Tunnel Details:

- **Dimensions:**
  - Length = 133 ft.
  - Diameter at wide angle diffuser end = 51.67 ft
  - Diameter at annular diffuser end = 41.25 ft
- **Material A516 Grade 70 steel**
- **Load: Internal pressure = 5 atm (73.0 psi)**



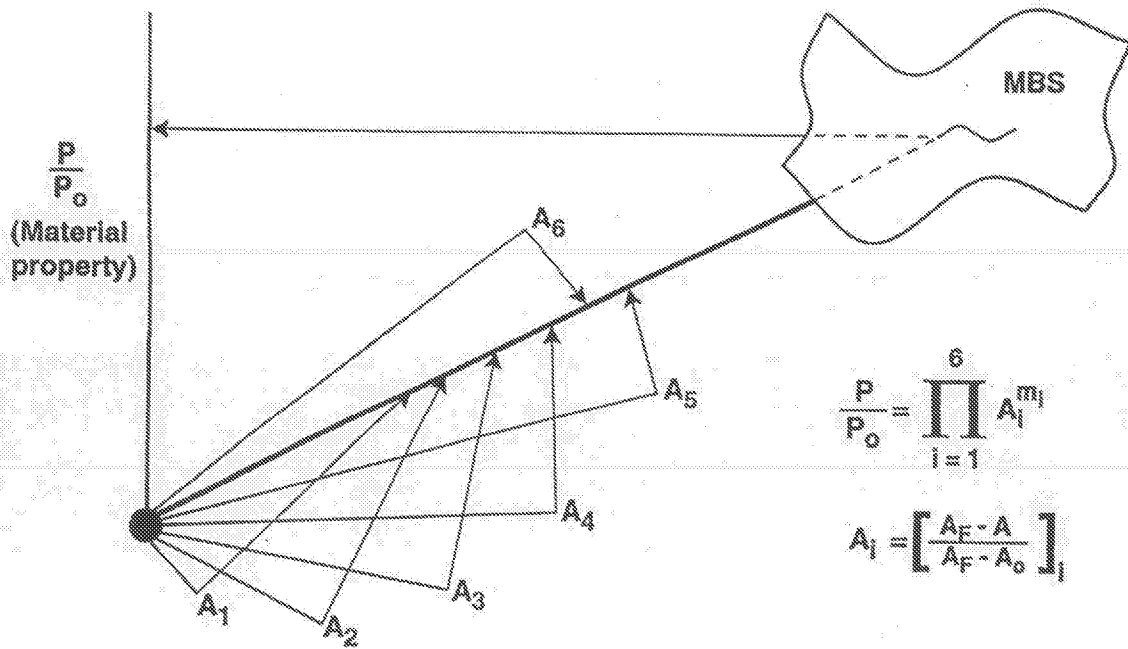
## **SUMMARY:**

- Probabilistic fatigue by computational simulation is doable and can be adapted throughout the design practice.
- One way to enhance its implementation is to identify appropriate staff and task them to do it.
- The method constitutes a “virtual” statistical desk-top laboratory applicable at all stages of the design, development and service life cycle.
- Probabilistic fatigue evaluations rely on computational simulation results while statistic methods rely on experimental data.

## WHAT IS THE FUTURE OF PROBABILISTIC FATIGUE METHODS?

- The future is an exponential use of Probabilistic Fatigue Methods because the drive for Better-Cheaper-Faster engines necessitates quantification of risk for the utilization of unproven:
  - Design Concepts
  - Material
  - Processes
  - Etc.
- Probabilistic Fatigue Simulation is the most effective formal method to quantify risk and justify commitment of required resources.

# Multi-Factor Interaction Model for Material Behavior Space (MBS)



$$\frac{p}{P_0} = \prod_{i=1}^6 A_i^{m_i}$$

$$A_i = \left[ \frac{A_F - A}{A_F - A_0} \right]_i$$

Factors influencing material behavior