Probabilistic Life and Reliability Analysis of Model Gas Turbine Disk

Frederic A. Holland, Matthew E. Melis and Erwin V. Zaretsky
National Aeronautics and Space Administration
Glenn Research Center
Cleveland, Ohio 44135
Ph: 216–433–8367
Email: Frederic.a.Holland@grc.nasa.gov

In 1939, W. Weibull developed what is now commonly known as the "Weibull Distribution Function" primarily to determine the cumulative strength distribution of small sample sizes of elemental fracture specimens. In 1947, G. Lundberg and A. Palmgren, using the Weibull Distribution Function developed a probabilistic lifing protocol for ball and roller bearings. In 1987, E. V. Zaretsky using the Weibull Distribution Function modified the Lundberg and Palmgren approach to life prediction. His method incorporates the results of coupon fatigue testing to compute the life of elemental stress volumes of a complex machine element to predict system life and reliability. This paper examines the Zaretsky method to determine the probabilistic life and reliability of a model gas turbine disk using experimental data from coupon specimens. The predicted results are compared to experimental disk endurance data.

NASA/CP—2002-211682

Glenn Research Center

Probabilistic Life and Reliability Analysis of Model Gas Turbine Disk

Frederic A. Holland, Matthew E. Melis and Erwin V. Zaretsky

National Aeronautics and Space Administration
Glenn Research Center
Cleveland, Ohio

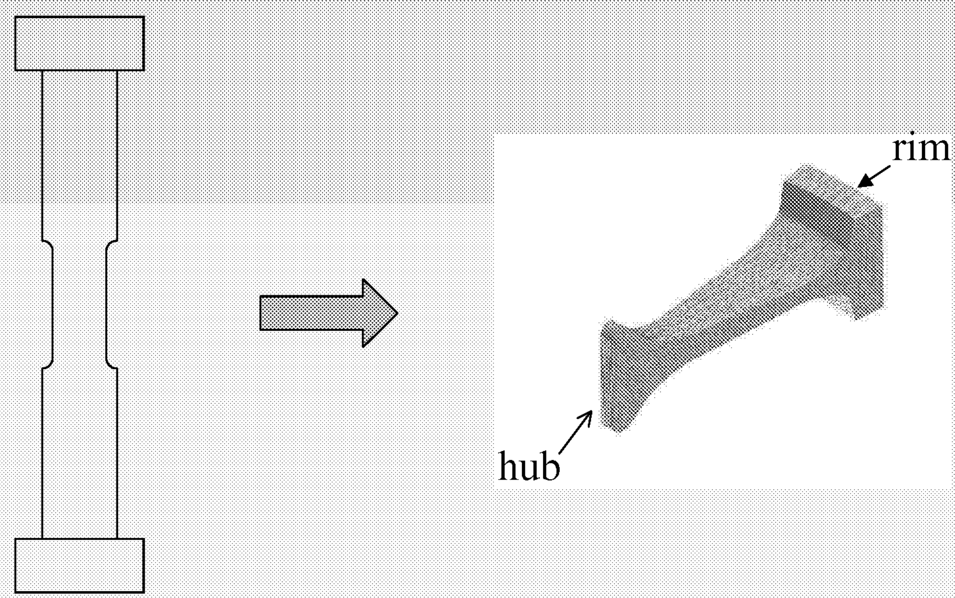
5th Annual FAA/Air Force/NASA/Navy Workshop
On The Application of Probabilistic Methods
for Gas Turbine Engines
June 13, 2001

Objective:

Predict Probabilistic Life and Reliability of
Model Turbine Disks From
A Statistical Material Database

Glenn Research Center

Use Simple Specimen Data to Predict Disk Life



Glenn Research Center

Weibull and Zaretsky Equations

From Weibull:

$$\ln\frac{1}{S} = \int_V f(X)\,dV$$

From Zaretsky:

$$f(x) = \tau^{ce} N^{e}$$

Zaretsky Modification of Weibull

$$\ln\frac{1}{S} = \tau^{ce} N^{e} V$$

For A Given Probability Of Survival S:

$$L = A\left(\frac{1}{\tau}\right)^{c}\left(\frac{1}{V}\right)^{\frac{1}{e}}$$

Material Life Factor

$$A = L_{ref}\,\tau_{ref}^{c}\,V_{ref}^{1/e}$$

Where:

S = Probability of Survival

τ = Critical Shear Stress

N = Life, stress cycles

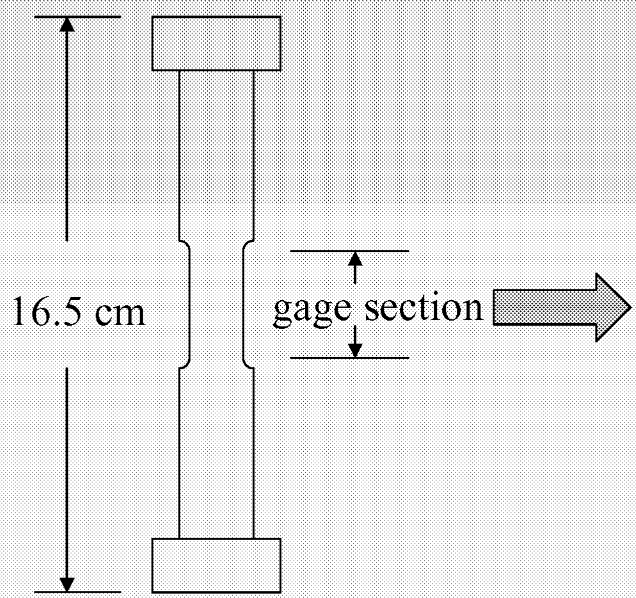
V = Stressed Volume

c = Stress-Life Exponent

e = Weibull Slope

Glenn Research Center

Rene' 88 LCF Specimen



$$A = L_{ref}\tau_{ref}^{c}V_{ref}^{1/e}$$

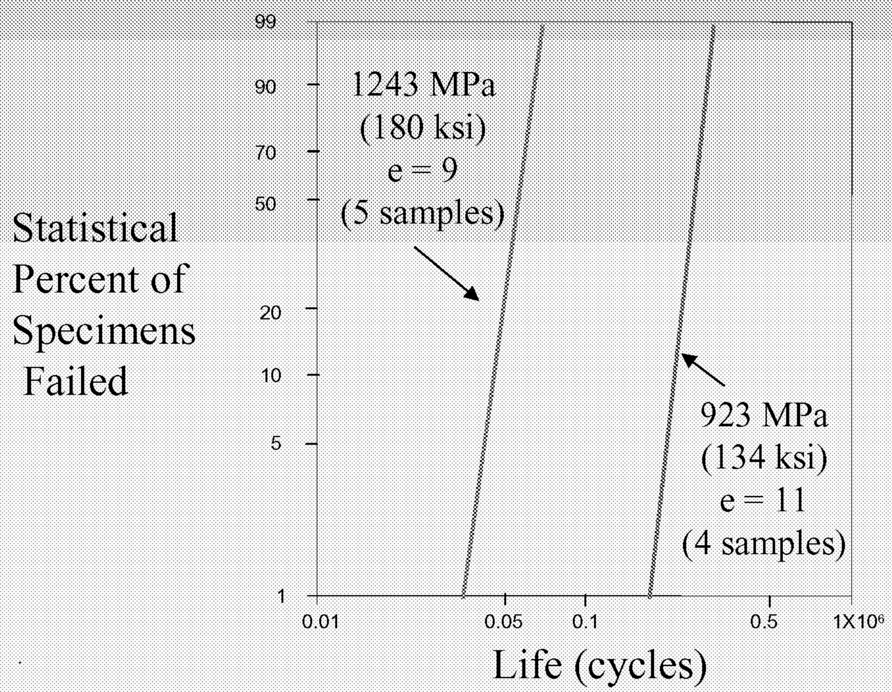
$$\tau_{ref} = \tau_{45}$$

$$L_{ref} = L_{10}$$

ref. volume, $V_{ref} = 1544\ mm^3$

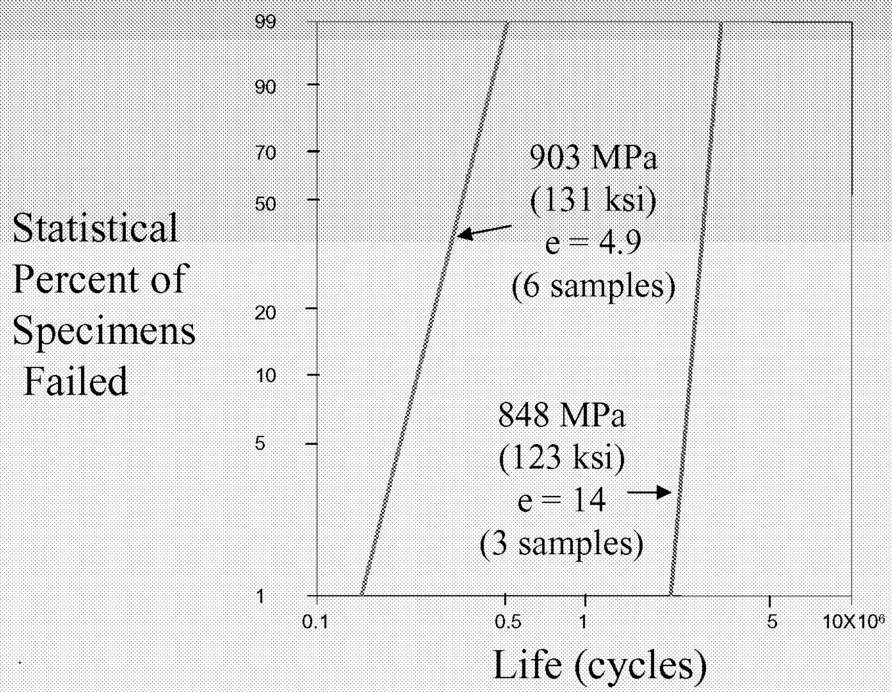
Glenn Research Center

Rene’ 88 Baseline Fatigue Data, 204°C (400°F)



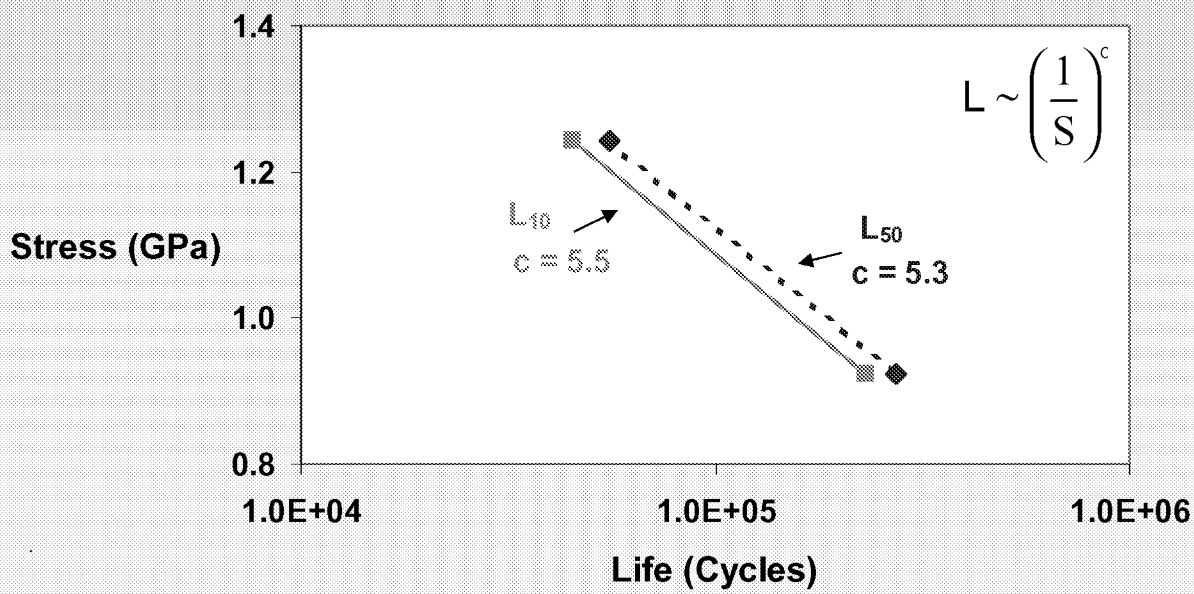
Glenn Research Center

Rene' 88 Baseline Fatigue Data, 649°C (1200°F)



Glenn Research Center

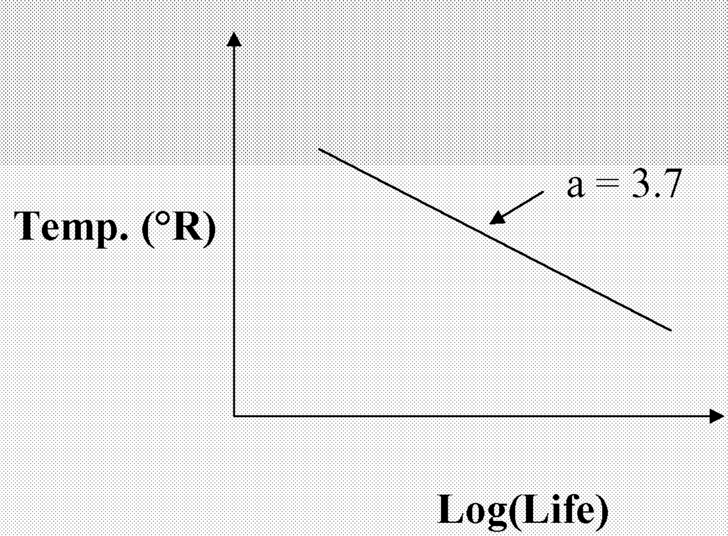
Rene’ 88 Stress-Life Relation, 204°C (400°F)



Glenn Research Center

Temperature-Life Relation

$$L \sim \left(\frac{1}{T}\right)^{a}$$

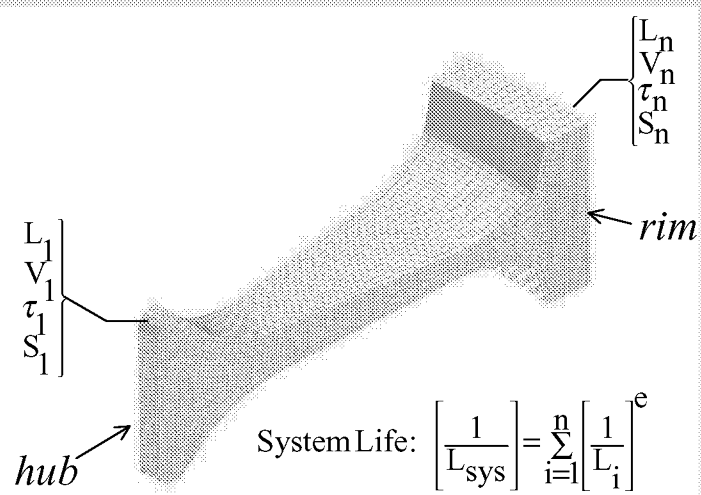


Material Parameters

Material:	Rene' 88
Elastic Modulus:	25,760 ksi
Poisson's Ratio:	0.323
Density:	0.78157×10^{-3} lbs/in.
Weibull Modulus, e	10
Stress-Life Exponent, c:	5.5
Ref. Stress, τ_{ref}:	0.129×10^{6} psi
Ref. Volume, V_{ref}:	1.427×10^{-6} $in.^{3}$
Ref. Life, L_{ref}:	1.2×10^{6} Cycles

Glenn Research Center

Finite Element Model of Model Disk



System Life: $\left[\frac{1}{L_{sys}}\right] = \sum_{i=1}^{n}\left[\frac{1}{L_i}\right]^e$

Glenn Research Center

Life Equations

Zaretsky Elemental Life:

$$L = L_{ref}\left[\frac{\tau_{ref}}{\tau}\right]^{c}\left[\frac{V_{ref}}{V}\right]^{\frac{1}{e}} \quad \text{or} \quad A\left[\frac{1}{\tau}\right]^{c}\left[\frac{1}{V}\right]^{\frac{1}{e}}$$

Where Material Life Factor: $A = L_{ref}\,\tau_{ref}^{\,c}\,V_{ref}^{\,1/e}$

System Life:

$$\left[\frac{1}{L_{sys}}\right] = \sum_{i=1}^{n}\left[\frac{1}{L_{i}}\right]^{e}$$

Glenn Research Center

Probability Equations

Elemental Probability of Survival:

$$S = S_{ref}^{\left(L_{ref}/L\right)^{e}}$$

System Probability of Survival:

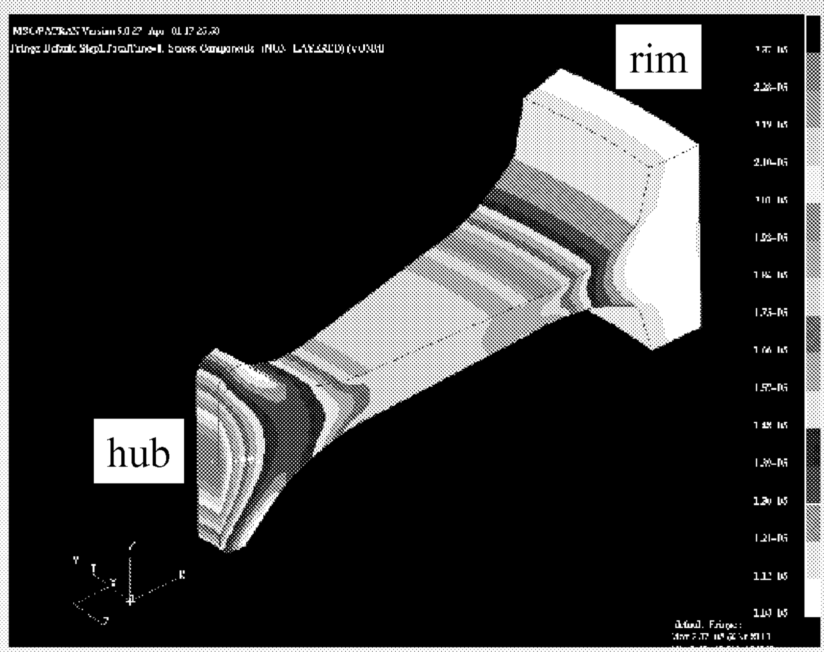
$$S_{sys} = S_1 \cdot S_2 \cdots S_n$$

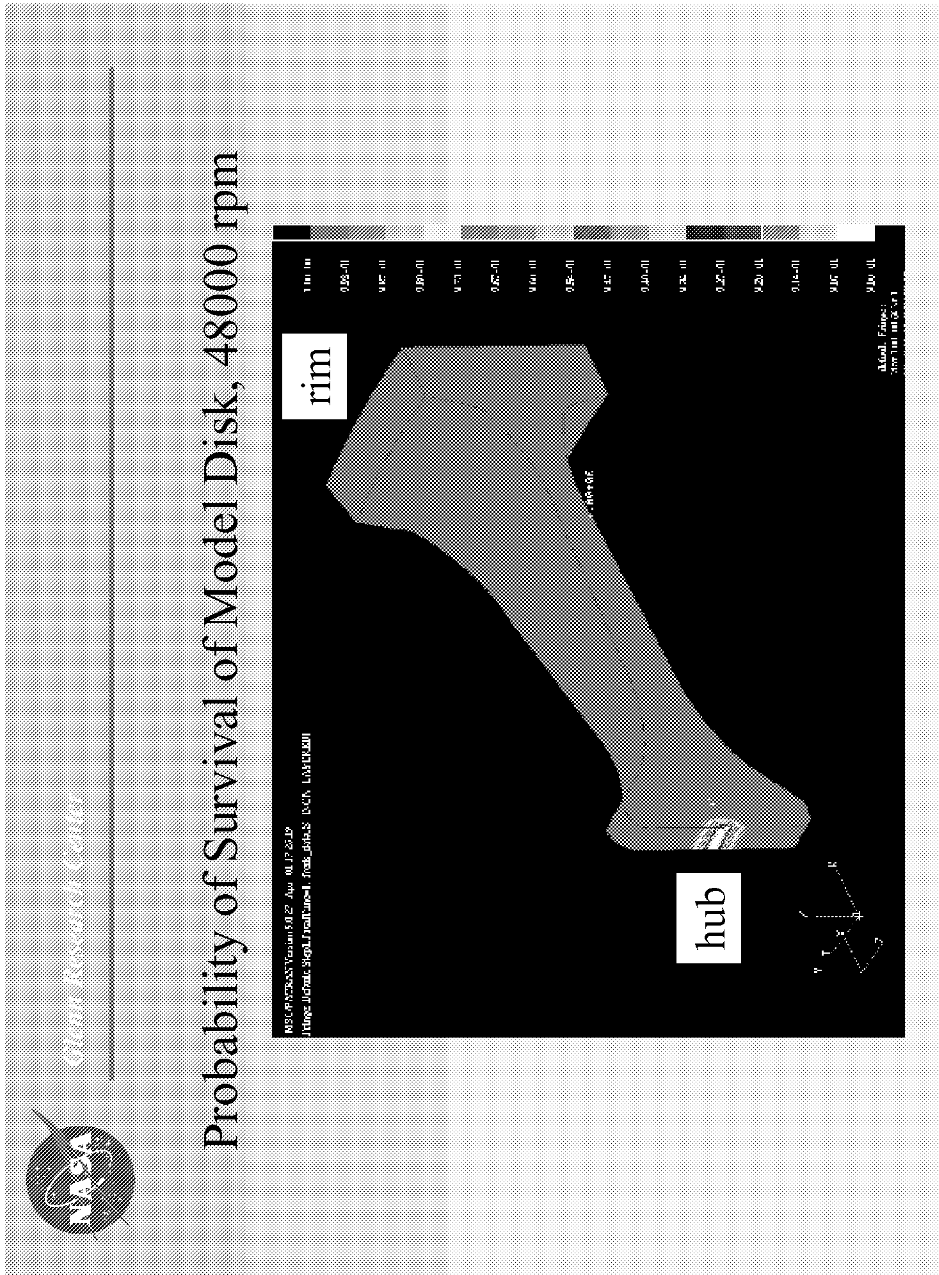
System Probability of Failure:

$$F_{sys} = 1 - S_{sys}$$

Glenn Research Center

Max. Shear Stress Distribution of Model Disk, 48000 rpm

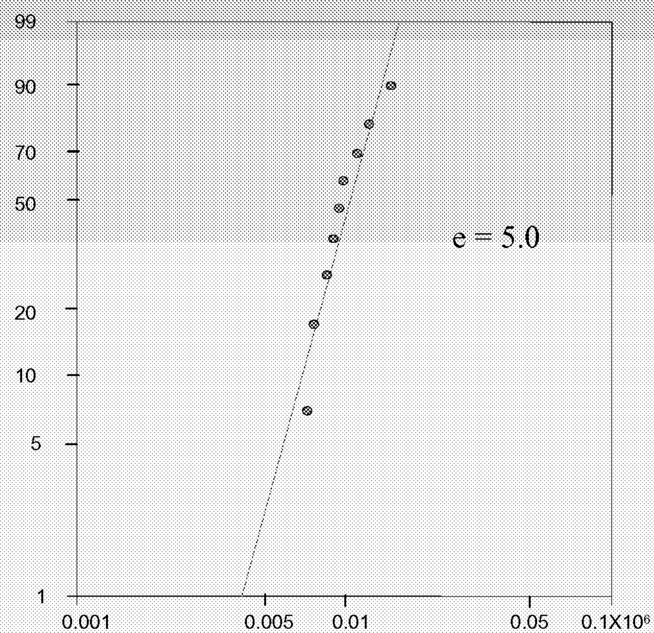



Glenn Research Center
Probability of Survival of Model Disk, 48000 rpm
rim
hub

Glenn Research Center

Rene' 88 Model Disk Test Data, 538°C (1000°F)

Statistical Percent of Specimens Failed



Life (cycles)

Speed: 48000 rpm

Glenn Research Center

Comparison of Experimental and Predicted L_{10} Disk Life

Cycles To Failure	
Predicted	Experiment
6,358	7,196

Glenn Research Center

Summary

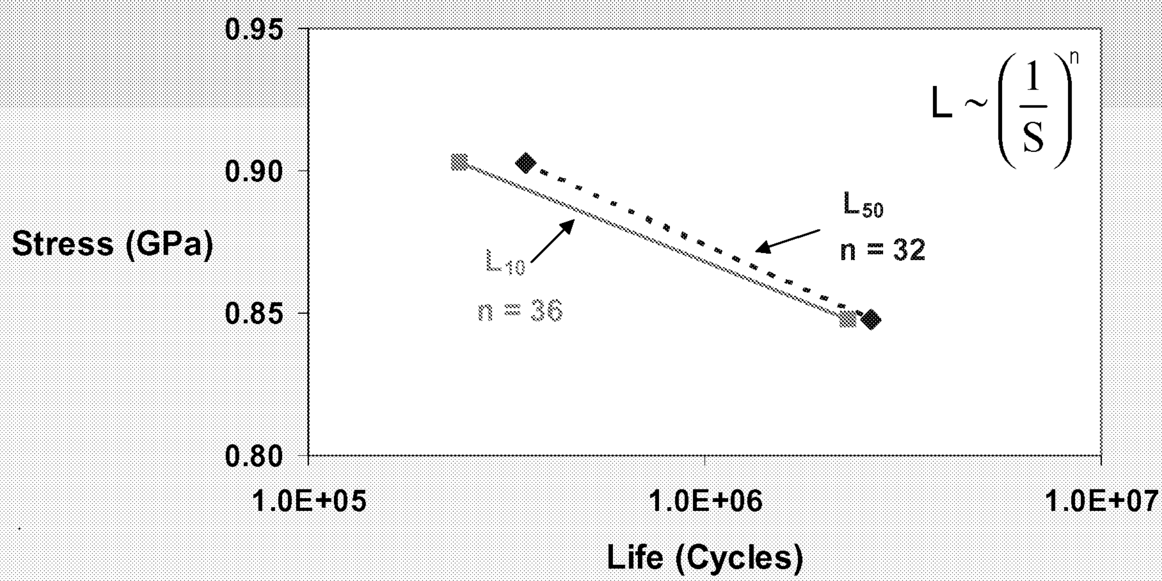
- Methodology gave a reasonably conservative prediction of L_{10} disk life from push-pull specimen data.

- Preliminary results suggest methodology is promising for accurately predicting fatigue life of metallic gas turbine disks.

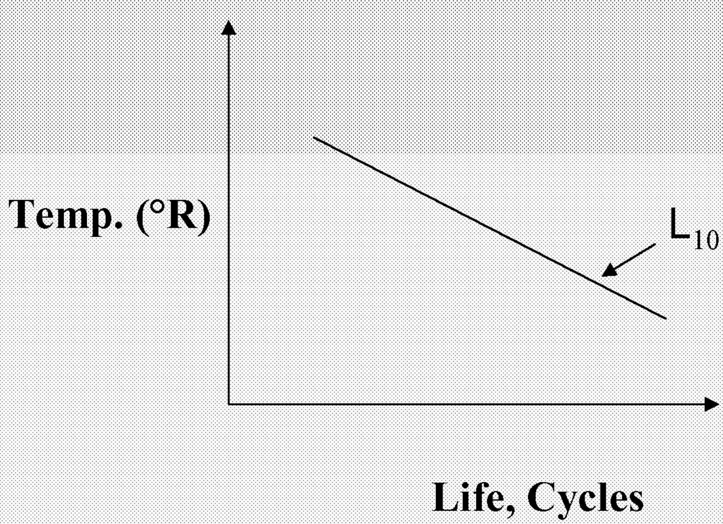
- More verification needed.

Glenn Research Center

Rene' 88 Stress-Life Relation, 649°C (1200°F)

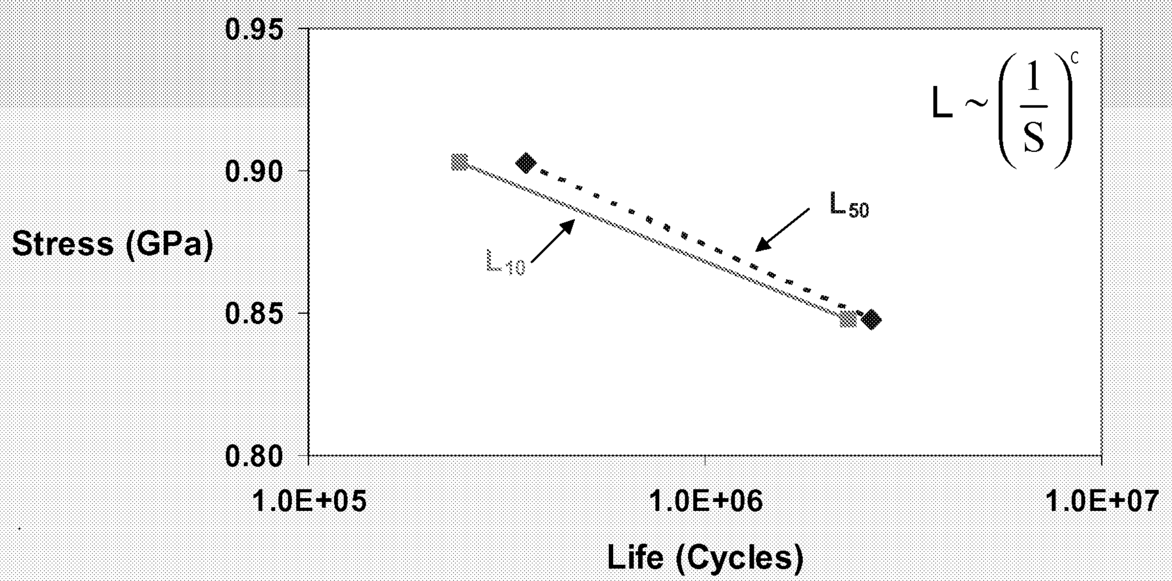


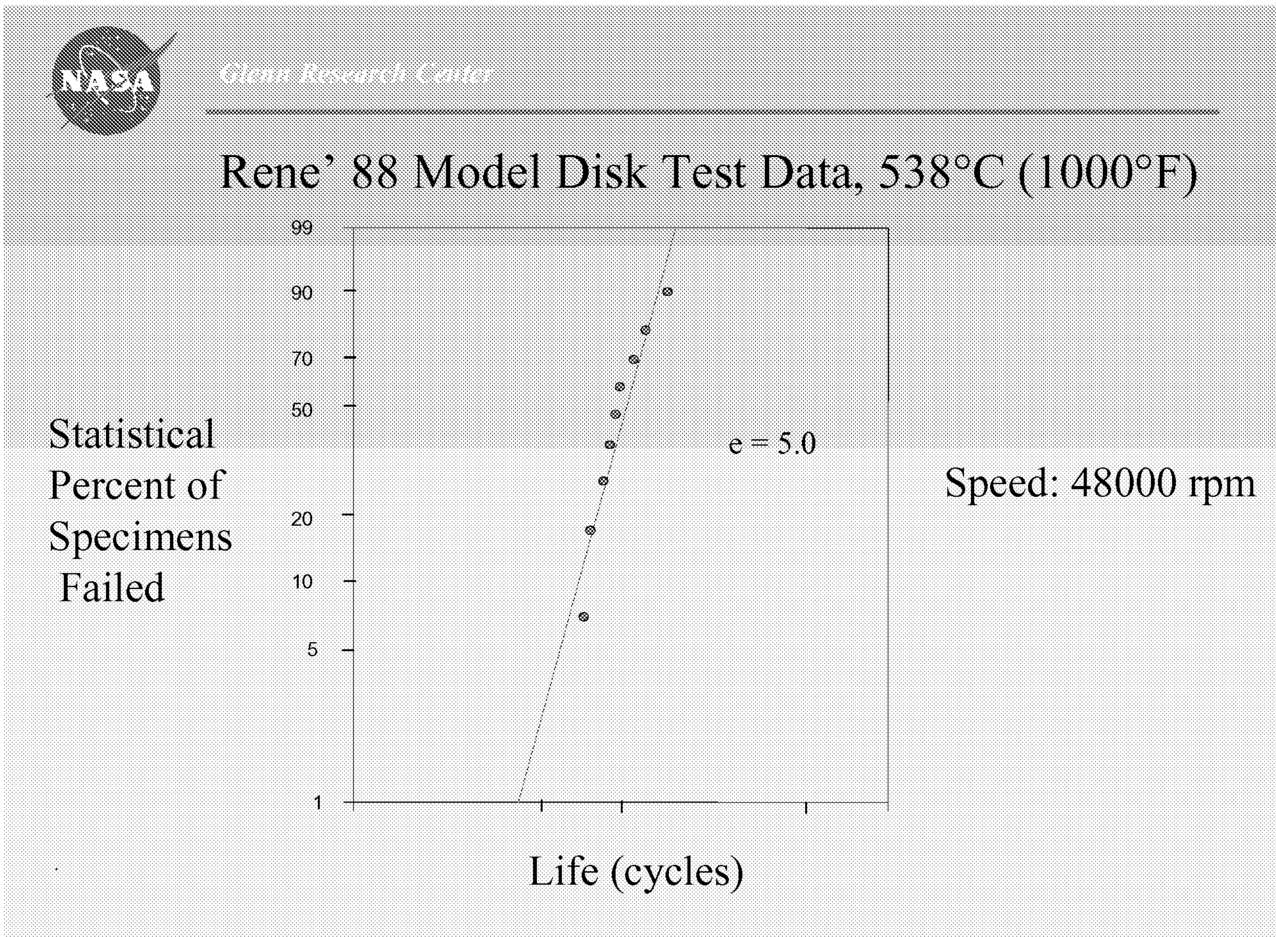
Temperature-Life Relation



Glenn Research Center

Rene’ 88 Stress-Life Relation, 649°C (1200°F)




Glenn Research Center
Rene' 88 Model Disk Test Data, 538°C (1000°F)
Statistical Percent of Specimens Failed
99
90
70
50
20
10
5
1
e = 5.0
Speed: 48000 rpm
Life (cycles)