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"Solar System Dynamics"

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Introduction

Grant NAG5-3915, from the Planetary Geology and Geophysics Program, supported my research in "Solar System Dynamics" during the period October 1, 1996 to September 30, 2002. It was the successor to grant NAGW-706, which spanned the period from October, 1, 1984 to September 1, 1996. Grant NAG5-11536 is the successor to NAG5-3915 and provides continuing support for my research in "Solar System Dynamics." In these 18 years, my research has touched every major dynamical problem in the solar system, including: the effect of chaotic zones on the distribution of asteroids, the delivery of meteorites along chaotic pathways, the chaotic motion of Pluto, the chaotic motion of the outer planets and that of the whole solar system, the delivery of short period comets from the Kuiper belt, the tidal evolution of the Uranian and Galilean satellites, the chaotic tumbling of Hyperion and other irregular satellites, the large chaotic variations of the obliquity of Mars, the evolution of the Earth-Moon system, and the resonant core-mantle dynamics of Earth and Venus. I have introduced new analytical and numerical tools that are in widespread use. Today, nearly every long-term integration of our solar system, its subsystems, and other solar systems uses algorithms that I invented. This research has all been primarily supported by this sequence of PGG NASA grants, for which I am very grateful.

During the period covered by Grant NAG5-3915 I completed and published major investigations of tidal evolution of the Earth-Moon system and of the passage of the Earth and Venus through non-linear core-mantle resonances. I also published a major innovation in symplectic algorithms: the symplectic corrector. I completed a paper on non-perturbative hydrostatic equilibrium. What follows are the abstracts of these papers. Detailed accounts of these investigations can be found in the progress reports for this grant.

Resonances in the Early Evolution of the Earth-Moon System

J. Touma and J. Wisdom 1998. *Astron. J.* **115**, 1653.

Most scenarios for the formation of the Moon place the Moon near Earth in low-eccentricity orbit in the equatorial plane of Earth. We examine the dynamical evolution of the Earth-Moon system from such initial configurations. We find that during the early evolution of the system, strong orbital resonances are encountered. Passage through these resonances can excite large lunar orbital eccentricity and modify the inclination of the Moon to the equator. Scenarios that resolve the mutual inclination problem are presented. A period of large lunar eccentricity would result in substantial tidal heating in the early Moon, providing a heat source for the lunar magma ocean. The resonance may also play a role in the formation of the Moon.

Nonlinear Core-Mantle Coupling
J. Touma and J. Wisdom 2001. *Astron. J.*, **122**, 1030.

We explore the nonlinear dynamics of a forced core-mantle system. We show that the free axisymmetric motion of uniform vorticity fluid core coupled to a rigid mantle (the Poincare-Hough) model is integrable. We derive an approximate Hamiltonian for the core tilt mode that includes the leading nonlinear contribution. We then include gravitational perturbations in the analysis. We identify the principal nonlinear prograde and retrograde resonances and the characteristic excitation associated with each. We compare the nonlinear excitation to the excitation expected in the corresponding linear model. The nonlinear model indicates that for each principal commensurability there are multiple overlapping resonances, and so varying degrees of chaotic behavior are predicted. Chaotic behavior at the principal core-mantle commensurabilities is confirmed with surfaces of section. We then present the results of numerical evolutions done with a generalization of the Lie-Poisson integrator of Touma and Wisdom (1994a) to allow for a Poincare-Hough core, core-mantle friction, and tidal dissipation. We use our analytical and numerical models to explore the evolution of the Earth through the prograde core-mantle resonances and to explore the evolution of Venus through the retrograde resonances.

Symplectic Correctors
J. Wisdom, M. Holman, and J. Touma 1996. *Fields Inst. Com.* **10**, 217.

Symplectic integration algorithms typically yield trajectories that exhibit spurious oscillation in energy and state variables. In the delta function formulation of symplectic integration these oscillations have a clear origin, and canonical transformations can be made to remove them. The accuracy of symplectic integrators is substantially improved when combined with these symplectic correctors. The methods developed here are generally applicable to the integration of perturbed dynamical systems, but illustrated here by applications to the planetary n-body problem.

Non-perturbative Hydrostatic Equilibrium
J. Wisdom

A non-perturbative treatment of hydrostatic equilibrium is presented. We find that the widely used third order Zharkov-Trubitsyn theory is not adequate to model the interiors of Jupiter and Saturn. We use the method to generate abstract objective interior models of the Jovian planets, with no input other than the observational data. The abstract objective models are in surprisingly good agreement with the physical models.

Articles Supported by Grant NAG5-3915

"Resonances in the Early Evolution of the Earth-Moon System," J. Touma and J. Wisdom 1998. *Astron. J.* **115**, 1653.

"Non-linear Core-Mantle Coupling," J. Touma and J. Wisdom 2001. *Astron. J.*, **122**, 1030.

"Symplectic Correctors," J. Wisdom, M. Holman, and J. Touma 1996. *Fields Inst. Com.* **10**, 217.

"Non-perturbative Hydrostatic Equilibrium," J. Wisdom 1996.

Articles Supported by predecessor Grant NAGW-706

"Evolution of the Earth-Moon System: Part I," J. Touma and J. Wisdom 1994. *Astron. J.* **108**, 1943.

"The Chaotic Obliquity of Mars," J. Touma and J. Wisdom 1993. *Science 259*, 1294.

"Lie-Poisson Integrators for Rigid Body Dynamics in the Solar System," J. Touma and J. Wisdom 1993. *Astron. J.*, **107**, 1189.

"Dynamical Stability of the Outer Solar System and the Delivery of Short Period Comets" M. Holman and J. Wisdom 1993. *Astron. J.* **105**, 1987.

"Symplectic Maps for the N-Body Problem: Stability Analysis," J. Wisdom and M. Holman 1992. *Astron. J.* **104**, 2022.

"Chaotic Evolution of the Solar System," G.J. Sussman and J. Wisdom 1992. *Science* **257**, 56.

"Long Term Evolution of the Solar System," J. Wisdom 1992, proceedings of the IAU symposium on "Chaos, Resonances, and Collective Dynamical Phenomena in the Solar System" held in Angra Dos Reis, Brazil, 1991.

"Symplectic Maps for the N-Body Problem," J. Wisdom and M. Holman 1991. *Astron. J.* **102**, 1528.

"Simple Dynamical Models of Neptune's Great Dark Spot," L.M. Polvani and J. Wisdom with E. DeJong and A.P. Ingersoll 1990. *Science* **249**, 1393.

"Is the Solar System Stable? and Can We Use Chaos to Make Measurements?" J. Wisdom 1990. in *Chaos*, proceedings of the "Joint Soviet-American Chaos Conference" held at Woods Hole, June, 1989.

"On Chaotic Flow around the Kida Vortex," L.M. Polvani and J. Wisdom 1990. in *Topological Aspects of Fluid Mechanics*, H K. Moffat, Ed. (Cambridge).

"Chaotic Lagrangian Trajectories Around an Elliptical Vortex Patch Embedded in a Constant and Uniform Background Shear Flow," L.M. Polvani and J. Wisdom 1990. *Physics of Fluids A*.

"Tidal Evolution of the Uranian Satellites. Part III, Evolution through the Miranda-Umbriel 3:1, Miranda-Ariel 5:3, and Ariel-Umbriel 2:1 Mean Motion Commensurabilities." W. Tittemore and J. Wisdom 1990. *Icarus* **85**, 394-443.

"Tidal Evolution of the Uranian Satellites. Part II: Explanation of the Anomalously Large Inclination of Miranda." W. Tittemore and J. Wisdom 1989. *Icarus* **78**, 63.

"Numerical Evidence that the Motion of Pluto is Chaotic." G.J. Sussman and J. Wisdom 1988. *Science* **241**, 433.

"Rotational Dynamics of Irregularly Shaped Natural Satellites." J. Wisdom 1987. *Astron. J.* **94**, 1350.

"Tidal Evolution of the Uranian Satellites. Part I: Passage of Ariel and Umbriel through the 5:3 Mean-Motion Commensurability." W. Tittemore and J. Wisdom 1987. *Icarus* **74**, 172.

"Urey Prize Lecture: Chaotic Dynamics in the Solar System." J. Wisdom 1987. *Icarus* **72**, 241.

"Chaotic Behavior in the Solar System." J. Wisdom 1987. *Proc. R. Soc. Lon. A* **413**, 109.

"Canonical Solution of the Two Critical Argument Problem." J. Wisdom 1986. *Celes. Mech.* **38** 175.

"The Outer Solar System for 200 Million Years," J. Applegate, M. Douglas, Y. Gursel, G.J. Sussman, J. Wisdom, 1986. *Astron. J.* **92**, 176. reprinted in *Lecture Notes in Physics #267 - Use of supercomputers in stellar dynamics*, Springer Verlag, 1986.

"A Perturbative Treatment of Motion Near the 3/1 Commensurability." J. Wisdom 1985. *Icarus* **63**, 272.

"Meteorites May Follow a Chaotic Route to Earth." J. Wisdom 1985. *Nature* **315**, 731.