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AN EXPERIMENTAL INVESTIGATION OF INCOMPRESSIBLE RICHTMYER-MESHKOV INSTABILITY

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ABSTRACT

Richtmyer-Meshkov (RM) instability occurs when two different density fluids are impulsively accelerated in the direction normal to their nearly planar interface. The instability causes small perturbations on the interface to grow and eventually become a turbulent flow. It is closely related to Rayleigh-Taylor instability, which is the instability of a planar interface undergoing constant acceleration, such as caused by the suspension of a heavy fluid over a lighter one in the earth's gravitational field. Like the well-known Kelvin-Helmholtz instability, RM instability is a fundamental hydrodynamic instability which exhibits many of the nonlinear complexities that transform simple initial conditions into a complex turbulent flow. Furthermore, the simplicity of RM instability (in that it requires very few defining parameters), and the fact that it can be generated in a closed container, makes it an excellent test bed to study nonlinear stability theory as well as turbulent transport in a heterogeneous system. However, the fact that RM instability involves fluids of unequal densities which experience negligible gravitational force, except during the impulsive acceleration, requires RM instability experiments to be carried out under conditions of microgravity.

This experimental study investigates the instability of an interface between incompressible, miscible liquids with an initial sinusoidal perturbation. The impulsive acceleration is generated by bouncing a rectangular tank containing two different density liquids off a retractable vertical spring. The initial perturbation is produced prior to release by oscillating the tank in the horizontal direction to produce a standing wave. The instability evolves in microgravity as the tank travels up and then down the vertical rails of a drop tower until hitting a shock absorber at the bottom. Planar Laser Induced Fluorescence (PLIF) is employed to visualize the flow. PLIF images are captured by a video camera that travels with the tank. Figure 1 is as sequence of images showing the development of the instability from the initial sinusoidal disturbance far into the nonlinear regime which is characterized by the appearance of mushroom structures resulting from the coalescence of baroclinic vorticity produced by the impulsive acceleration. At later times in this sequence the vortex cores are observed to become unstable showing the beginnings of the transition to turbulence in this flow. The amplitude of the growing disturbance after the impulsive acceleration is measured and found to agree well with theoretical predictions. The effects of Reynolds number (based on circulation) on the development of the vortices and the transition to turbulence are also determined.

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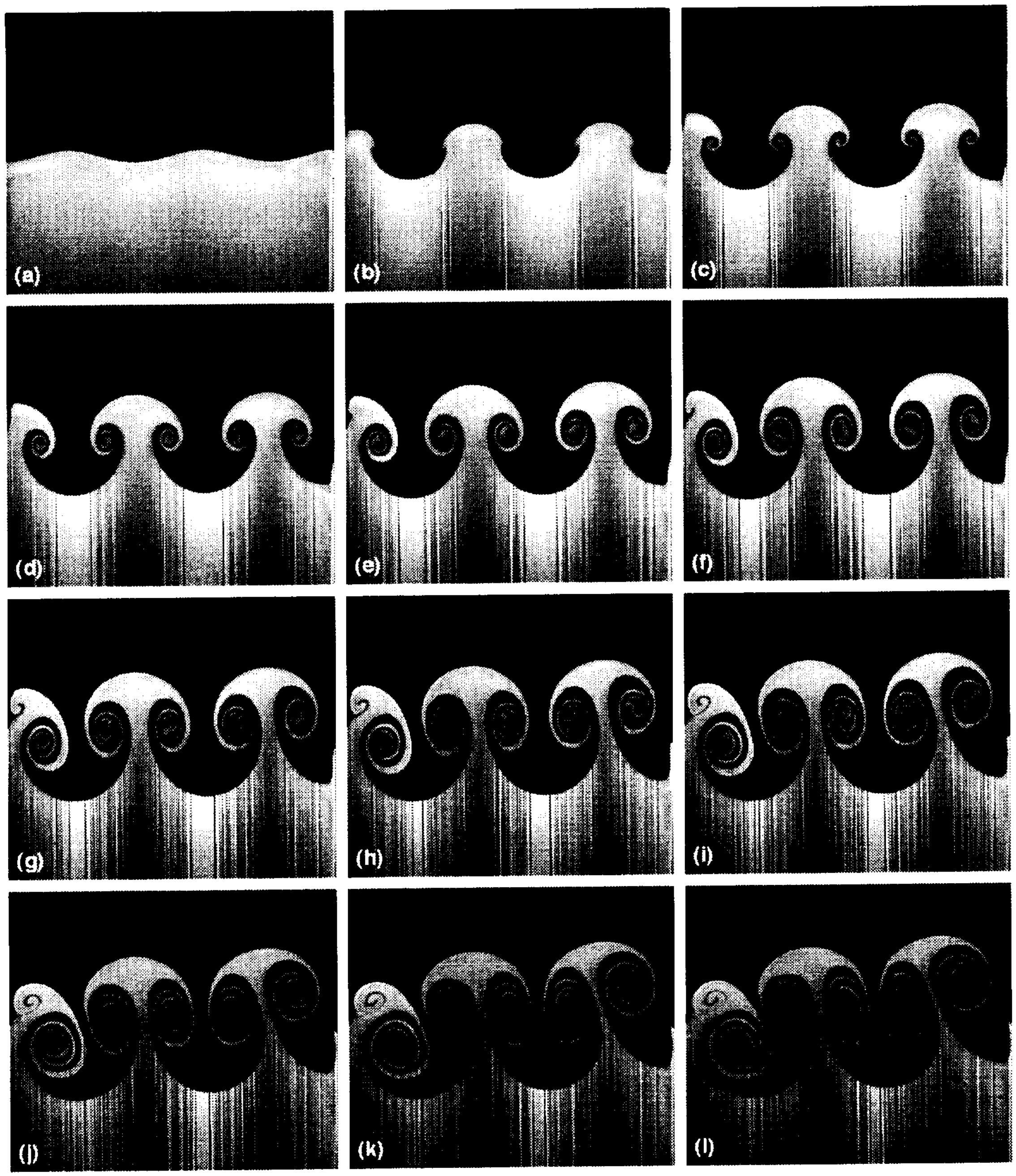

Figure 1. A sequence of images from an experiment showing the development of the RM instability. The initial sinusodial perturbation inverts (due to the acceleration direction) and then evolves into a mushroom pattern that is nearly symmetrical due to the moderate density difference (Atwood number of 0.15). At late times and high Reynolds number (2400 for this experiment based on circulation), an instability develops in the vortex cores, as can be seen beginning in (i). Times relative to the midpoint of spring impact are (a) −28 ms, (b) 88 ms, (c) 172 ms, (d) 255 ms, (e) 339 ms, (f) 422 ms, (g) 505 ms, (h) 589 ms, (i) 672 ms, (j) 756 ms, (k) 839 ms, and (l) 906 ms.

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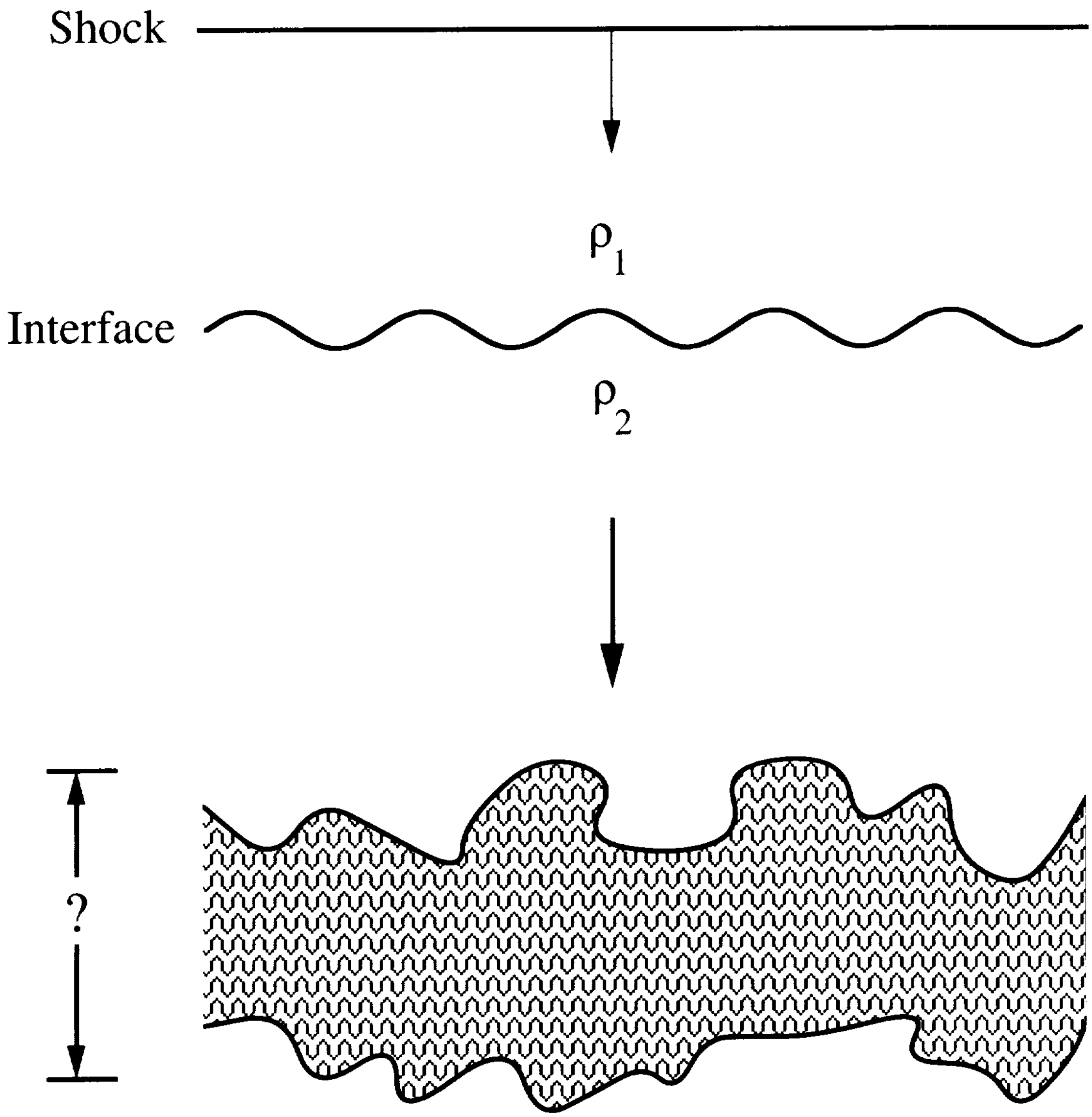
Richtmyer-Meshkov Instability



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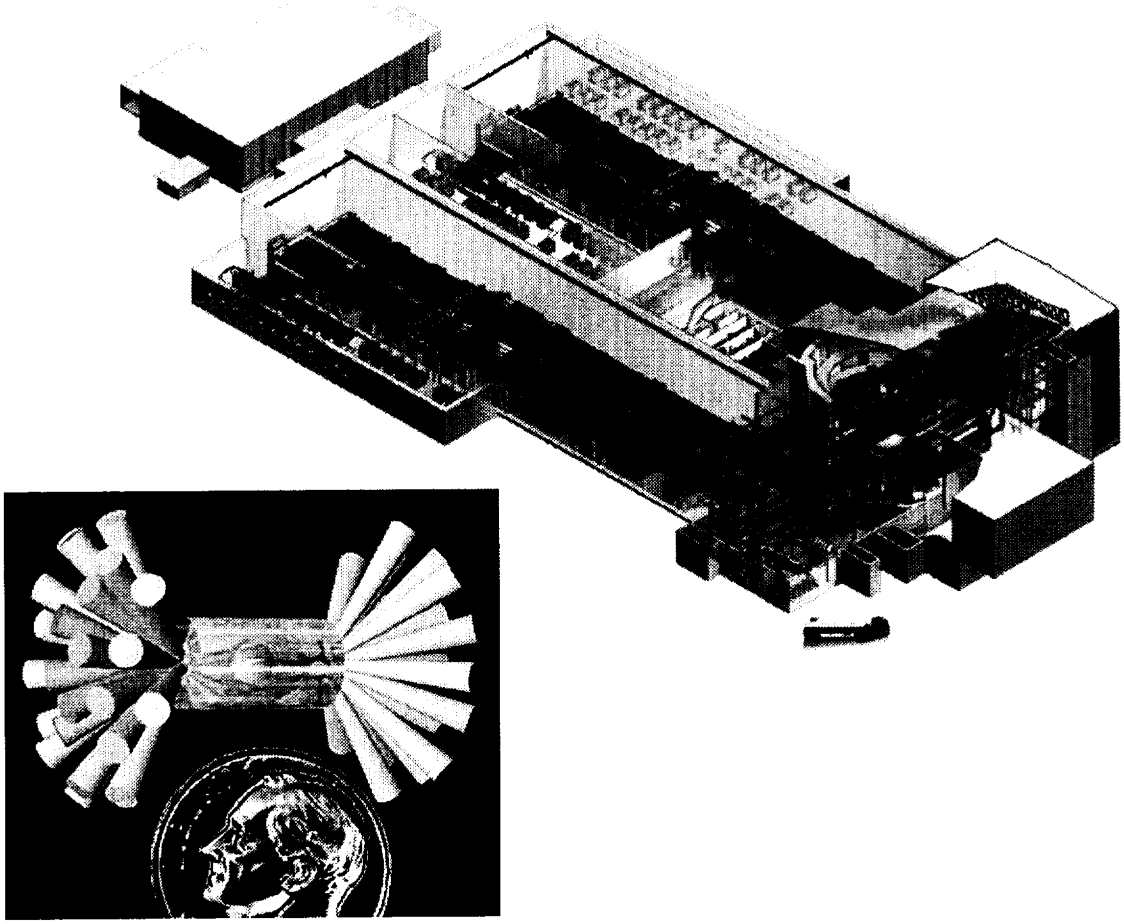
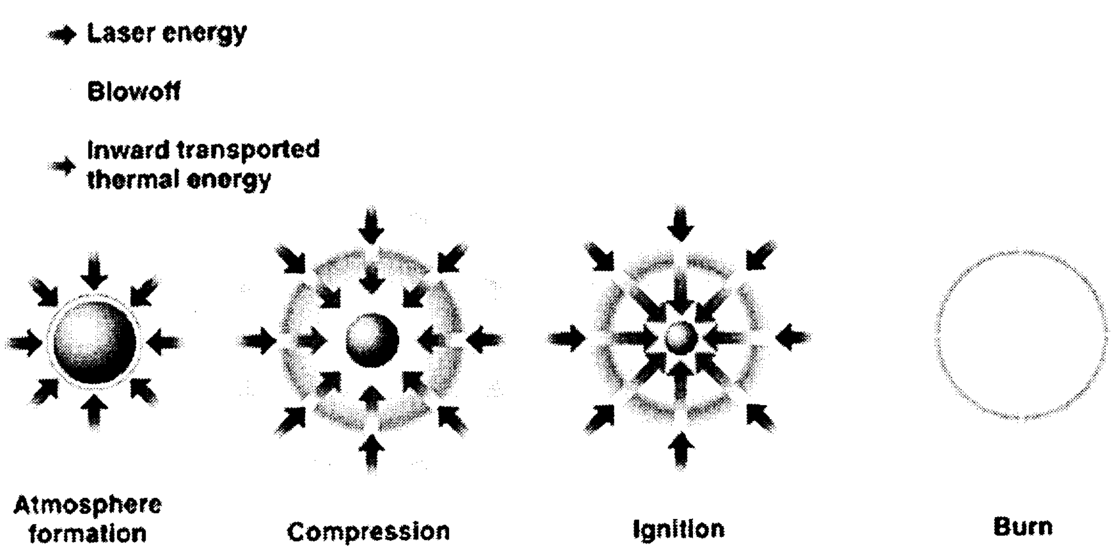


The Inertial Confinement Fusion Concept



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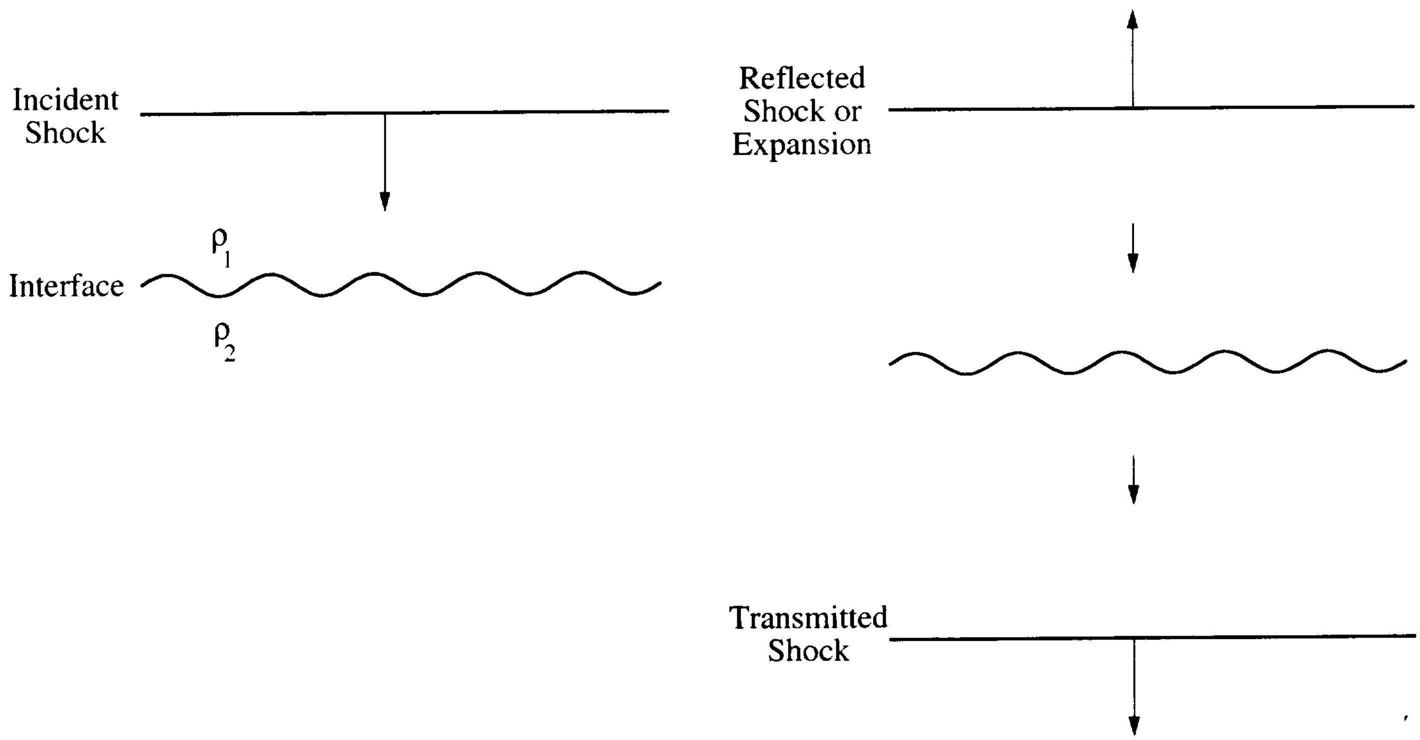
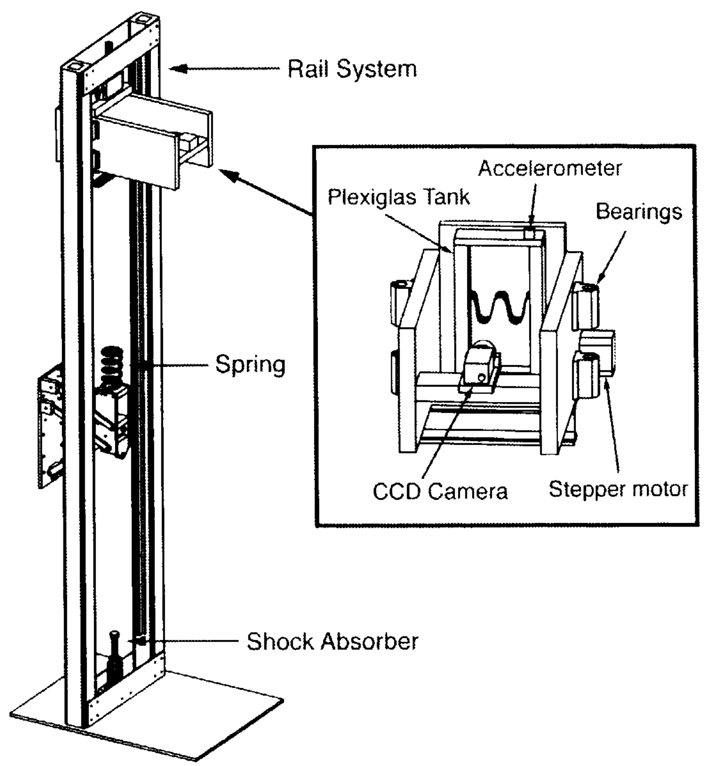
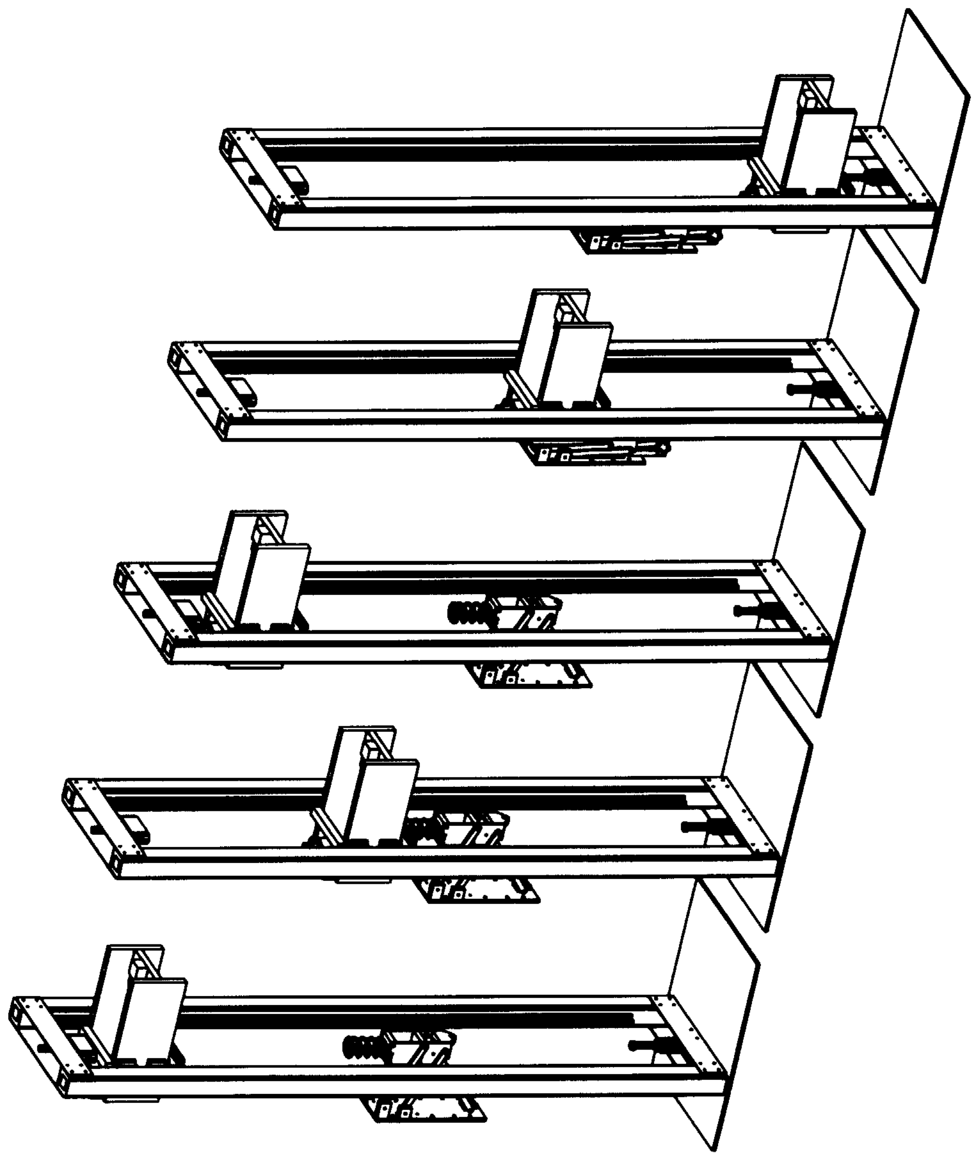
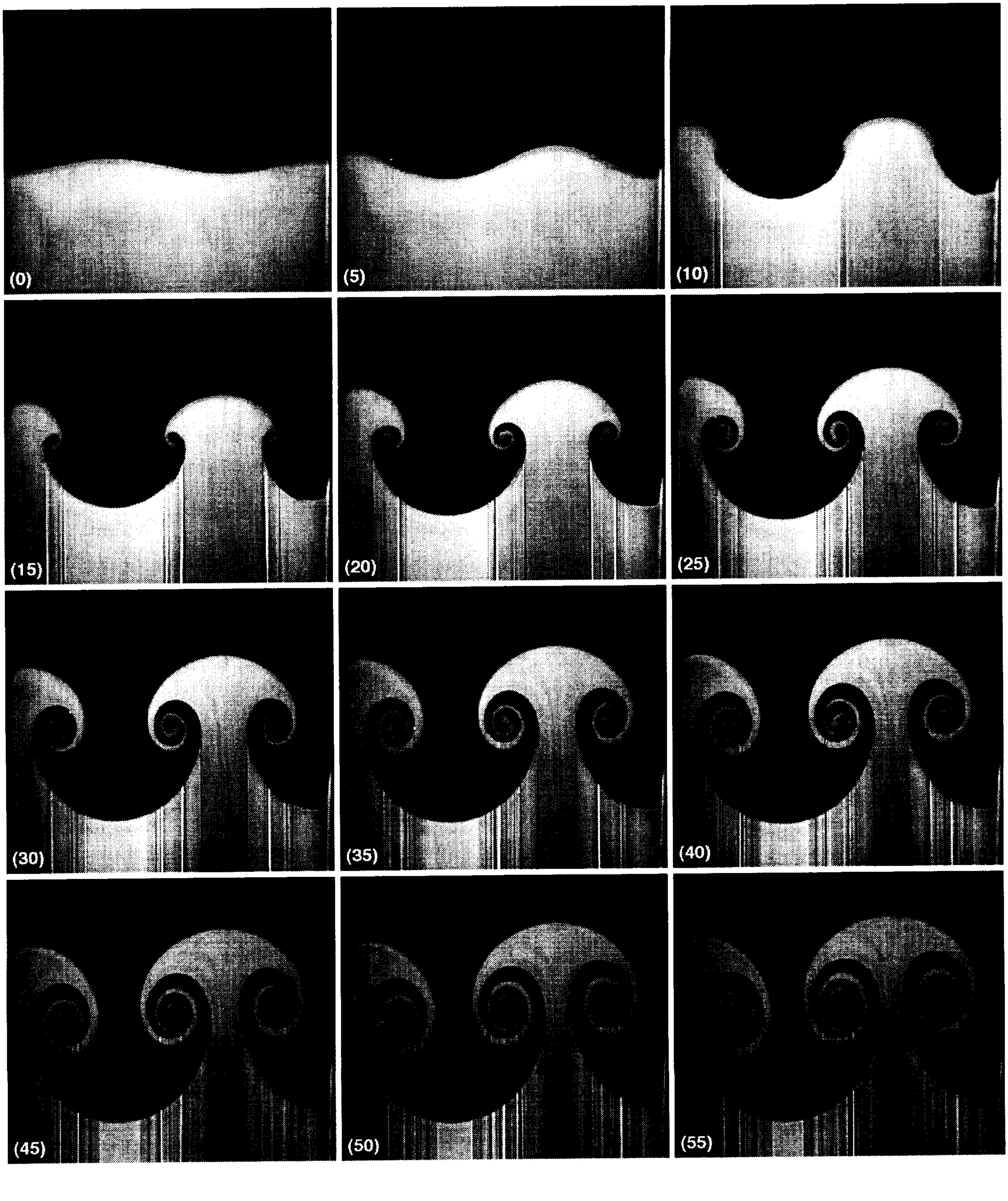
Shock Interaction with a Density Interface



Incompressible RM Apparatus







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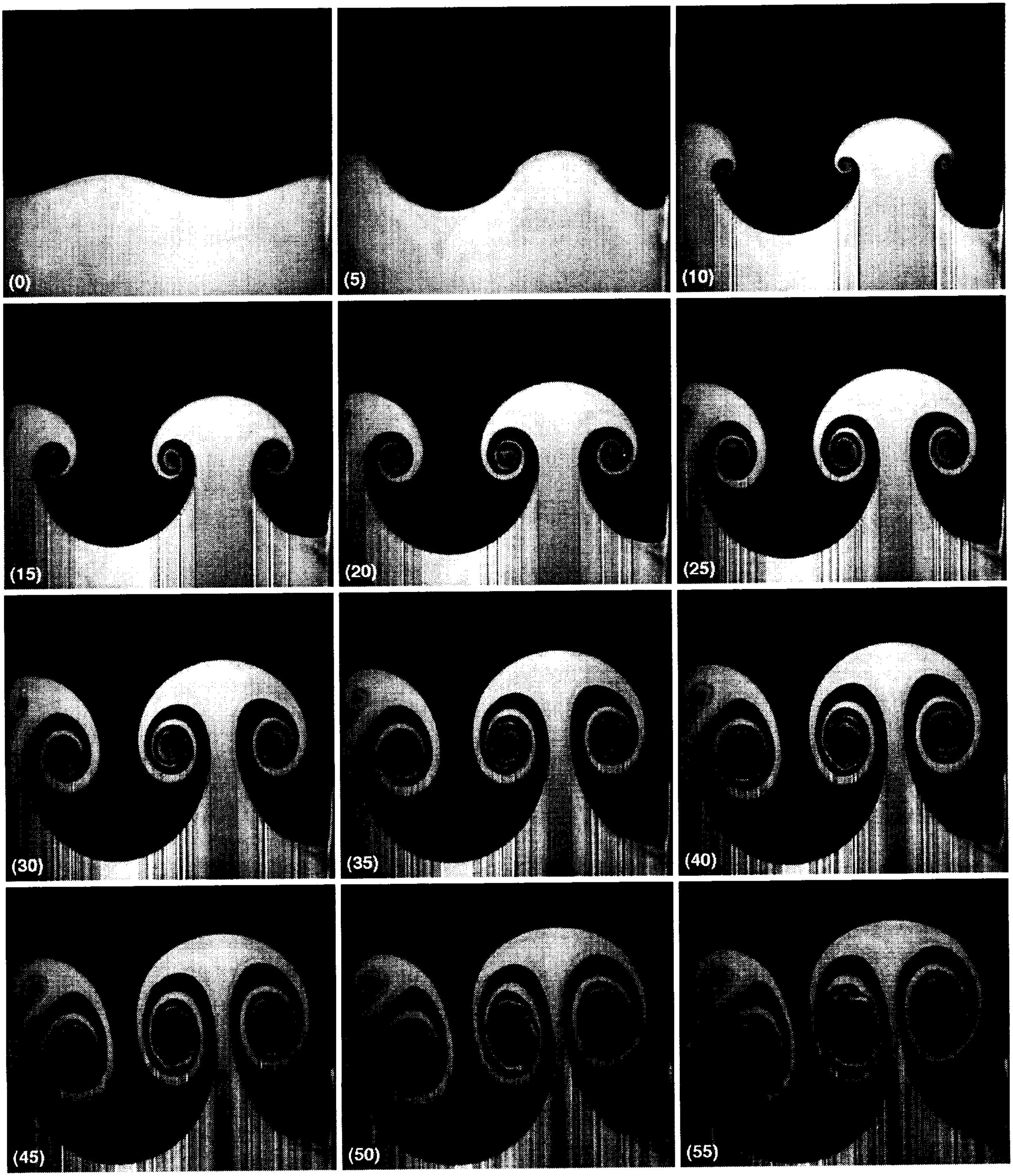
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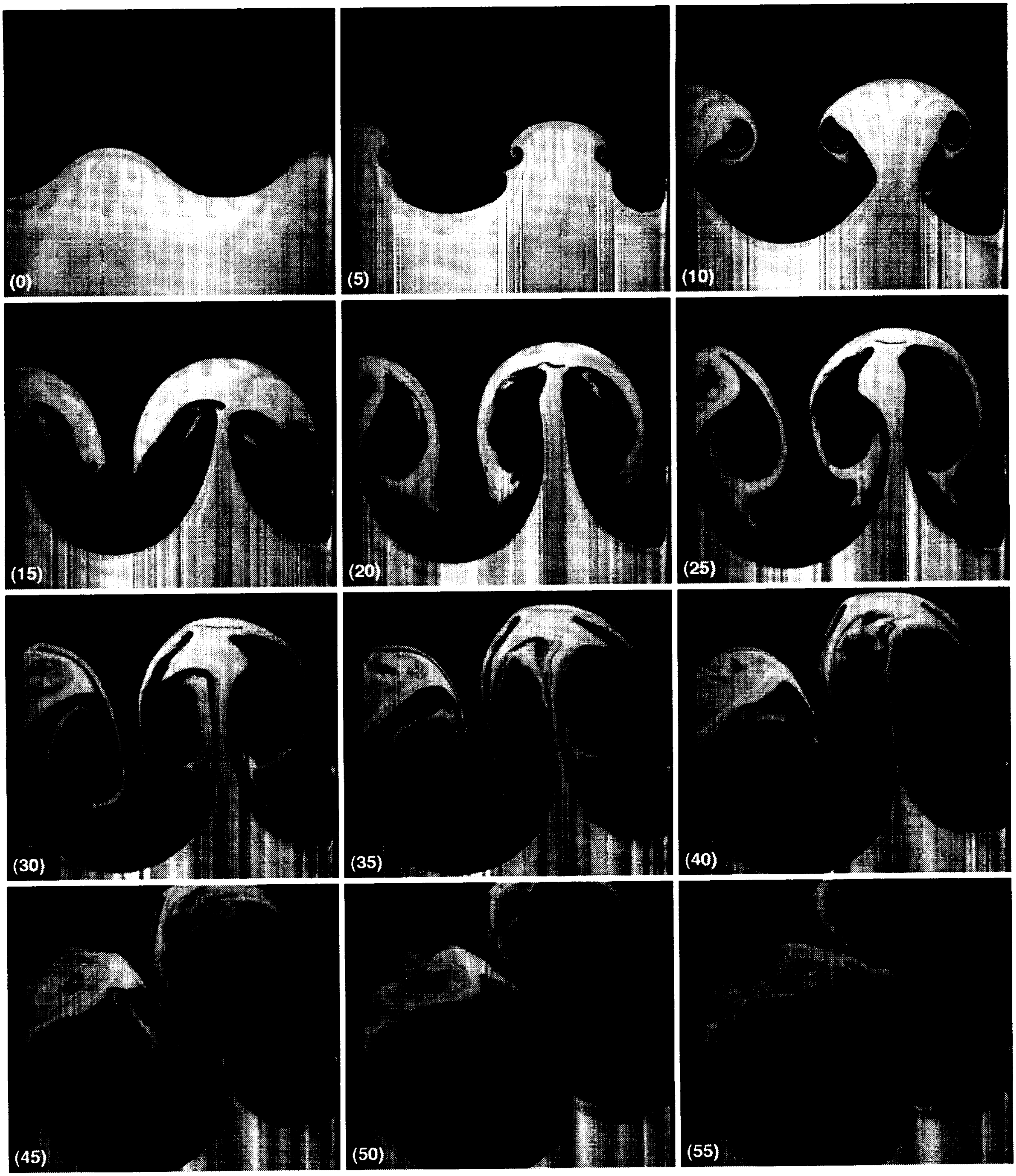
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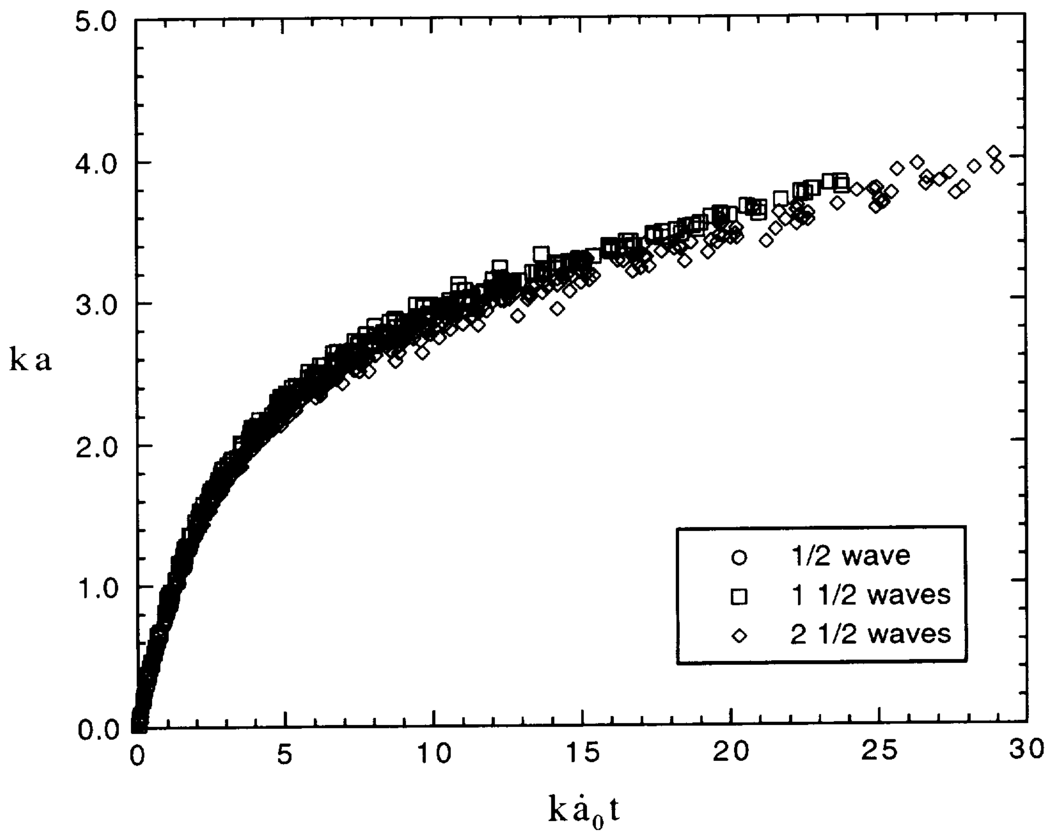
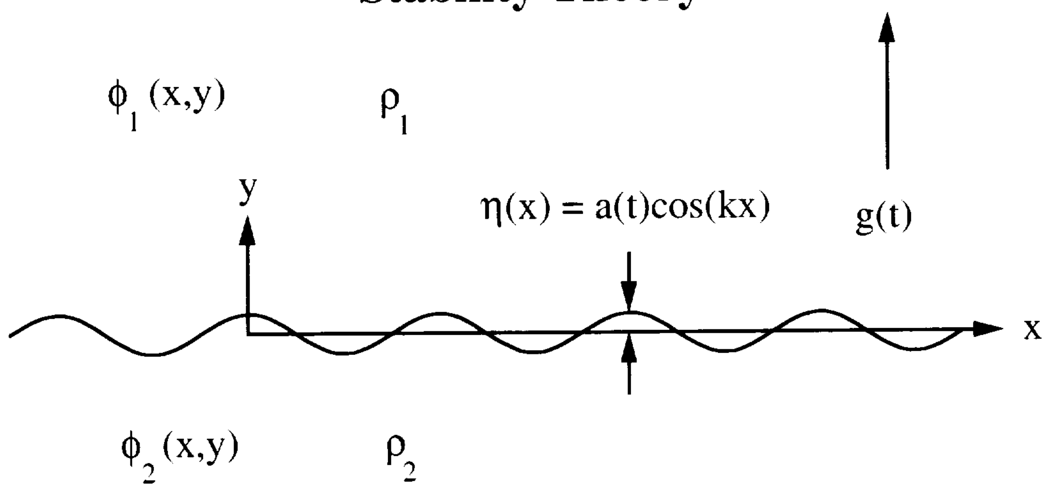
Amplitude Measurements



Stability Theory



Linear Theory (Taylor 1950, Richtmyer 1960)

$$\boxed{\dot{a} = k\,a_0\,A\,\Delta V \equiv \dot{a}_0}$$

Atwood number: $$A = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

Weakly Nonlinear Theory (Zhang & Sohn, 1997)

$$\boxed{\dot{a} = \dot{a}_0\left[1 - (a_0 k)\dot{a}_0 k t + (A^2 - \tfrac{1}{2})\dot{a}_0{}^2 k^2 t^2 - \cdots\right]}$$

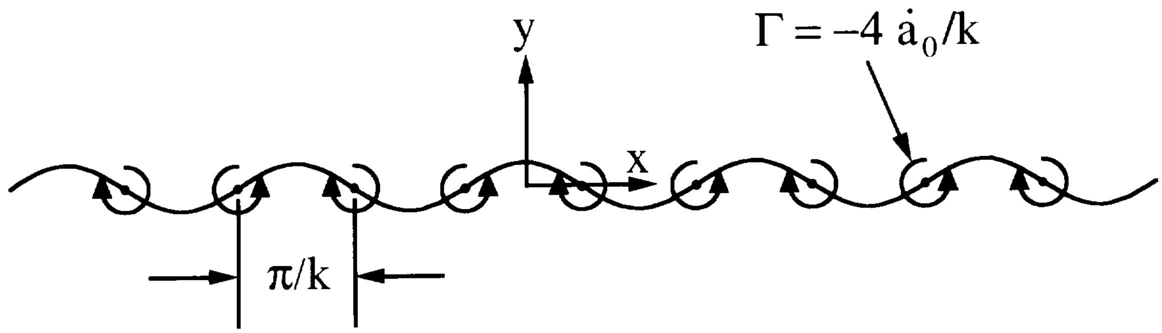
Padé approximant,

$$\boxed{\dot{a} = \dot{a}_0\left[\frac{1}{1 + (a_0 k)\dot{a}_0 k t + \max\left\{0, a_0{}^2 k^2 - A^2 + \tfrac{1}{2}\right\}\dot{a}_0{}^2 k^2 t^2}\right]}$$

$$\text{when } A \le \frac{1}{2}, \quad \dot{a} \to \frac{1}{t^2} \quad \text{as} \quad t \to \infty$$

Nonlinear Models

Vortex Model (Jacobs et al. 1995)



$$a(t) = \frac{1}{k}\sinh^{-1}\left[\frac{2}{\pi}k\,\dot{a}_0\,t + \sinh(k\,a_0)\right]$$

$$\boxed{\dot{a} \to \frac{1}{k\,t} \quad \text{as} \quad t \to \infty}$$

Empirical Model (Sadot el al. 1998)

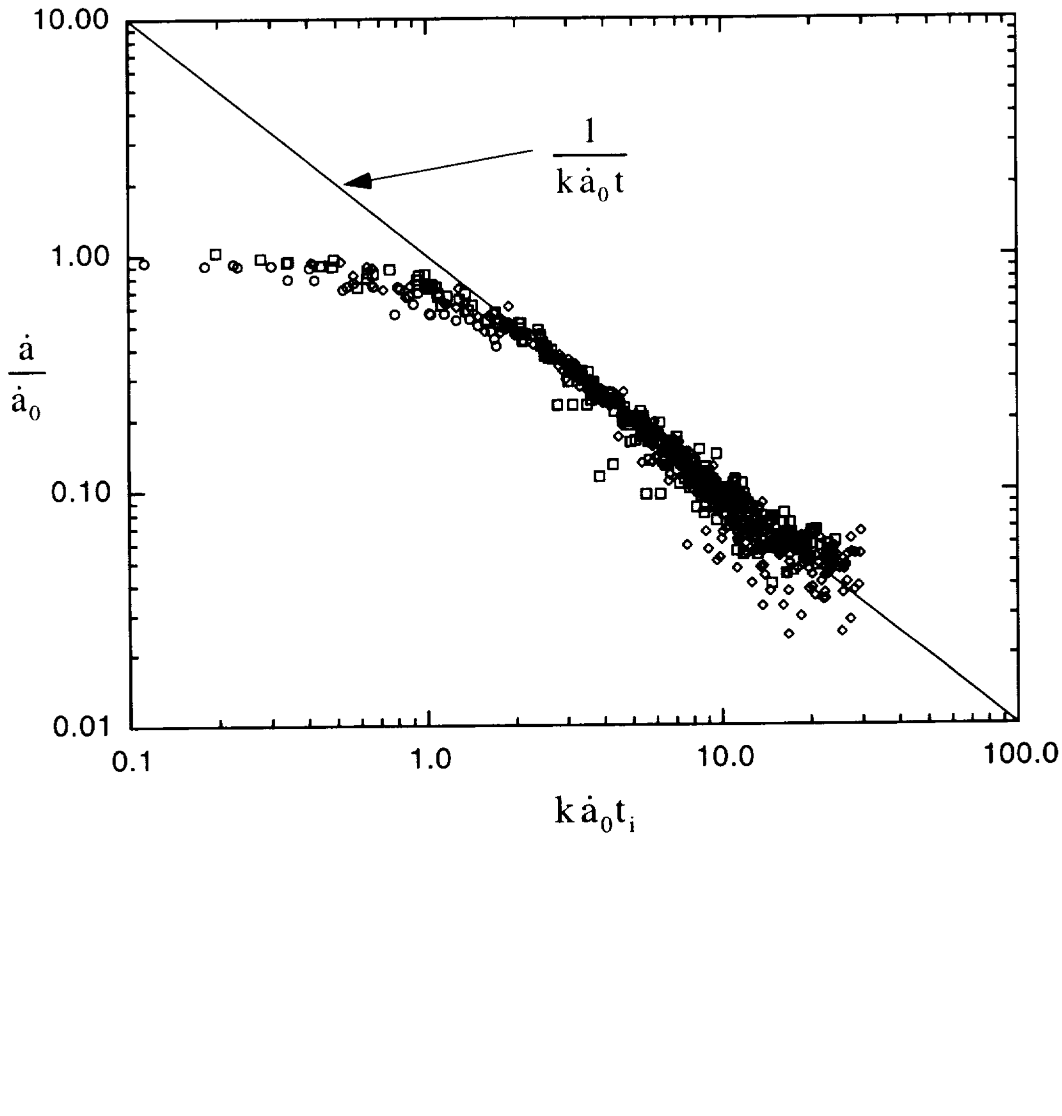
$$\boxed{\dot{a}_{b/s} = \dot{a}_0 \frac{1 + \dot{a}_0 k\,t}{1 + (1 \pm A)\dot{a}_0 k\,t + \left[\frac{1 \pm A}{1 + A}\right]\frac{1}{2\pi C}\dot{a}_0^{\,2} k^2\,t^2}}$$

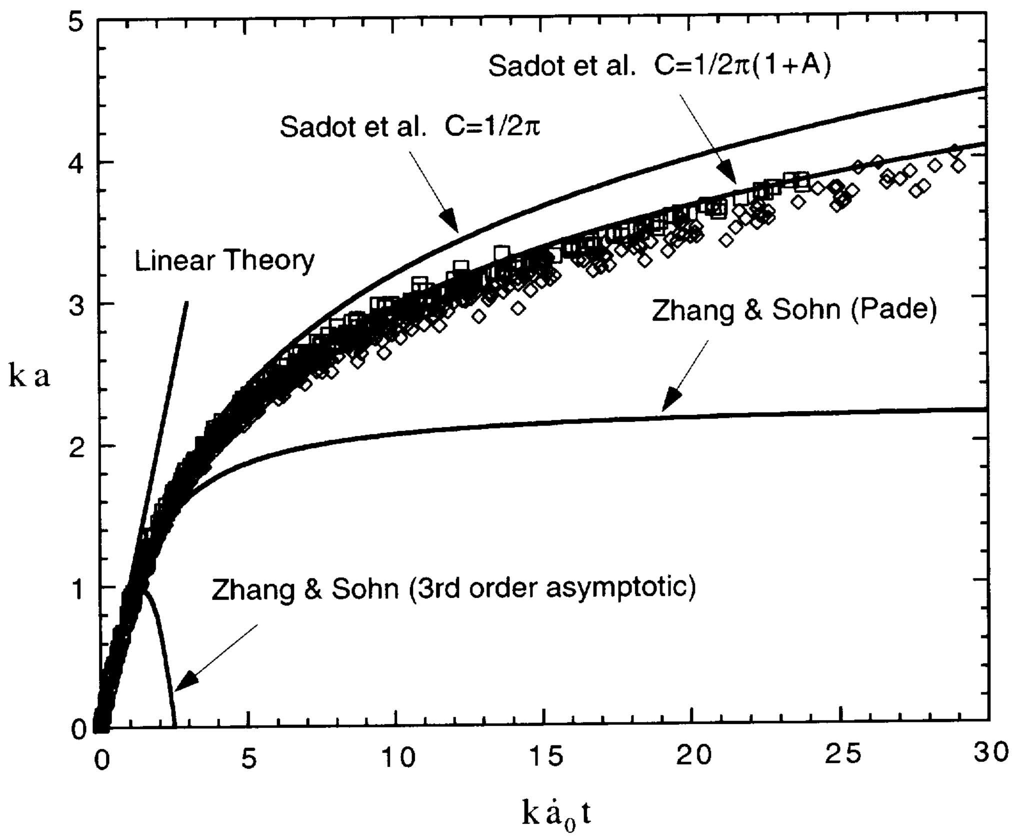
$$C = \begin{cases} 1/3\pi & A \geq 0.5 \\ 1/2\pi & A \to 0 \end{cases}$$

Small Atwood Number Correction (Niederhaus & Jacobs 2002)

$$\boxed{C = \frac{1}{2\pi(1 + A)} \quad \text{when} \quad A \to 0}$$

Growth Rate Measurements




Sadot et al. C=1/2π(1+A)
Sadot et al. C=1/2π
Linear Theory
Zhang & Sohn (Pade)
Zhang & Sohn (3rd order asymptotic)
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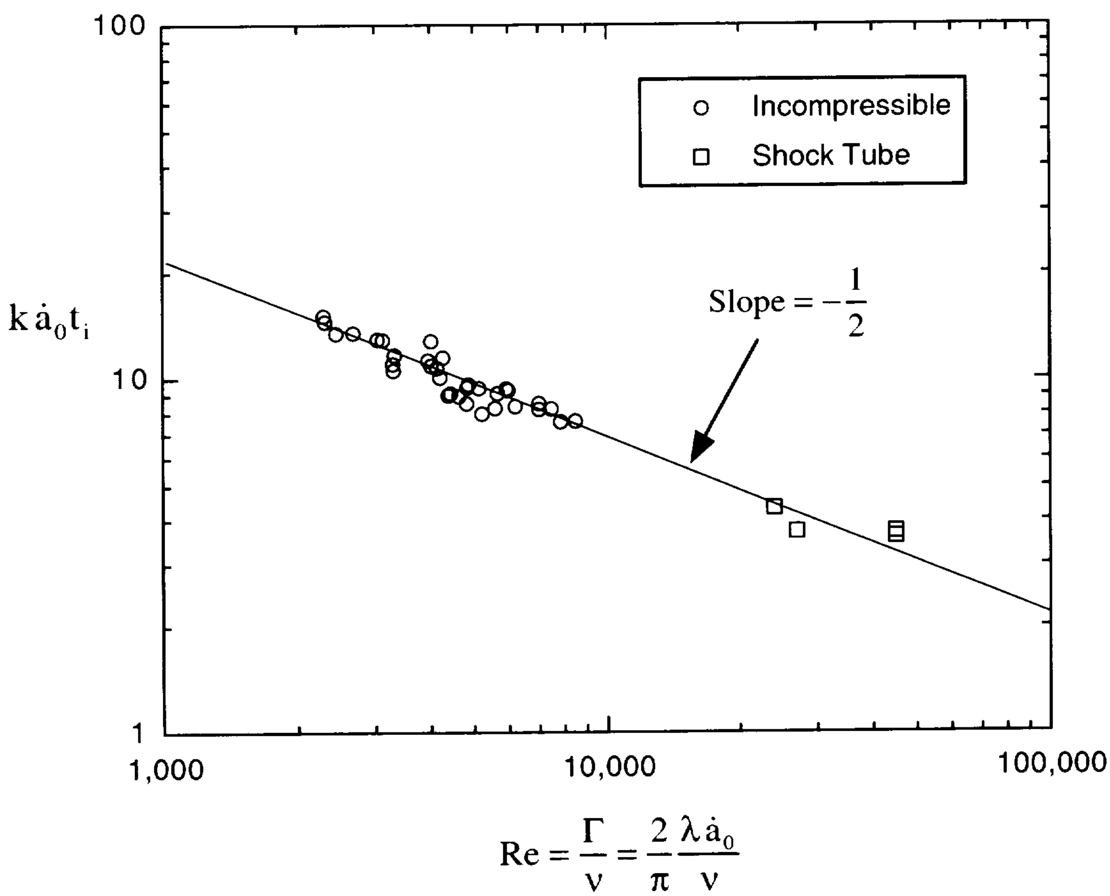
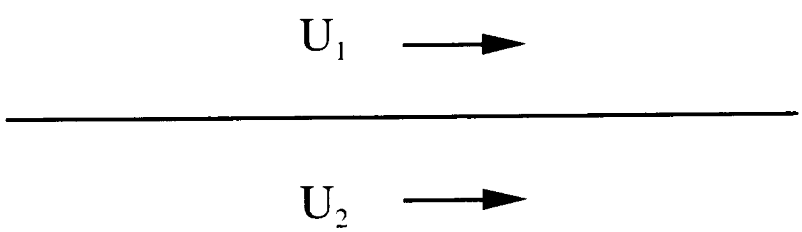
Vortex Instability Time of Appearance



Kelvin-Helmholtz Instability



Kelvin-Helmholtz growth rate:

$$\sigma_{K-H} \sim \frac{U_1 - U_2}{\lambda_{K-H}} \sim \frac{\dot{a}_0}{\lambda_{K-H}} \sim \frac{1}{t_i}$$

Fastest growing wavelength:

$$\lambda_{K-H} \sim \text{Vorticity Thickness} \sim \sqrt{\nu/\varepsilon}$$

Strain rate:

$$\varepsilon \sim \frac{\dot{a}_0}{\lambda}$$

Dimensionless time of appearance:

$$\boxed{k\,\dot{a}_0\,t_i \sim \sqrt{\frac{\nu}{\dot{a}_0\lambda}} \sim Re^{-1/2}}$$

Conclusions

- ❑ Incompressible experiments have proven to be an excellent method for studying the late-time development of RM instability.
- ❑ Late-time amplitude measurements show Reynolds number independence and are in excellent agreement with nonlinear models.
- ❑ Late-time images show the instability of the vortex cores resulting in the transition to turbulence.
- ❑ Dimensionless vortex instability occurrence times for incompressible and compressible experiments show the same Reynolds number dependence.
- ❑ Future experiments in the GRC 2.2 Second Drop Tower will allow even later-time observations of the instability.