ACOUSTIC EXPERIMENT TO MEASURE THE BULK VISCOSITY OF NEAR-CRITICAL XENON IN MICROGRAVITY

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ABSTRACT

We plan a rigorous test of the theory of dynamic scaling by accurately measuring the bulk viscosity of xenon in microgravity 50 times closer to the critical temperature $T_c$ than previous experiments. The bulk viscosity \( \zeta \) (or “second viscosity” or “dilational viscosity”) will be determined by measuring the attenuation length of sound \( \alpha_\lambda \) and also measuring the frequency-dependence of the speed of sound. For these measurements, we developed a unique Helmholtz resonator and specialized electro-acoustic transducers. We describe the resonator, the transducers, their performance on Earth, and their expected performance in microgravity.

Any fluid very near its liquid-vapor critical point contains highly-correlated density fluctuations characterized by a long relaxation time \( \tau \). When \( \omega \tau \geq 1 \) [where \( \omega = 2\pi f \) (period of sound wave)] the attenuation of the sound greatly exceeds the attenuation calculated from the shear viscosity and thermal conductivity. The excess sound attenuation is associated with dilatational motion of the fluid and is accounted for by the bulk viscosity. The excess attenuation encountered in monatomic, near-critical, xenon somewhat resembles the excess attenuation encountered in polyatomic molecules that have a long relaxation time characterizing energy exchange between translational and internal degrees of freedom.

According to the theory of dynamical scaling, both the attenuation length \( \alpha_\lambda \) and the sound-speed dispersion become universal functions of \( \omega \tau \) as the reduced temperature \( \theta \equiv 0 \). [Here \( \theta \equiv (T / T_c) - 1 \).] The best prior tests of the theory were conducted in near-critical xenon and helium \([1,2]\) at frequencies of 0.5 MHz and higher. These tests encountered the condition \( \omega \tau = 1 \) at \( \theta \equiv 10^{-3} \). The experimenters introduced empirical parameters to describe how their \( \alpha_\lambda(\omega \tau) \) data might cross over from the experimentally accessible region to the expected asymptotic region at smaller values of \( \theta \). Even with such parameters, the data disagreed with theory, within combined uncertainties. Because we are using lower frequencies (130 Hz to 1300 Hz) and because we will use microgravity to evade the stratification of near-critical xenon in the Earth’s gravity, we expect to encounter \( \omega \tau < 1 \) at \( \theta \equiv 2 \times 10^{-5} \), 50 times closer to \( T_c \).

For these measurements, we designed the acoustic resonator shown in Figs. 1a and 1b. It is composed of two chambers connected by a small tube. The chambers are cylindrical with perpendicular axes. The chambers have nearly equal volumes (~10 cm³), but very different aspect ratios. One is 48 mm long with a 16 mm ID; the other is 22.2 mm long with a 23.5 mm ID. The connecting tube is 15 mm long with a 4 mm ID. This resonator has two low-frequency acoustic modes that are well separated from each other and all other modes. The lowest-frequency mode is a Helmholtz mode in which the gas oscillates between the chambers through the small tube. In the next mode, the gas oscillates along the length of the longer chamber. For these two modes in microgravity, Figure 2 shows the expected contributions to the acoustic losses \( Q^{-1} \) from the bulk
viscosity, the shear viscosity, and the thermal conductivity. On Earth, the stratification of the xenon’s density from the top to the bottom of the resonator will prevent accurate measurements of the bulk viscosity closer than 150 mK to the critical point ($t < 5 \times 10^{-4}$).

Two types of transducers have been developed to both excite and detect sound. The first type uses piezo-ceramic disks epoxied to the outside of a 2.5 mm thick diaphragm at each end of the chambers. Flexure of the diaphragms couples to the radial displacement of the ceramics and, therefore, to the voltages that are applied or detected at the electrodes. The advantages of this technique are (1) electrical feedthroughs are not necessary, (2) no foreign materials come in contact with the xenon, and (3) coupling to the xenon is independent of temperature.

The second type of transducer is immersed in the xenon. It consists of closely-spaced (< 10 μm), interdigitated electrodes deposited on a quartz substrate. (The electrodes are the lighter areas in Fig. 3.) For detecting sound, the electrodes are biased with a DC voltage. An impinging acoustic wave changes the density of the xenon near the electrodes. The resulting change in capacitance generates an AC signal. To generate sound, an AC voltage $V$ at frequency $f$ is applied to the interdigitated electrodes. The resulting electric field creates a pressure (proportional to $V^2$) that pulls the xenon towards the electrodes at frequency $2f$. This phenomenon is called electrostriction. The advantages of the interdigitated transducers are (1) the frequency response is straightforward to model, (2) low dissipation, (3) no cross talk between drive and detection circuits, and (4) the sensitivity increases with the isothermal compressibility as $\rho_0$.

BVX: An Acoustic Experiment to Measure the Bulk Viscosity of Near-critical Xenon in Microgravity

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We plan a rigorous test of the theory of dynamic scaling by accurately measuring the bulk viscosity of xenon in microgravity 50 times closer to the critical point than previous experiments. The bulk viscosity $\chi$ (also “second viscosity” or “dilational viscosity”) will be determined from measurements of sound attenuation $\alpha_\lambda$ and dispersion. For these measurements, we have developed a unique acoustic resonator (Greenspan viscometer) and specialized electro-acoustic transducers.

The resonator is an asymmetric double-chamber Helmholtz resonator designed so that the 2 lowest frequency modes are isolated and non-degenerate. The lowest mode is a Helmholtz mode, characterized by alternating gas flow from one chamber to the other through a small connecting tube. This mode has a very long wavelength (0.5m) and low frequency (120 Hz). The next mode is the 1st longitudinal mode of one chamber, which occurs when the wavelength (0.1m) is twice the chamber’s length.

With these two modes, we will measure the dissipation and dispersion between $t = 1 \times 10^{-3}$ and $1 \times 10^{-5}$ in microgravity. Initial ground-based experiments will demonstrate the saturation of thermal boundary layer dissipation. These measurements and others will provide a means to optimize the resonator design so that the most accurate measurements over the widest temperature range will be possible in microgravity.
Motivation

• The attenuation length $\alpha_\lambda$ and the sound-speed dispersion become universal functions of $\omega \tau$ as $\tau \to 0$, according to dynamic scaling theory.

• The best prior tests of the theory were conducted on Earth in near-critical xenon and helium at frequencies of 0.5 MHz and higher. These tests encountered the condition $\omega \tau = 1$ at $\tau \approx 10^{-3}$.

• Empirical parameters were used to describe how their $\alpha_\lambda(\omega \tau)$ data might cross over from the experimentally accessible region to the expected asymptotic region at smaller values of $\tau$. The data did not agree with theory within the combined uncertainties.

Our approach

• We use lower frequencies (120 Hz to 1300 Hz) and plan to use microgravity to evade the stratification of near-critical xenon in Earth’s gravity. We expect to encounter $\omega \tau < 1$ at $\tau \approx 2 \times 10^{-5}$, 50 times closer to $T_c$ than previous measurements.

• Initial ground-based experiments will verify the saturation of thermal boundary layer dissipation for $\tau \approx 5 \times 10^{-4}$. Gravity will limit bulk viscosity measurements for $\tau \approx 1 \times 10^{-3}$.

• Micro-interdigitated electrode capacitors are being developed as electro-acoustic transducers based on the principle of electrostriction.
The critical point is the highest temperature where liquid and vapor can coexist in the same container. For xenon, $T_c = 16 \, ^\circ\text{C}$, $P_c = 58 \, \text{Atm}$. 

![Graph showing the critical point on a temperature-pressure diagram. The critical point is marked with a solid circle. The phases are indicated as solid, liquid, and vapor.](image-url)
Critical Fluctuations

The sizes of the fluctuations are characterized by the correlation length $\xi$. If the density is exactly the critical density

$$\xi = \xi_0 \left( \frac{T - T_c}{T_c} \right)^{-\gamma} = \xi_0 t^{-\gamma} = 1.84 \times 10^{-10} t^{-0.63} \text{ m}$$

Thus, $\xi$ reaches 1.1 $\mu$m at $t = 10^{-6}$, corresponding to 0.3 mK above $T_c$.

Large $\xi$ implies intense light scattering, critical opalescence.

$\xi$ is a natural measure of the distance from the critical point and governs thermodynamic behavior in the critical region.
Bulk Viscosity

- Bulk viscosity $\zeta$ of a fluid represents a resistance in the relationship between density and pressure that arises from the conversion of kinetic energy between translational and internal degrees of freedom governed by one or more relaxation times $\tau_i$.

- Sound waves are oscillating pressure/density fields that will exhibit attenuation and dispersion due to bulk viscosity when $2\pi f \tau_i = 1$

$$\alpha_\lambda = \frac{\pi \omega}{c^2} \left( (\gamma - 1) \frac{\lambda}{\rho C_p} + \frac{4 \eta}{3 \rho} + \frac{\zeta}{\rho} + (\gamma - 1) \frac{c^2}{\omega} \frac{C_{relax}}{C_p} \frac{\omega \tau}{1 + (\omega \tau)^2} \right)$$

- Important for polyatomic fluids at low densities
Near critical points, monatomic fluids behave like polyatomic fluids with an infinite number of slowly relaxing modes. The distribution of relaxation times is characterized by:

\[ \tau \equiv 6\pi\eta\xi^3 / (k_B T_c) = \tau_0 t^{-\nu(3+\eta)} = \tau_0 t^{-1.93}, \quad t \equiv (T - T_c) / T_c \]

\[ \zeta \approx R_B \tau p_c c^2 \propto \zeta^{2.89} \propto t^{-1.82} \]

Kogan and Meyer, 1998

Measurements and theory do not agree. Limitations of high frequency and gravity.

\[ \omega^* = 2\pi f \xi^2 / D_T \]
Chamber 1

2r₁ = 16 mm
L₁ = 48 mm

Transducers

Helmholtz mode

\[ f_{H} \approx \frac{c}{2\pi} \sqrt{\frac{A_d}{L_d} \left( \frac{1}{V_1} + \frac{1}{V_2} \right)} \quad [120 - 240 \text{Hz}] \]

\[ Q_{H}^{-1} = \frac{2g}{f} \approx \frac{\zeta}{\rho c} \sqrt{\frac{A_d}{L_d} \left( \frac{1}{V_1} + \frac{1}{V_2} \right)} \]

\[ + (\gamma - 1) \frac{\delta r S_c}{2V_c} \quad [0.01 - 0.3] \]

Longitudinal mode

\[ f_{L} \approx \frac{c}{2L_1} \quad [650 - 1300 \text{Hz}] \]

\[ Q_{L}^{-1} \approx (\gamma - 1) \left( 1 + \frac{2r_1}{L_1} \right) \delta r \frac{\pi \zeta}{L_1 \rho c} \quad [0.01 - 0.3] \]
Expected Results

Helmholtz mode  Longitudinal mode

Shear Viscosity

\( t = \frac{(T - T_C)}{T_C} \)

\( (\delta_r \rho C_p)_\text{solid} = (\delta_r \rho C_p)_\text{gas} \)
Measured Spectrum of BVX Resonator

Argon 3.7 MPa

1st Longitudinal Mode

2nd Longitudinal Mode

Frequency (Hz)

Amplitude (µV)
Longitudinal mode

$Q_x - Q_{th}$

density (kg/m$^3$)

argon
xenon

Scale factor = 1.018
Greenspan mode

Scale factor = 1.027

argon
xenon

$Q_{ax}^{-1} - Q_{mx}^{-1}$

density (kg/m$^3$)
The graph shows the relationship between density (kg/m³) and the ratio of long-term to greenspan. The graph includes data points for argon and xenon, with xenon having a higher density range and a notable trend of decreasing density with increasing ratio.