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AIAA 2002-4233

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38th AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit

7-10 July 2002
Indianapolis, Indiana

TRAJECTORIES FOR HIGH SPECIFIC IMPULSE HIGH SPECIFIC POWER DEEP SPACE EXPLORATION

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Preliminary results are presented for two methods to approximate the mission performance of high specific impulse high specific power vehicles. The first method is based on an analytical approximation derived by Williams[1] and Shepherd[2] and can be used to approximate mission performance to outer planets and interstellar space. The second method is based on a parametric analysis of trajectories created using the well-known trajectory optimization code, VARITOP[3]. This parametric analysis allows the reader to approximate payload ratios and optimal power requirements for both one-way and round-trip missions. While this second method only addresses missions to and from Jupiter, future work will encompass all of the outer planet destinations and some interstellar precursor missions.

Symbols

M_i	Initial Spacecraft Mass
M_p	Propellant Mass
M_t	Tankage Mass
M_{ps}	Power/Propulsion System Mass
M_n	Net Mass
I_{sp}	Specific impulse
P	Total Electrical Power To Propulsion System
P_j	Jet power
P_{sp}	Specific Power (kW/kg)
α	Power/Propulsion System Specific Mass
α_j	Power/Propulsion System Jet Power Specific Mass
α_{pwr}	Specific Power
η	Power/Propulsion System Efficiency
k_t	Tankage Fraction
n_E	Mean Motion, or average angular rate of motion, of Earth
n_J	Mean Motion of Jupiter

Introduction

There are several fusion propulsion concepts currently proposed in the literature. Most of these concepts are being actively pursued at research centers around the world. These propulsion concepts are nearly always identified for deep space missions, that is missions to the outer planets and to interstellar space. The merits of these concepts are quantified in terms of the potentially attainable Specific Impulse (I_{sp}), the Lawson number, which is the product of the confinement time and plasma density confined, and the energy return, Q, which is the ratio of energy out to energy input. This work quantifies the possibilities of these propulsion concepts in terms of vehicle design criteria for deep space vehicles, i.e. time of flight, or trip time, for the vehicle and the ratio of the payload mass delivered to the final destination to the initial mass of the vehicle.

Flight times and deliverable masses for electric and fusion propulsion systems are difficult to approximate. Numerical integration is required to accurately predict the performance of these continuous thrust systems. However, two reasonably accurate methods of predicting the performance of these vehicles that do not require the reader to use a complex numerical integration code are presented in this paper.

The first method identifies the optimum conditions for time of flight and payload mass ratio using analytical techniques. Achieving these optimum conditions can sometimes even exceed the capabilities of proposed fusion systems so sensitivities to non-optimum conditions are also quantified. These sensitivities illustrate that I_{sp} is not the only important variable in interplanetary performance.

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The second method is based on data generated by the well-known trajectory simulation and optimization code, VARITOP[3]. VARITOP is a trajectory optimization code that performs numerical integration based on calculus of variations. Several charts were developed that illustrate the dependence of net mass ratio, M_n/M_i, and jet power, P_j, on time of flight, I_{sp}, and specific power, α_{pwr}. These charts are intended to be a tool by which people in the propulsion community can explore possible missions for their propulsion system concepts.

The intent of this paper is to provide two simple methods to calculate these mission parameters for a given propulsion concept. In this manner the propulsion engineer can have at his fingertips the data necessary to guide his design. The data will assist the engineer in making trades in overall performance and mass necessary to optimize his design. These results are preliminary, illustrating the methodology to be used in the completion of this project. The final result, to be reported in a future paper, will explore the options to all of the outer planets and preliminary estimates for interstellar missions.

Definitions

At this point it is necessary to spend a moment discussing the definitions of the various symbols that have been introduced. The nomenclature is based on a NASA standard[4] generated in 1969. In this paper, it is assumed that the spacecraft initial mass (M_i) is made up of four component masses. The propellant mass (M_p) is simply the mass of the propellant, and the tankage mass (M_t) is the mass of the tanks required to contain that propellant. In this analysis, tankage fraction (k_t) is assumed to be 10% of the total propellant mass, and is intended to cover the mass of propellant tank structure, valves, and plumbing. The rest of the vehicle mass must be accounted for in the remaining two component masses, the power/propulsion system mass (M_{ps}) and the net mass (M_n). The power/propulsion system mass should contain the masses for any subsystem that scales with power. In addition to the nuclear reactor/thruster, this term may include shielding, a thermal control system, and/or a power conditioning system. It is from this term that the α_{pwr} and power/propulsion system specific mass, α are defined.

$$\alpha_{pwr} = \frac{P}{M_{ps}} \qquad (\#)$$

$$\alpha = \frac{M_{ps}}{P} \qquad (\#)$$

A more useful quantity for this analysis is the power/propulsion system jet power specific mass, α_j.

$$\alpha_j = \frac{M_{ps}}{P_j} \qquad (\#)$$

Where P_j, is defined as

$$P_j = \eta \times P \qquad (\#)$$

The α_j is preferred in the analyses to follow because its use allows users to define a propulsion system efficiency (η) specific to their design.

The vehicle mass that is not included in the three component masses discussed above must be accounted for in the net mass term. Payload mass can only be established after further detail designs identify what portion of the net mass must be used for additional structure, communications, thermal control, and other essential systems. It is only after all necessary subsystem masses have been allotted that a true payload mass can be identified.

Background

Propulsion Concepts Considered

There are several fusion propulsion concepts currently on the drawing board. A partial list of these concepts and their documented predicted performance is listed in Table 1. Any attempt to compare these concepts would be futile since none have come to fruition. Each has their own advantages and disadvantages for attaining confinement and sustainable fusion reactions. Some sacrifice performance for the promise of shorter development time to achieve fusion. This paper attempts to determine the trade of the critical trajectory variables to make these concepts most useful for interplanetary flight.

Other concepts are also listed in Table 1. The Laser Augmented Plasma Propulsion System (LAPPS), first proposed by Karnmash et al.[5] exists on the high end of the I_{sp} spectrum. Conversely, existing and near term nuclear electric propulsion concepts populate the lower end. These concepts are included to round out the range of I_{sp} used in continuous thrust systems. Note that propulsion systems that use external energy for propulsion, such as the various solar, laser and magnetic sails are not included due to their extremely low thrust, thrust as a function of distance to the sun or laser, and radial thrust profiles makes the underlying assumptions for these calculations untenable.

Table 1. Fusion Propulsion Concepts

	I_{sp} (ksec)	T (kN)	P_j (GW)	M_{ps} (mt)	α_j (kg/kW)
Magnetic Mirror, Gas Dynamic Mirror (GDM)	112.9	49.7	27.5	7228.3	0.26
Pulsed Field Reversed Configuration (FRC)	100	0.3	0.15	9.0	0.06
Inertial Confinement Fusion (ICF)	12.6	185.0	11.4	979	0.09
Magnetic Confinement Fusion (MCF)	35.4	27.8	4.83	473	0.10
Magnetic Targeted Fusion (MTF)	77	16.5	6.23	41.34	0.01
Laser Acc. Plasma Prop. Sys. (LAPPS)	7754	0.013	0.49	5	0.01

Previous analytical trajectory work

A review of the literature reveals several attempts to address high I_{sp} constant thrust trajectories. Williams[1] discussed several papers and works by Cole[6], Moeckel[7] and Shepherd[2] as genesis of his work. This paper builds on the efforts of both Shepherd and Williams in high I_{sp} - high thrust calculations.

The Shepherd -Williams equations calculate time of flight as a function of I_{sp}, distance and specific power. The equation for payload ratio is derived and differentiated to find the optimum relationship between payload ratio and specific power. The resulting equation is then included in the derivation of time of flight. The use of an optimum payload ratio implies a limitation of the range of specific powers. This limitation can drive the required specific power to untenable values, even for the most advanced fusion concepts. Williams also developed two-burn rendezvous and four-burn round trip calculations assuming coast time.

There is some disagreement on the need for coast time in optimum interplanetary trajectories. Irving and Blum calculate that no coast time should be included in the trajectory. However, Moeckel states that a coast time of 1/3 the total trip time should be included for an optimum trajectory. Their calculations are compared and contrasted below.

Method I: Analytical Approximation

Using the calculations made by Shepherd, Williams, Moeckel and Irving/Blum as a starting point this work derives time of flight and payload ratio as a function of distance, specific impulse, power ratio, and ratio of total power to initial mass. Optimum points are also determined separately but are not included in time of flight calculations in order to allow for non-optimum operation at more realistic specific impulse- specific power combinations.

Trajectory Assumptions

Method one assumes straight line trajectories, and requires only a distance and time of flight as inputs.

Calculations

This analysis is based on three base equations. These base equations are the mass equation, ideal rocket or Tsiolkovsky equation and distance equation. These equations are combined in the analysis below to meet the objective of an approximate analysis method.

The mass convention for this analysis divides the vehicle mass into three parts; payload, propellant and structural mass. The summation of these masses comprises the total inert mass of the vehicle as illustrated in the equation below.

$$m_i = m_p + m_s + m_{pay} \qquad (1)$$

The Tsiolkovsky equation is displayed here without derivation

$$\Delta V = -c\ln\left(\frac{m_i}{m_f}\right) - \overline{g\cos(\theta)}T \qquad (2)$$

The second term above is the average gravitational loss times the burn time. The final mass is the sum of the structural and payload mass

$$m_f = m_s + m_{pay} \qquad (3)$$

For a constant thrust system the mass flow rate is constant and is related to the propellant mass

$$\dot{m}_p = \frac{m_p}{T} \qquad (4)$$

Combining equations (1) through (4) and defining *t* as the instantaneous mission time yields

$$\Delta V = V - V_o = -c\ln\left(1 - \frac{\lambda t}{T}\right) - \overline{g\cos(\theta)}\mathrm{t} \qquad (5)$$

where the propellant ratio is

$$\lambda = 1 - \frac{m_f}{m_i} = \frac{m_p}{m_i} \qquad (6)$$

The distance equation is given as

$$S = \int_0^T V dt \qquad (7)$$

If gravitational losses is neglected then substitution of equations (2) into (7) yields

$$S = V_o T + cT\left(\frac{1-\lambda}{\lambda}\ln(1-\lambda) + 1\right) - \frac{1}{2}\overline{g\cos(\theta)T} \qquad (8)$$

It is convenient to replace the propellant ratios with the mass components. Also the initial velocity and gravitational loss is considered to be negligible for the propulsion systems of interest. Solving for trip time

$$T = \frac{S}{c}\left(\frac{1 - \left(\frac{m_{pay}}{m_i} + \frac{m_s}{m_i}\right)}{\left(\left(\frac{m_{pay}}{m_i} + \frac{m_s}{m_i}\right)\left(\ln\left(\frac{m_{pay}}{m_i} + \frac{m_s}{m_i}\right) - 1\right) + 1\right)}\right) \qquad (9)$$

At this point it is necessary to develop relationships for the two mass ratios in the equation above. The combination of equations (1) through (3) and using the minimal gravity loss assumption gives the following

$$\Delta V = -c\ln\left(\frac{m_i}{m_s + m_{pay}}\right) \qquad (10)$$

Simplifying yields

$$\frac{\Delta V}{c} = \ln\left(\frac{1 + \frac{m_s}{m_p}}{\frac{m_{pay}}{m_i} + \frac{m_s}{m_p}}\right) \qquad (11)$$

Further simplification requires the definition of a couple new variables. The jet power is defined as

$$P = \frac{1}{2}\dot{m}_p c^2 \qquad (12)$$

and the specific power is

$$\alpha = \frac{P}{\eta m_s} \qquad (13)$$

where η is the conversion efficiency. Combining equations (4) and (13) yields

$$\frac{m_s}{m_p} = \frac{c^2}{2\eta\alpha T} \qquad (14)$$

also

$$\frac{m_s}{m_i} = \frac{m_s}{P}\frac{P}{m_i} = \frac{1}{\alpha}\frac{P}{m_i} \qquad (15)$$

Solving for the payload ratio in equation (11) and inserting (14) yields

$$\frac{m_{pay}}{m_i} = 1 - \frac{2T}{c^2}\frac{P}{m_i} - \frac{1}{\alpha}\frac{P}{m_i} \qquad (16)$$

This very important equation allows for the calculation of the maximum payload for a given initial mass. The power to initial mass ratio is an independent variable in this analysis. Other papers have eliminated this variable using an optimization equation but the required power levels and specific power can exceed reasonable values. The payload ratio is maximized when the exhaust velocity and specific power are maximized and the required trip time and power to initial mass ratio is minimized.

It is now time to finish reducing equation (9). Combining with equations (15)and (16) and working through a significant amount of algebra yields

$$T = \frac{S}{c}\left(\frac{1-\left(1-\frac{2T}{c^2}\frac{P}{m_i}\right)}{\left(1-\frac{2T}{c^2}\frac{P}{m_i}\right)\left(\ln\left(1-\frac{2T}{c^2}\frac{P}{m_i}\right)-1\right)+1}\right) \qquad (17)$$

This equation is implicit and complicated. If the logarithmic expression in the denominator on the right hand side could be simplified it might yield a simpler solution. A binominal expansion of the logarithm yields

$$\ln\left(1-\frac{2T}{c^2}\frac{P}{m_i}\right) = -\frac{2T}{c^2}\frac{P}{m_i} - 2\left(\frac{T}{c^2}\frac{P}{m_i}\right)^2 - \frac{8}{3}\left(\frac{T}{c^2}\frac{P}{m_i}\right)^3 - 4\left(\frac{T}{c^2}\frac{P}{m_i}\right)^4 \cdots \qquad (18)$$

Replacing the logarithm in equation (17) with the first term of the expansion yields

$$T = \frac{S}{c}\left(\frac{1-\left(1-\frac{2T}{c^2}\frac{P}{m_i}\right)}{1-\left(1-\frac{2T}{c^2}\frac{P}{m_i}\right)\left(1+\frac{2T}{c^2}\frac{P}{m_i}\right)}\right) \qquad (19)$$

Inclusion of the expansion and solving for trip time yields

$$T = \sqrt{\frac{cS}{2\frac{P}{m_i}}} \qquad (20)$$

Due to the binominal expansion the above equation is only good when the following condition holds

$$2T\frac{P}{m_i} << c^2 \qquad (21)$$

Example

Insert an example use of this method here. Possibly compare with parametric analysis to follow.

Method II – Parametric Analysis

Problem Statement

It has been shown that time of flight is a function of distance traveled, specific impulse and the ratio of power to initial mass (P/M_i). The delivered net mass to initial mass (M_n/M_i) is then a function of the time of flight and specific power of the power system (P/M_{ps}).

A second method of estimating performance of fusion-class vehicles is presented below. A range of I_{sp} and α_j are used to generate curves that describe the M_n/M_i ratio and optimal P/M_i ratio as a function of these two variables. This process was repeated for various flight times. One-way and round-trip performance for missions between Earth and Jupiter can be approximated using this method of simple algebraic expressions with some iteration.

Performance data was generated using VARITOP, a general purpose low-thrust trajectory optimization program developed at the Jet Propulsion Laboratory. This program performs numerical integration of the state and co-state equations to solve the two-point boundary value problem that satisfies user-defined terminal constraints. The optimization is based on the calculus of variations. The program is designed to optimize trajectories where the thrust-to-weight ratio is too small for an impulsive ΔV approximation, but large enough that perturbative techniques do not apply.

Assumptions

To eliminate the dependence on departure date, the planet orbits are assumed to be circular and co-planar. For the Jupiter missions, this is not an outrageous assumption. Jupiter's eccentricity is only 0.048, about half that of Mars, and it's inclination 1.3°.

Also, outbound and return trajectories of the same flight time, I_{sp} and α_j are assumed to be symmetric, therefore, mass and power ratios defining the performance in one direction can be used for the reverse trajectory.

In the trajectory parametrics created with VARITOP, each trajectory starts from the sphere of influence of the Earth with zero hyperbolic excess velocity, and terminates at the sphere of influence of the destination planet with zero hyperbolic excess velocity. Since the plots only reflect the interplanetary portion of the trajectory, the user can apply this data to any mission. One needs only to correct the mass fractions to reflect the additional ΔV required if lower altitude departure and arrival conditions are desired.

VARITOP selected the jet power and departure date that would optimize each transfer. Circular and coplanar orbits were assumed for the Earth and Jupiter, so the optimum departure date has no relevance except to ensure that the optimal travel angle is used.

Results

For each of these trajectories, the propulsion system jet power specific mass (α_{jet}) was varied from 0.01 – 0.14 kg/kW (or until no net mass was possible).

For each α_j analyzed, the specific impulse, I_{sp}, was varied from 10,000 seconds to 450,000 seconds (or until no net mass was possible). Figure XX shows the ratio of net mass (m_n) to initial mass (m_0) for a 100-day trip from Earth to Jupiter. Using this chart, and the associated jet power ratio chart, Figure XX, one can determine the maximum net mass that can be delivered with a particular propulsion system.

One-way Example Case

Let us consider the Magnetized Target Fusion propulsion system outlined in reference ##. The predicted performance of this system is 77,000 seconds I_{sp} and 16,500 N of thrust. The jet power can be calculated from these two values, and is found to be 6.2 GW. The propulsion system mass (including radiators and power handling hardware) is predicted to be 41,340 kg. From this we determine that the Propulsion System Jet Power Specific Mass is approximately 0.01 kg/kW. Using this α_j, and the I_{sp}, we can see from the charts that the net mass ratio will be about 0.6, and the jet power ratio about 12 kW/kg. Beginning with the jet power ratio and multiplying by the 6.2 GW identified above, we see that the initial mass that results in the optimal thrust to weight for this I_{sp} and trip time is 74,400,000 kg. From this the net mass is found to be 44,640,000 kg.

Round-Trip Missions

Round trip mission performance can be calculated similarly. An additional constraint is applied in this case however; the vehicle's final location and the Earth's location at that time must be the same. Selecting the appropriate stay time at Jupiter satisfies this constraint. The angular velocity of the two planets and the travel angles of the outbound and return trajectories are all that is needed to calculate this time period.

Adding Departure and Arrival Details

Trajectories were calculated for different trip times from Earth to Jupiter and from Jupiter to Earth. The missions were designed to start at the sphere of influence of the departure planet and stop at the sphere of influence of the arrival planet with zero hyperbolic excess velocity at both. This was done to maintain the broad applicability of the data generated. User's have the flexibility to select their own departure and arrival orbits around Earth or Jupiter, approximate the required ΔV for the escape or capture spiral from that orbit using reference ## or any other method of their choice, and combine that with the heliocentric portion of the trajectory to create a complete and unique mission design.

Example use of these charts:

Starting with the known performance parameters, I_{sp} and α_j, a mission trip time must be selected that will result in a positive M_n/M_i. The M_n/M_i ratio and corresponding P_j/M_i ratio can then be identified directly from the appropriate chart or through interpolation between two chart values if the time of flight selected lies between the values of time of flight plotted. When one of the three variables that make up those ratios, M_n, M_i, or P_j, is selected the mission design is completely specified. It is expected that many users will select a desired net mass to be delivered. Staying with this assumption, the remaining vehicle component masses can be determined through the following simple relationships.

$$M_i = \frac{M_n}{M_n / M_i}$$

$$P_j = M_i \times P_j / M_i$$

$$M_{ps} = \alpha_j P_j$$

and since

$$M_i = M_n + M_{ps} + M_p + M_t$$

and

$$M_p + M_t = (1 + k_t) M_p = M_i - M_n - M_{ps}$$

$$M_p = \frac{M_i - M_n - M_{ps}}{1.1}$$

$$M_t = k_t \times M_p = 0.1 \times M_p$$

The round-trip mission analysis is understandably more complex.

Table 2. Optimal Travel Angles for Earth-Jupiter Transfers

Trajectory	Δθ
50-day	23°
100-day	50°
150-day	66°
200-day	85°

$$n_E(t_1 + t_{stay} + t_2) = \theta_0 + \Delta\theta_1 + n_J(t_{stay}) + \Delta\theta_2 \quad \text{(\#\#)}$$

Draft. All technical content is complete, but Some asthetic/gramatical/etc. changes will be made prior to final release.

Figures

Figure 1. 50-day Transit Between Earth & Jupiter
Net Mass Ratio vs. Specific Impulse

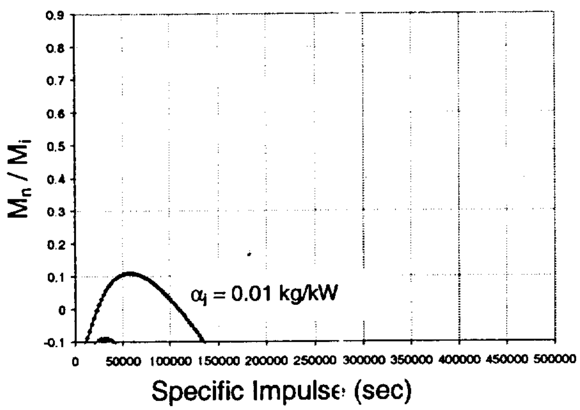


Figure 2. 100-day Transit Between Earth & Jupiter
Net Mass Ratio vs. Specific Impulse

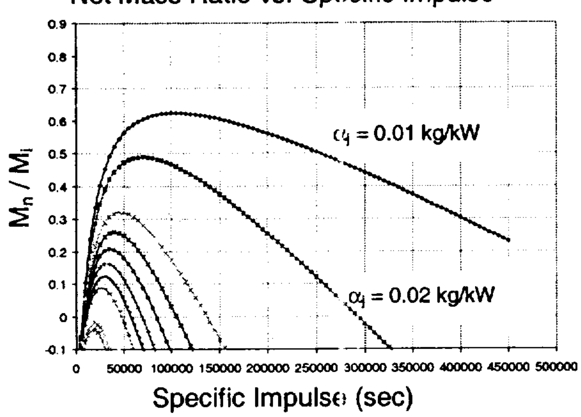


Figure 3. 150-day Transit Between Earth & Jupiter
Net Mass Ratio vs. Specific Impulse

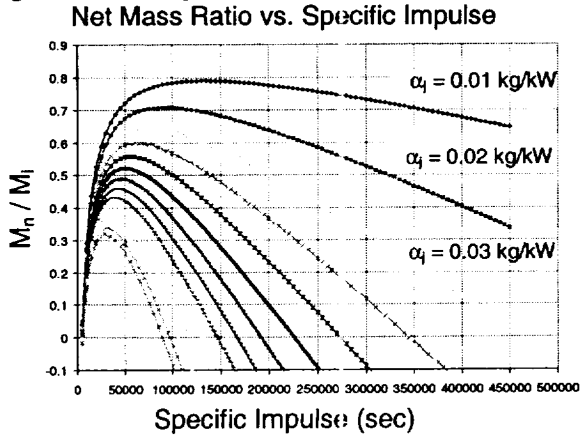


Figure 4. 200-day Transit Between Earth & Jupiter
Net Mass Ratio vs. Specific Impulse

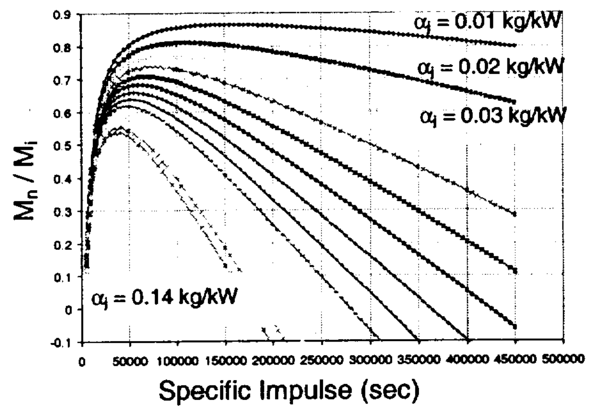


Figure 5. Optimal Power/Initial Mass Ratio vs.
Specific Impulse For All Flight Times

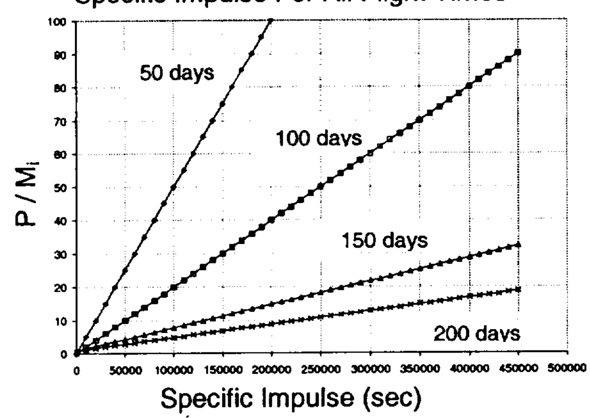
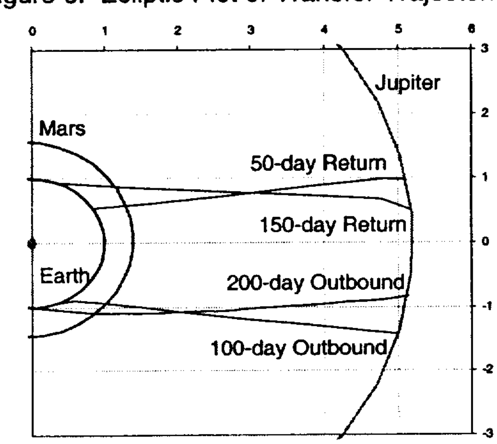


Figure 6. Ecliptic Plot of Transfer Trajectories



Future Work

The analytical approximation, method I, will be modified in the future to account for coast phases and round trip missions. Additionally the straight-line approximation will be addressed. Finally, results will be compared to method II.

In the future, method II will be completed to include a characterization of the outbound and return trajectories for all of the outer planets, as well as some analysis of interstellar precursor missions. Interstellar missions will be simulated with Pluto flybys in VARITOP.

Conclusions

This paper provides readers with a means of approximating vehicle performance for missions to Jupiter. This method has high degree of accuracy without complex computer codes.

Acknowledgements

The authors would like to thank Carl Sauer of NASA's Jet Propulsion Laboratory for assistance in using the VARITOP software.

References

1. Williams, C. H., "An Analytic Approximation to Very High Specific Impulse and Specific Power Interplanetary Space Mission Analysis", AAS 96-151, 12-15 Feb 1996

2. Shepherd, *Aerospace Propulsion*, American Elsevier Publications, NY, 1972

3. C.G. Sauer, "A User's Guide to VARITOP – A General Purpose Low Thrust Trajectory Optimization Program", Jet Propulsion Laboratory.

4. "Electric Propulsion Mission Analysis Terminology and Nomenclature", Prepared by: NASA Office of Advanced Research and Technology, Nuclear Electric Propulsion Systems Analysis Task Group, 1969, National Aeronautics and Space Administration, Washington, D.C.

5. Kammash, T., Flippo, K., Umstadter, D., "Laser Accelerated Plasma Propulsion System (LAPPS)", AIAA 2001-3810, 8-11 July 2001

6. Cole, D. M., "Minimum Time Interplanetary Orbits", *Journal of Astronautical Sciences*, 1959, pp 31-38

7. Moeckel, W. E., "Comparison of Advanced Propulsion Concepts for Deep Space Exploration", AIAA Journal of Spacecraft, Vol. 9, No. 12, December 1972

8. See Geoff about getting the reference he pulled MTF numbers from: Magnetized Target Fusion (Thio et al, AIAA-1999-2703)

9. P. Hill, C. Peterson, Mechanics and Thermodynamics of Propulsion, second edition, 1992, Addison-Wesley Publishing Company, Inc., Pages 504-508

10. Irving, J. H., Blum, E. K., "Comparative Performance of Ballistic and Low-Thrust Vehicles for Flight to Mars", *Vistas in Aeronautics II*, Pergamon Press, New York, 1959

11. Moeckel, W. E., "Propulsion Systems for Manned Exploration of the Solar System", Astronautics and Aeronautics, Vol 7, No 8, Aug 1969, pp 66-67

12. Williams, C et al., "Realizing 2001: A Space Odyssey: Piloted Spherical Torus Nuclear Fusion Propulsion", AIAA 2001-3805, 8-11 July 2001.

13. Kammsh, T. et al., "High-Performance Fusion Rocket for Manned Space Missions", Fusion Energy in Space Propulsion, American Institute of Aeronautics and Astronautics, Washington DC, 1995.

14. Chakrabarti et al., Impact of Energy Gain and Subsystem Characteristics on Fusion Propulsion Performance Balances, AIAA 2000-36790, 16-19 July 2000.