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Nomenclature

b_o	initial vortex separation
g	acceleration due to gravity
N	Brunt-Vaisala frequency, $[(g/\theta)\,\partial\theta/\partial z]^{0.5}$
N^*	nondimensional N, $2\pi N (b_o)^2/\Gamma_o$
r	radius from vortex center
R	nondimensional radius, r/b_o
t	time coordinate
T	nondimensional time, $t\,V_o/\,b_o$
T_L	nondimensional time to vortex linking
V^*	vortex descent velocity normalized by V_o
V_o	initial vortex descent velocity, $\Gamma_o/\,(2\,\pi\,bo)$
z	vertical coordinate
Z	nondimensional vertical coordinate, $z/\,b_o$
Γ^*	vortex circulation at $r=b_o$ normalized by Γ_o
$\overline{\Gamma}$	normalized average circulation
Γ_o	initial vortex circulation
ε	turbulence (eddy) dissipation rate
ε^*	nondimensional ε, $(\varepsilon\, b_o)^{1/3}\, V_o^{-1}$
θ	potential temperature

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Abstract

This paper proposes two simple models to predict vortex transport and decay. The models are determined empirically from results of three-dimensional large eddy simulations, and are applicable to wake vortices out of ground effect and not subjected to environmental winds. The results from the large eddy simulations assume a range of ambient turbulence and stratification levels. The models and the results from the large eddy simulations support the hypothesis that the decay of the vortex hazard is decoupled from its change in descent rate.

I. Introduction

The three-dimensional, large-eddy simulation (LES) model called the Terminal Area Simulation System (TASS)[1] has been used over the past six years to better understand aircraft wake vortex behavior.[2] This effort was a part of the Aircraft Vortex Spacing System (AVOSS) research conducted at NASA's Langley Research Center.[3] Results from TASS model simulations have guided the development of prediction algorithms that are part of AVOSS.[4,5] In this paper, wake vortex sensitivity to thermal stratification and ambient turbulence intensity is examined from TASS results, and two simple models are developed for independent prediction of wake vortex descent and decay.

The work presented previously in Switzer and Proctor[6] showed that stable stratification caused vortex circulation to decay much more rapidly at large radii than near the core ($r \leq 15\ m$). Hinton and Tatnall evaluated an averaged circulation in the near core region (10 to 15 *meters* radius) as an acceptable vortex strength hazard metric.[7] The circulation at radii on the order of the vortex separation controls the descent rate. Therefore, stable stratification reduces the descent rate more quickly than the strength of the hazard. This effect may add to the difficulty of one-equation prediction models, such as proposed by Greene[8] and Sarpkaya[5], for predicting both vortex descent and strength of the hazard. This decoupling of circulation decay near and far from the core motivates solving the prediction problem by treating each in a separate but related manner.

TASS has demonstrated good agreement with observational data,[1,2,6,9,10,11] and can be a useful tool for quantifying wake vortex sensitivity to atmospheric variables. This paper utilizes TASS results in the development of two predictive models. The first is for the prediction of vortex descent, and the second is for the prediction of hazard. The models also present some additional insight into the mechanisms active for each process.

TASS Model Description

All of the TASS results for this report were three-dimensional with a fixed size and grid resolution. Periodic boundary conditions are assumed in all coordinate directions and ground effects are ignored. The mean ambient wind velocity is assumed zero; thus any effects from crosswind shear are ignored. The vortex parameters also remained unchanged for all simulations. The chosen variations of stratification and turbulence intensity encompass a variety of atmospheric conditions. Refer to Switzer and Proctor[6] for details about the vortex parameters, domain size and resolution, boundary conditions, stratification, and ambient turbulence generation.

TASS Environmental Parameters

Table 1 shows the three stratification levels chosen for model guidance. The first level, $N^* = 1.0$, represents a very stable stratification. The last level, $N^* = 0$, corresponds to neutral stratification.

Following Han *et al*[10] and Switzer and Proctor[6], a homogeneous isotropic turbulence field is generated prior to wake vortex injection. Vortex decay from molecular diffusion is negligible since all simulations assume a rotational Reynolds number of (Γ_o/ν) of ~ 10^7, as is the case for atmospheric wake vortices. Numerical simulations were performed with a range of turbulence intensities (Table 2) representing values typically found in the atmospheric boundary layer.

Table 3 shows the matrix of TASS runs that that were utilized in the development of the predictive models. The chosen runs give a wide variation in the turbulence intensity level for neutral stratification while addressing the stratification effects at the moderate and weak turbulence intensity levels.

Table 1. *Ambient stratification levels used for the predictive model development.*

N^*	N (s^{-1})	$\frac{\partial\theta}{\partial z}$ (°C/km)
1.0	4.42×10^{-2}	54.2
0.5	2.21×10^{-2}	13.6
0.0	0.0	0.0

Table 2. *Ambient turbulence intensity levels used for the predictive model development.*

Turbulence Intensity	ε (m^2/s^3)	ε^*
Strong	3.02×10^{-3}	0.30
Moderate	1.35×10^{-3}	0.23
Weak	4.0×10^{-5}	0.07
Very Weak	1.0×10^{-7}	0.01

Table 3. *Matrix of stratification and turbulence intensity levels used for the predictive model development.*

Turbulence Intensity	Strongly Stable (N*=1.0)	Moderately Stable (N*=0.5)	Neutral (N*=0.0)
$\varepsilon^* = 0.30$			X
$\varepsilon^* = 0.23$	X	X	X
$\varepsilon^* = 0.07$	X	X	X
$\varepsilon^* = 0.01$			X

TASS runtime parameters

For developing the simple predictive models the vortex circulation and height are diagnosed from each simulation as described in Switzer and Proctor[6]. Circulation averaged between radii of 10 and 15 meters represents strength of the vortex hazard[7].

II. Predictive Model Development

Based on data from the TASS simulations described above, two empirical models are proposed.

The first model predicts vortex transport, while the second predicts vortex hazard. They are developed as follows.

Vortex Transport Model

The circulation at radii of the order of the vortex separation controls the descent rate. The rate of change in the normalized circulation at $r = b_o$ can be modeled by:

$$\frac{d\Gamma^*}{dT} = \frac{d\Gamma_L^*}{dT} + \frac{d\Gamma_D^*}{dT} + \frac{d\Gamma_S^*}{dT}$$

The terms on the righthand side are the contributions from linking (and other *3-D* instabilities), turbulence diffusion, and stratification, respectively.

Based on guidance from TASS results, the change in circulation from linking (long-wave instability) is modeled by a hyperbolic tangent function of the following form:

$$\Gamma_L^* = \frac{1}{2}\left[1 - \tanh\left(\beta(T - T_L - \alpha)\right)\right]$$

The constants β and α assume the values of 0.5 and 1.3, respectively, and T_L is Sarpkaya's[5] vortex lifetime:

$$\begin{aligned} T_L &= 0.8039\,\varepsilon^{*3/4} \quad for \quad \varepsilon^* > 0.2535 \\ T_L^{1/4} e^{(-0.7T_L)} &= \varepsilon^* \quad for \quad 0.0121 < \varepsilon^* < 0.2535 \\ T_L &= -180\,\varepsilon^* + 9.18 \quad for \quad 0.001 < \varepsilon^* < 0.0121 \\ T_L &= -9 \quad for \quad \varepsilon^* < 0.001 \end{aligned}$$

The constant β controls the slope of the decay curve, while $T_L - \alpha$ defines a nondimensional time near the beginning of rapid decay. The rate of change of Γ_L^* is:

$$\frac{d\Gamma_L^*}{dT} = -\frac{\beta}{2}\operatorname{sec}h^2\left(\beta(T - T_L - \alpha)\right)$$

Turbulent diffusion assumes the model from Han et al:[10]

$$\Gamma_D^* = \exp\left\{-(c_1\varepsilon^*/R^2)T\right\}$$

with the nondimensional radius, R, equal to 1.0, and the constant c_1 is assumed as 0.08. The associated rate of decay for this term is:

$$\frac{d\Gamma_D^*}{dT} = -c_1\,\varepsilon^*\,\Gamma_D^*$$

The last contribution to this model comes from the change due to stratification and is identical to the term used by Greene[8]:

$$\frac{d\Gamma_S^*}{dT} = -A\left(N^*\right)^2 Sign(N^*)$$

with the constant, A, set equal to 0.2.

Substituting all three rate of decay terms into the total rate of change in circulation:

$$\frac{d\Gamma^*}{dT} = -\frac{\beta}{2}\operatorname{sec}h^2\left(\beta(T - T_L - \alpha)\right) - c_1\varepsilon^*\Gamma^* - A\left(N^*\right)^2 sign\left(N^*\right)$$

This equation predicts the normalized circulation at $r = b_o$. If we assume that the vortex separation remains constant, then the normalized descent rate, V^*, is equivalent to Γ^*, and

$$\frac{dV^*}{dT} = -\frac{\beta}{2}\operatorname{sec}h^2\left(\beta(T - T_L - \alpha)\right) - c_1\varepsilon^*\Gamma^* - A\left(N^*\right)^2 sign\left(N^*\right)$$

In the absence of environmental winds or the ground, this model predicts the vortex descent rate. The vortex system initially descends at the rate of V_o and starts to slow due to the affects of vortex linking instability, turbulence diffusion, and stratification.

Vortex Hazard Model

A similar approach is assumed for the hazard model. The average circulation between r = 10 to 15 m normalized by its initial value represents the strength of the vortex hazard. The rate of change of normalized average circulation is proposed as:

$$\frac{d\bar{\Gamma}}{dT} = \frac{d\bar{\Gamma}_{SS}}{dT} + \frac{d\bar{\Gamma}_D}{dT} + \frac{d\bar{\Gamma}_S}{dT}$$

The last two terms are modeled similar to those in the last section, but with different constants since the circulation near the vortex core decays at a different rate from that at larger radii. The first term on the right hand side represents the change in average circulation from small- and large-wave instabilities. The onset time for rapid circulation decay near the core is a function of both turbulence and stratification effects. The equation for this onset in nondimensional time units is:[12]

$T_{SS} = -(1.27\ln(\varepsilon^*) + 0.57)\exp(-1.15N^*)$, for $\varepsilon^* < 0.3$.

Unlike Sarpkaya's relationship for T_L (which is used in the transport model), the above relationship depends upon stratification as well as turbulence intensity. The dependency upon stratification is necessary since small-wavelength instabilities (and rapid decay) are induced earlier in more stable environments.[12]

A model for $\bar{\Gamma}_{SS}$ similar to that proposed for Γ_L^* is considered for representing the change in average circulation due to small- and large-wavelength instabilities:

$$\bar{\Gamma}_{SS} = \frac{1}{2}\left[1 - \tanh\left((\beta_1 + \beta_2 N^{*2})(T - T_{SS} - \alpha)\right)\right]$$

Based on TASS results, the constants β_1, β_2 and α are set to 0.75, 0.25 and 2.7, respectively. The differences between this term and the corresponding term from the transport model (Γ_L^*) are the dependence on stratification and the dependence on T_{SS} rather than T_L. These changes are based on TASS simulations, which show small-wavelength instabilities are more influential in reducing the circulation near the core than at large radii. The rate of change of the above equation gives:

$$\frac{d\bar{\Gamma}_{SS}}{dT} = -\frac{\beta_1 + \beta_2 N^{*2}}{2} \bullet \operatorname{sech}^2\left[(\beta_1 + \beta_2 N^{*2})(T - T_{SS} - \alpha)\right]$$

However, since the TASS data shows that the circulation decreases somewhat slower once the circulation has decayed to half its original value, the above term is modified by a time function. This modification gives:

$$\frac{d\bar{\Gamma}_{SS}}{dT} = -F(T)\frac{\beta_1 + \beta_2 N^{*2}}{2} \bullet \operatorname{sech}^2\left[(\beta_1 + \beta_2 N^{*2})(T - T_{SS} - \alpha)\right]$$

where the time function is specified as:

$$F(T) = \begin{cases} 1 & T \le T_{1/2} \\ 1 - \dfrac{T - T_{1/2}}{3} & T_{1/2} < T < T_{1/2} + 3 \\ 0 & T \ge T_{1/2} + 3 \end{cases}$$

In this formulation, the variable $T_{1/2}$ represents the nondimensional time that the circulation decays to half its original value; i.e., $\bar{\Gamma}(T_{1/2}) = 0.5$.

For rate of change due to turbulence diffusion, the constant c_1 is still 0.08, but R assumes the value of 0.5. The remaining term for the stratification assumes a much smaller value of 0.05 for constant A. The significance of this term is reduced from that in the transport model, since stratification (via baroclinic generation of opposite sign vorticity) is most effective in reducing the circulation at larger radii.

Combining the affects from the three respective terms gives:

$$\frac{d\bar{\Gamma}}{dT} = -F(T)\frac{\beta_1 + \beta_2 N^{*2}}{2} \bullet \operatorname{sech}^2\left[(\beta_1 + \beta_2 N^{*2})(T - T_{SS} - \alpha)\right] - c_1 \varepsilon^* \bar{\Gamma} - A\left(N^*\right)^2 \operatorname{sign}\left(N^*\right)$$

The above equation can then be integrated to arrive at a prediction for the normalized average circulation, which represents the vortex hazard.

III. Predictive Model Comparisons

Figures 1 through 3 show the comparisons of the vortex transport model with TASS results. Figure 1 shows the effect of turbulence intensity on vortex descent. The magnitudes of sink rate and maximum depth are reduced as the turbulence is increased. The comparison across the range of turbulence intensity shows very good agreement between TASS and model data. The next two figures evaluate the model's ability to handle the affects of stratification at two different turbulent intensity levels. All of the model results show a longer time to reach the maximum descent height, but the curves all closely match the TASS data.

Figures 4 through 6 show the comparisons of the vortex hazard model with TASS results. Again the model fits nicely to the TASS data. The weakest agreement occurs for very strong stratification (Figs. 5 and 6). In this case the model predicts the onset of rapid decay too slowly. However, for typical ranges of stratification, i.e. $N^* = 0.5$, the model does very well at matching the TASS decay rate. In all cases, the hazard model either matches the TASS results or predicts the hazard to last too long thereby giving a conservative estimate.

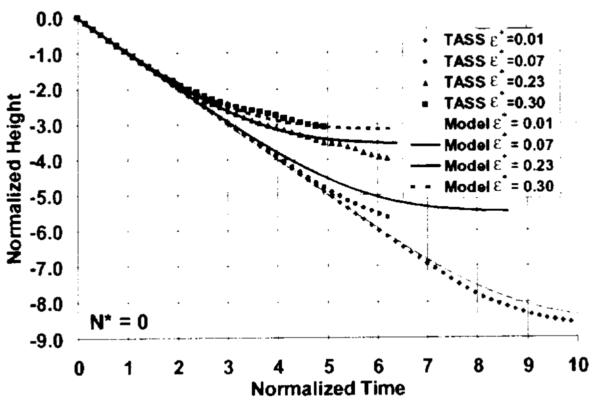

Figure 1. *Comparison of transport predictive model to TASS results for neutral stratification. The symbols and the lines with symbols refer to TASS and the predictive model, respectively.*

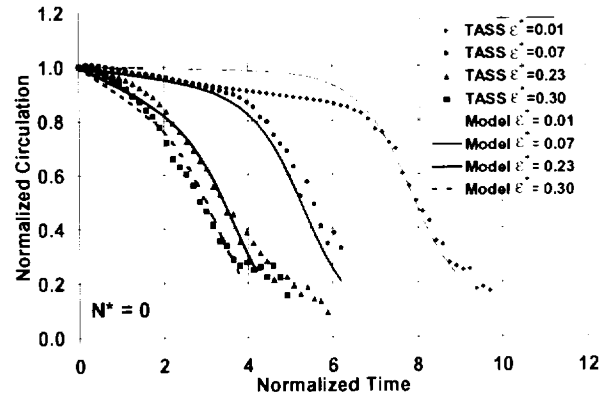

Figure 4. *Comparison of hazard predictive model to TASS results for neutral stratification. The symbols and the lines with symbols refer to TASS and the predictive model, respectively.*

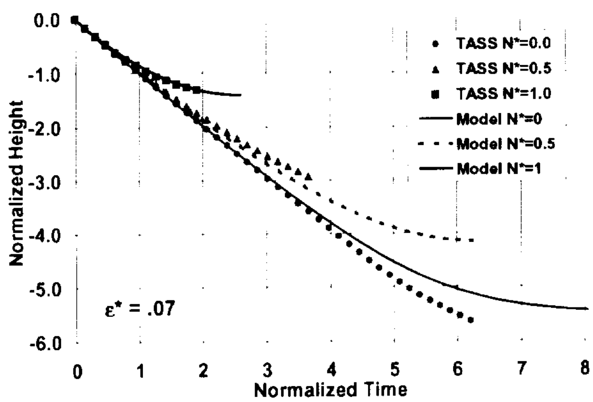

Figure 2. *Same as figure 1, but for the weak turbulence intensity level.*

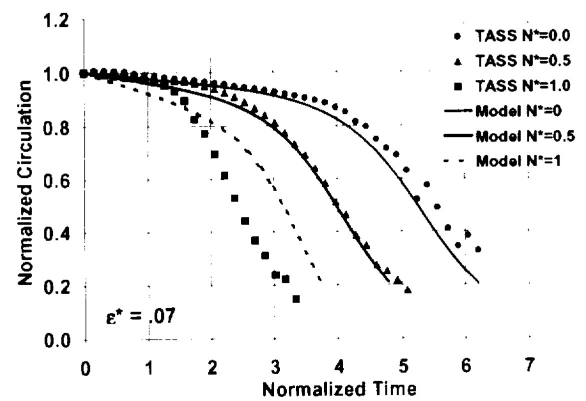

Figure 5. *Same as figure 4, but for the weak turbulence intensity level.*

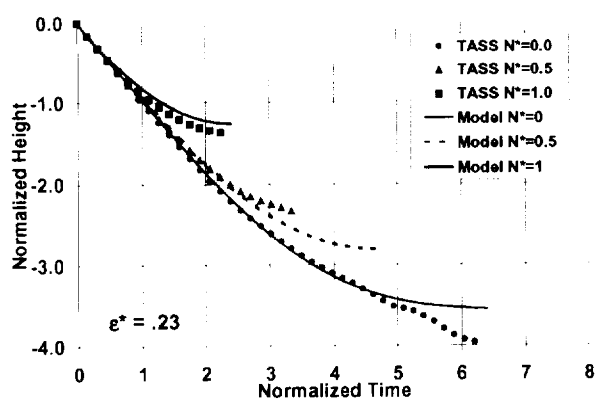

Figure 3. *Same as figure 1, but for the moderate turbulence intensity level.*

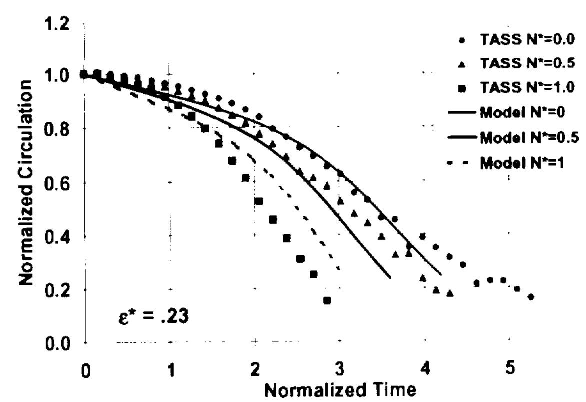

Figure 6. *Same as figure 4, but for the moderate turbulence intensity level.*

IV. Summary and Conclusions

This paper has presented a composite model to predict vortex transport and decay in conditions free from crosswinds and ground effects. There are two parts of this model arising from the dominance of different mechanisms at different distances from the vortex core. The models show very good agreement to TASS cases.

V. Future Work

The proposed models need to be evaluated further before application in an operational system. In future work, the models need to be compared and refined with LIDAR measurements of wake vortices, as previously done with Sarpkaya's[4] model. Lastly, effects of windshear and ground effect need to be considered.

Acknowledgements

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