Statistical distribution of inflation features on lava flows: Analysis of flow surfaces on Earth and Mars

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Abstract

Introduction

The surface morphology of a lava flow results from processes that take place during the emplacement of the flow. Certain types of features, such as tumuli, lava rises and lava rise pits, are indicators of flow inflation or endogenous growth of a lava flow (e.g., Walker, 1991). Tumuli in particular have been identified as possible indicators of tube location (e.g., Guest et al., 1984; Calvari and Pinkerton, 1998; Duraiswami *et al.* 2001; Duncan et al., 2002), indicating that their distribution on the surface of a lava flow is a function of the internal pathways of lava present during flow emplacement. However, the distribution of tumuli on lava flows has not been examined in a statistically thorough manner.

In order to more rigorously examine the distribution of tumuli on a lava flow, we examined a discrete flow lobe with numerous lava rises and tumuli on the 1969-1974 Mauna Ulu flow at Kilauea, Hawaii (Figure 1). The lobe is located in the distal portion of the flow below Holei Pali, which is characterized by hummocky pahoehoe flows emplaced from tubes (e.g., Swanson, 1973). We chose this flow due to its discrete nature allowing complete mapping of surface morphologies, well-defined boundaries, well-constrained emplacement parameters, and known flow thicknesses (e.g., refs). In addition, tube locations for this Mauna Ulu flow were mapped by Holcomb (1976) during flow emplacement. We also examine the distribution of tumuli on the distal portion of the hummocky Thrainsskjoldur flow field provided by Rossi and Gudmundsson (1996).

Analysis of the Mauna Ulu and Thrainsskjoldur flow lobes and the availability of high-resolution MOC images motivated us to look for possible tumuli-dominated flow lobes on the surface of Mars. We identified a MOC image of a lava flow south of Elysium Mons with features morphologically similar to tumuli (Figure 2). The flow is characterized by raised elliptical to circular mounds, some with axial cracks, that are similar in size to the tumuli measured on Earth (e.g., Walker, 1991; Duncan et al., 2002). One potential avenue of determining whether they are tumuli is to look at the spatial distribution to see if any patterns similar to those of tumuli-dominated terrestrial flows can be identified.

Since tumuli form by the injection of lava beneath a crust, the distribution of tumuli on a flow should represent the distribution of thermally preferred pathways beneath the surface of the crust. That distribution of thermally preferred pathways may be a function of the evolution of a basaltic lava flow. As a longer-lived flow evolves, initially broad thermally preferred pathways would evolve to narrower, more well-defined tube-like pathways (Self et al., 1998; Anderson et al., 1999). The final flow morphology clearly preserves the growth of the flow over time, with inflation features indicating pathways that were not necessarily contemporaneously active. Here, we test using statistical analysis whether this final flow morphology produces distinct distributions that can be used to readily determine the distribution of thermally preferred pathways beneath the surface of the crust.

Background

Walker (1991) examined tumuli on Hawaiian pahoehoe flows, and defined them as features whose total cross-profile width is equal to or less than the total width of the structure. He found that tumuli tend to form on shallow slopes, in lava undergoing a modest amount of extension. Walker (1991) defines three types of tumuli: shallow-slope tumuli, moderate-slope tumuli and flow-lobe tumuli. Shallow-slope tumuli were described as often large an forming in clusters or trains. Walker 91991) did not find any association of tumulus trains and tubes, but suggested that they may form over lesser, more transient tubes.

Self et al. (1998) have defined two types of pahoehoe flows: sheet flows and hummocky flows. Sheet flows are composed of sheet-like lobes that form from relatively continuous, rapid emplacement over shallowly sloping, smooth surfaces (Self et al., 1998). Hummocky flows have many discrete tumuli, and form on rougher surfaces on steeper slopes, with relatively slow, discontinuous emplacement (Self et al., 1998). Self et al. (1998) suggests that tumuli form over depressions, indicating that the distribution of tumuli should reflect the pre-existing topography.

Rossi and Gudmundsson (1996) studied tumuli on the Thrainsskjoldur flow field in Iceland, defining several types of tumuli based on their distance from the vent, and modeling their formation. They calculated magma overpressures needed to lift the surface of the crust to form a tumulus, suggesting that tumuli can be used to study the variations in overpressure within the inflating flow. This has been applied to flows at Hawaii (Anderson et al., 2000) and Mt. Etna, Sicily (Duncan et al., 2002). Duraiswami *et al.* (2001) describe tumuli as being common on both hummocky and thicker sheet lobes in the Deccan volcanic province. They interpret the alignment of tumuli to indicate they have developed along anastomosing tube systems, and are similar in their general morphology to tumuli in Hawaii.

Anderson et al (1999) said that there is a network of thermally preferred pathways under the crust of a flow that could give rise to inflation features such as tumuli or lava rises on hummocky flows. This would imply that tumuli could reflect more transient pathways, as well as longer-lived tube systems. Duncan et al. (2002) related location of tumuli to series of pathways on the 1983 flow at Mt. Etna, and found that larger, more complex tumuli (focal tumuli) lay directly over major feeder tubes, while satellite and distributary tumuli lie over more transient, lesser pathways.

Byrnes and Crown (2001) tried to relate the surface morphology of flows at the Mauna Ulu flow field to the tube system mapped by Holcomb (1976). They concluded that the units they mapped, which included a rougher, inflated unit, did not correlate to major tubes, and therefore may be related to a smaller-scale distributary network.

Method

In order to statistically analyze the distribution of inflated features on the flow lobe at Mauna Ulu, we documented the height, planform shape, location, and major fractures using a Trimble ProXR real-time differential GPS, logging positions every 1 second. We mapped the perimeters of every inflated feature with well-defined margins that had 1 or more meters of relief. In addition, we measured the highest point on the tumulus and the lowest point along the perimeter by acquiring at least 15 seconds of data (positions logged every second) at each point. The acquisition of real-time data from the nearest working and available base station (Kopoho, Hawaii, XXX km from field site transmitting at xxxx Hz) was inconsistent owing to distance and topography between the base station and field site. Therefore, the GPS field data were differentially corrected with Kopoho base station data (Long?/Lat?) using the Trimble Pathfinder Office 2.8 post-processing utility and downloaded Kopoho base station data. After post-processing, the average horizontal precisions for each tumulus/lava rise ranged from 0.29-0.44 m, and average vertical precisions ranged from 0.44-1.07 m.

Results

Our objective is to assess whether or not there is any systematic behavior involved in the formation of inflation features (including tumuli and lava rises) indicated by their spatial distribution on the flow surface. For example, are the tumuli clustered along a tube, near the margins, or near the break in slope? We first assess the distribution of only tumuli on the Mauna Ulu flow, then consider the combined population of tumuli and lava rises on the Mauna Ulu flow and the Thrainsskjoldur flow in Iceland, and on suspected inflated flow southwest of Orcus Patera on Mars.

Mauna Ulu

Figure 3 shows the locations of 76 tumuli and 12 lava rises measured near Mauna Ulu. The margins for the flow lobe that was investigated are also indicated. The flow unit containing these tumuli was chosen because it is relatively simple compared to other units in the area. The flow overlies much older material not associated with the Mauna Ulu eruption. The easternmost margin of the field area is the lateral margin of the flow unit. The western margin is distinct and indicates where a later flow unit has covered the western extent of the tumuli-dominated flow lobe.

Based on visual inspection in the field, it appeared that tumuli within this unit may have tended to cluster near the lateral margin of the flow unit, and we suspected that statistical analysis would show a non-random distribution. In order to test for randomness of the spatial distribution of the tumuli, we have compared the tumulus locations to the Poisson distribution. If the spatial distribution of inflation features within the study area is significantly different from the Poisson, we can conclude that there is some systematic behavior controlling their occurrence. Alternatively, if the spatial distribution is indistinguishable from the Poisson, we must conclude that the inflation features occur randomly.

The Poisson distribution is the limiting form of the binomial used to describe random events in time or space that are relatively rare (Snedecor and Cochran, 1967). Requirements for using the Poisson distribution are that (1) the probability of at least one occurrence of the event in a given spatial interval is proportional to the area of the interval, (2) the probability of two or more occurrences of the event in a very small area is negligible, and (3) the occurrence of an event within one spatial interval has no effect on the occurrence or nonoccurrence in another nonoverlapping interval of the same size (Larson, 1974).

To compare the spatial distribution of tumuli to the Poisson, it is necessary to impose an arbitrary grid containing cells of equal area. Because the flow unit is an irregular shape, it is not immediately obvious how this should be done. The flow area can be grossly described by a large trapezoid (shown in Figure 1) that includes 74 of the 76 measured tumuli. This trapezoid includes a minimal amount of area that is not part of our flow unit, and all inflation features meeting the criteria described above have been identified.

Figure 2 illustrates an example of an equal area trapezoidal grid. To create the equal area grid, we allowed the dimensions of the cells to vary by row. We attempted to find a combination of parameters that resulted in row heights that are as close to the same as possible. The grid shown in Figure 2 contains 77 cells, each with an area of 1,383 m^2. In comparing the spatial distribution of tumuli to the Poisson, we assume that each tumulus is a single point. In fact, tumuli have a range of areal sizes. However, we are careful to choose our grid sizes such that they are sufficiently large to contain multiple tumuli, while still maintaining the rare nature of occurrence ($<=$ 1 tumulus/grid on average). We note here that the geometric mean value (appropriate for distributions with long tails) of the tumulus areas is ~ 107 m^2, significantly smaller than the grid area used in the analysis.

For our comparison, we did not use the three cells at the right hand end of the bottom row, as this area is almost entirely beyond the boundary of the flow unit. Thus, in this example, we have 74 tumuli divided among 74 equal area cells. As can be seen in Figure 2, some of these cells contain no tumuli, some contain 1 tumulus, some 2 and some more.

The Poisson probability distribution for k discrete events (tumuli) occurring within some spatial area a is given by

$$p(k) = \frac{(\lambda a)^k e^{-\lambda a}}{k!} \tag{1}$$

For the grid established in Figure 2, λ = 1.0 tumuli/grid cell and a = 1 grid cell. The probabilities for finding 0, 1, 2 or >= 3 tumuli in any grid are shown in the second column of Table 1. Note that the sum of all the probabilities is equal to 1 indicating that all possible choices are represented. The third column of Table 1 shows how many grid cells we would expect to find with k tumuli, if they are indeed spatially random as described by the Poisson distribution. The fourth column shows how many cells in Figure 2 actually contain k tumuli.

Table 1.

		λ = 1.0 tumuli/grid cell	
k	p(k)	p(k) * 74	Actual Number of Cells
0	0.3679	27	26
1	0.3679	27	31
2	0.1839	14	11
>= 3	0.0803	6	6

While the actual distribution of tumuli appears similar to that predicted by the Poisson, there are some differences. We can quantitatively evaluate how well the Poisson describes the distribution of tumuli by performing a χ^2 hypothesis test. The χ^2 test uses the differences between the predicted and actual occurrences to estimate a test statistic, U. The hypothesis to be tested is that the spatial distribution of tumuli is random. If U is less than a critical value, we must accept this hypothesis. The critical value is found from the χ^2 distribution. For 2 degrees of freedom (appropriate for the 4 bins identified in Table 1) and a significance level of 5% (5% probability of rejecting the null hypothesis, even if it is true), the critical value is 5.99. For the case given in Table 1, $U = 1.08$. We cannot, therefore, preclude the possibility that the tumuli occur randomly in space within the flow unit.

It is not clear, however, that the grid cell size resulting in $\lambda = 1$ is the most appropriate choice. It may be that different choices show more clustering of tumuli (near the margins for example). Thus we have investigated both smaller and larger grid cells to test the sensitivity of our conclusion. Tables 2 and 3 show the results of these comparisons. Note that while the test statistics, U, are greater than for the $\lambda = 1$ case, they are still significantly less than the critical value (= 5.99). Thus, we still cannot preclude a random spatial distribution.

Table 2.

$\lambda = 2.06$ tumuli/grid cell; $U = 2.05$			
k	p(k)	p(k) * 36	Actual Number of Cells
0	0.128	5	5
1	0.2632	9	13
2	0.2705	10	8
>= 3	0.3384	12	10

Table 3.

$\lambda = 0.52$ tumuli/grid cell; $U = 1.84$			
k	p(k)	p(k) * 143 cells	Actual Number of Cells
0	0.596	85	88
1	0.3084	44	40
2	0.0798	11	11
>= 3	0.0157	2	4

For completeness, we have also conducted the analyses described for all inflation features within the investigation area, including the 12 lava rises. Table 4 shows the results of this analysis when we use the 74 equal area grids (same grid as shown in Figure 2). In this case, we have 86 features and □ = 1.16 features/grid. The result is a test statistic $U = 0.364$. Not only is this U significantly less than the critical value, it is also less than that calculated based only on tumuli. This indicates that the Poisson is an even better fit when all inflation features are included.

Table 4.

	$\lambda = 1.16$ features/grid cell; $U = 0.364$		
k	p(k)	p(k) * 74	Actual Number of Cells
0	0.3128	23	25
1	0.3635	27	25
2	0.2112	16	15
>= 3	0.1124	8	9

Based on the analyses above, we must conclude that the tumuli are randomly distributed in space. However, while walking around on the flow field at Mauna Ulu, it is clear that tumuli vary dramatically in size. Figure 3 shows a histogram of tumulus heights. The distribution is clearly asymmetric, but there are substantial differences between the data and the lognormal distribution plotted for comparison. However, the χ^2 test for goodness of fit indicates that the lognormal cannot be precluded (U = 7.38 is less than the critical value of 11.1 for 5 degrees of freedom and a 5% significance level). Although there are few more large tumuli than might be expected for a lognormal distribution, there are not enough to indicate a bi-modal population (as we might have suspected when we were walking around in the field).

Our next step is to see if, perhaps, there is some clustering among the tumuli of different size ranges. Figure 4 shows all the tumuli where 8 different symbols have been assigned to those tumuli that fall into the 8 bins in Figure 3. We then used the grid containing 37 cells to perform the comparison with the Poisson.

Table 5.

	Bin 1 $\lambda = 0.108$		Bin 2 $\lambda = 0.568$		Bin 3 $\lambda = 0.649$		Bin 4 $\lambda = 0.351$		Bin 5 $\lambda = 0.162$		Bins 6, 7, & 8 $\lambda = 0.162$	
k	Pred	Act	Pred	Act	Pred	Act	Pred	Act	Pred*	Act	Pred*	Act
0	33	33	21	21	19	19	26	26	31	32	31	31
1	4	4	12	12	13	13	9	9	5	4	5	6

2	0	0	3	3	4	4	2	2	0	1	0	0
>= 3	0	0	1	1	1	1	0	0	0	0	0	0

* note that the predicted numbers of grids does not sum to 37 due to rounding to the nearest whole number for each k.

The results in Table 5 are almost scary! Each individual size range is distributed almost identically to that expected from the Poisson distribution.

From looking at Figure 1, one can almost convince oneself that there might be some clustering near the margin of the flow unit, and that perhaps the lack of tumuli in the flow interior is somehow compensating for this in the Poisson comparisons of Tables 1 - 3. To investigate this, we have divided the trapezoidal grid in Figure 2 (74 cells) into two regions: margin (last two cells of each row), and interior (all other cells). When divided in this way the "margin" contains 29 tumuli in 16 equal area cells (λ = 1.81 tumuli/grid cell), and the "interior" contains 45 tumuli in 58 equal area grids (λ = 0.776 tumuli/grid cell). Table 6 shows the results of the comparison to the Poisson. Again, the Poisson still cannot be precluded.

Table 6.

k	Margin, $U = 0.532$		Interior, $U = 1.41$	
	Predicted	Actual	Predicted	Actual
0	3	2	27	24
1	5	6	21	25
2	4	4	8	7
>= 3	4	4	2	2

Another way to look at this issue of spatial variability is to use the Nearest Neighbors technique (Clark and Evans, 1954). This technique eliminates the arbitrary nature of the grid sizes and simply uses the distance to the nearest neighbor for each data point. The average of the measured nearest neighbor distances is then compared to the expected average nearest neighbor distance for spatially random points with the same density (# of features/unit area).

For the nearest neighbor approach we use the 74 tumuli contained in the large trapezoid in Figure 1 (with total area = 106,491.6 m^2). Thus, the density of tumuli is ρ =

6.9×10^{-4} tumuli/m^2. The mean value of the nearest neighbor distances is 1.04 times that expected for a truly random distribution with the same density. This difference is NOT significant at the 5% level (c = 0.712 < 1.96 - critical value). Thus, we must again conclude that the tumuli are randomly distributed in space.

Iceland flow

Mars flow

Figure 2 is a MOC image (MOC image 20-01192, centered at 1.86°N, 186.11°W) of a portion of a lava flow to the southwest of Orcus Patera. Based on Viking data, the flow extends for over 60 km; we have mapped a 3 x 4.5 km section of the flow. The flow itself is relatively dark with an irregular surface, little apparent mantling, and few impact craters. It appears to superpose a bright unit to the southeast. The surface of the flow has many positive relief features that are at the same scale (<10 – 50 m) as terrestrial inflation features (tumuli, lava rises). Every positive relief feature of at least 3 pixels was mapped on the flow surface; relief was determined by shading. Many of the larger positive relief features have central depressions or clefts. The outlines of each feature were used to find its center, and Figure 5 shows the center locations, with an arbitrary grid superimposed. There are 801 features and 735 grid cells inside the thick black line, thus, □ ~ 1.09. The 2nd and 3rd columns of Table 7 indicate the predicted and actual number of cells containing k features. The test statistic for this scenario is 22.01, and is significantly greater than the critical value of 5.99. Thus, we must conclude that the features within the image are NOT randomly distributed in space.

Table 7.

k	Entire Area (□ ~ 1.09), U = 22.01		Interior (□ ~ 1.42), U = 1.142	
	Predicted	Actual	Predicted	Actual
0	247	287	129	120
1	269	220	183	192
2	147	136	130	131
>= 3	72	92	91	89

However, we note that the MOC image is an arbitrary slice through the lava flow that happens to capture one margin. The 4th and 5th columns in Table 5 illustrate the results when we look only at the features within the interior of the flow (i.e., from the top thick black line down to the red line). In this case there are 754 features within 532 grid cells (□ = 1.42), and $U = 1.14$. Thus, we must conclude that the features within the interior of the flow are distributed randomly and consistent with the Poisson.

Discussion

Lori need some stats discussion here.

we need to summarize how we tried to make them be non random, as our impression in the field had been that they were not. We need to say how rigorous these results are

In the Mars case, if we included the margin of the flow that had few positive relief features, the distribution was not random. However, the distribution of positive relief features was random in the interior of the flow, consistent with our terrestrial results. Either the margin of the flow lobe did not inflate or inflated in a more-sheet-like manner. Are these positive relief features on the Mars flow tumuli and/or lava rises? Their distribution, morphology and size are consistent with them being inflation features, however, we cannot rule out that they have been produced by differential erosion of the flow surface. We do not favor this interpretation because the morphology of this flow is different that another, more clearly eroded flow in the region, there are few craters on the flow surface and they are not highly eroded, and the flow has a well-defined boundary.

More discussion of randomness?

Conclusions

1. Tumuli provide information on the growth of a flow field:
   - the flow has inflated
   - the flow has been active long enough for secondary processes like inflation to modify the flow surface

- likely eruption rates????

Tumuli must form above pathways capable of producing overpressures sufficient for inflation. On some lava flows, linearly aligned tumuli clearly tap well-defined tubes (Duncan et al., 2003). However, the random distribution of tumuli and lava rises on the flow lobes in this study shows that the location of inflated features cannot always be used to infer major tube locations. We suggest this random distribution of tumuli and lava rises on hummocky flow lobes result from formation of tumuli above transient thermally-preferred pathways and tubes that change position over the growth period of the lobe.

Although many major tubes are relatively long-lived features (Jim K papers), the growth of hummocky pahoehoe lobes is linked to a network of anastamosing thermally preferred pathways that migrate beneath a cooled crust (Anderson et al., 1999; 2000). Since inflation is essentially an intrusive process, Anderson et al. (1999, 2000) suggested that these pathways behave as "pods", or "viscous fingers" (Saffman and Taylor, 1958; Feder, 1988), of injecting lava that are analogous to the 3-5 m wide "fingers" of magma found along the margins of laccoliths that penetrate the host rock (Pollard et al., 1975). We suggest that viscous fingering results from fluid instabilities (Saffman and Taylor, 1958; Bruno et al., 1992; 1994) in the inflating flow, and that overpressure in these pathways result in the local inflation of the overlying crust. As these instabilities propagate during the emplacement of the flow, inflation occurs in "pulses" (Anderson et al., 1999; 2000) where discrete sections of the flow lobe in the vicinity of the advancing instability inflate as other portions of the active lobe stagnate. Therefore, tumuli formation is tied to the spatial and temporal migration of these pathways, and the random spatial distribution of tumuli on hummocky lava flows marks the locations of transient pathways, rather than those that have fixed positions (such as long-lived major tubes) with time.

Self et al. (1998) suggested that hummocky flows 'invert' pre-existing topography, with tumuli forming over depressions. If correct, tumuli distribution could be used to 'map' the pre-flow surface. The random distribution of tumuli on the flows studied here may indicate a random distribution of depressions on the pre-flow surface. However, if the pre-flow surface had non-random depressions, such as a channel or a flow margin, then our random distribution of tumuli indicates that the relationship between underlying topography and tumulus formation is not as simple one

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