Size Effects in Impact Damage of Composite Sandwich Panels

Alan Dobyns - Sikorsky Aircraft
Wade Jackson - Army VTD at NASA Langley

ABSTRACT

Panel size has a large effect on the impact response and resultant damage level of honeycomb sandwich panels. It has been observed during impact testing that panels of the same design but different panel sizes will show large differences in damage when impacted with the same impact energy. To study this effect, a test program was conducted with instrumented impact testing of three different sizes of sandwich panels to obtain data on panel response and residual damage. In concert with the test program, a closed form analysis method was developed that incorporates the effects of damage on the impact response. This analysis method will predict both the impact response and the residual damage of a simply-supported sandwich panel impacted at any position on the panel. The damage is incorporated by the use of an experimental load-indentation curve obtained for the face-sheet/honeycomb and indentor combination under study. This curve inherently includes the damage response and can be obtained quasi-statically from a rigidly-backed specimen or a specimen with any support conditions. Good correlation has been obtained between the test data and the analysis results for the maximum force and residual indentation. The predictions can be improved by using a dynamic indentation curve. Analyses have also been done using the MSC/DYTRAN finite element code.

INTRODUCTION

The application of composite materials to helicopter structure often includes use of thin face sheet honeycomb structure (e.g. RAH-66 Comanche). These structures must be designed to sustain barely visible impact damage at ultimate load, which can result in a reduction of 50% or more in the design strength relative to an undamaged structure. The design strength of sandwich structure is usually related to a face-sheet instability known as wrinkling. Since the failure mechanics associated with face sheet wrinkling combined with impact damage are not well understood, the design allowables for sandwich structure are normally obtained empirically using coupon tests (ref. 1, 2, 3).

However, the damage produced and the residual strength in actual sandwich structure due to a given impact energy can be quite different from that produced in the small coupon specimens. This is primarily because the larger panel is more flexible than a small panel of the same basic design. The most important parameter determining the structural response to an impact is the effective spring constant at the impact point. This is influenced by the panel size, panel

boundary conditions, and the impact location on the panel.

This paper discusses impact testing of sandwich panels and an impact analysis for sandwich structures which is able to predict the impact damage resulting from a given impact energy in panels of different size with user specified impact locations. The sandwich impact analysis entails an extension of an existing closed-form plate transient analysis method (ref. 4) to account for the unique force-indentation behavior of honeycomb sandwich. In order to model the softening type nonlinear load-indentation behavior shown by the sandwich, the impact analysis was modified to accept an experimental load-indentation curve instead of the power law Hertz equation normally used for solid laminates. This method implicitly accounts for the face sheet failure and core crushing seen in sandwich impact tests. It can also be used with dynamic load-indentation Curves obtained from panel or coupon impact tests.

To provide data for correlation of the analysis, impact tests were conducted on various pieces removed from a 3' x 5' sandwich panel cut from a scrapped component. Impact tests at several impact energies have been conducted on the original 3' x 5' panel, and the panel was then cut into smaller panels to obtain 3' x 2.5', 1.5' x 2.5' panels. These panels were impacted to give a range of panel sizes and impact energies for the analysis development. Further impact testing is scheduled on 12" x 12", and 6" x 6" panels to complete the panel size study. Static indentation

tests were performed on rigidly-backed specimens to obtain load vs indentation behavior. A dynamic force-indentation curve was also extracted from several of the impact tests and proved to be slightly different from the static curve. Use of the dynamic curve in the analysis gave improved correlation with the test data.

Sandwich low velocity impact tests were performed at the Army Vehicle Technology Directorate (VTD) at NASA Langley, with impact analysis development at Sikorsky. After the impact testing is complete, compression after impact tests of specimens cut from the impacted panels will be performed to provide data on the effect of panel size on compression strength as related to barely visible impact damage.

SANDWICH PANEL IMPACT TESTING

To provide data for correlation with the plate impact analysis, a series of impact tests were conducted on three sizes of sandwich panels. A 7.00-lbm instrumented falling-weight impactor was used to obtain force-time data using the Instron Dynatup 930-I Data Acquisition System. The impactor consisted of a cylindrical mass attached to an Instron Dynatup 5000-lb capacity tup which was guided inside a slotted tube. A 1" diameter hemispherical indentor was used for all the impact tests. Impactor displacement data was obtained by twice integrating the force-time data. On some specimens, a non-contacting fiber optic sensor was placed underneath the impact site to record the back face displacement history using a digital oscilloscope. The indentation response was obtained by subtracting the back face displacement from the impactor displacement. In the static indentation tests, a sandwich coupon placed on the solid test machine platen was loaded while recording the force and indentation.

A 5' x 3' flat sandwich panel was cut from a scrapped test component and impact tested in the locations shown in Figure 1. Impacts were made to determine the impact energy to produce threshold of visibility damage (BVID) and to obtain force-time data. The panel was simply-supported on I-beams to provide a support size of 59" x 35". The impact sites were spaced 6" apart to provide for 6" x 6" compression specimens to be obtained at each impact site. The panel was then cut in half and the remaining 36" x 30" panel was impacted at points 7-10 at several energy levels. The panel was cut in half

again to 30" x 18" size and impacted at points 11 & 12. Additional 12" x 12" and 6" x 6" panels will be cut from the intact scraps and impacted to complete the study.
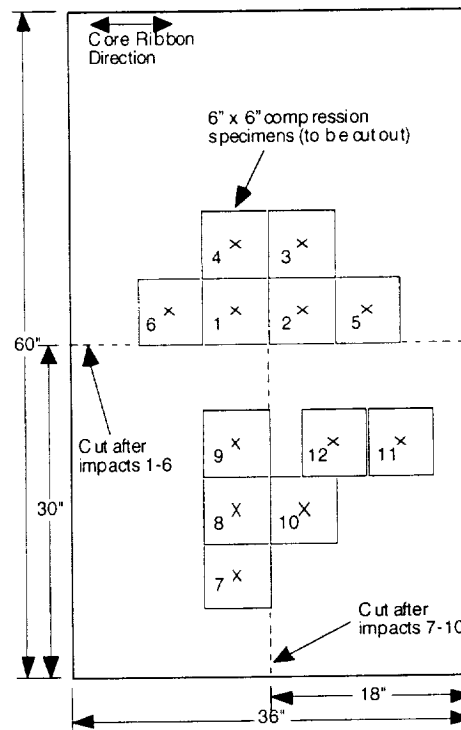

Figure 1. Panel Impact Locations

Table 1 gives the face sheet laminates and core of the honeycomb panel. The face sheets are G30-500/5225 graphite epoxy fabric which is an intermediate strength graphite/epoxy with properties generally similar to the popular AS4/3501. A "full ply" has a thickness of .015", while a "half ply" is .0075" thick.

Table 1. Sandwich Panel Design

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Face Sheet 1:
   $[(0/90)(0/90)(0/90)_{1/2}]$
   G30-500/5225 Gr/ep
   t =.0375"   (2 1/2 Plies)

Core:
   4 PCF Nomex core - 1/8" cell size
   t = 1.0"

Face Sheet 2:
   $[(0/90)_{1/2}(0/90)(0/90)]$
   G30-500/5225 Gr/ep
   t = .0375"   (2 1/2 Plies)

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SANDWICH IMPACT ANALYSIS

Closed Form Analysis

Low velocity impact analysis methods had been previously developed for solid laminate composites (ref 4, 7). In order to extend the methodology to analysis of sandwich impacts, the unique force-indentation behavior of honeycomb sandwich required attention. Solid laminate composites exhibit a progressive stiffening of the load-indentation curve with increasing load, whereas sandwich shows a linear load-indentation curve up to the point of face sheet fracture. The curve then abruptly drops to a lower load as the core begins crushing. The indentation into the panel continues at approximately a constant load with numerous peaks and valleys of load until the core crushes almost to the back face. Figure 2 shows typical static and dynamic load-deflection curves produced by our Nomex cored panel. The figure shows load-indentation curves for both a 1/2" and a 1" diameter indentor. For thin facesheets, the out-of-plane stiffness is primarily a function of core stiffness as shown by the nearly identical curves for the two indentor sizes (quasi-static loading) prior to face sheet damage. There is a slight increase in the out-of-plane stiffness for the dynamic impact loading.
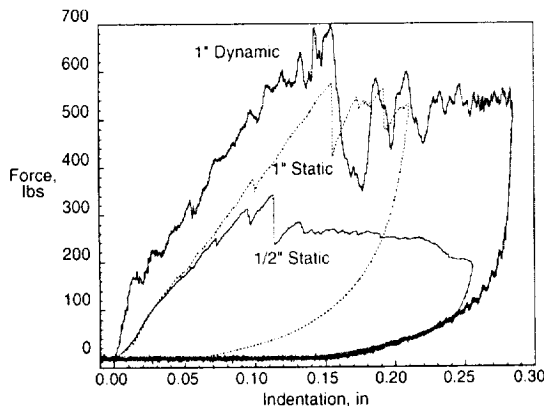

Figure 2. Typical Sandwich
Force-Indentation Curves

The nonlinear load-indentation behavior was modeled with a curve through points on the experimental load-indentation curve obtained from the static indentation test, instead of the power law Hertz stress equation used for solid laminates. A straight line was used up to the maximum load, followed by a drop to the load of continuous crushing and a uniform load during crushing. Some later analyses added several points in the crush region for more definition.

The transient response of plates and beams subject to a transverse impact has been the subject of much work. For a rigid impact (no indentation of the impactor into the plate) Goldsmith (ref. 5) gives the equation as:

$$v_o t - \frac{1}{m} \int_0^t dt \int_0^t F\, dt = w_1(c) \qquad (1)$$

where t is time, $w_1(c)$ is the transient response of a beam or plate at the contact point as a function of time, m and v_o are the mass and initial velocity of the impactor, and F is the impact force.

Timoshenko developed a method for including the effect of the indentation of a flexible impactor into a beam as:

$$\alpha = w_2 - w_1(c)$$
$$= v_o t - \frac{1}{m} \int_0^t dt \int_0^t F\, dt - w_1(c) \qquad (2)$$

where α is the difference between the displacement of the impactor (w_2) and the deflection of the beam at the contact point $w_1(c)$, as shown in Figure 3. Hertz's contact law gives a relationship between the contact force and the indentation as:

$$F = k_2 \alpha^{\frac{3}{2}} \qquad (3)$$

where α is the indentation of the impactor into the plate and k_2 is the Hertz stiffness parameter determined by the type of bodies in contact. See Goldsmith (ref. 5) or Timoshenko (ref. 6) for a discussion of the derivation of equation 3. This relation is valid for impact on solid laminates but not for sandwich structures since it produces a stiffening (less indentation per given increase in load) as the force is increased. Substituting equation (3) into equation (2) gives:

$$\left(\frac{F}{k_2}\right)^{\frac{2}{3}} = v_o t - \frac{1}{m} \int_0^t dt \int_0^t F\, dt - w_1(c) \qquad (4)$$

The equation for the transient response of a beam or plate to a force at a point can be inserted into equation (4) to give the complete analysis for low velocity impact. The equation for the transient response at a point in a rectangular simply-supported specially orthotropic Mindlin type plate subject to a time varying concentrated force at a given point is:

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$$w(x,y,t) = \frac{1}{P} \sum_{m} \sum_{n} \left(\frac{q_{mn} \sin\left(m\pi \frac{x}{a}\right) \sin\left(n\pi \frac{y}{b}\right)}{\omega_{mn}} \right.$$

$$\left. * \int_0^t F(\tau) \sin(\omega_{mn})(t-\tau) \, d\tau \right) \qquad (5)$$

where

$$P = \int_0^h \rho \, dz$$

ρ = mass density

for a concentrated load P_z at point ζ, η

$$q_{mn} = \frac{4 P_z}{ab} \sin\left(m\pi \frac{\zeta}{a}\right) \sin\left(n\pi \frac{\eta}{b}\right)$$

ω_{mn} = plate natural frequencies - considering out of plane and shear deformations z, ψ_x, ψ_y (ref. 4, 7)

where ω_{mn} are the plate natural frequencies, x, y is the analysis location, ζ, η is the impact location, a & b are the plate dimensions, and m & n are the half wave numbers in each direction. The complete derivation of equation 5 is given in references 4 and 7 and will not be repeated here due to its length. Equation 5 has been modified to consider the force to be a uniform force over the contact area or as a cosine shaped force over the contact area, as given in reference 4.
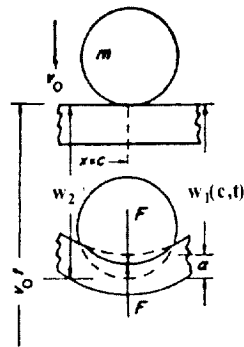

Figure 3. Impact Indentation

A computer program has been written to solve equations (4) and (5) using an iteration method developed by Timoshenko (ref 8) to determine the transient response of a simply-supported rectangular composite plate subject to a low velocity impact at any point in the plate. The program can consider the force as a concentrated force, a uniform force over a small patch, or a

cosine shaped force over a small patch. The cosine force model was used for the analyses given here as it more accurately represents the impact force applied to the panel.

Equation (4) as shown above does not apply to sandwich panels since the Hertz equation is used for the force-indentation behavior. The program has been modified to replace the Hertz indentation function with a user supplied force-indentation curve. This requires that indentation tests be performed to obtain force-indentation curves for the face-sheet / sandwich combination of interest, using the same indentor used in the impact test. Modifying equation (4) to consider a user specified force-indentation definition $\alpha(w)$ gives:

$$\alpha(w) = v_0 t - \frac{1}{m} \int_0^t dt \int_0^t F \, dt - w_1(c) \qquad (6)$$

Equation (6) is solved using a Newton method to determine the indentation and plate response at each time step.

DYTRAN Analysis

In concert with the closed form impact analysis discussed above, MSC/DYTRAN (ref 10) finite element impact analyses were performed on several of the test impacts to give further insight into the impact event. The DYTRAN Sandwich Plate Models were constructed with 3 layer face sheets using PCOMP, MAT8 and MAT8A cards. The core was modeled with hex elements with the DYMAT 26 material model, using 4 or 5 elements through the thickness to show any variation in crush with depth. This allows a crush force vs deflection curve to be input to model the core crushing. The core crush curve used was linear up to start of core crushing, followed by a drop to half of the initial crush load, with continued crushing at that load, as shown in Reference 11. A mesh size of 0.1 x 0.1" was used in the impact region, increasing to 3" at the panel edges. A 1" diameter rigid hemispherical impactor model was used with master-slave contact. The impactor was given an initial velocity to cause it to impact the panel with the required energy.

SANDWICH IMPACT - TEST & ANALYSIS CORELATION

Closed Form Analysis

Impact testing has been completed on the 60" x 36", the 36" x 30", and the 30" x 18" panels. Table 2 gives a summary of these tests with the

Table 2. Sandwich Panel Impact Test Summary

Impact No.	Impact Location In	Impact Energy ft-lbs	Test Max Force Lbs	Max Defl In	Res Dent in	Calc Max Force lbs	Max Defl in	Res Dent in
60" x 36"	**Panel Size**							
1	26.5 x 20.5	10	394	0.739	Hole	350	0.430	0.430
2	26.5 x 14.5	5	413	0.205	0.004	385	0.241	
6	26.5 x 26.5	5	490	0.201	0.007	446	0.230	0
3	20.5 x 14.5	10	573	0.316	0.033	569	0.342	0.008
5	26.5 x 8.5	10	634	0.287	0.050	567	0.339	0.06
4	20.5 x 20.5	15	676	0.383	0.058	570	0.453	0.17
4b						650	0.411	0.08
36" x 30"	**Panel Size**							
7	14.5 x 8.5	5	516	0.197	0.004	512	0.232	0
8	14.5 x 14.5	10	634	0.293	0.030	570	0.348	0.13
9	14.5 x 20.5	15	755	0.377	0.129	569	0.457	0.25
9a						750	0.413	0.17
10	20.5 x 14.5	20	654	0.521	0.280	570	0.571	0.36
10a						749	0.513	0.28
10b						650	0.526	0.32
30" x 18"	**Panel Size**							
11	5.5 x 11.5	5	575	0.194	0.010	541	0.220	0
11b						586	0.201	0
12	11.5 x 11.5	10	653	0.303	0.082	570	0.338	0.13
12b						649	0.308	0.08
13	5.5 x 17.5	15	ringing	0.439	0.170	570	.457	.251
13b						649	.416	.202
14	11.5 x 17.5	20	ringing	0.611	.3+	569	.576	.369
14b						649	.521	.316

Impact 1 Impactor dia = 1/2"   Impactor wt = 6.685 lb
Impacts 2-12   Impactor Dia = 1",   Impactor wt = 6.997 lb
Calc 1-12 used static load-indentation curve - max load - 570 lbs
Calc 9a & 10a used modified load-indentation curve - max load = 750 lbs
Calc 4b, 10b, 11b & 12b, 13b, 14b used dynamic load-indentation curve obtain from test 10

panel size, impact location, impact energy, maximum impact force, indentor deflection, and the residual indentation. Both test and calculated values are shown. All of the tests were analyzed with the static load-indentation curve, and several of the later tests were analyzed with modified load-indentation curves to improve correlation, as discussed below. Figure 4 shows a plot of the impact energy vs the maximum impact force for the tests. Figure 5 gives a plot of the impact energy vs the residual dent in the panel, while Figure 6 gives a plot of the impact energy vs the maximum indentor displacement. The 60" x 36", 36" x 30", and 30" x 18" panels are designated A, B, and C, respectively, in the figures. The calculated values were in generally good agreement with the test data. There does not appear to be much of a size effect in the max force (Figure 4) and max deflection (Figure 6) data. This is due to the thin facesheets and the crushing behavior of the honeycomb core, which provides a limitation on the maximum impact force that can occur. The maximum force generally occurred as the indentor began to penetrate the facesheet and ranged from 500 to 700 lbs. Core crushing also accounts for most of the motion of the impactor, especially for impacts at the higher energies.

However, ringing in the impactor, particularly at higher impact energies, sometimes resulted in force oscillations that created larger maximum forces at other points. Consequently, the measured maximum forces were slightly higher than the predicted values. The residual dent depth plot (Figure 5) shows a marked size effect. Smaller panels require less impact energy to show a residual dent than larger panels. For a given impact energy, a small panel will show a deeper residual dent than a larger panel.
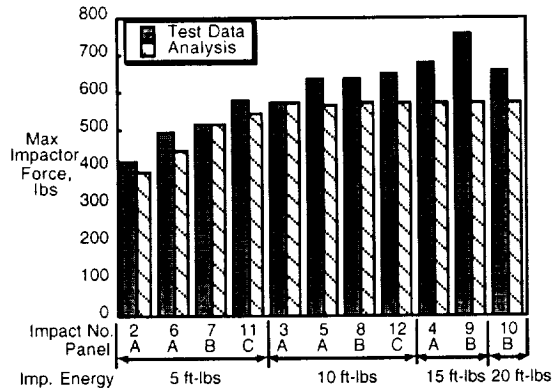
Figure 4. Maximum Force Predictions
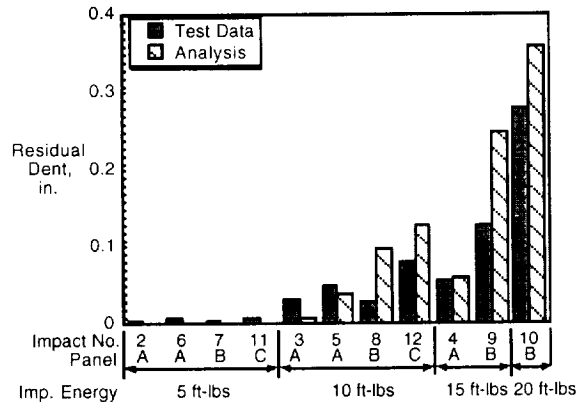
Figure 5. Residual Dent Depth Predictions

Figure 7 gives a plot of residual dent depth vs impact energy, showing that the larger panels require more energy to produce a given dent depth.

Figures 8-11 give the test and calculated load-time and indentor deflection-time histories for two tests. Figures 8 and 9 show test 2 with a 1" impactor at 5 ft-lbs of impact energy. This test produced very little residual damage, which can be seen from the shape of the load-time curve, as there is no abrupt drop in load indicating facesheet damage and the start of core crushing.
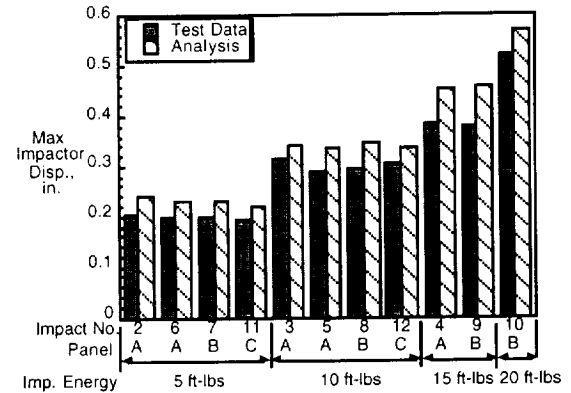
Figure 6. Maximum Impactor Displacement Predictions
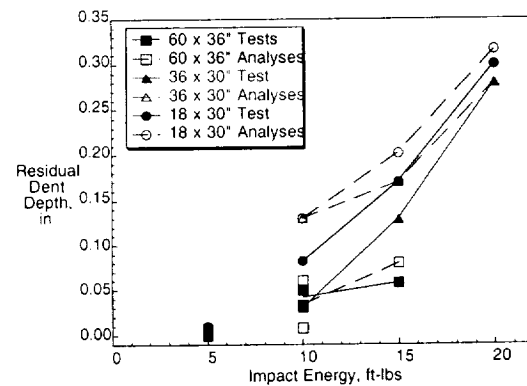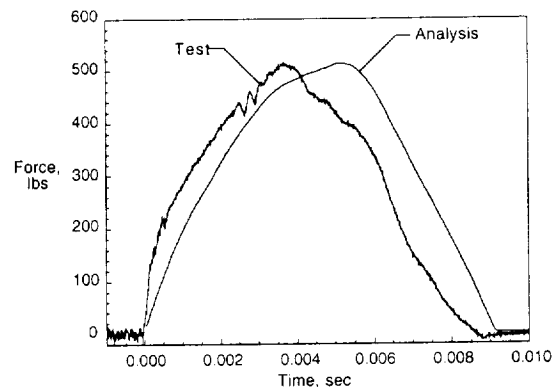
Figure 7. Impact Energy vs Residual Dent Depth

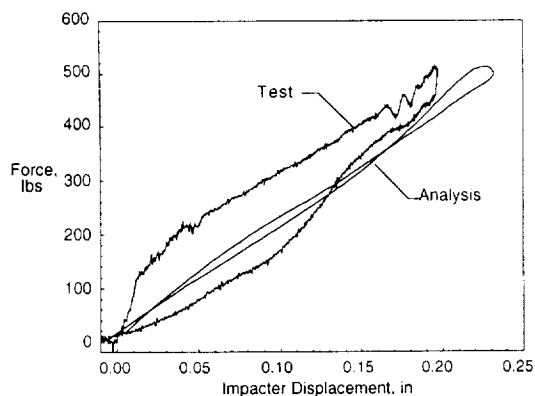Figure 8. Force History Response (Impact 7, 5 ft-lbs., 36 x 30" Panel)

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Figure 9. Load vs Deflection Response
(Impact 7, 5 ft-lbs., 36 x 30" Panel)

Generally when visible damage is seen in the panel the load-time curve shows a peak and then an abrupt drop when the face sheet breaks, followed by holding the load for a time at a lower load as the impactor begins crushing the honeycomb. Figures 10 and 11 give an example of a test at 15 ft-lbs impact energy with load drop and residual damage (test 4). The load-indentor deflection curve in Figure 11 shows a typical behavior after the initial load drop where the indentor is punching into the panel with a uniform load until it runs out of energy. The indentor then returns to near zero deflection at approximately the same slope as the initial part of the curve. The area enclosed by the load-indentor deflection curve is an indication of the amount of damage done to the panel. Figure 9, at 5 ft-lbs of impact energy, shows fairly little area enclosed by the curve and not much residual damage. Figure 11, at 15 ft-lbs of impact energy shows much more area under the curve and more residual damage.
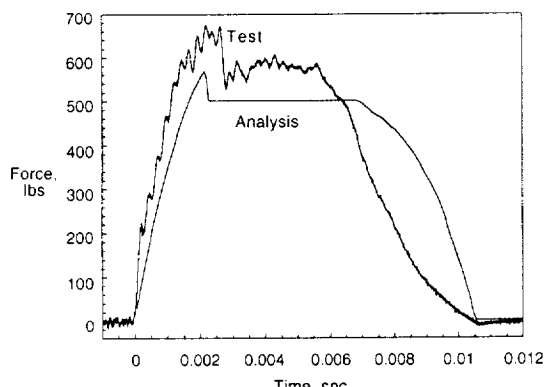

Figure 10. Force History Response
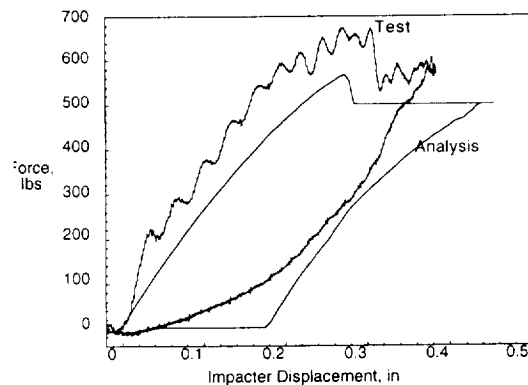(Impact 4, 15 ft-lbs., 60 x 36" Panel)

Figure 11. Load vs Deflection Response
(Impact 4, 15 ft-lbs., 60 x 36" Panel)

Impact tests 9 - 12 employed a fiber-optic apparatus to measure the panel deflection vs time on the back face under the impact point, which allowed the panel indentation to be calculated. Figure 12 shows a test displacement vs time plot for Impact 10, showing the back face displacement, the indentor displacement and the panel indentation. Figure 13 gives an analysis prediction for Impact 10, showing similar behavior for the initial loading phase of the response, although the crushing phase of the response occurs faster than what was measured in the test. For this test, most of the indentor displacement is due to core crushing behavior. For panels impacted at lower energies where the core does not crush, the indentation and back face displacement are more equal.
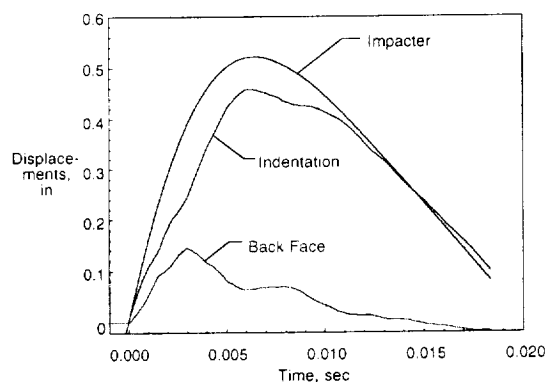

Figure 12. Deflection vs Time - Experimental
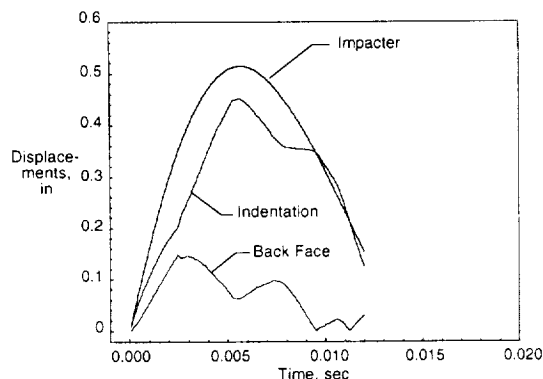(Impact 10, 20 ft-lbs., 36 x 30" Panel)

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Figure 13. Deflection vs Time - Analysis
(Impact 10, 20 ft-lbs., 36 x 30" Panel)

Load vs indentation plots for tests 9 and 10 are given in Figures 14 and 15 for comparison with the static indentation curves in Figure 1. It can be seen that the peak impact force is higher than the static result for the 1" diameter indentor, but the indentation at the initial load drop is about the same, at .13"-.15". The force-indentation response was very similar for all tests where indentation was measured.

The analysis program generally produced good predictions for the peak load and the maximum deflection, but was not as successful at predicting the residual indentation. This appears to be due to the tendency of the face sheet to spring back and close up the dent after the impact even though the core under the impact point is completely crushed. Impacts at higher energies produce a more complete bending failure of the face sheet and thus less spring back.
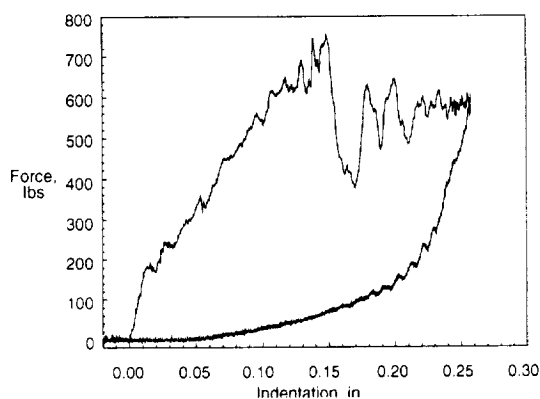

Figure 14. Indentation Response
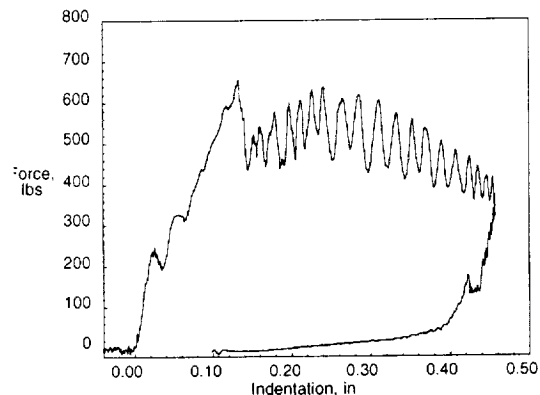(Impact 9, 15 ft-lbs., 36 x 30" Panel)

Figure 15. Indentation Response
(Impact 10, 20 ft-lbs., 36 x 30" Panel)

Although the peak impact loads were well predicted in impacts that did not reach the core crush load, the load at the start of face sheet failure and core crushing was found to have a significant variation, with the static crush strength (Figure 2) being a lower bound. The seven impacts listed in Table 3 reached the point of breaking the face sheet and producing an abrupt drop in load at the peak.

Table 3. Impact Tests Producing Abrupt Load Drops

Impact No.	Energy, ft-lbs	Peak Load, lbs
3	10	573
4	15	676
5	10	634
8	10	634
9	15	755
10	20	654
12	10	653

When the static load-indentation curve was used in the analysis the program would not let the peak load go higher than 570 lbs since the load would start dropping at that point, whereas the actual peaks varied from 573 to 755 lbs. In order to obtain more accurate correlation with the test data it was necessary to use a load-indentation curve with the break set to the peak load seen in the test.

Analyses 9a and 10a in Table 2 were performed using a modified load-indentation curve with the peak load set to 750 lbs, while analysis 10b, 11b, and 12b used the dynamic load-indentation curve from Impact 10 (Figure 15). Using the dynamic load-indentation curve in the analysis gives better correlation with the test data. Figures 16 and 17 show calculated load-time and

load-deflection plots for Impact 10 using the load-indentation curve from Impact 10, showing very good correlation with the test data. The load-indentation curve used in the analysis included the first three oscillations in the test curve and it can be seen that the first several oscillations are very well modeled by the analysis.
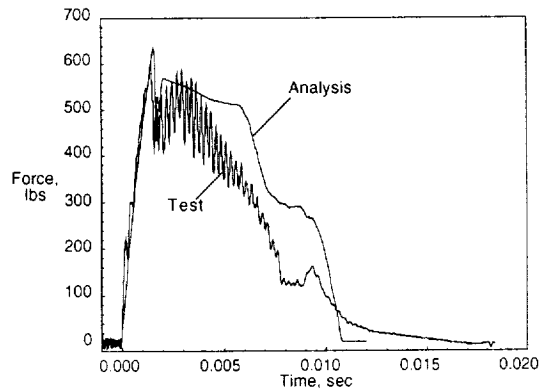

Figure 16. Force History
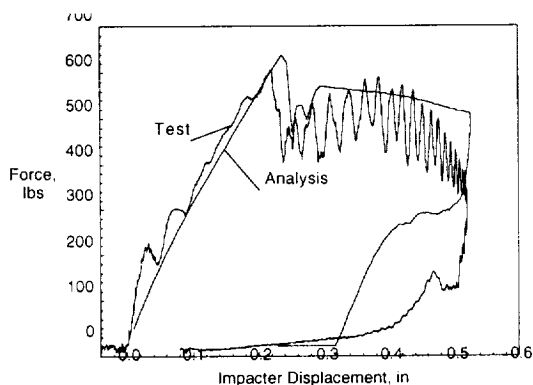(Impact 10b, 20 ft-lbs., 36 x 30" Panel)

Figure 17. Load vs Deflection Response
(Impact 10b, 20 ft-lbs., 36 x 30" Panel)

DYTRAN Analysis

An impact has been analyzed for each of the three panels. Figure 18 shows the core crush behavior for a 60 x 36" panel impacted with 15 ft-lbs of energy, while Figure 19 gives the force-time behavior and Figure 20 gives the force-indentation behavior for this case. The predictions of force-time and force-indentation are not as good as for the analytical method. The rebound phase is not modeled as well as with the closed form solution discussed earlier, as the indentation remains near the max value and does not close up with decreasing load, as is seen in the tests. This results in predictions of residual dent that are too high when compared

with the test data. This effort is continuing, and the analysis will be further refined to attempt to get better correlation.
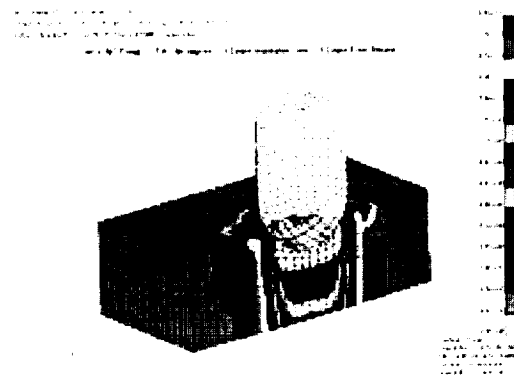

Figure 18. Dytran Analysis Model
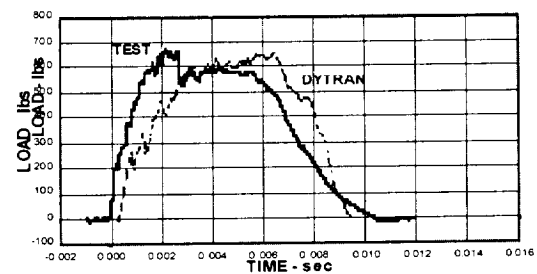(60 x 36" Panel, 15 ft-lb Impact)

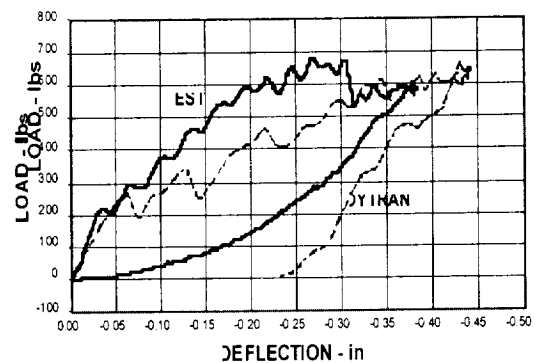Figure 19. DYTRAN Analysis – Force History
(60 x 36" Panel, 15 ft-lb Impact)

Figure 20. DYTRAN Analysis – Indentation Response (60 x 36" Panel, 15 ft-lb Impact)

CONCLUDING REMARKS

A test and analysis program is being conducted to study the effects of low velocity impact on sandwich composite panels. An existing closed-form impact analysis program has been modified to include an experimental load-indentation curve. This curve includes the nonlinear and

discontinuous effects of facesheet penetration and sandwich crushing. This experimental curve can be obtained quasi-statically or dynamically on a rigidly-backed specimen or with any other support conditions. Impact tests have been performed on sandwich panels of various sizes to obtain data for correlation with the analysis.

The closed-form impact analysis provided good correlation with the impact test data for maximum impact load, maximum deflection, maximum indentation, and residual indentation. The predictions improved when a load-indentation curve derived from the impact testing was used in the analysis. The predictions also improved when a dynamic load-indentation curve was used. The DYTRAN analyses have been less successful at this point, although development work is continuing.

The program will continue with impact testing of 12" x 12" and 6" x 6" sandwich panels. At the conclusion of the impact testing, a number of 6" x 6" compression specimens will be cut from the impacted panels and tested to obtain the compression after impact (CAI) strength. Analytic and empirical models will be evaluated to relate the CAI strength to the impact parameters such as maximum indentation, residual indentation, maximum deflection, and maximum impact load.

The combined impact and CAI strength model will then provide an analytic means to tailor the design of sandwich structure as a function of panel size and geometry to the design criteria. These design criteria generally require ultimate load capability for barely visible impact (BVI) damage that is unlikely to be detected and repaired, and limit load capability for a maximum impact threat that should be quickly repaired.

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