Application of Local Discretization Methods in the NASA Finite-Volume General Circulation Model

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Abstract

We present the basic ideas of the dynamics system of the finite-volume General Circulation Model developed at NASA Goddard Space Flight Center for climate simulations and other applications in meteorology. The dynamics of this model is designed with emphases on conservative and monotonic transport, where the property of Lagrangian conservation is used to maintain the physical consistency of the computational fluid for long-term simulations. As the model benefits from the noise-free solutions of monotonic finite-volume transport schemes, the property of Lagrangian conservation also partly compensates the accuracy of transport for the diffusion effects due to the treatment of monotonicity. By faithfully maintaining the fundamental laws of physics during the computation, this model is able to achieve sufficient accuracy for the global consistency of climate processes. Because the computing algorithms are based on local memory, this model has the advantage of efficiency in parallel computation with distributed memory. Further research is yet desirable to reduce the diffusion effects of monotonic transport for better accuracy, and to mitigate the limitation due to fast-moving gravity waves for better efficiency.

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1. Introduction

The activities of the atmosphere can be simulated with computers by programming approximate solutions to the physical laws that govern the atmosphere, which is represented by a set of grid boxes in the computers. An atmospheric model is usually divided into two parts: a dynamical core that solves a set of partial differential equations for an idealized atmosphere, and a set of physical parameterizations that provide realistic forcing to the idealized atmosphere, such as the precipitation in the atmosphere, the friction on Earth's surface and the radiation from the Sun. These two parts are usually treated separately, and we will focus on discussing the numerical methods that are employed to construct approximate solutions for the idealized atmosphere, where there are no sources or sinks of physical quantities implied by the physical parameterizations.

As distributed memory has become a trend of modern computer design, the efficiency of the implementation of parallel computation with distributed memory has become an important issue for designing computing algorithms for climate modeling which requires massive computation for long-term simulations. While numerical methods based on local memory have the advantage of parallel efficiency for distributed memory, there are fundamental issues concerning the application of these local methods to global climate modeling.

One of the key concerns for adopting local methods for climate modeling is the maintenance of global consistency among the climate phenomena. To achieve global consistency without using global information to constrain the numerical solutions, it is necessary for a local method to be as faithful as possible to the conservation laws of physics. The *finite-volume method*, which approximates the integral of a conservation law over the volume of each grid box, is thus considered viable for successful climate modeling because of its useful property of local conservation for maintaining physical consistency in the numerical solutions. Furthermore, the property of *Lagrangian conservation*, which conserves the physical quantities contained in a fluid element following the motion, can be used to improve the accuracy of the numerical solutions. Note that Lagrangian conservation is a special case of local conservation, and a physical quantity contained in a fluid element is conservative following the motion only when there are no sources or sinks during the transport process.

Another general issue of simulating atmospheric motions with computers is that spurious noise is often generated in the approximate numerical solutions. As spurious noise contaminates the simulation with unrealistic features, various techniques have been developed to mitigate the problem, yet it may be more desirable not to have the spurious noise generated in the first place. The spurious noise can indeed be prevented during the transport process by maintaining the *monotonicity* of the distributions of physical quantities in the sense that no new local maxima or minima are created by the numerical approximation. Unfortunately, monotonic transport schemes are often quite diffusive and therefore not necessarily more accurate than non-monotonic transport schemes in all cases. However, for scalar transport, in the absence of sources and sinks, the continuous governing equations dictate that the solutions be monotonic and it may be a desirable feature that the discrete model maintain this property in the solutions.

Based on the above considerations, we have developed a General Circulation Model (GCM) at the Goddard Space Flight Center of National Aeronautics and Space Administration (NASA), using monotonic finite-volume transport schemes with the choice of physical quantities that are conservative following the motion in the idealized smooth atmosphere. We have adopted a set of finite-volume schemes for the foundation of transport in one dimension, including the piecewise constant scheme of Godunov [1], a piecewise linear scheme of van Leer [2] and the Piecewise Parabolic Method (PPM) of Colella and Woodward [3]. The dynamical core of this model is based on the work of Lin and Rood [4,5,6,7] in 1990s, and the physical parameterizations are based on those of the Community Climate Model (CCM) of the National Center for Atmospheric Research (NCAR). In this article, we present the basic ideas of the NASA finite-volume dynamical core, concentrating on how to use the property of Lagrangian conservation to improve the accuracy of the approximate solutions for the idealized atmosphere.

2. The transport algorithms

The status of an idealized atmosphere can be uniquely specified with a set of state variables such as the air mass density, the air temperature and the wind velocity. Starting with a set of data of the state variables for the grid boxes that represent the idealized atmosphere, the objective of a numerical method is to predict the values of the state variables for the grid boxes in a short period of time, say 5-60 minutes, and a simulation is made with a sequence of such small time steps.

For the purpose of illustration, let us consider the PPM transport mechanism of the air in one dimension as shown in Fig. 1. Suppose we have the mean density of air mass

over the grid intervals at an initial time, and we wish to determine the mean density of air mass over the grid intervals after a short time of transport. We first notice that the air mass contained in a grid interval is simply the mean density times the length of the grid interval, and the mean density after transport is readily determined if we know the amounts of inflow and outflow of the grid interval during the transport. We also notice that the outflow of a grid interval across a grid interface is exactly the inflow of the adjacent grid interval across the same interface, they are the same amount of air crossing that interface during the transport, i.e., the *air mass flux*. The issue is then down to the determination of the air mass fluxes across the grid interfaces during the transport, and the accuracy of the approximate solution to the mean density after transport depends solely on how the air mass fluxes are calculated.

The PPM transport algorithm starts with the construction of a detailed structure of air mass density within each grid interval, using the given mean values of air mass density for the grid intervals. These detailed structures within the grid intervals will be called the *subgrid distributions* and the collection of the mean values will be called the *overgrid distribution* here for convenience of discussion. Figure 1 illustrates an example of overgrid distribution with black line segments and the corresponding subgrid distributions with pieces of red curves. Note that the subgrid distribution within a grid interval is constructed in such a way that the area under the curve is equal to the rectangular area under the line segment. In other words, the integral of a subgrid distribution over the corresponding grid interval is equal to the air mass contained in this grid interval. We also note that each subgrid distribution is monotonic within the corresponding grid interval, and adjacent subgrid distributions are monotonic across the grid interfaces. The monotonicity of the subgrid distributions is a consequence of the monotonicity of the given overgrid distribution.

Now, assume that the velocity of the air flow is known everywhere, then we know exactly which air particle crosses a grid interface during the transport. Suppose the air flow moves from the grid interval A to the grid interval B in Fig. 1, then we know how far upstream in grid interval A would an air particle reach the interface I_1 between A and B, and the air contained in this upstream range is exactly the flux across interface I_1 during the transport. In other words, the flux F_1 across interface I_1 is equal to the *volume integral* of the subgrid distribution over the upstream range in grid interval A, which corresponds to the shaded area under the curve in grid interval A in Fig. 1.

Similarly, we can determine the flux F_2 across the interface I_2 between the grid intervals B and C in Fig. 1. Suppose the air flow moves from B to C, then the flux F_2 is an outflow leaving B during the transport. Given the air mass M contained in grid interval B before transport, we see that $(M - F_2)$ is the amount of air remaining in B after transport, i.e., the volume integral of the subgrid distribution over the remaining section of B, and it corresponds to the blank area under the curve in grid interval B in Fig. 1. With F_1 being the inflow and F_2 the outflow during the transport, the air mass contained in grid interval B after transport is simply $M' = M + F_1 - F_2$, and the mean density over B after transport is obtained by dividing the resulted air mass M' with the length of the grid interval.

Note that the overgrid distribution of the mean values after transport is monotonic if the overgrid distribution of the mean values before transport is monotonic, the monotonicity is maintained by the monotonic construction of subgrid distributions. Furthermore, each mean value after transport is obtained by combining the neighboring volume integrals of the subgrid distributions that conserve the material within the corresponding grid intervals, the transported material is thus conserved locally. Since these constructed subgrid distributions are not exactly the actual ones for the smooth fluid to be simulated, errors have been introduced during the numerical transport. These errors would be greater if the transported material were not conservative following the motion in the smooth fluid system, for the actual distribution would have been changed during the transport—yet the approximate subgrid distributions were constructed with the overgrid distribution given at the initial time.

The errors in one-dimensional transport could be further amplified in multi-dimensional transport when using the one-dimensional schemes to estimate the fluxes in each dimension separately. Lin and Rood [4] designed a multi-dimensional transport algorithm to take advantage of the efficiency of dimensional splitting while reducing the associated errors by considering the contributions from other dimensions during the transport. This multi-dimensional algorithm also maintains the relationship between air mass and other transported physical quantities, and thereby promotes the physical consistency of the computational fluid system. To further improve both accuracy and efficiency in multi-dimensional transport, the *Lagrangian coordinates* [7] is adopted in the vertical direction, namely, the altitudes of horizontal material surfaces which evolve during the transport.
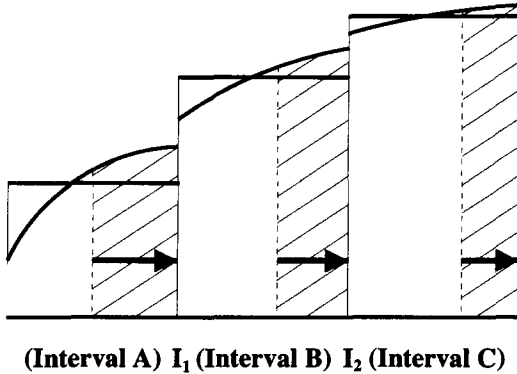
(Interval A) I₁ (Interval B) I₂ (Interval C)

Figure 1: PPM transport mechanism in one dimension. Detailed subgrid distributions (red curves) are first constructed from the given overgrid distribution of mean values (black line segments) for the grid intervals A, B and C. The fluxes crossing the grid interfaces during the transport are then calculated with volume integrals (shaded areas) of the subgrid distributions over the upstream ranges implied by the given velocity field (green arrows).

3. The dynamics system

As illustrated in the previous section, the property of Lagrangian conservation in the idealized atmosphere can be used to improve the accuracy of the approximate solutions by reducing the errors in the numerical transport processes. Not all physical quantities in the continuous dynamics system of the atmosphere are conservative following the motion. For instance, the momentum (mass times velocity) carried in an air parcel is not conservative following the motion because of the general existence of external forcing to the air parcel, such as the pressure-gradient force due to the pressure differences surrounding the air parcel. If the momentum were chosen to be conserved in the discrete numerical system through explicit transport with the wind velocity, the errors in inertial transport would be amplified by the interaction with the pressure-gradient force, and the conservation of momentum in the discrete system would not be localized as well as in the inertial case.

There are three physical quantities in the continuous dynamics system of the atmosphere that are conservative following the motion, they are the *air mass*, the *potential temperature* and the *absolute vorticity*. Potential temperature is a measure of molecular motions with respect to a reference thermodynamic state of the atmosphere. Absolute vorticity is a measure of the rotation of a fluid element relative to an inertial frame such as the space containing the Earth. These three quantities have been chosen to be conserved in the NASA finite-volume dynamical core, to promote the accuracy in transport and the localization of conservation.

Because they are directly related to primary variables, the properties of local conservation and monotonicity of air mass and potential temperature can be achieved directly by finite-volume transport. Absolute vorticity is, however, a quantity derived from the primary variable velocity. To achieve the local conservation and monotonicity of absolute vorticity with velocity being a primary variable, it is essential to apply the *Circulation Theorem* that the integral of vorticity over an area is equal to the integral of velocity along the boundary enclosing the area. By using the so-called D-grid configuration as shown in Fig. 2a, the primary variable velocity can be predicted directly along the boundary of the area while conserving absolute vorticity. Lin and Rood [5] demonstrated the advantage of such an approach in two dimensions with an excellent simulation of the Rossby-Haurwitz waves which has been recognized as a standard test for modeling atmospheric dynamics [8].

To extend the two-dimensional dynamics to three dimensions, we have adopted Lagrangian coordinates [7] in the vertical direction. Note that the vertical Lagrangian coordinates are the altitudes of horizontal material surfaces, and we may assume the *hydrostatic equilibrium* that the weight of the air confined between two material surfaces are balanced by the difference of the pressure at the material surfaces. The three-dimensional atmosphere is thus decoupled into a sequence of inertial horizontal layers, and the dynamics in each layer can be treated in a manner very similar to the two-dimensional dynamics, except the calculation of pressure-gradient force. The pressure-gradient force in a Lagrangian layer [6] is calculated with the *Green Theorem* that the integral of pressure gradient over the volume contained in a grid box is equal to the integral of pressure over the surface surrounding the grid box as shown in Fig. 2b. In other words, the net external force acting on a fluid element is obtained by integrating the surrounding pressure.
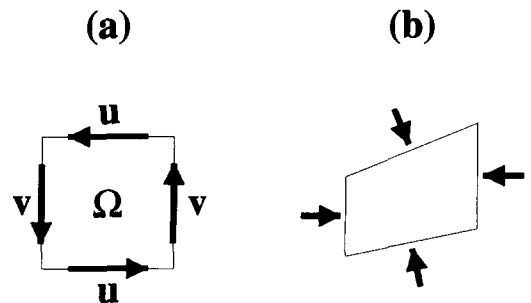
(a)   **(b)**

Figure 2: (a) D-grid configuration for applying the Circulation Theorem in two dimensions. The absolute vorticity (Ω) is a mean value over the grid box, and the wind components (u, v) are predicted along the boundary. (b) Schematic for applying the Green Theorem with Lagrangian coordinates in vertical direction. The net external force acting on a fluid element is obtained by integrating the surrounding pressure (blue arrows).

The NASA finite-volume dynamical core is in the end a collection of finite volumes where dynamical processes are calculated as integrals of simple locally continuous functions. Many of the inherent errors of finite-difference and spectral methods are reduced, and there is a high degree of consistency of primary variables and dynamical quantities. Figure 3 demonstrates the climatology of the NASA finite-volume GCM with the 500-hPa eddy height, i.e., the height at 500-hPa pressure level relative to its own zonal mean, for December, January and February. The simulation is produced with a 15-year run at the resolution of 2.5 degrees in longitude and 2.0 degrees in latitude, and the vertical dimension of the atmosphere is covered with 55 Lagrangian layers up to the pressure level of one Pascal. The size of time step is 7.5 minutes for the dynamical core, and it is 30 minutes for the physical parameterizations. The PPM is adopted for the foundation of transport in one dimension. Note that the 500-hPa eddy height represents the wave patterns and magnitudes in middle troposphere, and the simulated climatology is quite realistic according to the analysis from the European Center for Medium-range Weather Forecast (ECMWF). Additional results from this model are available at the web site <http://dao.gsfc.nasa.gov/NASanCAR>.

(this figure is provided in a separated file)

Figure 3: Climatology simulated with the NASA finite-volume GCM (upper panel) and the corresponding analysis from ECMWF (lower panel), demonstrated with an average of 500-hPa eddy height (m) over December, January and February. The simulation is produced with a 15-year run at the resolution of 2.5x2.0 degrees with 55 layers in vertical direction.

4. Future development

Our experience with the NASA finite-volume GCM indicates that the diffusion effects of the adopted monotonic transport schemes are often too strong for simulating small-scale phenomena in the atmosphere. We have developed new monotonic finite-volume transport schemes to address this issue, some of these new transport schemes have been incorporated in the model as options, some are yet to be published and implemented.

While the property of Lagrangian conservation can be used to enhance the physical consistency in the numerical solutions, the accuracy of transport depends largely on how the velocity is predicted in the dynamics system. Although Lin and Rood [5,6] provide an accurate way to predict the velocity while taking advantage of the Lagrangian conservation of absolute vorticity, the efficiency of their methodology is subject to the restriction on the size of time steps for the fast-moving gravity waves. Further research is desirable to enlarge the time step and hence improve the overall efficiency.

Furthermore, due to the adoption of the traditional longitude-latitude grid system, global memory is still needed in polar areas where the meridians converge, and this affects the efficiency of the implementation of parallel computation with distributed memory. A new design of the dynamical core with quasi-uniform grids is undergoing development, in order to refrain from using global memory, and thereby promote the parallel efficiency for distributed memory. It is yet to be proved that this approach would gain enough in efficiency while achieving sufficient accuracy for practical applications.

For more information about the NASA finite-volume GCM, the readers are referred to the documentation on the next-generation model provided at the web site <http://dao.gsfc.nasa.gov/pages/atbd.html>.

REFERENCES

1. Godunov, S. K., 1959: A difference scheme for numerical computation of discontinuous solutions of equations in fluid dynamics. *Mat. Sb.*, **47**, 271. Also: *Cornell Aero. Lab. Transl.*

2. van Leer, B., 1977: Toward the ultimate conservative difference scheme. Part IV: A new approach to numerical convection. *J. Comput. Phys.*, **23**, 276-299.

3. Colella, P., and P. R. Woodward, 1984: The piecewise parabolic method (PPM) for gas-dynamical simulations. *J. Comput. Phys.*, **54**, 174-201.

4. Lin, S.-J., and R. B. Rood, 1996: Multidimensional flux-form semi-Lagrangian transport schemes. *Mon. Wea. Rev.*, **124**, 2046-2070.

5. Lin, S.-J., and R. B. Rood, 1997: An explicit flux-form semi-Lagrangian shallow-water model on the sphere. *Q. J. R. Meteorol. Soc.*, **123**, 2477-2498.

6. Lin, S.-J., 1997: A finite-volume integration method for computing pressure gradient force in general vertical coordinates. *Q. J. R. Meteorol. Soc.*, **123**, 1749-1762.

7. Lin, S.-J., and R. B. Rood, 1999: Development of the joint NASA/NCAR General Circulation Model. *13th Conference on Numerical Weather Prediction, Amer. Meteorol. Soc., Denver, Colorado*, J2.5.

8. Williamson, D. L., J. B. Drake, J. J. Hack, R. Jakob, and P. N. Swarztrauber, 1992: A standard test set for numerical approximations to the shallow water equations in spherical geometry. *J. Comput. Phys.*, **102**, 211-224.

DJF 500−hPa Eddy Height

NASA/NCAR FVCCM3 AMIP II
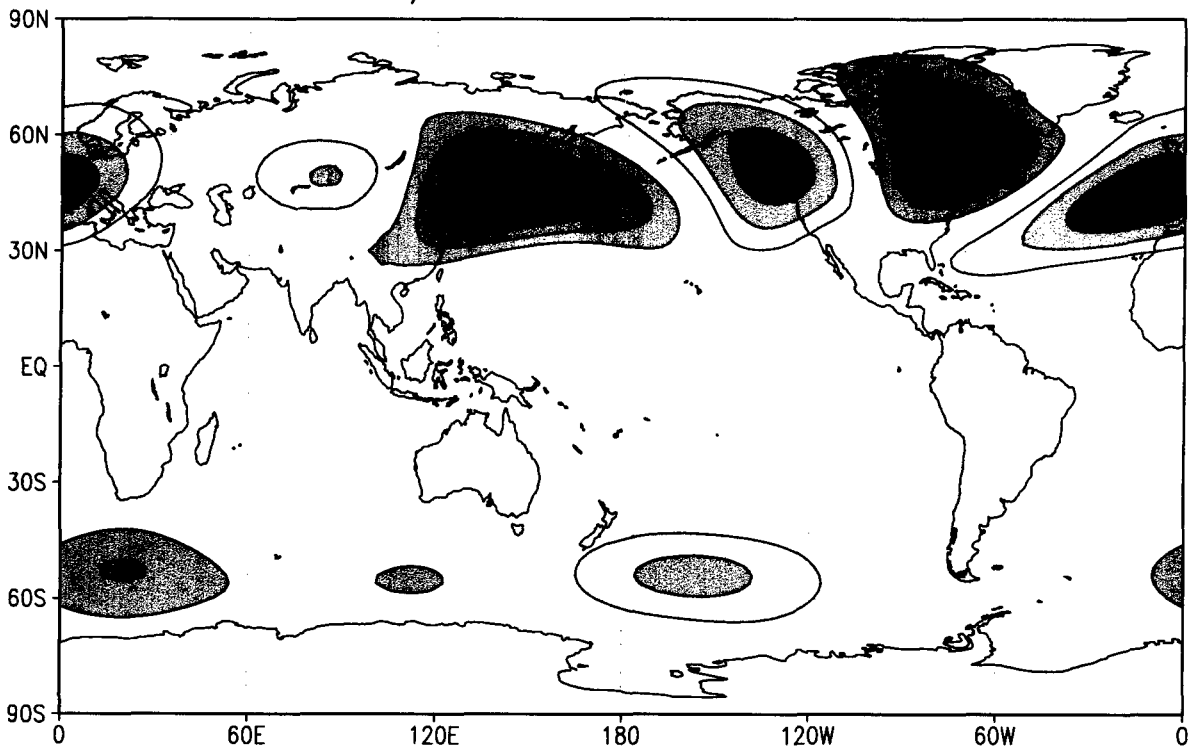
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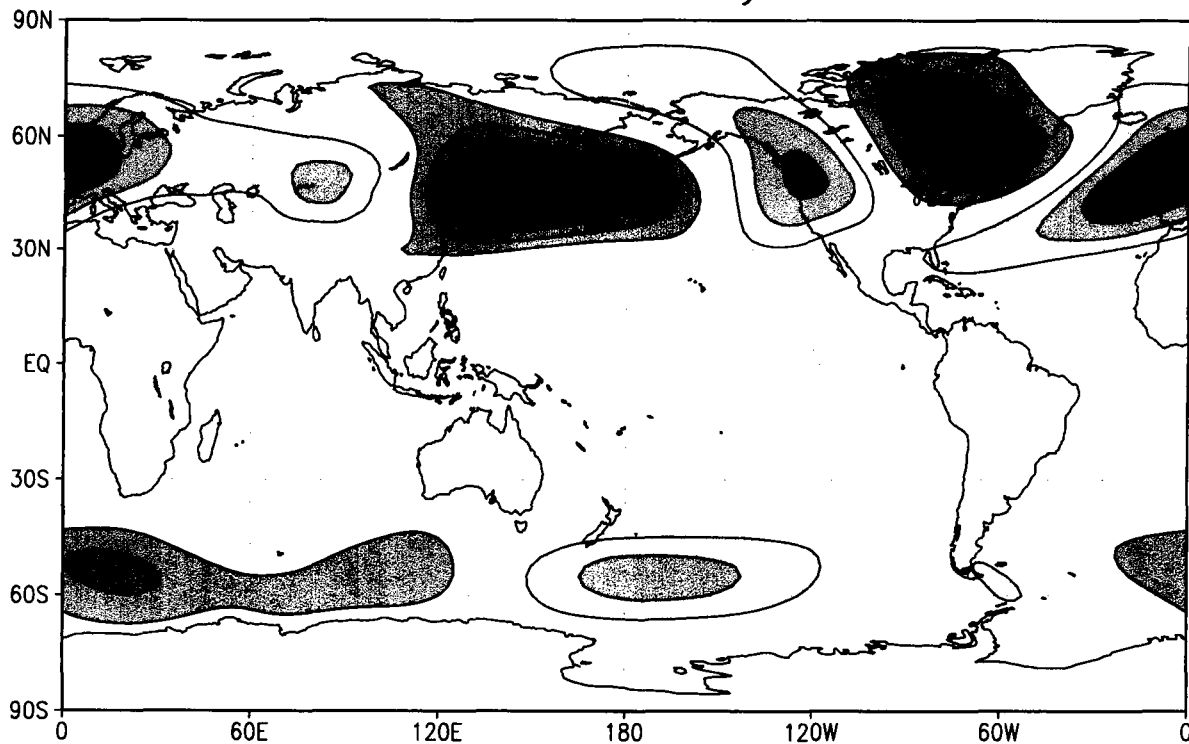
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