

Interchange Method in Compressible Magnetized Couette Flow: Magnetorotational and Magnetoconvective Instabilities

Dimitris M. Christodoulou,¹ John Contopoulos,² and Demosthenes Kazanas³

ABSTRACT

We obtain the general forms of the axisymmetric stability criteria in a magnetized compressible Couette flow using an energy variational principle, the so-called interchange or Chandrasekhar's method, which we applied successfully in the incompressible case. This formulation accounts for the simultaneous presence of gravity, rotation, a toroidal magnetic field, a weak axial magnetic field, entropy gradients, and density gradients in the initial equilibrium state. The power of the method lies in its simplicity which allows us to derive extremely compact and physically clear expressions for the relevant stability criteria despite the inclusion of so many physical effects. In the implementation of the method, all the applicable conservation laws are explicitly taken into account during the variations of a quantity with dimensions of energy which we call the "free-energy function."

As in the incompressible case, the presence of an axial field invalidates the conservation laws of angular momentum and azimuthal magnetic flux and introduces instead isorotation and axial current conservation along field lines. Our results are therefore markedly different depending on whether an axial magnetic field is present, and generalize in two simple expressions all previously known, partial stability criteria for the appearance of magnetorotational instability. Furthermore, the coupling between magnetic tension and buoyancy and its influence to the dynamics of nonhomoentropic magnetized flows becomes quite clear from our results. In the limits of plane-parallel atmospheres and homoentropic flows, our formulation easily recovers the stability criteria for suppression of convective and Parker instabilities, as well as some related special cases studied over 40 years ago by Newcomb and Tserkovnikov via laborious variational techniques.

Subject headings: accretion, accretion disks—hydrodynamics—instabilities—MHD—stars: magnetic fields—stars: mass-loss

¹Lucent Technologies, 1 Robbins Road, Westford, MA 01886. E-mail: dmc12@lucent.com

²ELVAL, 200 Akti Themistokleous Street, Piraeus 18539, Greece. E-mail: jcontop@elval.vionet.gr

³NASA/GSFC, Code 665, Greenbelt, MD 20771. E-mail: kazanas@milkyway.gsfc.nasa.gov