Libration Orbit Mission Design:
Applications Of Numerical And Dynamical Methods

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Abstract: Sun-Earth libration point orbits serve as excellent locations for scientific investigations. These orbits are often selected to minimize environmental disturbances and maximize observing efficiency. Trajectory design in support of libration orbits is ever more challenging as more complex missions are envisioned in the next decade. Trajectory design software must be further enabled to incorporate better understanding of the libration orbit solution space and thus improve the efficiency and expand the capabilities of current approaches.

The Goddard Space Flight Center (GSFC) is currently supporting multiple libration missions. This end-to-end support consists of mission operations, trajectory design, and control. It also includes algorithm and software development. The recently launched Microwave Anisotropy Probe (MAP) and upcoming James Webb Space Telescope (JWST) and Constellation-X missions are examples of the use of improved numerical methods for attaining constrained orbital parameters and controlling their dynamical evolution at the collinear libration points. This paper presents a history of libration point missions, a brief description of the numerical and dynamical design techniques including software used, and a sample of future GSFC mission designs.

INTRODUCTION

Sun-Earth libration point orbits serve as excellent locations for scientific investigations of stellar and galactic physics. These orbits are often selected to minimize environmental impacts and disturbances and to maximize observing efficiency. Trajectory design in support of such missions is challenging as more complex mission designs are envisioned. To meet these challenges, trajectory design software must be further enhanced to incorporate better understanding of the libration orbit solution space and to encompass new optimal methods. Thus the support community needs to improve the efficiency and expand the capabilities of current approaches. For example, invariant manifolds, derived from dynamical systems theory, have been applied to trajectory design over the past few years. The manifold approach offers new insights into the natural dynamics associated with the multi-body problem.[1,2] Overall it allows a more rapid and robust methodology to libration orbit and transfer orbit design when used in combination with numerical techniques. Trajectory design approaches should also include improved numerical targeting methods that allow optimization and a dynamical view of the state space allowing the user rapid intuitive feedback.

An Overview Of NASA Themes Involving Libration Orbits

The NASA Enterprises of the Space Sciences (SSE) and Earth Sciences (ESE) are a combination of several programs and themes that will benefit from the applications of improved numerical and dynamical approaches to meet mission trajectory designs[3]. The Space Sciences Enterprise includes themes such as Sun-Earth-Connections (SEC), Origins, the Structure and Evolution of the Universe (SEU), and Exploration of the Solar System (ESS). Each of these themes call for missions in libration point orbits. The attainment and maintenance of the particular orbits will be a challenge for the mission designer. For example, the desire to obtain a specific libration orbit and eliminate shadows and minimize fuel requirements while meeting specific payload needs will be a significant technology payoff.

SEC missions will use orbits that provide unique coverage for solar observation and Earth's environmental regions. Recent SEC missions included the L_1 libration point mission SOHO and the L_1/L_2 WIND mission; both missions used conceptual manifold implementations. The Living With a Star (LWS)

theme of the SEC may require mission design of trajectories that place spacecraft into heliocentric orbits and libration orbits at the L_1 and L_3 Sun-Earth libration points.

Other Space Science challenges include the Structure and Evolution of the Universe (SEU) program. Currently the Micro Arcsecond X-ray Imaging Mission (MAXIM) and Constellation-X missions of the SEU are libration point orbiters, with each mission a formation-flying concept.

As found on the Origins web site, "The Origins Program has embarked on a series of closely linked missions that build on prior achievements. As each Origins mission makes radical advances in technology, innovations will be fed forward from one generation of missions to the next. By the end of the decade, we will have combined the very best imaging, formation flying, and other visionary technologies, giving us the power of enormous telescopes at a fraction of the cost." A major goal of the Space Sciences' Origins Program is the launch of the James Webb Space Telescope (JWST) and The Terrestrial Planet Finder (TPF). Each mission is to the L_2 libration point. The mission design of JWST is currently employing the use of invariant manifolds to seed numerical targeting schemes.

The Triana mission of the Earth Science Enterprise (ESE) is the lone ESE mission not orbiting the Earth. It is a mission that has relied solely on manifolds for computation of its mission baseline to L_1. While this mission is a significant deviation from traditional low Earth orbiters, it represents the possibilities of other Earth observing mission at unique vantage points .

A major challenge to many of the above missions is the use of interferometry to form a virtual telescope. By placing telescope components on individual formation flying spacecraft, they would form a constellation or formation that would provide a powerful single telescope. Spacecraft carrying such instruments would have to fly in a precise formation, one that would provide us with the greatest possible information. Not only will spacecraft be separated across small to very large distances, they'll constantly be turning and pointing at different stars, expanding and contracting the distance between them. We'll need improved numerical and dynamical system applications and optimal control methods to monitor and maintain less than centimeter-sized changes in position in order to make the individual systems act as one large spacecraft.

Historical Missions

Even though libration orbits have become more mainstream and several missions to the Sun-Earth collinear libration points are now proposed, current NASA libration missions have been few in number totaling only seven with five true orbiters, International Sun-Earth Explorer (ISEE-3), Solar Heliospheric Observatory (SOHO), Advanced Composition Explorer (ACE), Microwave Anisotropy Probe (MAP), and Genesis and two that stayed briefly in a libration orbit, WIND and Geotail. The Flight Dynamics Analysis Branch (FDAB) of the Goddard Space Flight Center (GSFC) has designed and supported all but one of these missions.

While libration orbits share many dynamic properties, their diversity is revealed by how mission constraints are met. In a Sun-Earth rotating coordinate frame centered at the libration point, their geometric orientations and amplitudes may vary significantly. Also, the mathematical approach for representation of the reference orbit may differ. For ISEE-3 the complexity of mission design was handled through a combination of analytical and numerical methods to predetermine the reference libration orbit, along with proven operational numerical techniques for targeting and optimization.[4,5] The International Solar Terrestrial Program (ISTP) SOHO mission was the next true libration orbiter, with orbit amplitudes equal to that of the ISEE-3 mission.[6,7] While the SOHO transfer and mission orbit is similar to ISEE-3, the stationkeeping control method does not follow the ISEE-3 method of re-targeting back to a predetermined reference path. SOHO stationkeeping is performed to ensure that the orbit completes another revolution which has an added benefit of minimizing the ΔV required. The WIND spacecraft of the ISTP Program was originally planned as a libration orbiter, its trajectory was designed using multiple lunar gravity assist for rotation of the line of apsides to coincide with the Sun-Earth line before insertion into the L_1 lissajous[8]. ACE was designed differently in that a direct transfer orbit was adjusted to allow a capture into a small L_1 Lissajous orbit.[9] The most recent GSFC mission was MAP. MAP included a lunar gravity assist to obtain a small amplitude lissajous orbit.[10] The Genesis mission is similar to SOHO but utilizes invariant manifolds.

Future Mission Challanges

At GSFC, there is currently one mission awaiting launch, Triana, and two L_2 missions in design or formulation, the James Webb Space Telescope (JWST)[11] and Constellation-X. Future missions of formations such as Maxim and Stellar Imager are in their conceptual stage. These and other new missions

such as the Terrestrial Planet Finder or European Space Agency's Darwin drive designs of constrained transfer trajectories and mission orbits [12,13]. These missions are designed to meet orbit goals for specific Lissajous orbits, to minimize fuel or operational requirements, and to provide formation or constellation options. Traditionally, libration orbit design has been analyzed with a baseline trajectory concept set in place by project requirements or analytical boundary methods; that is, a trajectory had been baselined so that science requirements are met. Future mission design requires a more generalized approach as operational considerations require the launch window, gravity assist, transfer trajectories, final orbit geometry and orientation, and the number of spacecraft to be as flexible as possible to optimize science return while minimizing operational and launch requirements.

Upcoming missions also bring new challenges that individually may easily be met, but in combination they become problematic. These may include

- Biased Orbits when using large sun shades
- Minimal Fuel
- Constrained communications
- Shadow restrictions
- Very small amplitudes
- Limited ΔV directions
- Use of external libration orbits to L_4, L_5
- L_3 Co-linear orbits

- Reorientation to different planes and classes
- Earth-Moon libration orbits
- Low thrust transfers
- Solar sail applications
- Continuous control to reference trajectories
- Human exploration
- Quasi-stationary orbits
- Servicing of resources in libration orbits

A Brief History of Trajectory Design and Capabilities

GSFC libration point mission design capabilities have significantly improved over the last decade. The success of GSFC support is based on an accurate numerical computational regime. Before 1990, mainframe computers were the only resource to compute high fidelity trajectories for libration orbits. The software of choice at that time was the Goddard Mission Analysis System (GMAS). This software had complete optimization functionality as well as the capability to model all the required perturbing forces. The software was unique at the time since it allowed object modules to be linked into the run sequence as a way to allow the user access to data for trajectory analysis. During the early 1990's, the GSFC operational PC program called Swingby was developed.[14] Swingby was developed as a replacement for GMAS with an interactive graphical user interface to provide instantaneous feedback of the trajectory design in multiple coordinate systems. It was designed to be a generic tool to support a variety of missions including, lunar, planetary, libration, and deep space and of course gravity assisted trajectory designs. Swingby provides complete mission analysis and operations for the WIND, SOHO, ACE, and is currently being used for Triana analysis and as an independent check for MAP. Additionally, the lunar orbiter missions of Lunar Prospector and Clementine also used Swingby for mission design and maneuver planning. With the unprecedented success of Swingby, GSFC invested in a commercial program called Astrogator, produced by Analytical Graphics Inc. that is based on Swingby design and mathematical specifications[15]. Table-1 presents both historic and future planned and conceptual missions.

Numerical and Dynamical Targeting Methods

It is important that libration trajectories be modeled accurately. The software must integrate spacecraft trajectories very precisely and model both impulsive and finite maneuvers. Swingby and Astrogator allow this by incorporating various high order variable or fixed step numerical integrators (Runge Kutta, Cowell, and Bulirsch-Stoer). Precise force modeling includes up to 100x100 Earth and lunar gravity potentials, solar radiation pressure, multiple 3rd-body perturbation effects and an atmospheric drag model. Varying user-selected parameters to achieve the required goals performs trajectory targeting and optimization. A differential corrector (DC) is routinely used as the method of choice for targeting. Both programs use B-plane and libration coordinate targets. These software tools are also excellent for prelaunch analysis including error analysis, launch window calculations, finite engine modeling, and ephemeris generation.

Numerical Shooting Methods

Any trajectory design for libration orbit transfers and stationkeeping can be computed using GSFC's Swingby or Anaytical Graphic's (AGI) Astrogator software. Currently, both of these programs use

a direct shooting approach (forward or backward) for targeting and meeting mission goals. A shooting method using a differential corrector (DC) is widely used to achieve orbit goals in these programs although both provide the user with a limited Quasi-Newton / Steepest Descent method. All three methods use numerical partial derivatives to calculate the direction for convergence. The DC in Swingby uses the first derivative information. The partial derivatives are calculated by numerically propagating to the stopping condition, changing the independent variable with a small perturbation and re-propagating. The change in the goals divided by the change in the variables are used to compute the partials. The usual sequence of a forward shooting method is to vary the initial conditions though predefined perturbations. The initial conditions include the orbital initial conditions, an applied ΔV, or spacecraft design parameter to meet goals that include orbital parameters such as period, position, velocity, amplitude, etc.

Table 1 – Libration Orbit Missions

Mission	Location / Type	Amplitudes (Ax, Ay, Az)	Launch Year	Total ΔV Allocation (m/s)	Transfer Type
ISEE-3	L1 Halo/L2/Comet 1st mission	175000, 660670, 120000	1978	430	Direct
WIND+	L1 – Lissajous	10000, 350000, 250000	1994	685	Multiple Lunar Gravity Assist
SOHO	L1 – Lissajous	206448, 666672, 120000	1995	275	Direct
ACE	L1 – Lissajous 1st small amplitude	81775, 264071, 157406	1997	590	Direct (Constrained)
MAP	L2-Lissajous 1st L2 Mission	n/a, 264000, 264000	2001	127	Single Lunar Gravity Assist
Genesis	L1-Lissajous	25000, 800000, 250000	2001	540	Direct
Triana	L1-Lissajous Launch Constrained	81000, 264000, 148000	#	620	Direct
JWST*	L2-Quasi-Periodic Lissajous	290000, 800000, 131000	#	150	Direct
SPECs	L2-Lissajous Tethered Formation	290000, 800000, 131000	#	Tbd	Direct
MAXIM	L1 – Lissajous Formation	Large Lissajous	#	#	Direct
Constellation-X	L2 – Lissajous Loose Formation	Large Lissjaous	2010	150-250	Single Lunar Gravity Assist
Darwin	L1-Lissajous Large Lissajous	300000, 800000, 350000	2014	#	#
Stellar Imager	L2 – Lissajous ~30 S/C Formation	Large Lissajous	2015	#	Direct
TPF	L2 – Lissajous Formation?	Lissajous	#	#	#

* =This information represents concept only, Lissajous orbit as part of baseline trajectory. # =unknown at this time, + =WIND originally had a L_1 Lissajous orbit as part of baseline trajectory. (Red represents current missions; blue represents future mission concepts)

The general procedures used in developing a baseline L_2 direct transfer trajectory are:

- Target a trajectory that yields an escape trajectory towards a libration point with the Moon at the appropriate geometry
- Target the anti-Sun right ascension and declinations at the appropriate launch epoch
- Target the solar rotating coordinate system velocity of the Sun- Earth rotating coordinate x-z plane crossing condition to achieve a quasi-libration orbit, L_2 x-axis velocity ~ 0
- Target a second x-z plane crossing velocity which yields a subsequent x-z plane crossing, then target to a one period revolution at L_2

- In all above conditions, vary the launch injection C3 and parking orbital parameters (ω, Ω, parking orbit coast and inclination)
- Incorporate conditions to achieve the correct orientation of the Lissajous pattern

While the basic Swingby DC targeting procedures used in developing a baseline lunar gravity assist trajectory for L_2 were:

- Target the Moon at the appropriate encounter epoch to achieve an anti-Sun outgoing asymptote vector
- Target the lunar B-Plane condition to achieve gravity assist parameters and a perpendicular Sun-Earth rotating coordinate x-z plane crossing
- Target x-z plane crossing velocities which yields a second x-z plane crossing and target to a one period revolution at L2
- Re-target lunar B-plane conditions to achieve the correct orientation of the Lissajous pattern with respect to the ecliptic plane

In both procedures, target goals may include Time (epoch, durations, flight time), B-Plane conditions (B.T B.R angle, B magnitude, outgoing asymptote vector and energy), Libration sun-Earth line crossing conditions (position, velocity, angle, energy, or a mathematical computation (eigenvectors), or other parameters at intermediate locations. Targets may be single event string, nested, or branched to allow repeatable targeting. Maneuvers can be inserted were appropriate.

These procedures are duplicated for significant changes in launch date and to include phasing loop strategies. The phasing loop strategy allows time between launch and a lunar encounter, thus providing a longer launch window since the phasing loop periods can be adjusted by maneuvers to arrive at the chosen epoch and lunar phase angle with respect to the Sun-Earth line. Targeting to an opening Lissajous pattern assures that the spacecraft will not pass through the shadow for at least 3 years (assuming control of the unstable mode). Retarget conditions via addition of deterministic ΔVs can be used to achieve the correct orientation and Lissajous pattern size with respect to the ecliptic plane. This procedure is duplicated for significant changes in launch date or to include lunar phasing loop strategies. Targeting to an opening Lissajous pattern assures that the spacecraft will not pass through the shadow for multiple revolutions assuming control of the unstable mode.
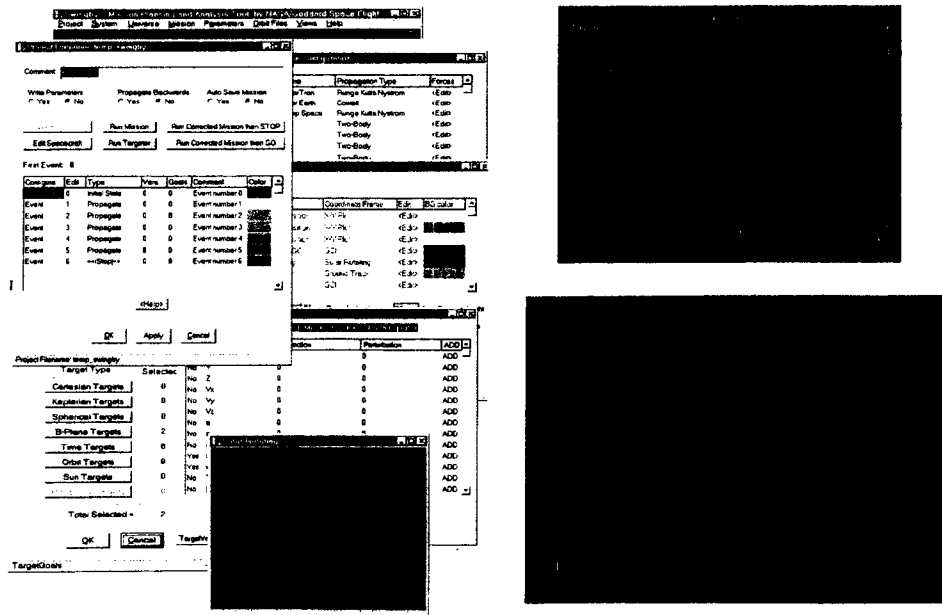

Figure 1. Sample Windows and DOS Versions of Swingby

While this procedure will achieve the required orbit, it is not robust for rapidly changing requirements. In order to decrease the difficulty in meeting mission orbit parameters and constraints in a direct targeting approach, the application of a dynamical system approach is investigated and incorporated into the overall trajectory design technique.

This procedure can also be used for backward targeting, that of using a predefined libration orbit and targeting backward in time to the launch / parking orbit conditions. This procedure also involves the use of a DC to provide maneuvers to attain the mission orbit and parking orbit constraints.

Using parametric scans, DC, and multiple targets, a more robust design can be achieved. Considerations are being given to new strategies that incorporate optimization routines into this scheme to ensure minimal fuel or time requirements can be met. Figure 1 presents sample output of the Windows and DOS versions of Swingby used to support GSFC missions.

Dynamical Systems Approach

As mission concepts become more ambitious, increasing innovation is necessary in the design of the trajectory. Design capabilities for libration point missions have significantly improved in recent years. The success of Swingby for construction of trajectories in this regime is evidence of the improvement in computational capabilities. Nevertheless, conventional tools do not currently incorporate any theoretical understanding of the multi-body problem and do not fully exploit dynamical relationships. An in depth discussion of the versatility of dynamical systems as they apply to libration trajectory design were previously presented and is summarized below with permission.[2]

Invariant Manifolds

Nonlinear dynamical systems theory (DST) offers insights in multi-body regimes, where qualitative information is necessary concerning sets of solutions and their evolution.[2] DST is, of course, a broad subject area with applications to many fields. For application to spacecraft trajectory design, it is helpful to first consider special solutions and invariant manifolds, since this aspect of DST offers immediate insights. Under a GSFC grant, Purdue University investigated various dynamical systems methodologies that now are included in software called Generator. In Generator, different types of solution arcs, some based on dynamical systems theory, are input to a process that differentially corrects the trajectory segments to produce a complete path in a complex dynamical model. A two level iteration scheme is utilized whenever differential corrections are required. This approach produces position continuity and then a velocity continuity for a given trajectory. An understanding of the solution space then forms a basis for computation of a preliminary libration and transfer orbit solution and the end-to-end approximation can then be transferred to a direct targeting methods like Swingby for final adjustments for launch window, launch vehicle error analysis, maneuver planning, or higher order modeling. Our current goal is to blend dynamical systems theory, which employs the dynamical relationships to construct the solution arcs into Swingby or Astrogator with strength in numerical analysis.

The geometrical theory of dynamical systems is based in phase space and begins with special solutions that include equilibrium points, periodic orbits, and quasi-periodic motions. Differential manifolds are introduced as the geometrical model for the phase space of dependent variables. An invariant manifold is defined as an n-dimensional surface such that an orbit starting on the surface remains on the surface throughout its dynamical evolution. So, an invariant manifold is a set of orbits that form a surface. Invariant manifolds, in particular stable, unstable, and center manifolds, are key components in the analysis of the phase space. Bounded motions which include periodic orbits such as halo orbits exist in the center manifold, as well as transitions from one type of bounded motion to another. Sets of orbits that approach or depart an invariant manifold asymptotically are also invariant manifolds (under certain conditions) and these are the stable and unstable manifolds, respectively, associated with the linear stable and unstable modes.

The periodic halo orbits, as defined in the circular restricted problem, are used as a reference solution for investigating the phase space in this analysis. It is possible to exploit the hyperbolic nature of these orbits by using the associated stable and unstable manifolds to generate transfer trajectories as well as general trajectory arcs in this L_2 region of space.

Lissajous-Manifold-Transfer Generation

The computation process of the stable and unstable manifolds, shown in Table 2, is associated with particular halo orbit design parameters and is accomplished numerically in a straightforward manner.

The procedure is based on the availability of the monodromy matrix (the variational or state transition matrix after one period of motion) associated with the lissajous orbit. A similar state transition matrix of this sort can be computed using the state equations of motion based on circular three-body restricted motion. This matrix essentially serves to define a discrete linear map of a fixed point in some arbitrary Poincaré section. As with any discrete mapping of a fixed point, the characteristics of the local geometry of the phase space can be determined from the eigenvalues and eigenvectors of the monodromy matrix. These are characteristics not only of the fixed point, but also of the lissajous orbit. The local approximation of the stable and unstable manifolds involves calculating the eigenvectors of the monodromy matrix that are associated with the stable and unstable eigenvalues. This approximation can be propagated to any point along the halo orbit using the state transition matrix.

Table 2 - Dynamical System Approach Segments

Utility	Input	Output
Phase (Generic Orbit)	User Data	Control Angles For Lissajous
Lissajous	Universe And User Data	Patch Point And Lissajous Orbit
Monodromy (Periodic Orbit)	Universe And Lissajous Output	Fixed Points And Stable And Unstable Manifold Approximations
Manifold	Universe And Monodromy Output	1-Dimensional Manifold
Transfer	Universe, User Selected Patch Points, Manifold Output	Transfer Trajectory From Earth To L_1 Or L_2

The first step is to generate the lissajous orbit of interest. This is indicated in Table 2 by "Lissajous". With this information, the monodromy matrix can then be computed (assuming periodic motion). Also, in the "Monodromy" block, the eigenvalues/eigenvectors associated with the nominal orbit are computed and near the fixed point, the half-manifold is determined to first order, by the stable eigenvector.

The next step is then to globalize the stable manifold. This can be accomplished by numerically integrating backwards in time. It also requires an initial state that is near but not necessarily on the halo orbit. A linear approximation is utilized to get this initial state displaced along the stable eigenvector. Higher order expressions are available but not necessary. A displacement is selected that avoids violating the linear estimate, yet the displacement is not so small that the time of flight becomes too large due to the asymptotic nature of the stable manifold. Note that a similar procedure can be used to approximate and generate the unstable manifold. The stable and unstable manifolds for any fixed point along a halo orbit are one-dimensional and this fact implies that the stable/unstable manifolds for the entire halo orbit are two-dimensional. This is an important concept when considering design options.

With the manifold as an initial guess, one can then perform differential corrections in the Transfer block that meet all the trajectory constraints while achieving an Earth access region.

Figure 2 Generator Menu Samples and Manifold Output

This final step provides the necessary conditions that are used in the numerical shooting process. Figure 2

5

presents some of the menus for the generation of invariant manifolds. This information is then transferred to the numerical operational GSFC tools for further refinement of the trajectory using the highest fidelity models available.

Applications of Numerical and Dynamical Methods

We now investigate the use of the above numerical and dynamical system approaches as they are applied to the JWST, Constellation-X, and two conceptual missions. These examples demonstrate the design of the libration orbit and the transfer orbit.

JWST Trajectory Design: Libration Orbit

The design of the JWST libration point trajectory begins with the generator dynamical system approach.[16] The required y-axis amplitude parameter of 800,000 km is input into the generation of a lissajous orbit. The resulting output as shown in Figure 3 is a result of the lissajous segment. This orbit reflects the use of multiple bodies, semi-analytical elliptical approximation of the orbit, and solar radiation pressure (SRP). The algorithms include parameters of a Richardson-Cary model as a first guess to obtain a differentially corrected orbit using a full planetary ephemeris. The orbit as shown meets all the JWST requirements because this is the starting point versus the end conditions of a shooting method. Figure 3 shows the JWST orbit in an Solar Rotating Coordinate (SRC) frame. It is a class I orbit that has an opening z-axis component. Figure 3 shows the complement of the Sun-Earth-Vehicle (SEV) angle. A maximum of 30^0 and minimum of 4^0 is achieved to meet all lighting constraints.

JWST Trajectory Design: Direct Transfer

Given a libration orbit with the above JWST requirements, a transfer trajectory is sought that will also minimize fuel requirements and incorporate possible JWST constraints. While a trajectory design approach similar to that used for SOHO or ACE can be pursued, the application of a dynamical system approach is investigated and is incorporated into the overall trajectory design technique. Using invariant manifolds and the JWST orbit parameters, libration orbits and transfer paths can be computed; a surface is projected onto configuration space and the three-dimensional plots appears in Figures 4 upside-down to show detail. This particular section of the surface is associated with the "Earth Access region" along the L_2 libration point orbit. An interesting observation is apparent as motion proceeds along the center of the surface. The smoothness of the surface is interrupted because a few of the trajectories pass close to the Moon upon Earth departure.

Lunar gravity assists were not incorporated into the approximation for the manifolds, but no special consideration was involved to avoid the Moon either. Using information available in Figure 4 the one trajectory that passes closest to the Earth is identified and used as the initial guess for the transfer path. The larger size of the Lissajous orbit reduces the Earth passage distance and minimizes any insertion ΔV. Given the initial guess, the transfer is differentially corrected to meet the requirements of achieving both the lissajous orbit and an Earth parking orbit. From this point, the solution is input directly into numerical tools and appears in Figure 5. Swingby/Astrogator and other tools are used for further visualization, analysis of launch vehicle and maneuver errors, midcourse corrections, and other design considerations.
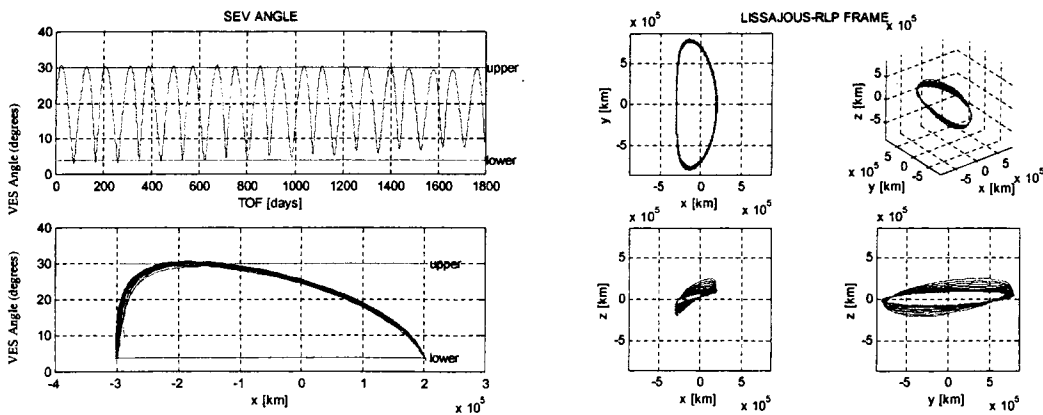

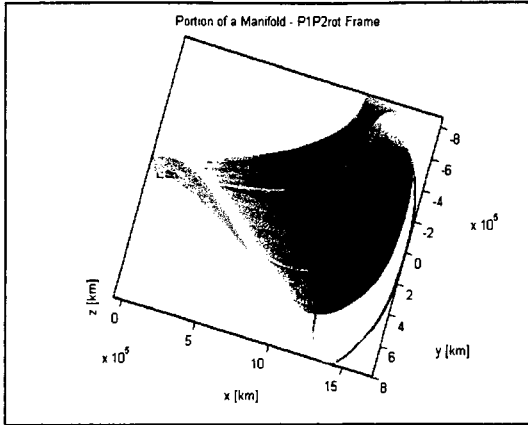**Figure 3 - Lissajous Pattern and Sun-Earth-Angles for 800,000 Km Orbit**
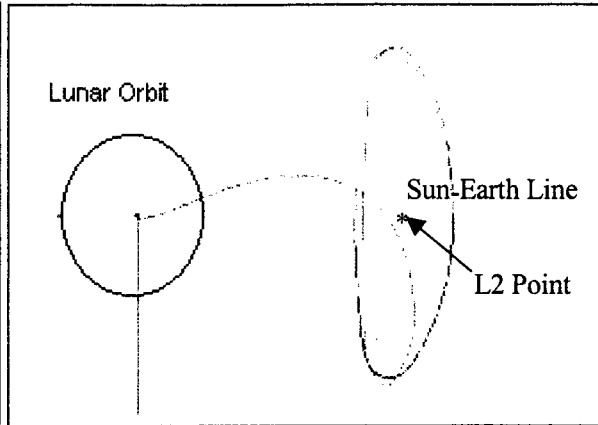
Figure 4. JWST 800km Y-axis Amplitude

Figure 5. Numerical Targeting

Continuous Low Thrust Options

Alternative JWST trajectory options have been investigated recently. They include a low thrust propulsion system and possible L2 servicing options. Low thrust trajectory solutions exist for the collinear libration points and have been analyzed.[17]   The trajectory generally consists of spiraling out to lunar orbit with periods of thrusting and coasting and targeting the post-lunar leg to insert into the periodic orbit by varying coast times. The thrust can be along the velocity vector or at an angle to it to achieve maximum efficiency.

The problem with most low thrust trajectory designs however is the extensive time-of-flight. This is amplified by the mass of JWST which in this analysis is in the vicinity of 10,000 kg. Most low thrust engines would take many years to raise the orbit to lunar distances. A recent investigation included the use of nuclear powered electric propulsion. The proposed system would provide 1.2 N of thrust at an Isp of 4800 sec. At this thrust level, an JWST trajectory design was completed with 510 days of continuous thrust followed by an 85-day coast into the nominal L2 orbit. Figure 6 shows the ecliptic plane view of the transfer. A small insertion maneuver is also required.
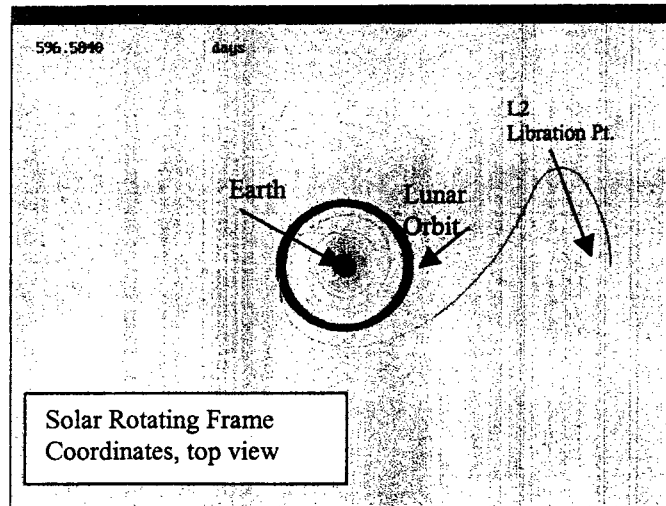

Figure 6. JWST Trajectory Design: Low Thrust Transfer

The design of this trajectory was accomplished using the numerical process of Swingby to model the continuous propulsion system, maintain the proper attitude profile, and perform a shooting method to achieve the libration orbit goals. While the DC method was chosen for this analysis, other utilities can be used to optimize transfer time (coast and finite burn sequence) or minimize fuel for the trajectory. Also generator can be used to setup initial conditions for the parking orbit and final targets.

Earth Return / Servicing missions

The possibility of L2 servicing brings up numerous scenarios. One such scenario studied recently is to return JWST to LEO to be serviced at the ISS. Unstable manifolds from the nominal L2 orbit that pass near the Earth are used as initial estimates. The manifold is targeted to meet inclination and dynamic pressure constraints. A large drag apparatus would be used to aerocapture at the Earth. After first perigee at the Earth, an apogee maneuver would be required to retarget perigee to the original 107 km altitude. After three perigees, perigee altitude remains constant and the spacecraft is aerocaptured within 4 days. Figures 7 and 8 show the transfer to and from L2 in the rotating frame and in the inertial frame near the Earth.
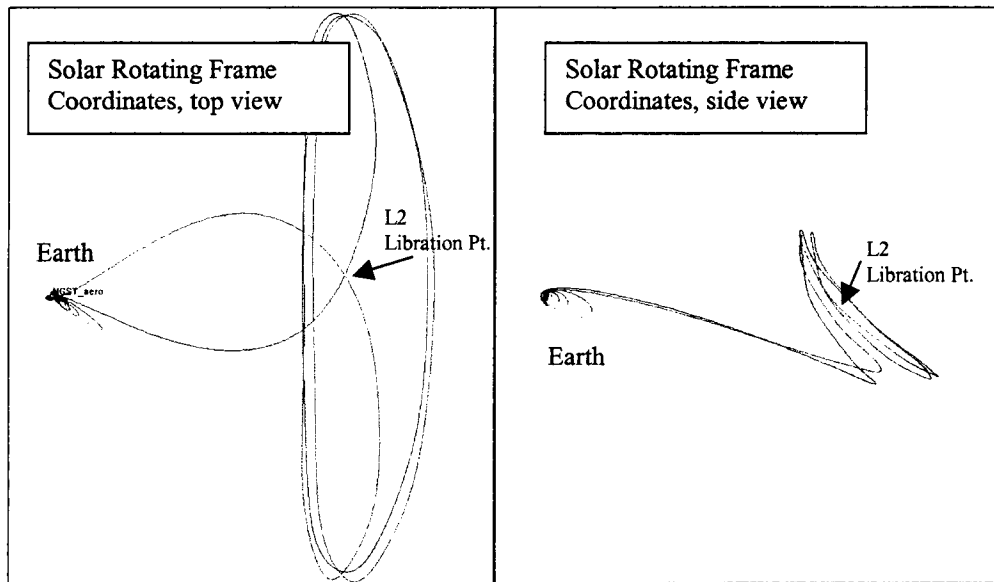

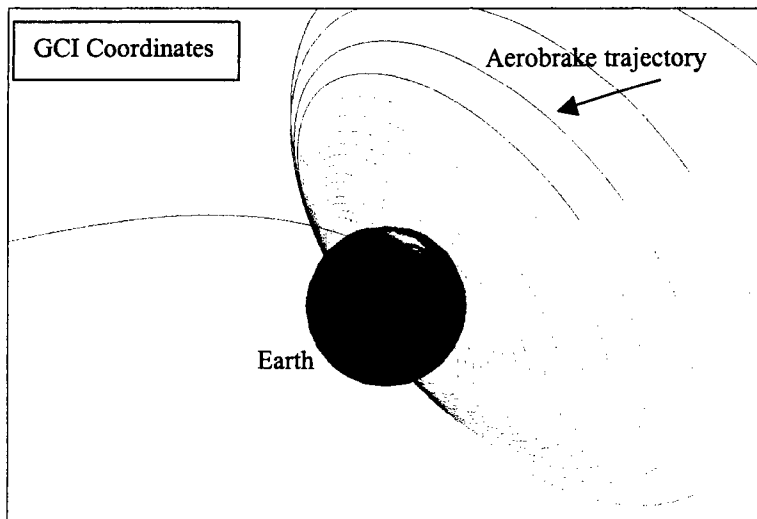**Figure 7. JWST Trajectory Design: Servicing Return with Aerobraking Transfer**

Figure 8. JWST Trajectory Design: Servicing Return with Aerobraking Transfer

Constellation-X

Constellation-X is more challenging. It involves a scientific desire to have four spacecraft in relative close proximity to one another while the transfer design requires a lunar gravity assist [18,19]. The mission separations are not determined as of this date, but initial goals indicate separations of greater than 50km but less than 50,000km. The spacecraft must maintain this separation throughout the mission. Thus part of the trajectory design challenge is to launch two spacecraft from one launch vehicle and perform a lunar gravity assist to attain a libration orbit that meets a relative formation requirement. The mission orbit was chosen primarily to meet the following sky coverage requirement, the mission orbit and attitude constraints must be such that 90% of the sky is accessible at least twice per year, with viewing windows not shorter than 2 weeks in duration; and 100% of the sky is available at least once a year with a minimum viewing window of one week.

The spacecraft will be inserted into the Lissajous orbit via a lunar swingby. The lunar swingby is necessary in order to reduce the amount of onboard ΔV and the C3 (launch energy) needed from the launch vehicle. Smaller (more negative) values of C3 yield a larger payload capability. In order to increase the number of launch opportunities, a number of phasing loops will be performed prior to the lunar swingby. Figure 9 shows the Constellation-X transfer trajectory and the characteristics of this approach assuming 3 ½ phasing loops. Different numbers of loops could be considered for various launch days to increase the number of launch opportunities.

Under a current concept, two Constellation-X spacecraft will be placed in a highly eccentric injection orbit by the launch vehicle. Maneuvers will be performed roughly centered on the phasing loop apses, using the spacecraft propulsion system, to properly time the spacecraft's encounter with the moon. The timing and geometry of the lunar encounter will be chosen to allow the spacecraft to be inserted into the L_2 Lissajous orbit with little or no maneuver required and still meet separation requirements. The trajectory design was computed using Swingby.

No requirement has been specified regarding the depth or duration of acceptable Earth or lunar shadows. In designing the nominal trajectory, an effort will be made to minimize shadows without significantly increasing total ΔV during all phases of the mission. However, some shadows may be unavoidable during design, or others may crop up during flight due to contingencies. Phasing loop maneuver adjustment will be used to mitigate cruise phase (after lunar swingby but prior to lissajous insertion) shadows post-launch. Unacceptable shadows in the lissajous orbit may be avoided through propulsive maneuvers, however no fuel has been budgeted for that purpose.

The total ΔV required in this particular example is approximately 160 m/sec per spacecraft. This includes correcting for launch vehicle errors, targeting the lunar swingby, mid-course correction maneuvers, Lissajous orbit insertion and station-keeping maneuvers. Table 3 details the ΔV budget for each spacecraft with a launch that assumes a C3 of -2.60km^2/s^2.
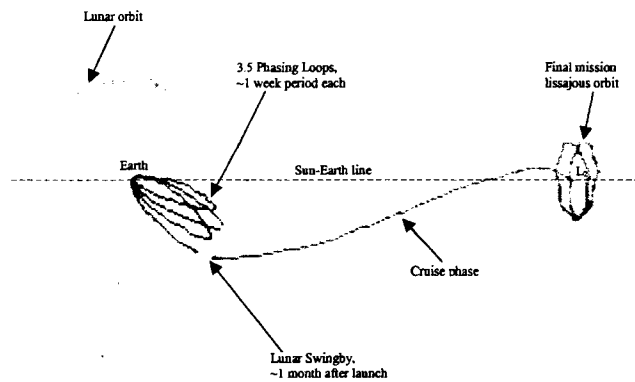

Figure 9. Constellation-X Transfer Trajectory

Conceptual Missions

GSFC's Flight Dynamics Analysis Branch performs many future mission studies in the conceptual phase. One recent analysis included a mission to study the distant geotail between the Earth-Sun L_2 point. The investigator wanted to obtain coincident measurements of the geotail over a wide spatial region from L_2 to about twice L_2. The resulting trajectory design uses the periodic orbits about L_2 to initiate slightly perturbed trajectory arcs that all cross into the geotail within a week of each other as shown in figure 10. A mother ship would release 16 small spacecraft into perturbed orbits, each cumulatively 10 m/s in along-

track ΔV off the periodic orbit. The mother ship would spend 333 m/sec in ΔV during the release phase over four days including returning to the periodic orbit. The small spacecraft would not have any propulsion system.

Table 3: Sample Constellation-X Delta-V Estimated Budget

|	Spacecraft-1 Delta-V (m/s)	Spacecraft-2 Delta-V (m/s)	Assumptions / Comments
LV Error Correction	20	20	Performed between TTI+8hrs to P2
Deterministic Phasing Loops	35	39	Lunar swingby timing
A1 ΔV	0	0	
P1 ΔV	25	25	
A2 ΔV	5	5	
P2 ΔV	0	3	
T3 ΔV	5	6	
P3 ΔV	0	0	
Midcourse Corrections	4	4	Post-lunar swingby
Libration Orbit Insertion	2	2	Assumes MidCourse Correction successfully remove LOI targeting errors
Station-keeping	40	40	Estimated at 4 maneuvers/yr for 10 years
Other	10	10	Calibration burns, etc.
Subtotal	**111**	**115**	
Finite Burn Losses, Momentum Dumping, ACS	22	23	~20% of above
Subtotal	**133**	**138**	
Contingency (Launch window, etc.)	13	14	~10% of above
Subtotal	**146**	**152**	
Cosine Losses	4	5	Assumes all thrusters canted 10°
Total	**151**	**157**	
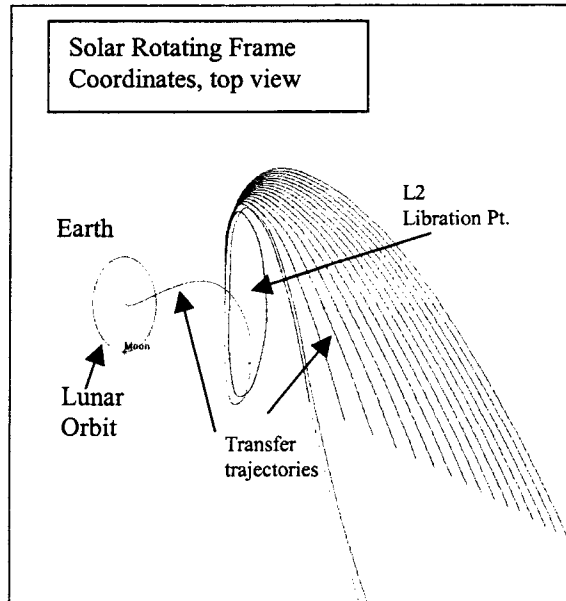
Figure 10 Conceptual Trajectory Design to Twice L2 Distance

LIBRATION FORMATION FLYING

In addition to the need for improved numerical and dynamical system approaches to libration trajectory design, capabilities are required to meet new multiple spacecraft mission goals of interferometer and optical measurements and need to include new methods for operational application to support the trajectory design. As seen in Table 1, there are at least five missions that require formations. In combination with dynamical and improved direct methods, algorithms are being developed for complete 'system' control of formation flying spacecraft. These methods employ linear and non-linear feedback control systems that can be managed to analyze cooperative spacecraft. Currently, formation flying spacecraft control is being extensively researched and has been demonstrated autonomously for in low Earth orbit. A growing interest in formation flying satellites demands development and analysis of control and estimation algorithms for stationkeeping and formation maneuvering. This development of controllers, such as discrete linear-quadratic-regulator control or non-linear approaches for formations in the vicinity of the co-linear sun-Earth libration point will be necessary. This development may include an appropriate Kalman filter as well.

Formation flying control can be performed in three ways—centralized, decentralized, or in combinations. With centralized control, one spacecraft or processor calculates and commands the motion of the entire formation. With decentralized control, each spacecraft, with input from the rest of the formation, processes its own control requirements. Speyer first introduced a decentralized linear-quadratic-Gaussian control method[20]. Folta and Carpenter applied this work to formation flying satellites, and further expanded it to deal with both time-invariant and time-varying systems[21]. Speyer's method produces identical results to the centralized linear-quadratic-Gaussian control method, and it also minimizes data transmission.

NASA has several distributed spacecraft libration missions planned for the next decade and beyond. The Stellar Imager[22], Constellation-X, and MAXIM will image stars and black holes while TPF will look for planets. They also rely on the capability of correctly modeling the dynamics of the libration region and the inclusion of the formation control method into the overall picture. For example in a recent libration formation flying research using the Stellar Imager mission that uses 31 spacecraft in close formation at L_2 as a baseline, the dynamics of the libration region were incorporated into the control state space to ensure accurate modeling and therefore more accurate control results. The modeling of the dynamics creates much more realistic Lissajous orbits than those derived from the circular restricted three-body problem. Also one can numerically compute and output the dynamics matrix, for a single/multiple satellite at each epoch. This matrix is not computed from the pseudopotential but is a monodromy matrix which incorporates the full perturbations and third body ephemeris data.

Stellar Imager Example

Stellar Imager (SI) is a concept for a space-based, UV-optical interferometer. The purpose of the mission is to view many stars with a sparse aperture telescope in an attempt to better understand the various effects of stars' magnetic fields, the dynamos that generate them, and the internal structures and dynamics of stars.

The leading concept for SI is a 500-meter diameter Fizeau-type interferometer composed of 30 small drone satellites that reflect incoming light to a hub satellite. The hub will recombine, process, and transmit the information back to Earth. As Figure 11 shows, in this concept, the hub satellite lies halfway between the surface of a sphere containing the drones and the sphere origin. Focal lengths of both 0.5 km and 4 km are being considered. This would make the radius of the sphere either 1 km

Figure 11. Stellar Imager Geometry

or 8 km. The type of orbit and location in space is an important part of mission design. The best orbit choice for the formation

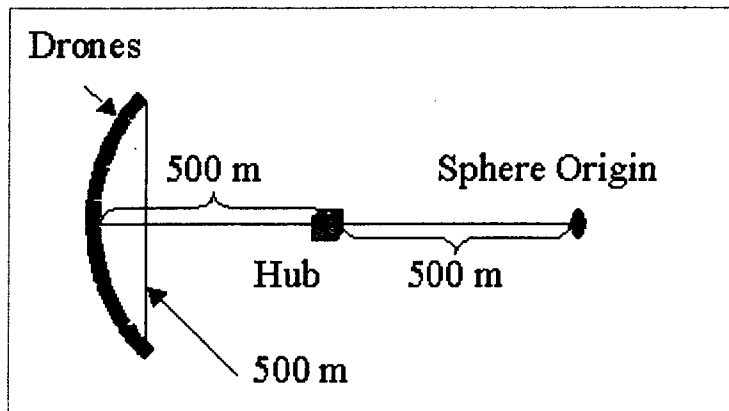able to cover the entire sky every half year while maintaining an aim perpendicular to the sun. For useful imaging, SI must aim within 10 degrees of perpendicular from the sun.

To function properly, SI will need to accommodate a wide range of control functions. In addition to maintaining its desired trajectory around L2, the formation must slew about the sky requiring movement of a few kilometers and attitude adjustments of up to 180 degrees. While imaging, though, the drones must maintain position within 3 nanometers of accuracy in the direction radial from the hub and within 0.5 millimeters of accuracy along the sphere surface. The accuracy required for attitude control while imaging is 5 milli-arcseconds tip and tilt (rotations out of the surface of the sphere). The rotation about the axis radial from the hub (rotation within the sphere) is a much less stringent 10 degrees.

SI Formation Flying Results

A common approximation in research of this type models the dynamics of a satellite in the vicinity of the sun-earth L2 point using the circular restricted three-body assumptions. These assumptions only account for gravitational forces from the sun and Earth. The moon is also included, but not as an independent body. The masses of the earth and moon are combined and assumed to be at the Earth-moon barycenter. The motion of the sun and the Earth-moon barycenter is also assumed to be circular around the system barycenter. There is extensive literature available on the application of linear quadratic regulator (LQR) control to formation flight in the two-body problem (2BP) and also a few examples of its application in the circular-restricted three-body problem (CR3BP). The numerical implementation of this type of control is often difficult because of the assumptions in its original development are based on two-body dynamics, general optimal control theory, and calculus of variations. The most significant of these assumptions is that the nonlinear system dynamics are linearized relative to a constant equilibrium solution. This results in a linear system that assumes constant matrices. In spite of this critical assumption, the available literature includes numerous examples in which this result is extended to time-varying systems, where the system matrices are actually time varying. Although the controller may appear to work on a case-by-case basis, most of the available research on formation flight to date provides no sound mathematical justification for this extension. Furthermore, since the most essential assumption in the development of this controller is violated, it is safe to say that the resulting controller is not truly optimal. To properly apply LQR control to formation flight in the three-dimensional CR3BP, particularly in the vicinity of periodic orbits near L_1 and L_2, it is necessary to account for the time-varying nature of the linearized dynamics; for a halo orbit, the system matrix is periodic.

This SI analysis uses high fidelity dynamics based on a dynamical systems simulation. This creates much more realistic Lissajous orbits than those derived from the circular restricted three-body problem. Using ephemeris files, we take into account the

Figure 12 Generator Reference Orbit

effects of eccentricity, an independent moon, the other planets of the solar system, and solar radiation pressure. The resulting Lissajous orbit can then be used as a more accurate reference orbit. In addition to providing the reference positions and velocities, the dynamics matrix is also numerically computed at each epoch. The SI reference orbit is shown in Figure 12 with the Earth as the origin. The X coordinate connects the two primary bodies, the Z coordinate is parallel to their angular velocity of the system, ω, and the Y coordinate completes a right-handed system.

Three different scenarios make up the SI formation control problem—maintaining the Lissajous orbit, slewing the formation to aim at another star, and reconfiguring the formation to take another snapshot of a star when necessary. Following the Lissajous orbit is not a problem of formation control, but rather a problem of maintaining an orbit. Therefore, only the hub satellite needs simulation to determine the
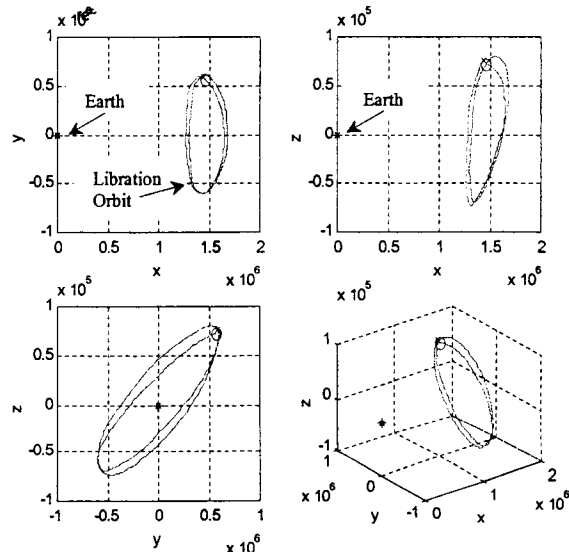
amount of control and fuel needed to maintain a Lissajous orbit. The results can be extended similarly to other satellites in the formation.

Formation Maintenance Results

Averaging the determined ΔV from a dozen simulations, the ΔV required to keep a satellite in a Lissajous orbit about L2 for two orbital periods is approximately 0.38 meters per second. A key part of the SI mission is to image many stars. Following a Lissajous orbit around L2, SI could view the entire sky approximately every half-year while slewing about just the radial (x) axis. This maintenance method will also maintain the aiming angle perpendicular to the sun. The formation slewing simulation follows a similar algorithm as the Lissajous orbit simulation. Figure 13 provides an image of the entire SI formation slewing 90 degrees, with a 0.5 km focal length.
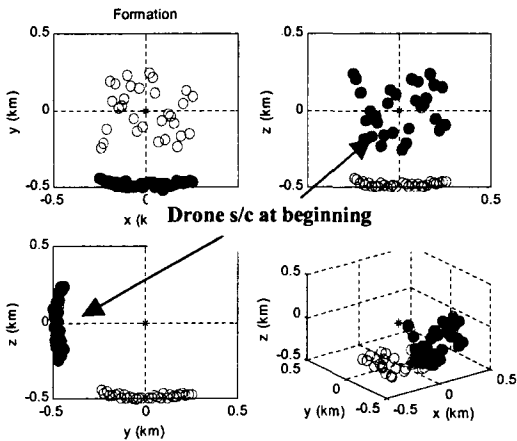

Figure 13 SI Slewing Configuration

The black dots represent drones at the beginning of the simulation, and the red circles represent drones at the end of the simulation. The hub is the black asterisk at the origin. The upper-right plot illustrates the Golomb[15] rectangle formation projected into the x-z plane. The lower-left plot clearly shows the drones slewing 90 degrees about the hub-centered x axis.

The formation slewing simulation runs for one day, with one maneuver per minute (1440 maneuvers), whereas the Lissajous orbit simulation runs for 359 days with one maneuver per day. Table 4 shows the average ΔV 's for a dozen simulations for the various scenarios. The larger the angle the formation slews through, the more ΔV is needed. Also, the larger the focal length, the more ΔV required.

Table 4. Average formation slewing ΔV 's

Focal Length (km)	Slew Angle (deg)	Hub ΔV (m/s)	Drone 2 ΔV (m/s)	Drone 31 ΔV (m/s)
0.5	30	1.0705	0.8271	0.8307
0.5	90	1.1355	0.9395	0.9587
4	30	1.2688	1.1189	1.1315
4	90	1.8570	2.1907	2.1932

FUTURE MISSION DESIGNS NEEDS

Improved Tools

If one includes search methods and optimization in numerical and dynamical approaches, a full system architecture can be made for design for libration missions that include single and multiple spacecraft, and Human Exploration and Development of Space (HEDS) missions. New search methods, such as genetic and simulated annealing algorithms, combined with indirect and direct optimization techniques can be applied to best meet scientific and HEDS libration formation flying requirements. These tools would provide the best-case scenario for the formation or orbit type, fuel cost, and transfers to minimize overall system cost. While the roadmap is still being worked, the outlook is very promising. Figure 14 gives a possible path to a level that incorporates all the best capabilities of numerical and

dynamical methods. The idea is to merge the best of current targeting, optimizing, and control applications. Additionally, these tools must be able to interface with one another.

Innovative Trajectory Design Concepts and Visualization

As numerical and dynamical systems are improved to incorporate high fidelity modeling of dynamics of libration points, new optimal targeting schemes that include direct and indirect methods or stochastic approaches, a new class of missions and capabilities will emerge just as over the past 25 years. There are several notions on the horizon that represent challenges. These incorporate research on quasi-stationary orbits[23], the use of weak stability boundary dynamics[24], heteroclinic/homoclinic trajectories[25], and improved numerical propagation schemes for formations of spacecraft, and stochastic optimization.

Also, as computer capabilities improve, the analysis and design of libration orbits should become more ordinary, just as the thought of designing an Earth orbiter was once viewed with awe. These improvements can be seen already in applications using 3-dimensional graphics which can be rotated to give the analyst a more intuitive approach to meeting mission requirements and goals. The use of intelligent systems that can be used in automation and multiple constraint checking is starting to find its way into everyday analysis, which will help the analyst.
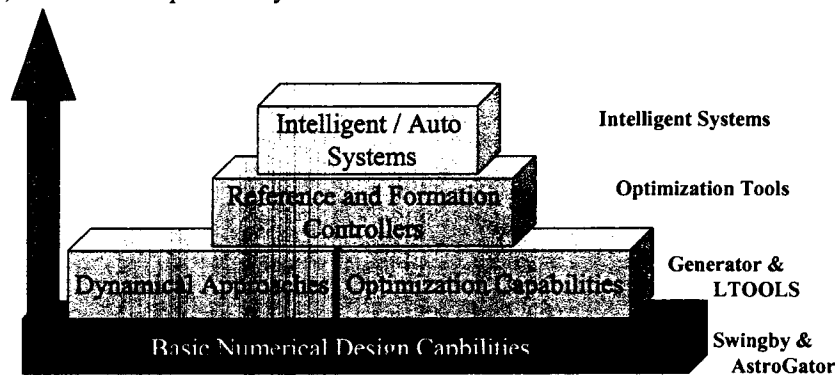

Figure 14 Required Capabilities

CONCLUSIONS

Trajectory design in support of libration missions is increasingly challenging as more constrained mission orbits are envisioned in the next few decades. Software tools for trajectory design in this regime must be further developed to incorporate better understanding of the solution space, improving the efficiency, and expand the capabilities of current approaches. Improved numerical and dynamical systems offers new insights into the natural dynamics associated with the multi-body problem and provide to methods to use this information in trajectory design. The goal of this effort is the blending of analysis from dynamical systems theory with the well-established NASA Goddard software programs such as Swingby to enhance and expand the capabilities for mission design and to make trajectories more operationally efficient.

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