INITIALIZATION OF FORMATION FLYING USING PRIMER VECTOR THEORY

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ABSTRACT – *In this paper, we extend primer vector analysis to formation flying. Optimization of the classical rendezvous or free-time transfer problem between two orbits using primer vector theory has been extensively studied for one spacecraft. However, an increasing number of missions are now considering flying a set of spacecraft in close formation. Missions such as the Magnetospheric MultiScale (MMS) and Leonardo-BRDF (Bidirectional Reflectance Distribution Function) need to determine strategies to transfer each spacecraft from the common launch orbit to their respective operational orbit. In addition, all the spacecraft must synchronize their states so that they achieve the same desired formation geometry over each orbit. This periodicity requirement imposes constraints on the boundary conditions that can be used for the primer vector algorithm. In this work we explore the impact of the periodicity requirement in optimizing each spacecraft transfer trajectory using primer vector theory. We first present our adaptation of primer vector theory to formation flying. Using this method, we then compute the ΔV budget for each spacecraft subject to different formation endpoint constraints.*

1 - INTRODUCTION

Currently more than a dozen missions, each involving flying a set of spacecraft in formation, are envisioned [Leit 02]. Clusters of spacecraft with a carefully designed relative motion enable a wide variety of applications ranging from co-observation of a location on the Earth (e.g. Leonardo-BRDF, Calipso, EO-1) to interferometry and improved coverage for communication and surveillance (TechSat21) to multi-point measurement of the Earth space environment (e.g. ClusterII, MMS, Aurora Lites, ST5). This concept has raised new challenges in designing and flying a mission. In the orbit design arena, three main topics have been identified: selecting the initial conditions for each spacecraft to best satisfy the science goals, initializing the formation from a common launching orbit, and maintaining the formation configuration in presence of the perturbations such as the Earth's gravitational field, atmospheric drag, etc. Even though this paper will focus on the initialization, the three areas listed above are, in most cases, coupled [Scha 99]. Increasing progress in nano-satellite technology has made multi-satellite launches feasible. The Cluster II mission successfully launched its four spacecraft formation using two dual-launches one month apart. A series of phasing maneuvers was used to initialize its tetrahedron-like formation [Dow 01]. The ST5 mission, composed of 3 nano-spacecraft, envisions sharing a launch payload with a regular size satellite to drive its launch cost down [ST5]. In this paper, we assume that all the spacecraft are launched simultaneously into a common orbit referred to as the *launching orbit* at a common epoch t_o. We define formation flying as a set of spacecraft with designed *periodic* collaborative dynamics. This periodicity requirement imposes constraints on the boundary conditions that can be used for the primer vector algorithm. Ideally, one would search for the subset of initial conditions that minimizes the initialization and maintenance fuel cost of the formation while satisfying the science goals. However, in the paper, we assume that this search has already been performed and a set of formation initial conditions found. We focus on the optimization of the ΔV cost to transfer from the launching orbit to these initial conditions using primer vector theory [Lawd 63], which is an optimization scheme based on calculus of variations. It has several appealing features including the indication of when and where to add an impulse to the trajectory and a visual assessment of the optimality of neighboring solutions. In the next section of this paper, we present our formulation of the minimum fuel problem with a brief review of primer vector history along with the main challenges encountered in its implementation. Then, we introduce our approach to solve the formation flying initialization problem using primer vector theory with various types of boundary constraints. In the following section, we discuss the preliminary results obtained using our in-house development Primer Vector Analysis Tool (PVAT) to initialize a 'Leonardo-type' formation of six spacecraft. Finally, we talk about the domain of applicability of such an approach for formation flying initialization and its promising marriage with a more global optimization methodology such as genetic algorithms.

2 - MINIMUM FUEL PROBLEM

2.1 - Primer Vector Theory and Implementation

In this section, we present a brief review of the primer vector theory as well as the main challenges involved in its implementation in PVAT [Guzm 02]. Primer vector theory takes its roots in calculus of variations and requires a set of first order *necessary* conditions, which the trajectory has to meet to be locally optimal. The necessary conditions, first derived by Lawden, are expressed in terms of the primer vector, which is defined as the adjoint to the velocity vector in the variational Hamiltonian formulation [Lawd 63]. If any of Lawden's conditions are violated, the transfer trajectory is not optimal and we can use the primer vector history to obtain information on how to improve its ΔV cost. Lawden, in his initial work, solved a fixed-time rendezvous and his theory was further extended by Lion and Handelsman and later, by Jezewski and Rozendal to solve the N-impulse optimal transfer problem [Lion 68][Jeze 67]. A more detailed derivation of the primer vector theory can be found [Hida 92]. As a prerequisite to the formation flying problem, we look at how to solve the *single* spacecraft "free-time" transfer problem while minimizing its mission ΔV cost subject to the mission constraints. Assuming an N-impulse trajectory, the problem translates to:

$$\text{Find} \quad Min\ \mathrm{J} = \sum_{i=1}^{N} \Delta V_i \qquad \textit{subject to}\ \ddot{\mathrm{x}}(\mathrm{t}) = f(\dot{x},x,t), \tag{1}$$

where $f(\dot{x},x,t)$ represents the spacecraft two-body problem equations of motion. Equation (1) models the "free-time" problem (*i.e.* there are no direct constraints on the departure epoch or the arrival epoch). The "fixed-time" problem represents a special case of Equation (1) with additional constraints on the initial and final time of the transfer. The problem defined, the primer vector equations can now be expressed using calculus of variations. The initial trajectory or first-guess, is labeled as the *reference* trajectory. Since primer vector is a first-order theory based on local variations, it will converge on local optimal neighboring trajectories of the *reference* trajectory. Therefore, the optimal solution is highly dependent on the reference trajectory but it will also depend on other design parameters discussed later in this paper. The primer vector obeys the second order canonical form of the Euler-Lagrange equation and its state cannot be integrated simultaneously as it is coupled with the spacecraft state. Solving for the primer vector history is equivalent to solving a two-point boundary value problem (TPBV). It can be demonstrated that the primer vector state $(\bar{p},\dot{\bar{p}})$ can be obtained from an initial state, $(\bar{p}_o,\dot{\bar{p}}_o)$, using $\phi(t,t_o)$ the satellite trajectory state transition matrix (STM). A STM developed by Der [Der 97], which is valid for arbitrary conics (i.e. the two-body problem), was implemented. Lawden derived four necessary conditions using the primer vector for an optimal rendezvous trajectory (i.e. "fixed-time" problem): (1) the primer vector and its first derivative must be continuous for the entire history, (2) the magnitude of the primer must be less than one during a coasting phase and equal to one when an impulse is performed, (3) at the impulse time, the primer is a unit vector in the thrust direction and (4) the derivative of the primer vector magnitude must be zero for all interior impulses (i.e. not the initial or final impulse). Once the primer vector history is computed, we can determine the optimality of the trajectory using the four Lawden necessary conditions. For a *non-optimal* primer vector history, two types of actions are possible to lower the ΔV cost: (I) moving the initial or final impulse (i.e. changing the time-of-flight) and (II) adding and/or moving an interior impulse. If the epoch of the endpoints is unconstrained, an action of type I is performed whenever the initial and/or the final primer vector magnitude slope is different from zero. If the primer vector magnitude goes above one during a coasting phase on a given segment, an impulse is added to the trajectory (action type II) to improve the overall ΔV budget. This impulse is examined by considering a neighboring path with an additional impulse. To get the most efficient decrease in cost, the impulse should be added at the time t_{max} where $|\bar{p}|$ takes on its maximum. A midcourse impulse is moved whenever the primer vector velocity and the Hamiltonian are discontinuous at the intermediate node (t_m, $\bar{r}_m$) being considered. This midcourse impulse is examined by considering a neighboring path with an altered mid-impulse position and time. A first-order gradient of the cost function J expressed in terms of primer vector parameters is used in conjunction with a Broyden-Fletcher-Goldberg-Shanno minimization technique to move the midcourse impulse (t_m, $\bar{r}_m$). The primer vector theory described above was implemented in PVAT. PVAT was developed in MATLAB and has a fully automated algorithm, which iterates following the primer vector principles to optimize a non-optimal reference trajectory. Usually a two-burn Lambert solution is input as a *reference* trajectory. Whenever multiple actions are possible, the algorithm always favors a change in time-of-flight versus adding an impulse to keep the number of burns to a minimum. However, there is no mathematical theory (of which we know) that determines the optimal sequencing of the actions and, in general, different sequencing will lead to different neighboring solutions as discussed in the results section. This paper will impose a constraint on the endpoints only, which translates into restricting potential improvement via an action type I. To better understand the impact of those constraints on the trajectory cost, we provide a more detailed derivation of the cost function J variation in terms of the primer vector parameters at the endpoints in the following section.

2.2 - Formation Flying Problem Formulation

In this section, we present our formulation of the boundary constraints to optimize a cluster of spacecraft using primer vector. After describing the different formation constraints envisioned, we provide a more detailed derivation of the cost function variation in terms of the primer vector parameters at the endpoints. Taking the first order gradient of the cost function enables us to better understand the implication of restricting the endpoint states and therefore, we can make wiser choices as to which constraint is more suitable for our problem at hand. In this paper, we assume that *all* the spacecraft are launched into a common orbit referred to as the *launching orbit* at a common epoch t_o. Consequently, no maneuvers can be performed earlier than t_o. Without loss of generality, the common launch epoch is set to zero and any subsequent epoch expressed as elapsed time from t_o. In general, when designing a formation, we must first define a reference spacecraft, fictitious or not, about which the formation spacecraft relative motion is specified at a given epoch t_{ref}. Because of its inherent periodic nature, knowing the formation state at some arbitrary time t_{ref} and its period P, enforces all subsequent formation states, which can be expressed as:

$$For\ t > t_{ref} \quad \forall\, i \in [1,\ldots Nsc] \quad \mathsf{F}_i(t_{ref}+P) = \mathsf{F}_i(t_{ref}) \tag{2}$$

where F_i represents the i^{th} spacecraft state relative to the reference, *Nsc* the total number of spacecraft in the formation, t_{ref} the time at which the formation initial conditions are specified and *P* the formation period. In this paper, the formation is to be first initialized at an arbitrary common epoch t_f, which implies that, all the spacecraft *reference* trajectories start with the same time-of-flight *tof*. Any given spacecraft reference trajectory will be constrained to the formation $\{\mathsf{F}_i\}$ specified at t_f. To summarize, the initial endpoint constraint is imposed by the launch vehicle jettison location at t_o and the final endpoint constraint is imposed by the formation periodicity. Once the initial common endpoints are defined, there are three possible scenarios: (constraint I) the cluster spacecraft must all leave at t_o and be initialized at t_f ('strict' rendezvous), (constraint II) the spacecraft must reach their initial state at t_f, and no maneuver should be performed later than t_f and (constraint III) the initialization of the spacecraft can happen at an epoch later than t_f, which is determined by PVAT to minimize the formation ΔV cost. In general, in the pre-launch phase, without any consideration of operational constraints, a type III constraint should always be considered. However, a type II or even type I constraint might be applied in case of an initialization maneuver failure or multiple operational and science data collection constraints. Figures 1a-d illustrate the constraint III concept through a simple scenario involving a formation of three spacecraft. Note that for constraint III, the formation will not be initialized until the last maneuver of the cluster set is performed. Providing that the formation flying constraints or *common initial endpoints* are set properly, each spacecraft trajectory can be optimized individually by PVAT. The final maneuvering history sequence as well as the formation initialization time will be fully known only when all the spacecraft have been optimized. To better understand how to handle the different constraints with primer vector theory, we examine the first-order cost variation δJ, as a function of changes in the departure time and in the arrival time:

$$dJ = -\dot{\bar{p}}_o^T \cdot \bar{p}_o \left|\Delta\bar{v}_o\right| \cdot dt_o - \dot{\bar{p}}_f^T \cdot \bar{p}_f \left|\Delta\bar{v}_f\right| \cdot dt_f \tag{3}$$

where $(\bar{p}_o, \dot{\bar{p}}_o)$ and $(\bar{p}_f, \dot{\bar{p}}_f)$ are the primer vector states at the departure and arrival orbit respectively. It was shown that $\dot{\bar{p}}^T \cdot \bar{p}$ is equivalent to the slope of the magnitude of the primer vector [Hida 92]. By inspection of Eq (3), if the initial slope of the primer magnitude is positive then a first-order decrease in ΔV (i.e. dJ < 0) is obtained by a positive change in initial time (i.e. the departure burn should be performed later) and vice-versa if the initial slope is negative. Therefore, in the "free-time" transfer problem, whenever the initial and/or final primer vector slope is different from zero, we can improve the total trajectory cost. Table 1 lists the allowed changes in initial and final epoch for the three different constraints.

Table 1. Endpoint Constraints Summary.

	dt_o		dt_f	
	+ (Burn Later)	- (Burn Sooner)	+ (Burn Later)	- (Burn Sooner)
Constraint I	*No*	*No*	*No*	*No*
Constraint II	Yes	*No*	*No*	Yes
Constraint III	Yes	*No*	Yes	Yes

Some action for improvements being forbidden by the boundary constraints, the three different constraints will have a different action sequence in the primer vector tool and in some cases, will converge to completely different neighboring trajectories with various numbers of burn. For example, constraint I, the strictest of all three, will only allow adding/moving an internal impulse. Note that in our problem we arbitrarily assumed two variables to be known: (1) the true anomaly (θ_o) at which the launch vehicle jettisons the spacecraft (2) the epoch at which the formation initial state is known (t_f). The first variable controls the initial geometry of the transfers and the second their initial energies. In mission such as MMS those variables are mostly fixed by other constraints in the mission such as dwell time in the magneto-tail, shadow minimization etc. However, other missions such as Leonardo-BRDF have currently more freedom in picking those parameters

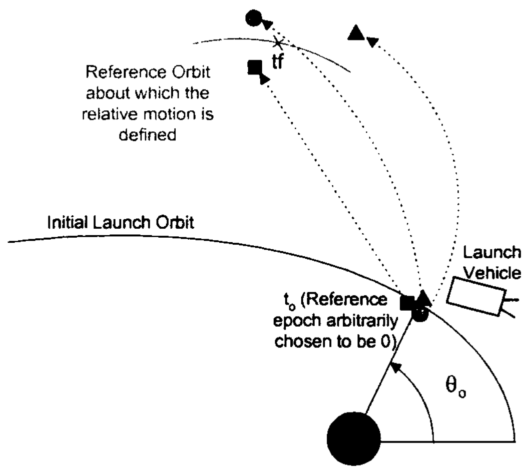

Fig. 1a Formation initialization for a given initial true anomaly in the launching orbit (θ_o) and final time (t_f) at which the formation initial conditions are defined. This initial boundary problem is used as a first-guess for the primer vector code PVAT using a two-impulse Lambert scheme. Each spacecraft is optimized individually as shown in this schematic with three spacecraft in formation with their relative motion defined about a reference orbit.

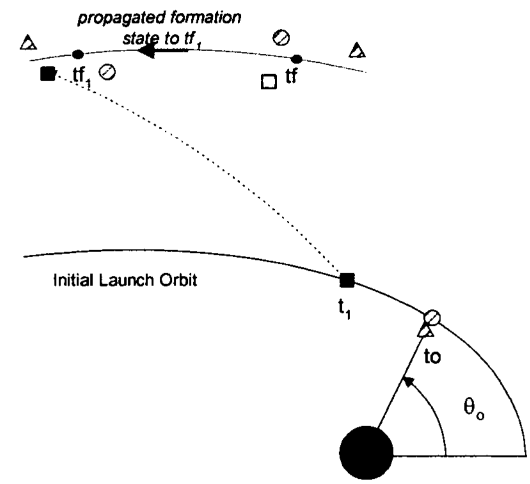

Fig. 1b Spacecraft 1 (represented as a square) initial two-impulse transfer is optimized using PVAT. The final neighboring optimal trajectory is composed of a first burn at $t_1 > t_o$ and a final burn at $tf_1 > tf$. This implies that the formation will not be fully initialized at any time earlier than tf_1. Spacecraft 2 (represented as a circle) and spacecraft 3 (represented as a triangle) are not yet optimized which is indicated by a shape with a stripe pattern. At tf, spacecraft 1 is not in formation yet which is represented in this schematic by a shape without pattern.

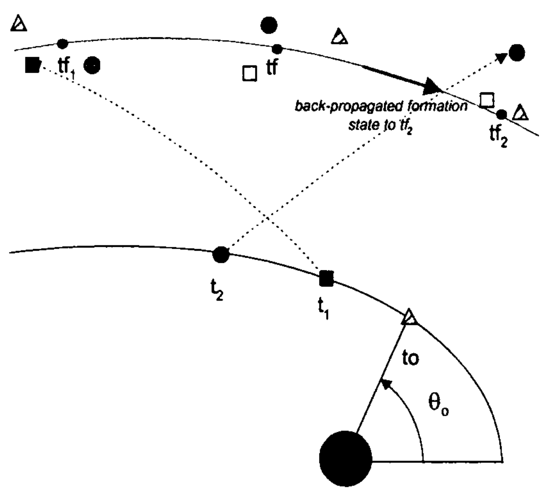

Fig. 1c Spacecraft 2 (circle) is optimized and the final neighboring optimal trajectory is composed of a first burn at $t_2 > t_1$ and a final burn at $tf_2 < tf$. Thus, spacecraft 2 will be in formation before spacecraft 1. The formation will not be initialized earlier than tf_1.

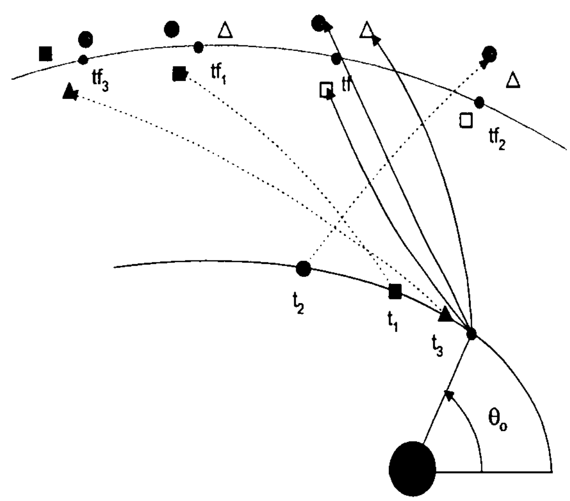

Fig. 1d Finally, Spacecraft 3 (triangle) performs its first optimally placed burn at $t_3 < t_1 < t_2$ and its final burn at $tf_3 > tf_1 > tf$. Once the last spacecraft is optimized, the entire timeline sequence is determined. Spacecraft 2 performs its first maneuver last but gets in formation first. Then, spacecraft 1 is initialized forming a sub-cluster with spacecraft 2. At tf_3, Spacecraft 3 joins the sub-cluster (1-2) and the formation is fully initialized.

3 - PRELIMINARY RESULTS

In this section we present some preliminary results to initialize a 'Leonardo-type' formation of six spacecraft using the approach discussed above. For our test case, we picked an arbitrary launch orbit (Sc_L) as Leonardo's is not yet defined. Each orbital element composing the launch orbit is an average of the six spacecraft in formation in a preliminary attempt to distribute the initialization ΔV cost among all the spacecraft. Future work will include a more detailed and thorough analysis on the best launch orbit to uniformly distribute the ΔV among the formation. The formation's six spacecraft and the launch orbital elements are listed in Table 2.

Table 2. Orbital Elements of the Formation.

Orbital Elements	Sc_L	Sc1	Sc2	Sc3	Sc4	Sc5	Sc6
a (km)	6803.13	6802.79	6804.84	6802.79	6802.79	6802.79	6802.79
e	0.00169	0.00117	0.00313	0.00136	0.00136	0.00136	0.00175
i (deg)	1.50841	3.6042	1.52049	0.10719	0.010719	0.10719	3.6042
Ω (deg)	278.395	256.791	124.344	320.196	319.49	3.6042	330.059
ω (deg)	329.589	284.266	230.026	53.894	363.17	330.059	50.372
θ	n/a	184.186	6.65201	352.978	39.1138	278.395	345.157

As mentioned earlier, the Leonardo mission does not have specific initialization period constraints at this stage in the mission. Therefore, the entire (θ_o, t_f) space is a valid search space for a solution. For this example, we chose a true anomaly of 175 degrees and a time-of-flight of 77 mins. For each spacecraft, a Lambert two-burn solution is computed with a first burn performed at t_o (arbitrarily set to 0) and a second burn 77 mins later. This two-burn initial solution is given to the primer vector code PVAT as a first-guess for the iterative optimization process. First, we set each spacecraft boundary problem with a type I constraint. Consequently, the spacecraft has to perform a first maneuver at t_o and the last maneuver to reach its final state has to occur at t_f. For constraint II, both the departure and arrival epoch are allowed to float within the initial [t_o, t_f] interval and no maneuver later than t_f will be performed in spite of an indication of potential ΔV savings by PVAT. Finally, constraint III is similar to constraint II but allows the final maneuver to occur at a later time than t_f. Table 3 summarizes the results obtained using the different constraints. The last row shows the average ΔV per spacecraft in the formation as well as the average number of burns needed for the trajectory to be initialized. The first column displays the ΔV cost associated with the reference trajectory prior to any optimization.

Table 3. Formation ΔV summary (for $\theta_o = 175°$ and $t_f = 77$ mins)

	Reference Traj.		Constraint I		Constraint II		Constraint III	
	# Burn	ΔV_{tot} (m/s)	# Burn	ΔV_{tot} (m/s)	# Burn	ΔV_{tot} (m/s)	# Burn	ΔV_{tot} (m/s)
Sc1	2	355.3	4	344.78	3	343.8	3	314.4
Sc2	2	754.6	5	401.96	2	395.6	2	395.6
Sc3	2	382.5	3	290.15	4	288.6	3	220.5
Sc4	2	336.4	4	208.12	3	203.2	3	193.1
Sc5	2	335.4	4	207.97	3	203.6	3	192.4
Sc6	2	536.0	3	451.75	3	433.5	3	433.5
Avg.	***2***	***450.0***	***3.83***	***317.45***	***3***	***311.4***	***2.83***	***291.6***

As expected, constraint I ($\mathbf{C_I}$) exhibits the higher number of burns which is often not desirable operationally. However, we observe a significant decrease of the average ΔV per spacecraft of about 132 m/s (i.e. about 35% decrease) over the 2-burn first guess (**Reference Traj.**). Constraint

II (C_{II}) has an average number of burns lower than constraint I but barely improves the average ΔV cost by about 6 m/s compared to constraint I. Constraint III (C_{III}) has, in average, the lowest number of burns and the lowest ΔV cost and saves an additional 25 m/s compared to constraint I. That is expected as constraint III has the most freedom in making the proper improvement to the trajectory when necessary. It is also very instructive to look at the individual spacecraft results. For example, Sc2 trajectory converged to an identical optimal 2-burn scenario under both constraint II or III by moving the epochs of the endpoints, which resulted in a 50% decrease from the *reference trajectory* ΔV. When under a constraint I which enforces burn at those non-optimal epochs, the primer vector theory reduces their magnitude close to zero and places additional internal burns at about the same location as the ones under constraint II and III (t_1 = 12 mins and t_2 = 59 mins). For this case, it appears that no matter what boundary constraint is imposed, PVAT tends to converge to the same local optimal trajectory. Table 4 lists the ΔV budget for Sc2. Figures 2a-c shows the primer vector magnitude history of Sc2 for the *reference trajectory* and the final optimized trajectories under the three constraints. The primer vector history is given as a function of elapsed time in seconds from the first burn of the transfer and each burn location is marked by a dot. On the other hand, in the case of Sc3, we observe that the different boundary constraints lead to distinctly different optimal trajectories. As seen in table 5, constraints I and III converged to two different 3-burn optimal transfer and constraint II converged to a 4-burn optimal trajectory. The different constraints limit the possible actions for ΔV improvement at a given iteration and in doing so, enforces a specific sequencing.

Table 4. Sc2 ΔV Budget for the different constraints.

	t_1	ΔV_1	t_2	ΔV_2	t_3	ΔV_3	t_4	ΔV_4	t_5	ΔV_5	ΔV_t
C_I	0	0.02	12.62	387.8	50.29	3.76	58.04	3.68	77.0	6.703	**401.9**
C_{II}	12.38	155.0	59.18	240.6	n/a	n/a	n/a	n/a	n/a	n/a	**395.6**
C_{III}	12.38	155.0	59.18	240.6	n/a	n/a	n/a	n/a	n/a	n/a	**395.6**

Table 5. Sc3 ΔV Budget for the different constraints.

	t_1	ΔV_1	t_2	ΔV_2	t_3	ΔV_3	t_4	ΔV_4	ΔV_t
C_I	0	120.21	57.00	135.45	77.0	54.48	n/a	n/a	**382.5**
C_{II}	7.36	185.9	49.59	6.1	56.7	27.8	77.0	68.8	**288.6**
C_{III}	13.12	84.2	53.42	5.5	97.06	130.8	n/a	n/a	**220.5**

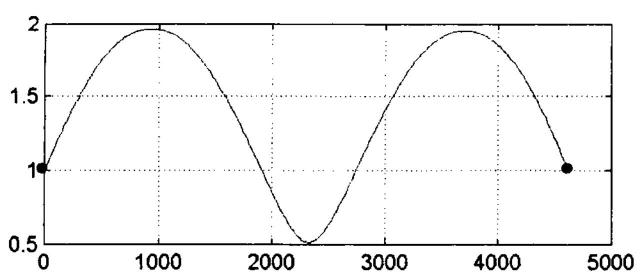

Fig. 2a Initial Primer Vector History for Sc2.

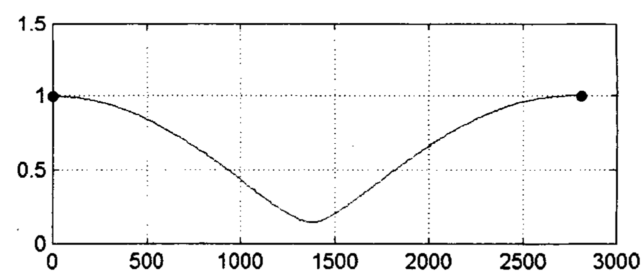

Fig 2c Final Primer Vector History for Sc2 (**C_{II} / C_{III}**)

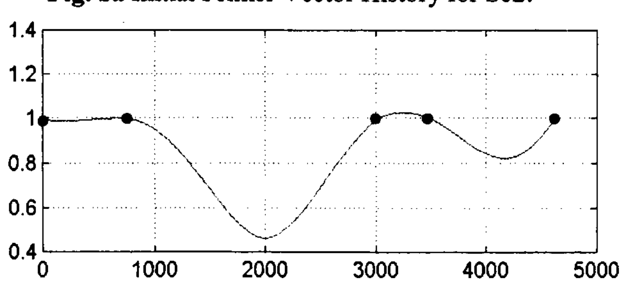

Fig 2b Final Primer Vector History for Sc2 (**C_I**)

4 - CONCLUSIONS

This paper presented a preliminary effort to use primer vector theory to initialize a formation of spacecraft from a common launching orbit. The classical primer vector theory was implemented in our Primer Vector Analysis Tool application (PVAT), which was developed in MATLAB. PVAT has a fully automated algorithm, which iterates following the primer vector principles to optimize a non-optimal *reference trajectory*. We enforced the formation flying constraints by initially setting all the spacecraft with identical boundary constraints. The initial endpoint constraint was imposed by the launch vehicle jettison location at an arbitrary epoch and the final endpoint constraint was imposed by the formation periodicity (assuming that we know the formation state at some reference time t_f). Once the *reference trajectory* common boundaries were defined, each spacecraft transfer was optimized *individually* by PVAT and we investigated three possible endpoint constraint scenarios, which spanned from a "strict" rendezvous case to allowing the epoch at which the formation is formed to be varied. As expected, the less restricted constraint led to optimal trajectories with the lowest ΔV and the least number of burns. In some cases, a decrease in ΔV of up to 50% was obtained as compared to the initial trajectory. In general, we showed each formation flying boundary constraints imposed a different prioritized action sequence for ΔV improvement and therefore, converged to a different neighboring path. However, in some instances, different constraints resulted in identical trajectories. Since primer vector is a first-order theory, it will converge on local optimal neighboring trajectories of the *reference trajectory*, which the optimal solution will highly depend on. The reference trajectory supplied in this paper was a two-burn Lambert transfer and we assumed that the true anomaly at which the spacecraft are launched into and the initial epoch at which the formation is formed were known. When those two variables are not determined by other mission constraints, or can be varied within some specific limits, we envision using a more global optimization technique such as a genetic algorithm to search the appropriate (θ_o, t_f) solution space as a higher hierarchy driver to the PVAT algorithm. In addition, PVAT can be applied to a wide range of transfers from highly eccentric to low Earth and the methodology developed in this paper can be used for various formation flying missions such as MMS as well as Techsat21 or Leonardo-BRDF. This work can also be extended to applications such as resizing or reconfiguration of a formation.

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