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Transient Growth Theory Prediction of Optimal Placing of Passive and Active Flow Control Devices for Separation Delay in LPT Airfoils

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May 2003

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TRANSIENT GROWTH THEORY PREDICTION OF OPTIMAL PLACING OF PASSIVE AND ACTIVE FLOW CONTROL DEVICES FOR SEPARATION DELAY IN LPT AIRFOILS

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Abstract

An analysis of the non-modal growth of perturbations in a boundary layer in the presence of a streamwise pressure gradient is presented. The analysis is based on PSE equations for an incompressible fluid. Examples with Falkner-Skan profiles indicate that a favorable pressure gradient decreases the non-modal growth while an unfavorable pressure gradient leads to an increase of the amplification. It is suggested that the transient growth mechanism be utilized to choose optimal parameters of tripping elements on a low-pressure turbine (LPT) airfoil. As an example, a boundary-layer flow with a streamwise pressure gradient corresponding to the pressure distribution over a LPT airfoil is considered. It is shown that there is an optimal spacing of the tripping elements and that the transient growth effect depends on the starting point. At very low Reynolds numbers, there is a possibility to enhance the transient energy growth by means of wall cooling.

Nomenclature

E	=	energy norm
G	=	E_{out}/E_{in} energy ratio
H	=	shape factor
H_L	=	$\sqrt{\nu L / U_{eL}}$
H_{Ls}	=	$\sqrt{\nu L_s / U_{exit}}$
H_{ref}	=	scale in y- and z- directions
L	=	length along the streamwise direction
L_{ref}	=	reference length
L_s	=	surface length
Re	=	Reynolds number
Re_L	=	$U_{eL} L / \nu$
T_w / T_{ad}	=	temperature factor (ratio of the wall temperature to the temperature of the adiabatic wall)
U	=	streamwise velocity component of the mean-flow velocity

U_{eL} = free-stream velocity at $x = L$
U_{ref} = reference velocity
V = normal-to-the wall velocity component of the mean-flow velocity
u = streamwise velocity disturbance
v = normal velocity disturbance
w = spanwise velocity disturbance
x = streamwise coordinate
y = coordinate normal to the wall
z = spanwise coordinate
β = spanwise wave number
β_H = Hartree parameter
ε = $\sqrt{\nu_\infty / U_{ref} L_{ref}}$
ν = kinematic viscosity
θ = momentum thickness
π = pressure disturbance

Superscripts
T = transposed

Subscripts
e = free stream
exit = exit conditions
in = starting point
out = ending point

1. Introduction

Laminar-turbulent transition in shear flows is still an enigma in the area of fluid mechanics. The conventional explanation of the phenomenon is based on the instability of the shear flow with respect to infinitesimal disturbances. The conventional hydrodynamic stability theory deals with the analysis of *normal modes* that might be unstable. The latter circumstance is accompanied by an exponential growth of the disturbances that might lead to laminar-turbulent transition. Nevertheless, in many cases, the transition scenario bypasses the stage of the exponential growth associated with the normal modes. This type of transition is called as *bypass transition*. Observations of laminar-turbulent transition in plane Couette flow, in circular pipe flow, in boundary layers at relatively low Reynolds numbers, etc., can serve as examples of bypass transition. An understanding of the phenomenon has eluded us to this day. One possibility is that bypass transition is associated with so-called *algebraic growth* of disturbances in shear flows.[1–3]

The phenomenon of algebraic growth of disturbances in shear flows has been of great interest during the last two decades as it may be associated with the bypass transition mechanism. Ellingsen and Palm[1] considered, in the inviscid case, an initial disturbance independent of the streamwise coordinate and found that the streamwise disturbance amplitude may grow in time, even though the basic flow does not posses an inflection point. Landahl[2] showed that all parallel inviscid shear flows are unstable to a wide class of three-dimensional disturbances. The result is independent of whether or not the shear flow is unstable to an exponential growth of wavelike disturbances. This type of instability that is not related to exponential growth is also referred to as "*non-modal growth.*" Mathematically, the effect of non-modal growth is associated with non-normality of the linearized Navier-Stokes operator and non-orthogonality of the eigenfunctions. In simple words, the essence of non-modal growth is the possibility of combining the exponentially decaying modes in such way that their sum will possess transient growth. One can find a vast bibliography on the topic in the Otto Laporte Award Lecture by Reshotko[3] and in a monograph by Schmid and Henningson.[4]

Numerical analysis of spatial non-modal growth within the scope of the linearized boundary-layer equations for an incompressible flow over a flat plate was carried out in Refs. 5 and 6. Spatial analysis within the scope of the linearized Navier-Stokes equations (quasi-parallel approximation of compressible and incompressible flows) was presented in Refs. 7 to 9. The main results of these theoretical models are as follows:

- A system of counter-rotating streamwise vortices, which are periodic in the spanwise direction, provides the strongest growth of the disturbance.
- There is an optimal spacing of the streamwise vortices, leading to the strongest effect.

The effect of pressure gradients on the transient growth mechanism was considered within the scope of temporal theory by Corbett and Bottaro[10] and within the scope of spatial theory by Tumin and Reshotko.[9] These results were based on the quasi-parallel flow assumption. Tumin[11] analyzed the pressure gradient effect for the Falkner-Skan profile within the scope of an analytical model when the spanwise wave number is very small. The pressure-gradient effect within the scope of spatial theory with nonparallel base flow and finite spanwise wave numbers has not been considered, yet.

Another motivation for the present work stems from separation flow control on low-pressure turbines (LPT). The performance of LPTs is strongly affected by the flow separation. There is a possibility of delaying the boundary-layer separation by tripping the boundary layer with the help of roughness elements or other devices. Usually, trial-and-error method is used to determine an appropriate placement of the control elements. This approach is time consuming and expensive. A recent investigation by Reshotko and Tumin[12] demonstrated that roughness-induced transition might be related to the transient growth mechanism.

Periodically spaced in the spanwise direction, roughness elements generate a system of counter-rotating streamwise vortices. Due to a secondary instability mechanism, the streamwise vortices can lead to earlier transition to turbulence. They also provide a mixing enhancement due to redistribution of the streamwise momentum. Consequently, optimization of the streamwise vortices for maximum energy growth leads to maximizing of the flow control effectiveness. In the present work, analysis of the optimal disturbances/streamwise vortices associated with the transient growth mechanism will be performed for boundary layers in the presence of a streamwise pressure gradient. The theory will provide the optimal spacing of the control elements in the spanwise direction and their placement in the streamwise direction.

2. Governing Equations

In the present work, we utilize the method developed in Ref. 19. Because the flows of interest have relatively low Mach numbers, we consider steady three-dimensional disturbances in an incompressible two-dimensional boundary layer. We choose the streamwise coordinate x along the surface. The coordinate y will measure distance from the wall. We define a small parameter $\varepsilon = \sqrt{\nu / U_{ref} L_{ref}}$, where ν, U_{ref}, and L_{ref} are viscosity, reference velocity, and reference length, respectively. The streamwise coordinate is scaled with L_{ref} while the vertical coordinate y and spanwise coordinate z are scaled with $\sqrt{\nu L_{ref} / U_{ref}}$. The following scaling is assumed for the velocity disturbances u, v, and w, and the pressure π:

$$u \sim U_{ref}, \quad v \sim \varepsilon U_{ref}, \quad w \sim \varepsilon U_{ref}, \quad p \sim \varepsilon^2 \rho U_{ref} \tag{1}$$

This scaling of the linearized Navier-Stokes equations and neglecting of the curvature effects lead to the governing equations for Görtler instability with the Görtler number equal to zero. We look for a periodic solution in the spanwise direction with the corresponding wave number β. The governing equations for the amplitude functions can be written in dimensionless form as follows:[5,6]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \beta w = 0 \tag{2}$$

$$\frac{\partial}{\partial x}(Uu) + V\frac{\partial u}{\partial y} + v\frac{\partial U}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \beta^2 u \tag{3}$$

$$\frac{\partial}{\partial x}(uV + vU) + \frac{\partial}{\partial y}(2Vv) + \beta V w + \frac{\partial p}{\partial y} = \frac{\partial^2 v}{\partial y^2} - \beta^2 v \tag{4}$$

$$\frac{\partial}{\partial x}(Uw) + \frac{\partial}{\partial y}(Vw) - \beta p = \frac{\partial^2 w}{\partial y^2} - \beta^2 w \tag{5}$$

where $U(x,y)$ and $V(x,y)$ are the streamwise and normal velocity components of the base flow, respectively (in addition, the latter is divided by ε). The following boundary conditions are applied to the solutions:

$$y = 0: \qquad u = v = w = 0 \tag{6a}$$

$$y \to \infty: \qquad u, w, \pi \to 0 \tag{6b}$$

The equations (2) to (5) can be solved subject to boundary conditions (6a) and (6b) with prescribed initial velocity perturbations at $x = x_0$.

The governing equations can be recast as follows:

$$\left(\mathbf{Af}\right)_x = \mathbf{B}_0\mathbf{f} + \mathbf{B}_1\mathbf{f}_y + \mathbf{B}_2\mathbf{f}_{yy} \tag{7}$$

where $\mathbf{A}$, $\mathbf{B}_0$, $\mathbf{B}_1$, and $\mathbf{B}_2$ are 4×4 matrices (one can find them in the Appendix; see also Ref. 5) and $\mathbf{f} = (u, v, w, p)^T$. The superscript "$T$" stands for "transposed," and the subscripts "x" and "y" denote differentiation with respect to x and y, respectively .

3. Optimization of Energy Growth

The authors of Refs. 5 and 6 employed an iterative procedure to find the optimal disturbances in terms of the maximum of the energy growth ratio $G = E_{out} / E_{in}$, where E_{in} and E_{out} stand for the input and output energy norms. Andersson et al.[5] used the same definitions of E_{in} and E_{out} as for the disturbance energy

$$E = \int_0^{y_{\max}} \left(u^2 + \varepsilon^2 v^2 + \varepsilon^2 w^2\right) dy \tag{8}$$

whereas Luchini[6] employed the knowledge that the optimal disturbances are represented by streamwise vortices with corresponding output as the streamwise velocity streaks,

$$E_{in} = \varepsilon^2 \int_0^{y_{\max}} \left(v^2 + w^2\right) dy \tag{9a}$$

$$E_{out} = \int_0^{y_{\max}} u^2 dy \tag{9b}$$

$$G = \frac{E_{out}}{E_{in}} = \varepsilon^{-2} \frac{\int_0^{y_{\max}} u^2 dy}{\int_0^{y_{\max}} \left(v^2 + w^2\right) dy} \tag{9c}$$

As was shown in Ref. 5, the two definitions of the optimal disturbances lead to the same results at Reynolds numbers of 10^4 and higher. Because the iteration procedure based on the optimization of ratio (9c) provides significant simplification, we adopt it for the following analysis. Because Eqs. (2) to (5) are independent of ε, the value of $\varepsilon^2 G$ is invariant with respect to the Reynolds number.

4. Numerical Results

4.1 Falkner-Skan Base Flow

We consider a Falkner-Skan family of boundary-layer profiles with free-stream velocity distribution $U_e = Cx^m$ and corresponding Hartree parameter $\beta_H = 2m/(m+1)$. For the purpose of convenience, we have used the velocity scale $U_{ref} = U_{eL} = CL^m$ and the length scale $L_{ref} = L/(m+1)$. The latter allowed the use of the conventional scaling of boundary-layer solutions with $H_{ref} = \sqrt{\nu L/(m+1)U_{eL}} = \sqrt{\nu L_{ref}/U_{ref}}$.

Figure 1 shows the scaled the energy ratio versus spanwise wave number β for three Hartree parameters, β_H = –0.1, 0.0, and 0.1. The starting and the ending points, x_{in}/L and x_{out}/L, are equal to 0.2 and 1.0, respectively. The Reynolds number Re_L in Fig. 1 and what follows is defined as $U_{eL}L/\nu$. One can see that an unfavorable pressure gradient $(\beta_H < 0)$ leads to an increase in the energy growth while a favorable pressure gradient $(\beta_H > 0)$ leads to suppression of the transient growth mechanism. The latter is consistent with results obtained within the scope of parallel flow approximation.[5]

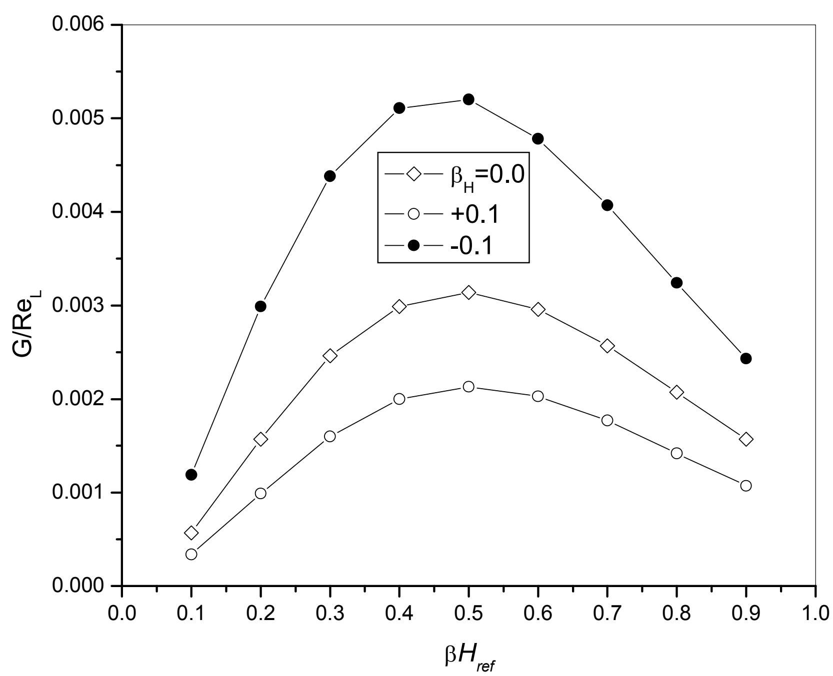

Fig. 1. Effects of the spanwise wave number β and the Hartree parameter β_H on transient growth (starting point $x_{in}/L = 0.2$).

Figures 2 and 3 show similar results, but the starting points are $x_{in} / L = 0.4$ and 0.6, respectively. respectively. A comparison of Figs. 1 to 3 indicates that there is a spanwise wave number, $\beta H_{ref} = 0.5 - 0.6$, and a starting point, x_{in} / L, that maximize the energy growth. These parameters correspond to optimal spanwise spacing and streamwise placing of perturbators for maximum flow control effectiveness. Figure 4 demonstrates the energy ratio versus the downstream coordinate x / L at $\beta H_{ref} = 0.5$ and x_{in} / L = 0.2.

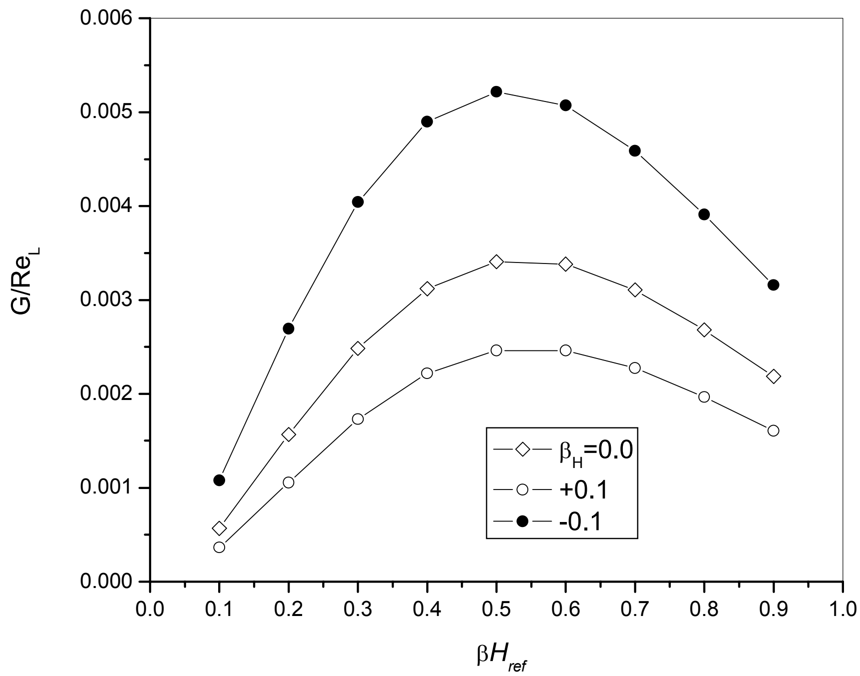

Fig. 2. The same as Fig. 1, with starting point $x_{in} / L = 0.4$.

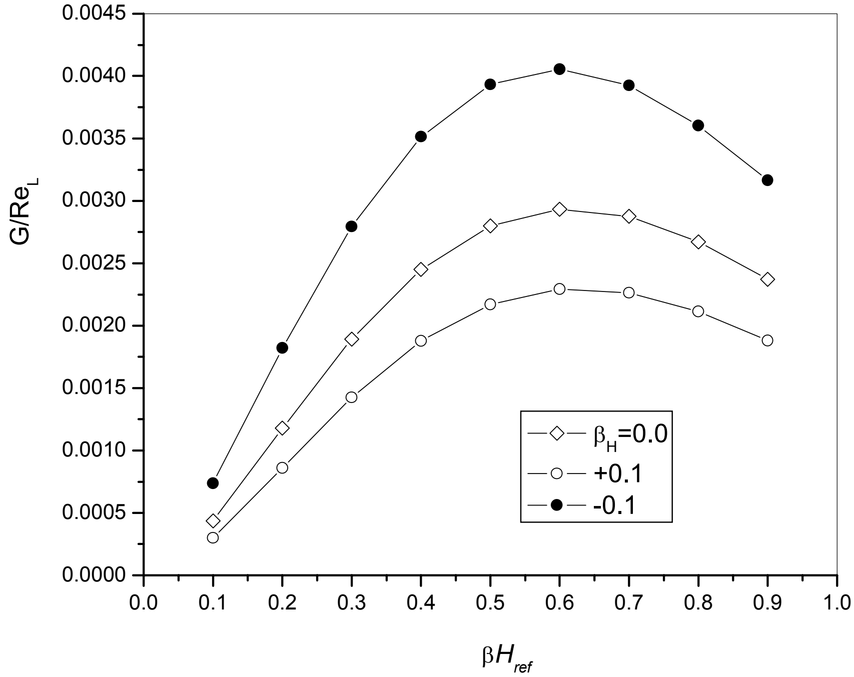

Fig. 3. **The same as Fig. 1, with starting point** $x_{in} / L = 0.6$.

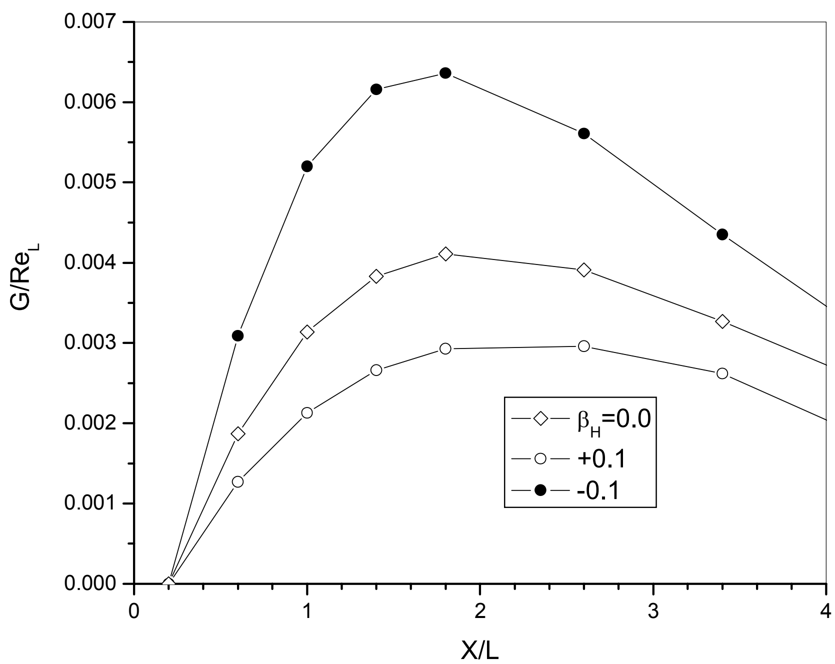

Fig. 4. **Optimal energy ratio versus the downstream coordinate at three Hartree parameters (** $x_{in} / L = 0.2$ **,** $\beta H_L = 0.5$ **).**

4.2 Example No. 1 of LPT Conditions

Volino[13] simulated low-pressure turbine (LPT) airfoil conditions in a low-speed wind tunnel. The test section was designed as a passage between two airfoils. The local free-stream velocity at a favorable pressure-gradient region was closely approximated by the following equation:

$$\frac{U_e}{U_{exit}} = 1.48\left(\frac{x}{L_s}\right)^{0.214} \qquad (10)$$

where L_s is the suction surface length and U_{exit} is the nominal exit free-stream velocity based on the inviscid solution. The distribution (10) corresponds to the Falkner-Skan flow with the Hartree parameter $\beta_H = 0.353$.

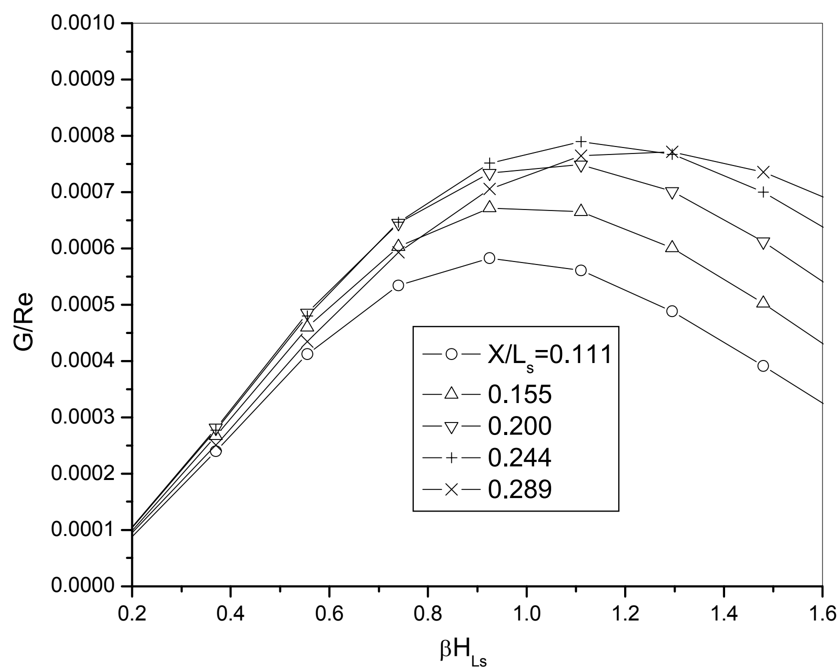

Fig. 5. Effects of the spanwise wave number β and the starting point x_{in}/L on transient growth at conditions of the experiment in Ref. 13 ($\beta_H = 0.353$, x_{out}/L_s=0.444).

Figure 5 demonstrates the energy ratio scaled with the Reynolds number Re_{exit} = $U_{exit}L_s/\nu$ versus the spanwise wave number scaled with $H_{Ls} = \sqrt{\nu L_s/U_{exit}}$. The ending point was prescribed at $x_{out}/L_s = 0.444$ while the starting points varied from 0.111 to 0.289. The streamwise velocity perturbation at $x_{out}/L_s = 0.444$,

$x_{in} / L_s = 0.111$, and βH_{Ls} = 0.925 is shown in Fig. 6, and the corresponding optimal profiles of v and w are presented in Fig. 7.

The results indicate that we are dealing with a very strong favorable pressure gradient that suppresses the transient growth mechanism. If the Reynolds number is about 50,000, the transient growth will provide an energy amplification of less that 50. If we take into account that the real perturber will not produce the optimal inflow field, there is little likelihood that the bypass transition via a secondary instability of the velocity streaks will prevent the flow separation.

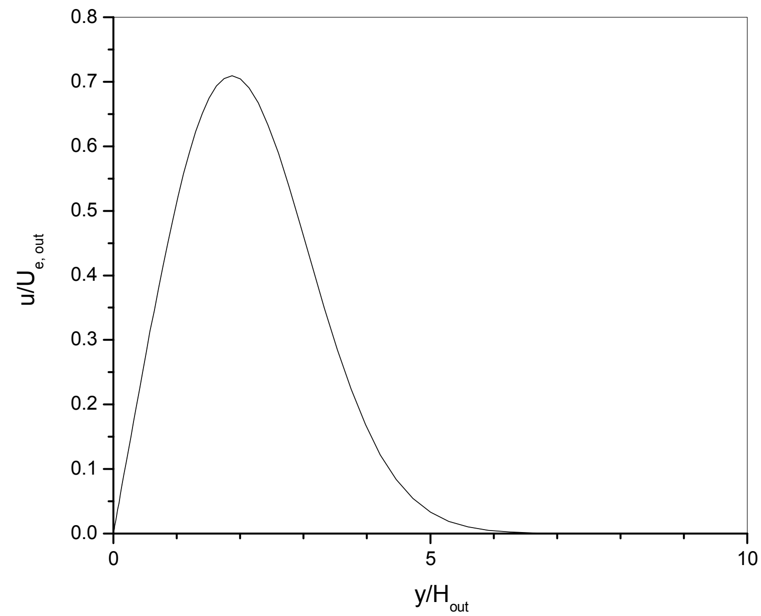

Fig. 6. The streamwise velocity perturbation at the ending point x_{out} / L_s = 0.444. The parameters correspond to the experimental conditions in Ref. 13 (x_{in} / L_s = 0.111, βH_{Ls} = 0.925).

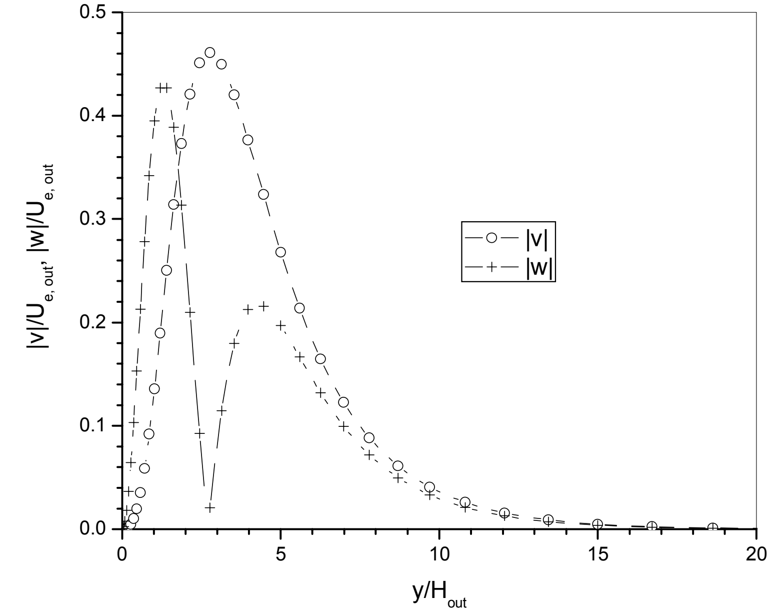

Fig. 7. **The optimal velocity perturbations at** x_{in}/L_s **=0.111 and** βH_{Ls} **= 0.925 corresponding to the streamwise velocity perturbation at** x_{out}/L_s **= 0.444 shown in Fig. 6**.

There is a possibility of enhancing the transient growth mechanism by means of wall cooling. The effect of wall cooling was investigated by Tumin and Reshotko[9] within the scope of a parallel flow approximation. In order to estimate possible increases of the energy ratio on a cold wall at a high favorable pressure gradient, we utilize the method of Ref. 9 for a compressible flow with local Mach number 0.5 and Hartree parameter 0.353. The results are shown in Fig. 8. One can see that cooling of the wall might provide a tenfold increase in the energy ratio.

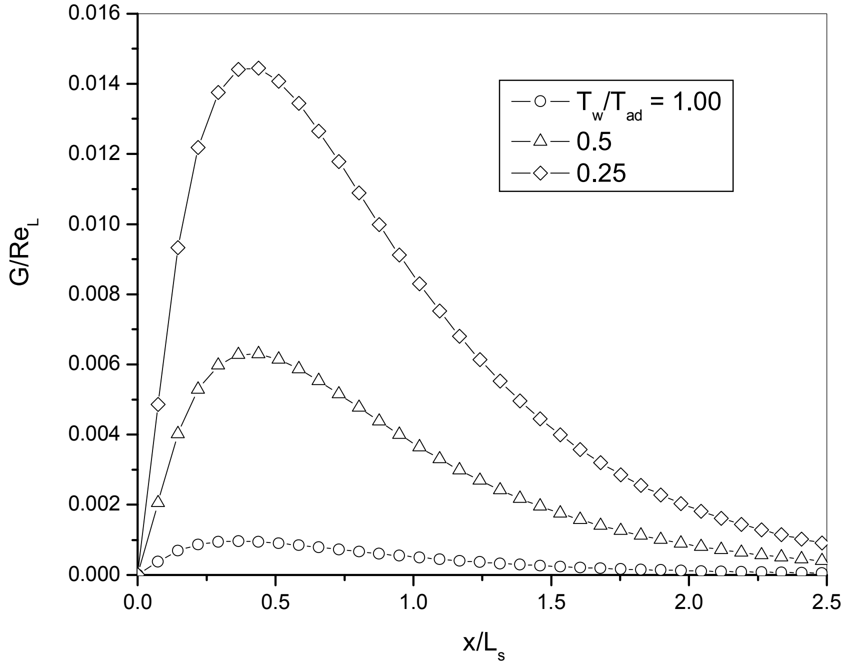

Fig. 8. Effect of the temperature factor on energy growth at the experimental conditions of Ref. 13.

4.3 Example No. 2 of LPT Conditions

Experiments on a flat plate with a pressure gradient corresponding to a modern low-pressure airfoil were carried out by Volino and Hultgren[14] at the NASA Glenn Research Center. Figure 9 shows a comparison of the theoretical and experimental free-stream velocity distributions along the flat plate (one can find more details in Ref. 14). Although there is a discrepancy between the theoretical and experimental data for the region of the favorable pressure gradient, we utilize a theoretical distribution in the following evaluations. The numerical methods for solving the boundary layer equations are described elsewhere.[15,16] Figures 10 and 11 show comparisons of the theoretical and experimental shape factors and the momentum thickness, respectively. One can see that the theoretical distribution of the free-stream velocity provides reasonable agreement between the theoretical and experimental data for the boundary-layer parameters.

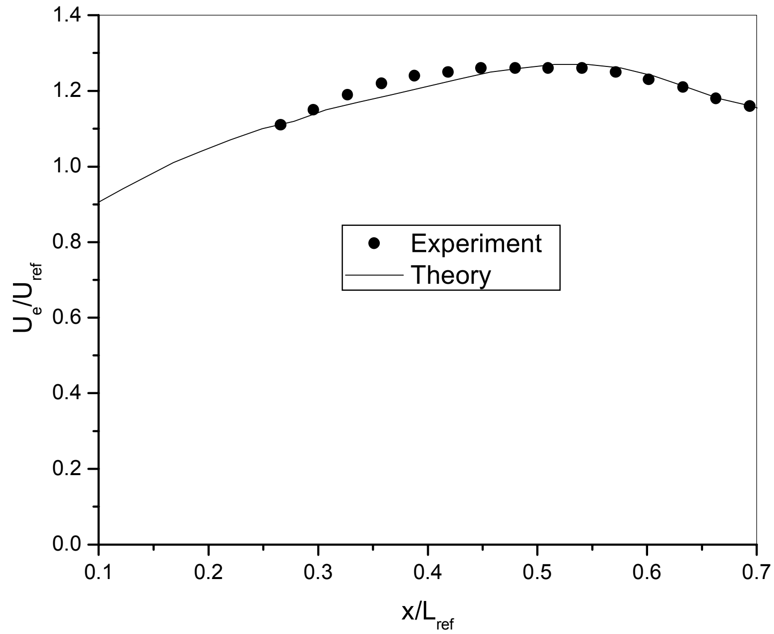

Fig. 9. **Comparison of theoretical and experimental[14] free-stream velocity distribution along the plate.**

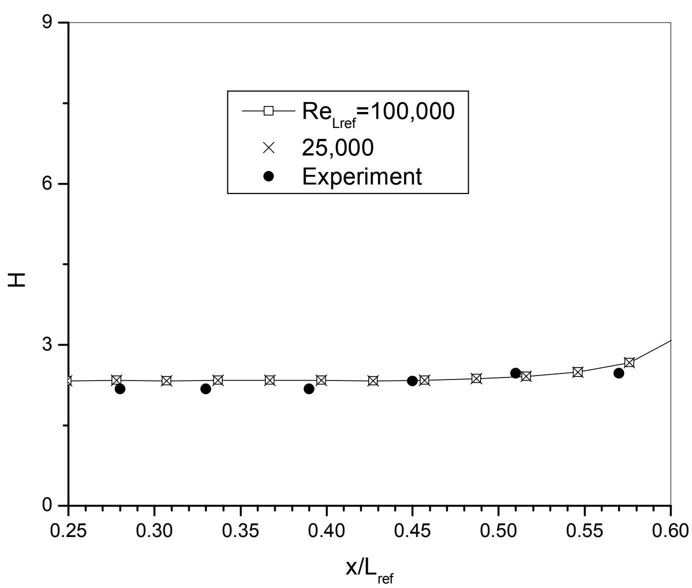

Fig. 10. **Comparison of theoretical shape factors at two Reynolds numbers with the experimental data in Ref. 14.**

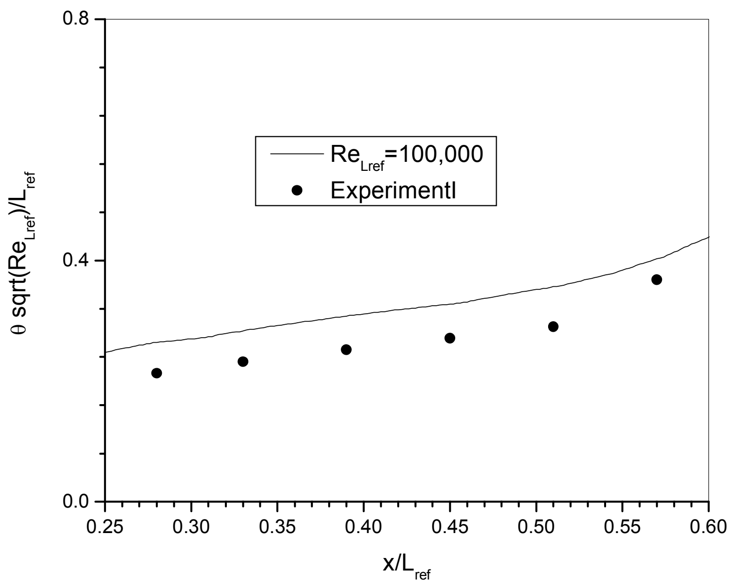

Fig. 11. Comparison of the theoretical momentum thickness with the experimental data in Ref. 14.

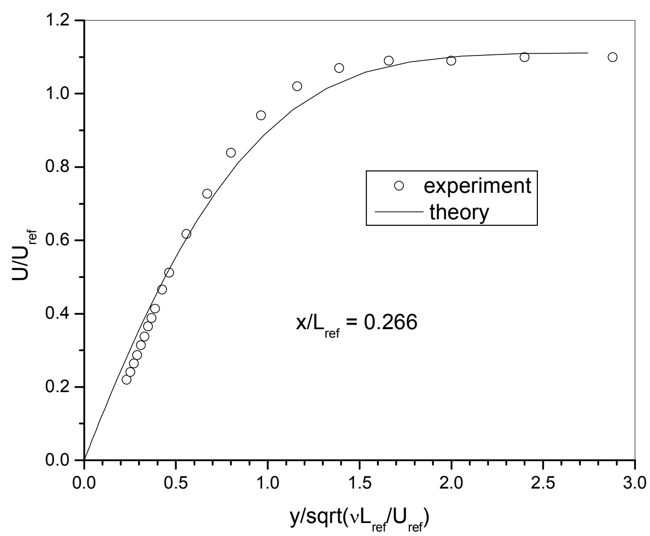

Fig. 12. Comparison of the theoretical and experimental[14] velocity profiles at x/L_{ref} = 0.266.

Comparisons of the theoretical and experimental velocity profiles are shown in Figs. 12 and 13. Because there are some discrepancies between the theoretical and experimental data, one should consider the following transient growth analysis as an estimation under the experimental conditions.

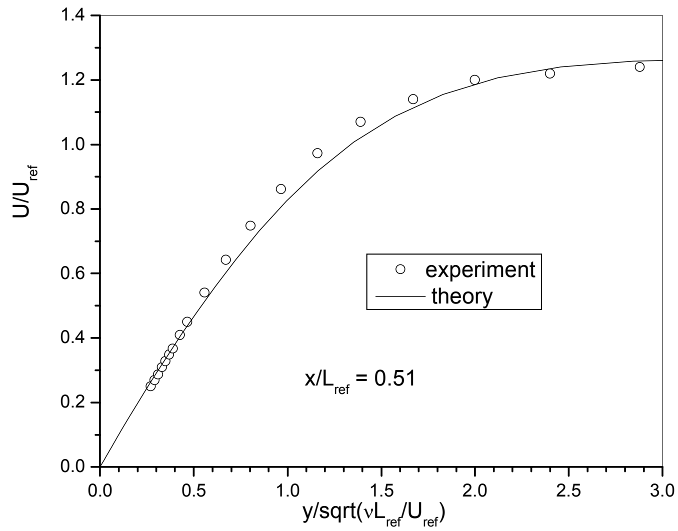

Fig. 13. **Comparison of the theoretical and experimental[14] velocity profiles at x/L_{ref} = 0.51.**

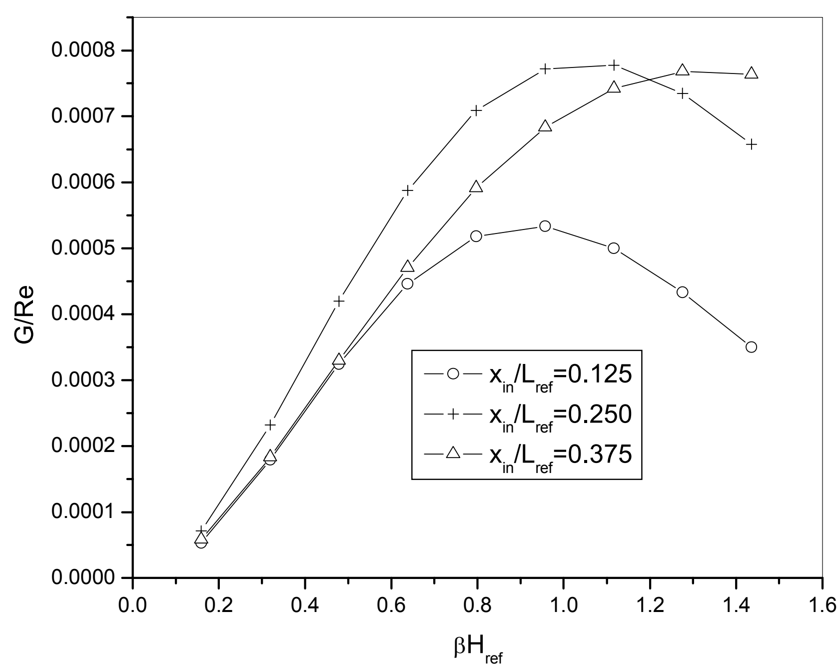

Fig. 14. **Effects of the spanwise wave number, β, and the starting point, x_{in}/L_{ref}, on the energy growth at the experimental conditions of Ref. 14.**

Figure 14 demonstrates the energy ratio versus the spanwise wave number at x_{out}/L_s=0.5 and three starting points, x_{in}/L_s = 0.125, 0.250 and 0.375. One can see that the results are similar to the previous case with a Hartree parameter of 0.353.

5. Summary

The results for the transient growth phenomenon within the scope of the linearized boundary-layer equations in the presence of a streamwise pressure gradient are consistent with previous results obtained within the scope of the parallel flow approximation and linearized Navier-Stokes equations.[9] A favorable pressure gradient decreases the non-modal growth while an unfavorable pressure gradient leads to an increase of the amplification.

The example of a Falkner-Skan flow with a Hartree parameter β_H = 0.353 corresponds to the experimental data[13] and simulates the flow over a low-pressure turbine airfoil upstream of the separation point. At this pressure gradient, the transient growth mechanism appears to be weak, and the latter, together with the low Reynolds number allows us to speculate that tripping of the boundary layer via a secondary instability of streaky structures is likely to fail. This conclusion should be verified by further work. Estimation of the transient growth at experimental conditions on a flat plate with a streamwise pressure gradient[14] shows that the results are similar to the example of Falkner-Skan flow at $\beta_H = 0.353$. The theory of the transient growth mechanism predicts the possibility of enhancing the energy growth by means of wall cooling. The example within the scope of the parallel flow theory[9] demonstrates that cooling of the wall might provide a tenfold increase in the energy ratio. Future experiments on boundary layer tripping accompanied by wall cooling will contribute to our understanding of the bypass transition mechanism.

Theoretical analysis of the optimal disturbances predicts that there is an optimal spacing between perturbers and their optimal location from the leading edge. The latter results can be utilized in future experiments with tripping of the boundary layer over the LPT airfoil. This type of experiment can be carried out at the NASA Glenn Research Center,[14] at the US Naval Academy,[13] and at the University of Notre Dame.[17]

Consideration of the optimal velocity perturbations in Fig. 7 indicates that they are spreading across the boundary layer. This means that an array of generators localized on the wall will not provide excitation of the optimal disturbances. Therefore, the question of realizability of the optimal disturbances arises. For example, one can solve the receptivity problem for an array of generators on the wall and evaluate their shapes (or other parameters), providing velocity disturbance profiles closest to the optimal ones. Another option is to solve the receptivity problem for distributed generators upstream from the starting point, x_{in}, to find their distribution, leading to the optimal disturbances. The next option is to design a disturbance generator that provides impact on the flow inside the boundary layer instead of a perturbation of the near-wall region only. For example, it might be a focused laser beam proceeding from the wall, where it could be delivered by a fiber-optic system.[18] These fundamental issues should be addressed in future research programs on the application of bypass transition mechanisms to separation flow control at low Reynolds numbers.

Appendix

For purpose of consistency of the formulation, we repeat the main features of the numerical scheme described in Ref. 5. Matrices $\mathbf{A}$, $\mathbf{B}_0$, $\mathbf{B}_1$, and $\mathbf{B}_2$ in Eq. (7) are as follows:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ U & 0 & 0 & 0 \\ V & U & 0 & 0 \\ 0 & 0 & U & 0 \end{pmatrix}, \quad \mathbf{B}_0 = \begin{pmatrix} 0 & 0 & -\beta & 0 \\ -\beta^2 & -U_y & 0 & 0 \\ 0 & -2V_y - \beta^2 & -\beta V & 0 \\ 0 & 0 & -V_y - \beta^2 & \beta \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -V & 0 & 0 & 0 \\ 0 & -2V & 0 & -1 \\ 0 & 0 & -V & 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

The scheme utilized for Eq. (7) in the streamwise direction is

$$(\mathbf{Af})^1 - (\mathbf{Af})^0 = \Delta x\left[(\mathbf{B}_0 f)^1 + (\mathbf{B}_1 f_y)^1 + (\mathbf{B}_2 f_{yy})^1\right]$$

$$\frac{3}{2}(\mathbf{Af})^{n+1} - 2(\mathbf{Af})^n + \frac{1}{2}(\mathbf{Af})^{n-1} = \Delta x\left[(\mathbf{B}_0 f)^{n+1} + (\mathbf{B}_1 f_y)^{n+1} + (\mathbf{B}_2 f_{yy})^{n+1}\right], \quad n \geq 1$$

where n stands for the step number along the coordinate x. At each streamwise position, the one-dimensional boundary-value problem is solved using a spectral collocation method based on Chebyshev polynomials. Usually, we used 100 intervals along the coordinate x and 100 Chebyshev polynomials for the solution approximation.

To find the optimal perturbations corresponding to the maximum of the energy ratio in (9c), the forward solution of Eq. (7) is accompanied by the backward solution of the adjoint problem.[5,6] In the present work, the adjoint system was discretized, i.e., the discretized adjoint equations were employed. Another approach is to utilize the adjoint form of the discretized forward equations (see discussion in Ref. 5). An arrangement of the iterations is described elsewhere.[5,6] Usually, 2 to 3 iterations were enough to achieve convergence at the 0.1 percent level.

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13. ABSTRACT ***(Maximum 200 words)***

An analysis of the non-modal growth of perturbations in a boundary layer in the presence of a streamwise pressure gradient is presented. The analysis is based on PSE equations for an incompressible fluid. Examples with Falkner-Skan profiles indicate that a favorable pressure gradient decreases the non-modal growth while an unfavorable pressure gradient leads to an increase of the amplification. It is suggested that the transient growth mechanism be utilized to choose optimal parameters of tripping elements on a low-pressure turbine (LPT) airfoil. As an example, a boundary-layer flow with a streamwise pressure gradient corresponding to the pressure distribution over a LPT airfoil is considered. It is shown that there is an optimal spacing of the tripping elements and that the transient growth effect depends on the starting point. At very low Reynolds numbers, there is a possibility to enhance the transient energy growth by means of wall cooling.

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