2002 NASA/ASEE SUMMER FACULTY FELLOWSHIP PROGRAM

JOHN F. KENNEDY SPACE CENTER
UNIVERSITY OF CENTRAL FLORIDA

FLUID DYNAMICS OF SMALL, RUGGED VACUUM PUMPS OF VISCOUS-DRAG TYPE

JOHN M. RUSSELL
Associate Professor, Aerospace Engineering
Florida Institute of Technology
NASA colleague: FREDERICK W. ADAMS, YA-C3

ABSTRACT

The need to identify spikes in the concentration of hazardous gases during countdowns to space-shuttle launches has led Kennedy Space Center to acquire considerable expertise in the design, construction, and operation of special-purpose gas analyzers of mass-spectrometer type. If such devices could be miniaturized so as to fit in a small airborne package or backpack then their potential applications would include integrated vehicle health monitoring in later-generation space shuttles and in hazardous material detection in airports, to name two examples. The bulkiest components of such devices are vacuum pumps, particulary those that function in the low vacuum range. Now some pumps that operate in the high vacuum range (*e.g.* molecular-drag and turbomolecular pumps) are already small and rugged. The present work aims to determine whether, on physical grounds, one may or may not adapt the molecular-drag principle to the low-vacuum range (in which case ***viscous-drag principle*** is the appropriate term). The deliverable of the present effort is the derivation and justification of some key formulas and calculation methods for the preliminary design of a single-spool, spiral-channel viscous-drag pump.

FLUID DYNAMICS OF SMALL, RUGGED VACUUM PUMPS OF VISCOUS-DRAG TYPE

JOHN M. RUSSELL

1. INTRODUCTION

Consider the geometry illustrated in Fig. 1.1 nearby:

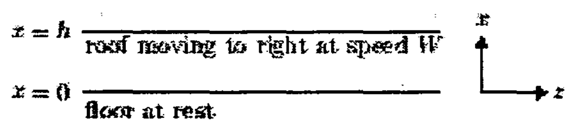

Fig. 1.1 Concept of a viscous-drag pump

The *no-slip* boundary condition of a viscous fluid asserts that a viscous fluid adheres to any solid with which it is in contact. The fluid that is in contact with the roof thus moves with the roof and the fluid in contact with the floor remains stationary. The distribution of longitudinal fluid velocity, w, with respect to the cross sectional coordinate, x, is an unknown of the problem. The no-slip condition furnishes boundary conditions for this w-distribution. If the channel is blocked at the ends then there can be no net mass transport through it. In this case the w-distribution must reverse sign at some value of x, as illustrated in Fig. 1.2.

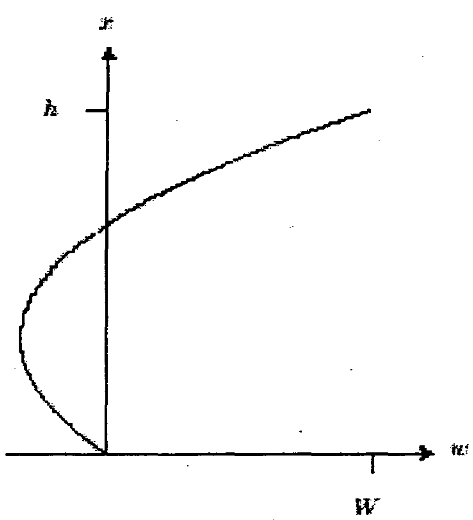

Fig. 1.2 Sample velocity profile with backflow

One may understand principle of operation of a viscous-flow pump by considering two extreme cases in turn. In the first case there is no imposed pressure difference between the two ends of the channel and the velocity distribution reduces to a simple linear interpolation between the boundary values. Following custom, I will refer to this case as COUETTE flow. In COUETTE flow there is nonzero mass transport in the direction of the wall motion but neither compression nor expansion. In the second extreme case there is a nonzero imposed pressure difference between the two ends of the channel (higher at the right end, say) but there is no wall motion. This latter case corresponds to capillary flow, in which the mass transport is nonzero and in direction from greater to lesser pressure (*i.e* in the negative z-direction in this example). In the capillary-flow case the fluid expands in the direction of its motion.

If, now, there is an imposed difference between the pressures at the ends of the channel and if there is wall motion in the direction of increasing pressure then the COUETTE mechanism—which favors mass transport in the direction of wall motion—and the capillary-flow mechanism—which favors mass transport in the opposite direction—are in competition. Whenever the COUETTE mechanism is the dominant of the two mechanisms (*i.e.* mass transport is in direction of wall motion), the moving-wall channel acts as a *compressor*. I will refer to such a compressor as a *viscous-drag pump*. A molecular-drag pump is, of course similar in concept, except that the channel width, h, in the latter case is large enough compared to the mean free path of the molecules to violate the usual continuum hypothesis of fluid mechanics.

One may imagine bending the straight channel illustrated in Fig. 1 into the shape of a corkscrew. One may produce such a channel by cutting a screw thread from the outer surface of an initially smooth cylindrical drum. By rotating such a drum within a smooth circular cylinder one may simulate, approximately, the conditions described in the foregoing paragraphs and thus generate a crude viscous-drag pump. In the following, I will provide detailed formulas for estimating the performance of a viscous-drag pump in according to such a model.

Consider a definite material point, $\mathcal{P}$, on the rotating drum situated at a some station along the channel. A observer fixed to $\mathcal{P}$ would report that the outer cylinder is in motion and that the local velocity of the cylinder has nonzero components both tangent and perpendicular to the screw-thread. In the foregoing discussion I did not consider the effects of a nonzero component of wall motion perpendicular to the screw-thread. An interesting and important question, which I will address only partially in this report, is: ***How does wall motion in the direction perpendicular to the screw thread affect the formulas for predicting of pump performance?*** Further work to address this latter question is a fit topic for follow-on work.

2. METHODS EMPLOYED; CALCULATION EXAMPLES

Equations of motion. Typical scales. The operation of a viscous drag pump involves the physical effects of compressibility and viscosity, so I will begin by writing down the general form of the equations of motion of a viscous gas. I will suppose *a-priori*, that the coefficients of shear and bulk viscosity (denoted μ and κ, respectively) depend only upon the absolute temperature, T, and that the latter is uniform throughout the flow. Let ρ denote the mass density of the gas; let p denote the equilibrium (or thermodynamic) pressure as defined by the equation of state of an ideal gas, *i.e.*

$$p = \rho RT \;, \tag{2.1}$$

(in which R is the gas constant for the particular gas under consideration). Let $\mathbf{v}$ denote the fluid velocity vector and let $\mathbf{g}$ denote the local gravitational force-per-unit-mass. Then there are two equations of motion, namely the equation of conservation of mass,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \;, \tag{2.2}$$

and the equation for the rate of change of translational momentum,

$$\underbrace{\rho\frac{\partial \mathbf{v}}{\partial t}}_{1} + \underbrace{\rho(\mathbf{v}\cdot\nabla)\mathbf{v}}_{2} = \underbrace{-\nabla p}_{3} + \underbrace{(\kappa + \tfrac{1}{3}\mu)\nabla(\nabla\cdot\mathbf{v})}_{4} + \underbrace{\mu\nabla^2\mathbf{v}}_{5} + \underbrace{\rho\mathbf{g}}_{6} \;. \tag{2.3}$$

Equation (2.3) asserts that the mass (per unit volume) times the acceleration of an infinitesimal fluid element equals the force (per unit volume) exerted on the element by its surroundings. Term 1 is the due to unsteadiness of the velocity field (if present). Term 2 is due to acceleration of a fluid element as it passes through a region where the velocity field is nonuniform. Following custom, I will refer to terms 1 and 2 as *inertial reactions* due to unsteadiness and *advection*, respectively. Term 3 is the part of the force (per unit volume) due to unequal surface pressures on opposite faces of an element. Terms 4 and 5 are the parts of to the force (per unit volume) due to unequal *viscous stresses* on opposite faces of an element and Term 6 is the *weight* force (per unit volume) on the element.

I will apply certain idealizations in the sequel. First among these is the assumption that the velocity field is steady in time [so Term 1 in (2.3) is identically zero]. Following custom, I will express the remaining idealizations in terms of *typical dimensional scales* of the problem. To this end, let ℓ denote a scale typical of the channel length; let h denote a scale typical of the channel width; let U denote a scale typical of the fluid speed and let $g := |\mathbf{g}|$ denote the scalar acceleration due to gravity.

Then the following list furnishes estimates of the order of magnitude of the various terms in (2.3):

$$\left.\begin{array}{l} |\text{Term } 2| \sim \rho U^2\ell^{-1} \;, |\text{Term } 4| \sim \mu U\ell^{-1}h^{-1} \;, \\ |\text{Term } 5| \sim \mu U h^{-2} \;, |\text{Term } 6| \sim \rho g \;. \end{array}\right\} \tag{2.4}$$

The lubrication-flow model. I will use the term *lubrication model* to describe the case when the following idealizations all hold:

$$\frac{h}{\ell}\cdot\frac{\rho U h}{\mu} \ll 1 \quad , \quad \frac{h}{\ell} \ll 1 \quad , \quad \frac{\ell g}{RT} \ll 1 \;. \tag{2.5}$$

Now the quantity RT/g has the dimensions of length. The length RT/g is called the *scale height of the atmosphere*. When the gas is air and T has the value 15° Celsius (the sea-level value for the U.S. Standard Atmosphere) the scale height has a value on the order of 10 km. Thus, as long as the channel length, ℓ, is small compared to 10 km the assumption $(2.5)_3$ will be accurate.

One may justify the neglect of the gravity term in (2.3) (*i.e.* Term 6) by forming the ratio of its order of magnitude to that of another term, say Term 3:

$$\frac{|\text{Term } 6|}{|\text{Term } 3|} \sim \frac{\rho g}{|\nabla p|} \sim \frac{\rho g/(RT)}{p/\ell} = \frac{\ell g}{RT} \ll 1 \;. \tag{2.6}$$

in which I have made use of (2.1) (in the second step) and $(2.5)_1$ (in the last). By similar reasoning (2.4) and (2.5) enable one to derive the following estimates:

$$\frac{|\text{Term } 2|}{|\text{Term } 5|} \sim \frac{h}{\ell} \cdot \frac{\rho U h}{\mu} \ll 1 \,, \tag{2.7}$$

$$\frac{|\text{Term } 4|}{|\text{Term } 5|} \sim \frac{h}{\ell} \ll 1 \,. \tag{2.8}$$

At this point, one has reason to neglect all of the terms in momentum equation, (2.3), except Terms 3 and 5. One concludes that these two terms balance each other in the lubrication approximation.

Lubrication flow with purely longitudinal roof motion. Here, and elsewhere, there will a be a velocity vector, $\mathbf{v}_{\text{roof}}$, which represents the motion of the roof of the channel relative to the floor. I will apply the phrase *purely longitudinal roof motion* to the case when the channel is straight and $\mathbf{v}_{\text{roof}}$ is parallel to its long axis. Let (x, y, z) be a set of cartesian coordinates arranged so that the positive z-axis is in the direction of $\mathbf{v}_{\text{roof}}$. Let (u, v, w) be the corresponding cartesian components of the velocity vector, $\mathbf{v}$. Then the form of (2.2) in steady flow is

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \,,$$

or, equivalently,

$$\frac{\partial(\rho w)}{\partial z} = -\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial y} \,. \tag{2.9}$$

Let V be a scale typical of the cross-channel velocity components, u and v. Then under assumptions stated earlier we have the estimates

$$\left|\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y}\right| \sim \frac{V}{h} \,, \quad \left|\frac{\partial(\rho w)}{\partial z}\right| \sim \frac{U}{\ell} \,.$$

Thus equation (2.9) implies that

$$\frac{U}{\ell} \sim \frac{V}{h} \,.$$

From this estimate and $(2.5)_2$, one concludes that

$$\frac{V}{U} \sim \frac{h}{\ell} \ll 1 \,. \tag{2.10}$$

Thus, in lubrication flow with purely longitudinal roof motion *the cross-channel velocity components, (u, v), of the velocity are small compared to the longitudinal component, w.*

Consider now the momentum equation, (2.3). If, for reasons stated above, one keeps only Terms 3 and 5, and one resolves the resulting vector equation into cartesian components one gets

$$\left.\begin{aligned} \frac{\partial p}{\partial x} &= \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \\ \frac{\partial p}{\partial y} &= \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \\ \frac{\partial p}{\partial z} &= \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \end{aligned}\right\} \,. \tag{2.11}$$

In view of (2.10) one may argue that the right members of the first two of these equations are small compared with the right member of the last. In view of the slenderness assumption, $(2.5)_2$, moreover, the second derivative of w with respect to the longitudinal coordinate, z, is small compared to the second derivatives of w with respect to the the cross-channel coordinates, x and y. The system (2.11) thus reduces to

$$\frac{\partial p}{\partial x} = 0 \,, \quad \frac{\partial p}{\partial y} = 0 \,, \quad \frac{\partial p}{\partial z} = \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \,. \tag{2.12}$$

The system (2.12) implies that *the pressure is effectively uniform over the channel.* This cross channel uniformity of the pressure is another feature of lubrication flow with purely longitudinal roof motion. In this context p is a function of the single variable, z, which I will sometimes denote by $p(z)$. The partial derivative $\partial p/\partial z$ thus reduces to an ordinary derivative [which I will sometimes denote by $p'(z)$] and the system (2.12) reduces to the single equation

$$p'(z) = \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \,. \tag{2.13}$$

Recall, now, the equation of state (2.1), which one may write in the form

$$\rho = \frac{p(z)}{RT} \,.$$

In the first paragraph of §2, I stated the assumption that T was uniform. In an ideal gas, so is R. One concludes from the above equation that ρ, like p, is a function of the single variable z [which I will sometimes denote by $\rho(z)$] in lubrication flow with purely longitudinal roof motion.

Plane flow. Consider the special case in which $v = 0$ identically and there is no variation of the other two velocity components with respect to the spanwise coordinate, y (*i.e.* the one perpendicular to the plane of Fig. 1.2). Then the fluid velocity vector is everywhere parallel to the z-x coordinate plane. Following custom, I will refer to a flow satisfying this condition as *plane flow*. In plane flow, then, (2.13) reduces to

$$p'(z) = \mu \frac{\partial^2 w}{\partial x^2} .$$

The second antiderivative of this equation with respect to x is

$$p'(z)\tfrac{1}{2}x^2 + C_1(z)x + C_2(z) = \mu w , \qquad (2.14)$$

in which $C_1(z)$ and $C_2(z)$ are arbitrary functions of z. If one applies the boundary conditions

$$w = 0 \quad \text{on} \quad x = 0 ,$$
$$w = W \quad \text{on} \quad x = h$$

(both of which are statements of the *no-slip condition*) to (2.14) one may solve for $C_1(z)$ and $C_2(z)$. If one substitutes these results back into (2.14) one gets

$$\frac{p'(z)}{2}(x^2 - hx) + \frac{\mu W x}{h} = \mu w(x,z) . \qquad (2.15)$$

A brief digression here to introduce the so-called *stream function* will expedite the statement of boundary conditions in the sequel. Under the restriction to plane flow equation (2.9) reduces to

$$\frac{\partial(\rho w)}{\partial z} = -\frac{\partial(\rho u)}{\partial x} .$$

A standard representation of the velocity components (w, u) which is compatible with the above equation is

$$\rho w = \frac{\partial \psi}{\partial x} , \quad \rho u = -\frac{\partial \psi}{\partial z} , \qquad (2.16)$$

in which $(x, z) \mapsto \psi(x, z)$ is any twice differentiable function of the variables (x, z). Following custom, I will refer to such a function, ψ, as a *stream function*.

One may interpret the stream function in terms of mass current (per unit span) across a contour in the plane of the flow. To this end, let (z, x) and $(z + dz, x + dx)$ be the coordinates of two neighboring points in the flow plane and let $\mathcal{C}$ be a contour drawn between them. In steady flow the net rate of transport of mass (per unit span) into or out of a small triangle with $\mathcal{C}$ as the hypotenuse and $(z + dz, x)$, say, as the vertex is zero. Now the expression $(\rho w)dx$ represents the transport across the vertical leg of the triangle (positive for outflow) and the expression $(\rho u)dz$ represents the transport across the horizontal leg of the triangle (positive for inflow). The *net outflow* across the two legs is thus

$$(\rho w)dx - (\rho u)dz ,$$

which must equal the inflow across $\mathcal{C}$. According to (2.16), however, the expression given above for the net rate of outflow across the legs is just

$$\frac{\partial \psi}{\partial x}dx - \left(-\frac{\partial \psi}{\partial z}\right)dz = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial z}dz ,$$

which is the expansion of the total differential, $d\psi$, of the stream function between the two ends of $\mathcal{C}$. One concludes that *the change in stream function between two points in a plane flow equals the rate of transport of mass (per unit span) across any contour drawn between those two points.*

This result is handy in the statement of boundary conditions. Thus, the condition that a wall is impermeable to mass reduces to the statement that *the stream function is constant on the wall.* This concludes the digression regarding the stream function and I now resume the main discussion at the point where I digressed, namely equation (2.15).

If one solves (2.15) for w and substitutes the result into $(2.16)_1$ one gets

$$\frac{\partial \psi}{\partial x} = \rho(z)\left[\frac{p'(z)}{2\mu}(x^2 - hx) + \frac{W}{h}x\right] .$$

The antiderivative of this equation with respect to x is

$$\psi = \rho(z)\left[\frac{p'(z)}{2\mu}(\tfrac{1}{3}x^3 - h\tfrac{1}{2}x^2) + \frac{W}{h}\tfrac{1}{2}x^2\right] + f(z) , \qquad (2.17)$$

in which $f(z)$ is an arbitrary function of z. Since the floor is impermeable, the stream function must be constant on it and it is convenient to choose zero for the value of the stream function on the floor. One thus arrives at the boundary condition $\psi = 0$ on $x = 0$ for all z. If one applies this boundary condition to (2.17) one concludes that $f(z) = 0$ and (2.17) reduces to

$$\psi = \rho(z)\left[\frac{p'(z)}{2\mu}(\tfrac{1}{3}x^3 - h\tfrac{1}{2}x^2) + \frac{W}{h}\tfrac{1}{2}x^2\right] . \quad (2.18)$$

Since the roof is also impermeable, the stream function must be constant on it as well. Let ψ_h denote the value of the stream function on the roof. If one evaluates (2.18) at $x = h$ (*i.e.* the roof elevation) one gets

$$\psi_h = \rho(z)\left[\frac{p'(z)}{2\mu}h^3(\tfrac{1}{3} - \tfrac{1}{2}) + \frac{W}{h}\tfrac{1}{2}h^2\right] .$$

But $\rho = \rho(z) = p(z)/(RT)$ [*cf.* (2.1) and the discussion following (2.13)], so the last equation implies that

$$\psi_h = \frac{p(z)}{RT}\left[-\frac{p'(z)}{2\mu}\frac{h^3}{6} + \frac{Wh}{2}\right] . \quad (2.19)$$

Equation (2.19) is an ordinary differential equation for the distribution of pressure down the channel. Two special cases serve as useful benchmarks. One may characterize the first special case by the condition $p'(z) = 0$, or $p = p_C$, in which p_C is a constant. This case—which I referred to as COUETTE flow in §1—corresponds to the physical situation when both ends of the channel are vented to the same reservoir. One may characterize the second special case by the condition $\psi_h = 0$. Now ψ_h represents the rate of mass transport per unit span through the channel so the condition $\psi_h = 0$ represents a blocked channel, *i.e.* one with no net mass transport. I will refer to this case as the *dead-head* case.

In the dead-head case the left member of (2.19) is zero by definition. In the mean time, expression outside the square brackets in the right member is nonzero. One concludes that in the dead-head case the expression inside the square brackets must be zero. It follows that

$$\frac{p'(z)}{2\mu}\frac{h^3}{6} = \frac{Wh}{2} ,$$

or

$$p'(z) = \frac{6\mu W}{h^2} . \quad (2.20)$$

Since the right member of (2.20) is constant in z one concludes that the pressure varies linearly along the channel in the dead-head case. Thus, $p'(z) = p_o/\ell_{dh}$, in which p_o is the pressure at the right end and ℓ_{dh} is a length, which I will call the *dead-head* length. If one writes p_o/ℓ_{dh} in place of $p'(z)$ in (2.20) and rearranges, one gets

$$\ell_{dh} = \frac{p_o h^2}{6\mu W} . \quad (2.21)$$

If the given channel length, ℓ, is greater than the dead-head length, ℓ_{dh}, then one may partition the channel into two parts, namely the part where $0 \le z \le \ell - \ell_{dh}$ (and the $p = 0$ identically) and the part where $\ell - \ell_{dh} < z < \ell$ (and the pressure ramps up linearly to the value, p_o, at the right end).

Some abbreviations will be convenient in the sequel. Thus I will denote by q the *pressure-volume* throughput (per unit span) through the channel. Let $\dot{\delta}$ denote the corresponding volumetric throughput (per unit span). Then the definition of q is $q := p\dot{\delta}$. According to (2.1), however, $p = \rho RT$. In the mean time, ψ_h is the mass throughput per unit span, so $\psi_h = \rho\dot{\delta}$. One concludes that

$$\psi_h RT = \rho\dot{\delta}RT = p\dot{\delta} = q . \quad (2.22)$$

One should note that q, like ψ_h, is constant in z under assumptions already introduced but $\dot{\delta}$ is not.

If one writes $\psi_h = q/(RT)$ in (2.19) one may arrange the resulting equation in the equivalent form

$$\frac{1}{6}\frac{d}{(z/h)}\left(\frac{ph}{\mu W}\right) = \frac{\dfrac{ph}{\mu W} - \dfrac{2q}{\mu W^2}}{\dfrac{ph}{\mu W}} , \quad (2.23)$$

in which each of the fractions illustrated is nondimensional. One may characterize three regimes of the flow as summarized in the following table:

$0 < \dfrac{ph}{\mu W} < \dfrac{2q}{\mu W^2}$,	favorable pressure gradient
$0 \le \dfrac{2q}{\mu W^2} < \dfrac{ph}{\mu W}$,	weak adverse pressure gradient
$\dfrac{2q}{\mu W^2} \le 0 < \dfrac{ph}{\mu W}$,	strong adverse pressure gradient

Table 2.1 Regimes of the solutions of (2.23)

I have applied the adjective *favorable* (or *unfavorable*) to the pressure gradient, $p'(z)$, if the pressures on the leading and trailing faces of a fluid element are unequal and urge the element to move *in the same direction as* (or *in the opposite direction* *from)* the wall motion, respectively. In flow with a favorable pressure gradient ($p'(z) < 0$), there is expansion rather than compression in the direction of wall motion. The case $p'(z) < 0$ is therefore irrelevant to the design of a viscous-drag pump. In flow with an adverse pressure gradient ($p'(z) > 0$), there will be compression in the direction of wall motion if the adverse pressure gradient is not strong enough to cause reversed flow ($\psi_h < 0$, or, equivalently, $q < 0$). Alternatively, if the pressure gradient *is* strong enough to cause reversed flow then the device will not act as a compressor in that case either. One concludes that the weak-adverse-pressure-gradient case in Table 2.1 above is the only one of interest in the design of a viscous drag pump.

One may rearrange (2.23) into a form suitable for solution in the case of a weak pressure gradient, *viz.*

$$6\frac{d(z/h)}{d\left(\frac{ph}{\mu W}\right)} = 1 + \frac{\frac{2q}{\mu W^2}}{\frac{ph}{\mu W} - \frac{2q}{\mu W^2}},$$

which now treats z/h as the dependent variable and $ph/(\mu W)$ as the independent variable. The antiderivative with respect to $ph/(\mu W)$ is

$$6\frac{z}{h} = \frac{ph}{\mu W} + \frac{2q}{\mu W^2}\ln\left(\frac{ph}{\mu W} - \frac{2q}{\mu W^2}\right) + C_3\,, \quad (2.24)$$

in which C_3 is an arbitrary constant. Let the stations $z = 0$ and $z = \ell$ correspond to the inlet and the outlet of the channel, respectively. Let p_o denote the pressure at the outlet. If one evaluates (2.24) at $p = p_o$ (where $z = \ell$) and subtracts the result from (2.24) one gets

$$6\frac{z-\ell}{h} = \frac{ph}{\mu W} - \frac{p_o h}{\mu W} + \frac{2q}{\mu W^2}\ln\left(\frac{\frac{ph}{\mu W} - \frac{2q}{\mu W^2}}{\frac{p_o h}{\mu W} - \frac{2q}{\mu W^2}}\right). \quad (2.25)$$

Figure 2.1 nearby shows a family of pressure distributions derived from (2.25) in the special case $p_o h/(\mu W) = 1{,}000$.

Two features of these results merit mention at this point. First, the curve with $q = 0$ is the dead-head case mentioned above. As stated there, the slope, $p'(z)$ is constant and the model predicts a perfect vacuum in the interval $0 \le z \le \ell - \ell_{dh}$. Second, each curve with $q > 0$ tends toward a horizontal asymptote in the upstream direction (see the left margins of the curves). Third, each pressure curve attains a value close to its upstream asymptotic value at a distance two or three dead-head lengths upstream of the outlet. This last observation suggests that any portion of the channel more that two or three dead-head lengths upstream of the outlet does little more than add weight to the pump while playing little or no role in compressing the gas.

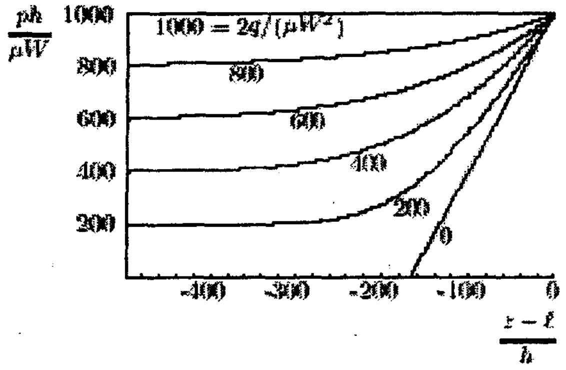

Fig. 2.1 Longitudinal pressure distributions down a channel predicted by (2.25) in the special case $p_o h/(\mu W) = 1{,}000$ (Plane-flow case).

That there should be an asymptote on a curve of constant q is apparent from (2.25) since the value of the natural logarithm tends to negative infinity as $ph/(\mu W) \to 2q/(\mu W^2)$. On such an asymptote the pressure is uniform and thus corresponds to the COUETTE state discussed earlier.

One may derive a simple formula formula for the volumetric throughput (per unit span), $\dot{\delta}_C$, on the upstream asymptote. This formula will prove useful in the sequel, so I will derive it now. In the text above equation (2.22), I pointed out that $\psi_h = \rho\dot{\delta}$. If one substitutes this result into the equation just before (2.19) one gets

$$\rho(z)\dot{\delta} = \rho(z)\left[-\frac{p'(z)}{2\mu}\frac{h^3}{6} + \frac{Wh}{2}\right].$$

If one cancels the common factor $\rho(z)$ in the left and right member and evaluates on the remote upstream asymptote (where $p'(z) \to 0$ and $\dot{\delta} \to \dot{\delta}_C$), one gets

$$\dot{\delta}_C = \frac{Wh}{2}. \quad (2.26)$$

As an application of the foregoing formulas to an initial pump design, consider the following specifications:

1. The fluid is air and its temperature inside the pump is 120° F;
2. The volumetric rate of transport, S, of gas into the inlet is 7 L/min [0.247 ft^3/(min)].*
3. The volumetric transport rate, S_C, on the remote upstream asymptote of the pressure distribution is such that $S_C = 1.1\,S$.;
4. The aspect ratio of the channel cross section, $\lambda := b/h$, in which b and h are the channel span and thickness, respectively, has the value $\lambda = 5$;
5. The inlet pressure, p_i, and the outlet pressure, p_o, equal 10^{-4} Atm and 1 Atm, respectively;
6. The spin rate of the drum, f, is in the range 30,000 to 90,000 RPM;
7. The radius of the drum, $\mathfrak{R}$, is in the range 1–3 cm.

With these specifications, one may calculate various parameters of a pump such as: the channel thickness, h, the channel length, ℓ, the altitude of the drum, L, and the area, A, of the drum (to name four examples). With the aim of calculating these quantities, one must first rewrite some of the equations already in hand to put them into a form convenient in this application.

From the foregoing definitions, we have $S_C = \dot{\delta}_C b$, or, in view of (2.26),

$$S_C = \frac{W b h}{2}\,. \tag{2.27}$$

Recall, next, that $q = p\dot{\delta}$ [*cf.* the equation before (2.22)]. But q is independent of position along the channel [*cf.* the text after (2.22)], so we may calculate q at any convenient station along it. If one takes the inlet (where $p = p_i$ and $\dot{\delta} = S/b$) as this reference station, one gets

$$q = \frac{p_i S}{b}\,. \tag{2.28}$$

Now evaluate (2.25) at the inlet (where $p = p_i$ and $z = 0$) and multiply the result by -1:

$$6\frac{\ell}{h} = -\frac{p_i h}{\mu W} + \frac{p_0 h}{\mu W} + \frac{2q}{\mu W^2}\ln\left(\frac{\dfrac{p_o h}{\mu W} - \dfrac{2q}{\mu W^2}}{\dfrac{p_i h}{\mu W} - \dfrac{2q}{\mu W^2}}\right).$$

* This value corresponds to the rated speed of a scroll pump (Air Squared model number P10H10/N4.0 or Spiradyn-003-DSP) now installed in a prototype gas analyzer (AVEMS) in the Hazardous Gas Detection Lab at KSC.

If one eliminates q by means of (2.28) one gets

$$6\frac{\ell}{h} = -\frac{p_i h}{\mu W} + \frac{p_0 h}{\mu W} + \frac{2p_i S}{\mu W^2 b}\ln\left(\frac{\dfrac{p_o h}{\mu W} - \dfrac{2p_i S}{\mu W^2 b}}{\dfrac{p_i h}{\mu W} - \dfrac{2p_i S}{\mu W^2 b}}\right).$$

If one eliminates b by means of (2.27) then one may show, after some reductions and rearrangement, that

$$\ell = \frac{p_o h^2}{6\mu W}\left\{1 - \frac{p_i}{p_o} + \frac{p_i S}{p_o S_C}\ln\left(\frac{\dfrac{p_o}{p_i}\dfrac{S_C}{S} - 1}{\dfrac{S_C}{S} - 1}\right)\right\}. \tag{2.29}$$

Note that the expression outside the large curly braces in the right member is equals the dead-head length, ℓ_{dh}, defined by (2.21).

If one substitutes $b = \lambda h$ into (2.27), one gets, after rearrangement,

$$h^2 = \frac{2S_C}{\lambda W}\,. \tag{2.30}$$

In the mean time, we know that.†

$$W = 2\pi\mathfrak{R}f\,. \tag{2.31}$$

If one eliminates W from (2.30) by means of (2.31) one gets

$$h^2 = \frac{2S_C}{\lambda 2\pi\mathfrak{R}f}\,. \tag{2.32}$$

If one substitutes (2.31) and (2.32) into (2.29) to eliminate W and h^2, respectively, one gets

$$\ell = \frac{p_o}{3\mu}\frac{S_C}{\lambda(2\pi\mathfrak{R}f)^2}\left\{1 - \frac{p_i}{p_o} + \frac{p_i S}{p_o S_C}\ln\left(\frac{\dfrac{p_o}{p_i}\dfrac{S_C}{S} - 1}{\dfrac{S_C}{S} - 1}\right)\right\}. \tag{2.33}$$

From the specification that the gas is air at $T = 120°$ F one can look up the corresponding shear viscosity, μ, from a handbook or calculate it from a standard empirical formula, such as the so-called *Sutherland law* [*cf. U.S. Standard Atmosphere 1976*, [1]. The latter gives

$$\mu = 1.91848 \times 10^{-4}\ \text{dyne s (cm)}^{-2}\,. \tag{2.34}$$

† Remember that f is in *cycles* per unit time rather than *radians* per unit time. This distinction accounts for the presence of the factor 2π in the result asserted.

With (2.34) and the data in the specifications above, equation (2.33) is in a form suitable for immediate substitution of numerical values, provided one takes due account of necessary unit conversions. As an example of the latter, one may note that

$$p_a = 1 \text{ Atm} = 1.01325 \times 10^6 \text{ dyne (cm)}^{-2}$$

which expedites the computation of the ratio p_a/p in (2.33). Thus, specifications 3 and 5 above are sufficient to determine the numerical value of the expression in curly braces above (It has the value 1.00096). The closeness of this factor to one is worth noting and is a consequence of the smallness of the factor p_i/p_a.

One may now tabulate the channel length, ℓ, by substituting the foregoing values of the parameters into the right member of (2.33). Table 2.1 furnishes results obtained in this way.

	ℓ (m)		
$\mathfrak{R}$ (cm)	$f = 3{\cdot}10^4$ (RPM)	$f = 6{\cdot}10^4$ (RPM)	$f = 9{\cdot}10^4$ (RPM)
1	45.12	11.28	5.01
2	11.28	2.82	1.25
3	5.01	1.25	0.557

Table 2.1 Dependence of channel length, ℓ, upon drum radius, $\mathfrak{R}$, and rotation rate, f.

The square root of equation (2.32) furnishes a formula for the h. Table 2.2 furnishes calculated values of h:

	h (mm)		
$\mathfrak{R}$ (cm)	$f = 3{\cdot}10^4$ (RPM)	$f = 6{\cdot}10^4$ (RPM)	$f = 9{\cdot}10^4$ (RPM)
1	1.278	0.904	0.738
2	0.904	0.639	0.522
3	0.738	0.522	0.426

Table 2.2 Dependence of channel width, h, upon drum radius, $\mathfrak{R}$, and rotation rate, f.

If, as stated earlier, one constructs a viscous drag pump by rotating a cylindrical drum in a stationary cylinder then one may estimate the inside area, A, of the cylinder by the product of the channel length, ℓ with the channel span, $b = \lambda h$. Of course, the cylinder area in a real pump must be larger to allow for the portion of the area taken up by seals between adjacent channels (or threads). Even, so the the product $\ell\lambda h := A$ is interesting since one expects that the mass of a hollow drum to scale with this quantity. Table 2.3 furnishes calculated values of A:

	A (cm)^2		
$\mathfrak{R}$ (cm)	$f = 3{\cdot}10^4$ (RPM)	$f = 6{\cdot}10^4$ (RPM)	$f = 9{\cdot}10^4$ (RPM)
1	2884.	509.8	185.0
2	509.8	90.12	32.70
3	185.0	32.70	11.87

Table 2.3 Dependence of cylinder area A, upon drum radius, $\mathfrak{R}$, and rotation rate, f.

If one divides the cylinder area, A, by the cylinder circumference, $2\pi\mathfrak{R}$, one gets the cylinder altitude, L. Table 2.4 furnishes calculated values of L:

	L (cm)		
$\mathfrak{R}$ (cm)	$f = 3{\cdot}10^4$ (RPM)	$f = 6{\cdot}10^4$ (RPM)	$f = 9{\cdot}10^4$ (RPM)
1	459.0	81.14	29.44
2	40.57	7.172	2.603
3	9.815	1.735	0.6296

Table 2.4 Dependence of cylinder altitude L, upon drum radius, $\mathfrak{R}$, and rotation rate, f.

I will conclude the discussion of numerical results by noting that the pressure-volume throughput, Q, defined by $Q := p_i S$, is a constant of the problem. Thus, since S and p_i are given by Specifications 2 and 5, respectively, the corresponding value of Q becomes 7×10^{-4} Atm L/min or

$$Q = 1.167 \times 10^{-2} \text{ Atm (cm)}^3\text{/s)}.$$

3. CONCLUSIONS

The present effort supports several conclusions including the following:

1. The assumptions of lubrication theory of a gas enable one to solve for the plane flow down a uniform channel (with roof motion parallel to the long axis of the channel) in terms of elementary functions;

2. One may identify three distinct regimes of the flow, namely favorable pressure gradient, weak adverse pressure gradient, and strong adverse pressure gradient, of which only one is compatible with flow in a compressor;
3. If one can spin the drum up to 9×10^4 RPM, then one can replicate the pumping speed (namely 7 L/min) of existing backing pumps of scroll- and diaphragm-type with a cylindrical drum of surface area less than 12 $(cm)^2$.

4. UNRESOLVED ISSUES; FUTURE OPPORTUNITIES

The foregoing conclusions give cause for optimism that miniaturization of backing and roughing pumps of viscous drag type may be possible. The relative simplicity of the concept also suggests that such a device may be ruggedized.

In the geometry considered thus far the velocity, v_{roof}, of the roof relative to the floor corresponds to the velocity of a stationary cylinder relative to a rotating drum interior to it. If the channel has the shape of a screw-thread then there will inevitably be a component of v_{roof}, say $v^{\perp}_{roof}$, perpendicular to the thread. The effects of a nonzero $v^{\perp}_{roof}$ include a two-dimensional *recirculating flow* in the plane of a typical cross section. The presence of such a recirculating flow may influence the performance of a pump in a number of ways. To discuss them, however, I must first describe the so-called *liftup effect*.

To set the stage for a description of the liftup effect, note that the recirculating flow will move some fluid particles closer to the roof and others closer to the floor. There will be some tendency for such particles to retain their translational momentum (and, hence, their streamwise velocity, w) as they move into a new environment. The velocity distribution given by (2.15) would thus be subject to warping. This warping of the streamwise velocity distribution is the liftup effect, a phenomenon discussed (for example) by LUDWIG PRANDTL in the 1920s. The liftup effect may affect pump performance in number of ways. Thus:

1. The pump performance estimates given above presume that the flow is not turbulent. The liftup effect typically destabilizes a nonturbulent flow and makes it more apt to become turbulent. The question of whether—or if—the flow trips to a turbulent state and, if it does, how the turbulence affects the pump performance, are serious questions worthy of further investigation.
2. Even if there is no turbulence, the redistribution of streamwise momentum in the cross section due to the liftup effect may affect the performance of the pump significantly. The derivation of appropriate formulas or computational models to predict such changes in performance is a important subject for follow-on work.
3. The recirculating flow in the cross section and, for that matter, the flow as a whole may be affected significantly by the leakage of fluid across seals that separate adjacent flow channels on the rotating drum. The derivation of appropriate formulas or computational models to predict such effects of seal leakage—and its effects on pump performance—is a fit subject for further investigation.

ACKNOWLEDGEMENTS

As usual, I am indebted to my NASA colleague RIC ADAMS (of the newly constituted Technology Implementation Branch, YA-C3, of the Spaceport Technology Development Office, YA-C, of the Spaceport Engineering and Technology Directorate, YA), for his suggestion of the problem addressed here, for inspiring conversations pertaining thereto and for continued moral and material support. I am also indebted to Dr. RICHARD ARKIN and GUY NAYLOR of Dynacs, Inc. for showing me their prototype Aircraft Volcanic Emissions Mass Spectrometer (AVEMS) and for furnishing me with technical data for both the scroll pump that the present device features and for a diaphragm pump under consideration as a replacement. I am indebted to Dr. BOB YOUNGQUIST for many useful conversations on technical subjects as well as on the mysteries of working for the government. I am, as usual, indebted to CASSIE SPEARS of the UCF/ASEE Faculty Summer Fellowship office and to Dr. TIM KOTNOUR for his assiduous care in managing of this summer's program.

REFERENCES

[1] ANONYMOUS *U.S. Standard Atmosphere 1976*. U.S. Committee on Extension of the Standard Atmosphere (COSEA), Superintendent of Documents, U.S. Government Printing Office, Washington, D.C.