NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM

No. 1091

THE EFFECT ON THE SPERRY DIRECTIONAL GYRO IN TURNING

By Rossello Rosselli Del Turco

Atti Di Guidonia
No. 18, October 10, 1939

Washington
August 1946

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 1091

THE EFFECT ON THE SPERRY DIRECTIONAL GYRO IN TURNING[1]

By Rossello Rosselli Del Turco

SUMMARY

The present report is concerned with an analytical treatment of the effects of the transverse inclination of an airplane in a turn on the indication of the directional gyro. It is found that the extreme inclinations which the airplane must necessarily assume for a correct turn in the approaches executed at high speed and small radius of curvature, renders the indications of the instrument worthless during such maneuvers.

INTRODUCTION

The use of the directional gyro during changes of direction is very useful to the pilot, because it enables him to follow the turn step by step, and to resume after completing the turn, the new course which he intends to follow. In fact, the resistance of the gyroscopic axis does not permit the graduated rim with which it is integral, to execute any oscillations during the maneuver, as opposed to what occurs on the compass where the oscillations due to the inertia of the magnetic equipment, as well as the errors imputed in relation to the vertical component of the terrestial field, do not afford the pilot a safe reading of the course during the maneuver or immediately after it. The pilot, however, can see that the directional gyro is not even in a position to indicate the course accurately during the maneuvers, if performed with considerable transverse inclinations. To illustrate: on departing from a 45° course he notices that the dial does not follow the motion of rotation of the airplane at the first instant but anticipates it by left-hand

[1] "Il Comportamento del Giroscopio Direzionale Tipo Sperry in Virata." Atti di Guidonia, No. 18, Oct. 10, 1939, pp.373-380.

turns, impedes it by right-hand turns. The relative errors of this phenomenon persist more or less during the directional change, but the pilot is not able to verify them directly; and they exceed in certain instances the maximum allowed to the directional gyro in normal setting.

In fact, the rule is that, 15 minutes after adjustment of the instrument by means of the appropriate adjusting lever, the errors due to the rotation of the earth, and the precision of the gyro in the azimuth due to various causes, should not exceed 3° to 4°; instead of that the instrument may introduce errors as high as 14° to 15°, depending upon the particular trim of the airplane.

Obviously, even putting aside the normal use of the directional gyro on an airplane, the previously outlined phenomena occur in all specific cases where the reading of the instrument during the maneuver of turning, is required.

So, the Instrument Section of the Guidon'a Laboratory made some experiments in which the instrument records during turning at various transverse inclinations were taken into account, and it was found that, at small inclinations, the experiments confirmed the results anticipated, while at great inclinations the discrepancies were considerable.

The behavior of the directional gyro in a turn was closely examined and a geometrical method developed parallel with 16 experiments, which confirmed the theoretical results.

GEOMETRICAL METHOD

The functioning of the instrument is referred to a system of axes moving with the airplane and its setting to a system of axes fixed in space.

Visualize (fig. 1) a system of axes x, y, z, integral with the hypothetical airplane for simplicity, as being coincident with the principal axes of inertia in such a way that axis x coincides with the longitudinal axis, positive backward, axis y with the transverse axis, positive to the left, and axis z with the axis of yaw, positive upward, and a system of axes ξ, η, ζ fixed in space, where ζ is vertically (zenith) upward, and ξ along a cardinal point, such as North, for instance. Employing Crocco's notation (reference 1) the plane xz is the plane of symmetry, plane zy the frontal plane, and plane yz the plane of the wing. The slope of the airplane is indicated by the angle ∂_0 between x and the intersection x_1, of the plane of symmetry with the horizontal center of gravity $\eta\zeta$, the obliquity of the airplane by the angle Γ_0 between z_1 perpendicular to x_1, in the plane of symmetry and the vertical center of pressure ζ, the direction of the airplane by the angle ζ_0 between x_1 and the horizontal axis of reference ξ.

Normally, the directional gyro is mounted in such a way that axis z coincides with the axis along which the supporting frame of the spinning disk cradle is bolted to the instrument casing, and the axis x with the diameter of the dial plate passing through the reference mark for the readings fixed to the box. Moreover, suppose that the gyroscopic axis is oriented along axis ξ by means of appropriately manipulated adjustment. This condition is maintained during the motion of the airplane as long as the two axes are fixed in space.

Consider, for simplicity (fig. 2), the settings of zero slope $(\partial_0=0)$, that is to say, turns in which the longitudinal axis of the airplane is kept on the horizontal plane $\eta\xi$ and in which obliquity (Γ_0) and direction (ζ_0) depart from 0. Such a setting coincides with that of horizontal flight, except for the angle between the longitudinal axis and the direction of the relative wind which is discounted in the present study. In such conditions x coincides with x_1 and z with z_1, and the directional gyro is constrained to indicate the direction ζ_0 similar to the angle between x and ξ. The actual reading at the dial is that of the indicated direction ζ_1 similar to the angle between x and the diameter 0° to 180° of the dial plate (figs. 2 and 4). For $\zeta_0 \neq \zeta_1$ the error of indication generally amounts to $\epsilon = \zeta_0 - \zeta_1$.

FUNCTION $\epsilon = F(\Gamma_0, \zeta_0)$

It is advantageous to settle one important characteristic of this error: namely, that ϵ is a function of the direction ζ_0, as well as of the obliquity Γ_0. For Γ_0 = constant, $\epsilon = \zeta_0 - \zeta_1$ = constant, and, with t signifying the time:

$$\frac{d\varepsilon}{dt} = \frac{d(\zeta_0 - \zeta_1)}{dt} = 0$$

or

$$\frac{d\zeta_0}{dt} - \frac{d\zeta_1}{dt} = 0$$

and where

$$\omega = \frac{d\zeta_0}{dt}$$

is the speed of turn of the airplane about the vertical axis ζ,

$$\omega_z = \frac{d\zeta_1}{dt}$$

the rate of rotation (of interest here) of the dial plate about axis z is

$$\omega = \omega_z$$

and in particular, if ω = constant (uniform circular motion of aircraft):

$$\omega_z = \text{constant}$$

This is not true in reality inasmuch as ω_z results from the decomposition of ω along axis z and along another axis m (fig. 2) defined by the condition that a rotation about it would be of zero effect on the directional indication. By reason of the fact that only rotations about the gyroscopic axis ξ and about axis n to which the cradle is bolted, (fig. 2) normal to ξ and z by construction have zero effect on the directional indications, axis m must lie in the plane ξn and, more precisely, must be the intersection of the said plane with the frontal plane ζ_z. Since, during the turn, the frontal plane rotates about ζ with respect to the fixed axes, the intersection m of the two planes rotates also in the frontal plane, that is, with respect to the system of moving axes x, y, z; hence, if the direction of one of the components of ω, in such a system changes the value of the other component ω_z must change also, even if its direction remains constant (always in the movable system) (Γ_0 = constant), the constancy of the resultant must be maintained.

The next step is to render the function $\epsilon = F(\Gamma_0, \zeta_0)$ in explicit form. It is evident that, according to the foregoing procedure, the decomposition of ω gives a relation of the type.

$$\omega_z = \omega f(\Gamma_0, \zeta_0)$$

or

$$\frac{d\zeta_1}{dt} = \frac{d\zeta_0}{dt} f(\Gamma_0, \zeta_0)$$

$$\zeta_1 = \int_0^{\zeta_0} f(\Gamma_0, \zeta_0)$$

but, since the error $\epsilon = \zeta_0 - \zeta_1$ is reduced to an inconvenient integration, it is advisable to abandon this concept of the decomposition of ω and to consider the relation between the angles direct.

The error is zero for zero values of the variable Γ_0 (z coincident with ζ) and in such conditions the directional gyro indicates the direction ζ_0: if, on the other hand, the instrument turns from Γ_0 about axis x (only the settings of zero slope being considered), its indication is ζ_1: that is, its variation from the angle $\zeta_0 - \zeta_1$ is equal to the error that is to be determined. To find ϵ, therefore, it is necessary to consider successively the orientation of the longitudinal axis x (to which the readings refer, since it passes through the reference mark at the box) with respect to the diameter 0° to 180° of the dial in one or the other setting.

Figure 3 shows the exact orientation of the axes for $\Gamma_0 = 0$ in plane xy coincident with the horizontal plane $\eta\xi$: n coincides with η, the gyroscopic axis ξ coincides with the diameter 0° to 180° of the dial plate, the axis x coincides with diameter ζ_0 to $180^\circ + \zeta_0$ It should be noted that this coincidence holds only for setting $\Gamma_0 = 0$; actually axis ξ and axis η are fixed, while n and the diameter 0° to 180° are integral with the dial plate the plane xy of which, for setting $\Gamma_0 \neq 0$, is not coincident with the fixed plane $\eta\xi$, since for settings $\Gamma_0 \neq 0$, x is not coincident with the diameter ζ_0 to $180^\circ + \zeta_0$ because of the error. The orientation of the axes, in this instance, will be in the plane xy of the dial plate, as represented in figure 4.

With p denoting the angle between axis x and the diameter 90° to 270° of the dial (fig. 4) and assuming its value to be known, there is obtained

$$90 - p = \zeta_1 \pm \zeta_0 - \epsilon$$

from which follows

$$\epsilon = p + \zeta_0 - 90^\circ \qquad (1)$$

Angle p is obtained from the solution of the spherical triangle PQR (fig. 5). In the latter $q = 90^\circ$ by the construction of the frame itself, the angle in $Q = \Gamma_0$, a straight section of the dihedral facing the horizontal plane $\eta\xi$ and the plane of the dial plate xy, the normals and z which form between them an angle Γ_0; $r = 180 - \zeta_0$, inasmuch as the angle $x\xi = \zeta_0$, as is seen from figure 3 (x and ξ are fixed axes and hence its reciprocal orientation for $\Gamma_0 = 0$ is maintained even in the setting $\Gamma_0 \neq 0$).

By conventional trigonometric formula

$$\cos q = \cos p \cos r + \sin p \sin r \cos \hat{Q}$$

$$0 = -\cos p \cos \zeta_0 + \sin p \sin \zeta_0 \cos \Gamma_0$$

or

$$\tan p = \frac{1}{\tan \zeta_0 \cos \Gamma_0} \qquad (2)$$

By equation (1)

$$\tan \epsilon = \cot\text{an}\,(p + \zeta_0) = \frac{1 - \tan p \tan \zeta_0}{\tan p + \tan \zeta_0}$$

or by equation (2)

$$\tan \epsilon = \frac{1 - \dfrac{\tan \zeta_1}{\tan \zeta_0 \cos \Gamma_0}}{\dfrac{1}{\tan \zeta_0 \cos \Gamma_0} + \tan \zeta_0}$$

whence

$$\epsilon = F(\zeta_0, \Gamma_0) = \tan^{-1}\left[\frac{1 - \cos \Gamma_0}{\text{cotan}\,\zeta_0 + \tan \zeta_0 \cos \Gamma_0}\right] \qquad (3)$$

which is the equation looked for.

A study of $F(\zeta_0, \Gamma_0)$ reveals that the error is zero at any obliquity Γ_0, when the direction of the airplane is 0°, 90°, 180°, 270°, the error is negative at any obliquity between 0° and 90°, when the direction of the airplane ranges between the first and third quadrant; it is positive when the direction varies between the second and fourth quadrant.

In a turn executed at constant obliquity, the error is maximum for the direction$_0$, which satisfy the equation

$$\tan \zeta_0 = \frac{1}{\sqrt{\cos \Gamma_0}}$$

obtained from

$$\frac{\partial F}{\partial \zeta_0} = 0$$

when the value given by equation (3) is substituted for F. That is to say, the maximum error, in a turn made with varying obliquity, shifts from the direction $\zeta_0 = 45^\circ$ (which has zero limit of obliquity) to $\zeta_0 = 90^\circ$ (which has a maximum point for $\Gamma_0 = 90^\circ$).

Figure 6 shows the curves of $F(\zeta_0, \Gamma_0)$ for various values of Γ_0. The curves coincide, at least for small and unavoidable differences, with those obtained experimentally manifesting the errors introduced by a Salmoiraghi-Sperry directional gyro subjected to rotation about the vertical axis with oblique setting.

On the other hand, figure 7 gives the error ϵ with respect to the indicated direction ζ_i, as well as ζ_0. This diagram is derived from figure 6.

The diagrams are plotted for obliquity not in excess of 60° for the reason that, when 60° are exceeded, the system of gimbal joints stops and the instrument ceases to function regularly.

CONCLUSIONS

The results obtained from the foregoing geometrical study together with the tests in the laboratory afford a legitimate appraisal of the phenomena accompanying the function of the directional gyro in a turn.

The errors of indication can be introduced by the instrument only to the extent that the airplane is maintained in oblique setting; the instrument is not likely to furnish accurate indications again after the airplane, upon completing the turn, has resumed its normal attitude. As regards the use of the instrument in an airplane, the problem therefore is reduced to an approximate appraisal of the instant when the new course is reached. To illustrate: while navigating with a directional gyro indicating 0°, it is desired to make a 50° right-hand turn, while the airplane has a $\Gamma_0 = 40^\circ$ obliquity, the turn is completed when the gyro indicates (cf. fig. 6) $42^\circ, 20'$ that is to say, $7^\circ, 40'$, within less then 50 effective degrees which are obtained. The airplane is not likely to resume normal attitude at the end of the turn when the directional gyro indicates exactly 50°. Thus, if the pilot wants to make allowance for these phenomena in order to effect a turn with a prescribed angle without having to make successive tentative approximations, he should take from figure 6 the error ($\epsilon = -7^\circ\ 40'$), which is appropriate for such an angle ($\zeta_0 = 50^\circ$ in the model problem) and stop the turn at a value of the indication $\zeta_1 = \zeta_0 + \epsilon$ or, in above example, at

$$\zeta_1 = 50^\circ - 7^\circ 40' = 42^\circ 20'$$

However, as such a correction of the course is inevitable even by imperfect maneuvers on the part of the pilot, the inconvenience will have no significant practical importance.

The permissible errors in the directional gyro are of the order of magnitude of 3° to 4°. This is generally accepted, even in respect to the phenomena of precession in the azimuth due to secondary causes. Now, errors of such order of magnitude may be introduced by the instrument even as a result of the airplane obliquity above 25° to 30°. It is at obliquity above 25° to 30°, therefore, that the indications of the instrument, during the maneuver, must be corrected by the factors developed in the present study.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

REFERENCE

1. Crocco, G. Arturo: Criteri di analisi del "Volo Strumentale." Memorie della Classe di Scienze Fisiche, Matematiche e Naturali, vol. 4, Ed. Roma, Reale Accademia d'Italia, 1933.

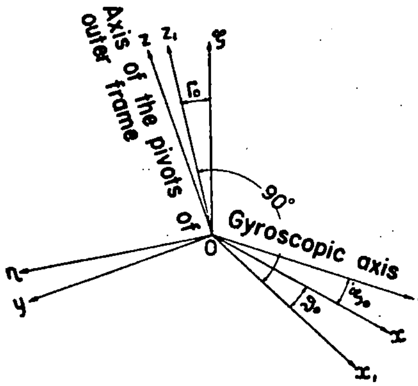

Fig. 1

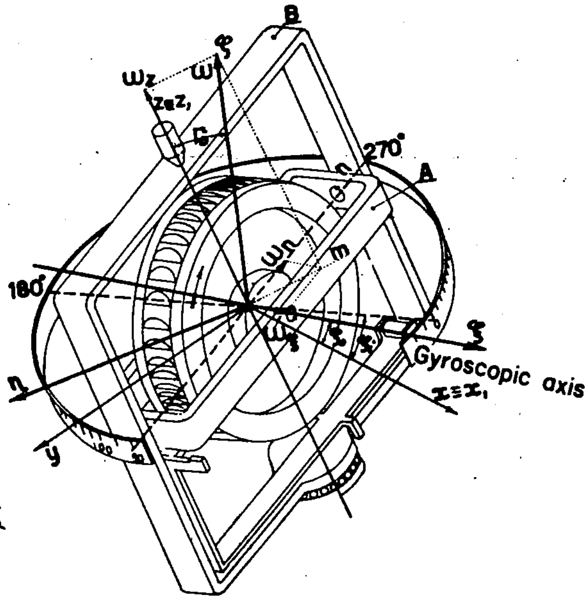

Fig. 2

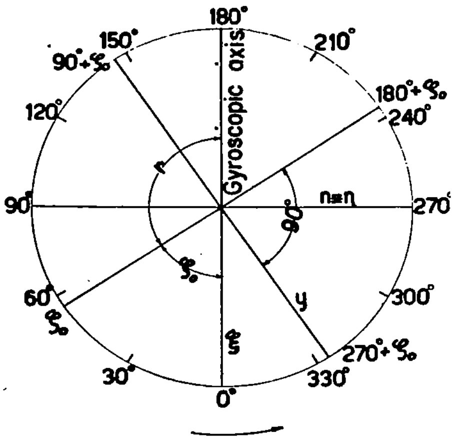

Fig. 3

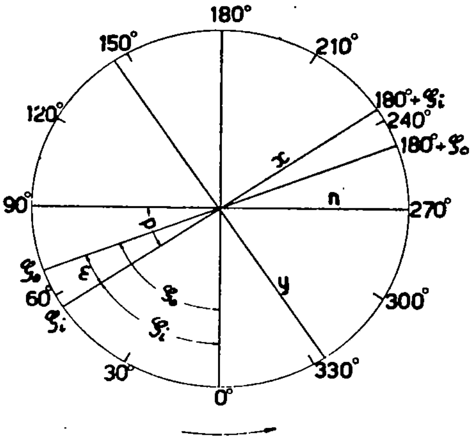

Fig. 4

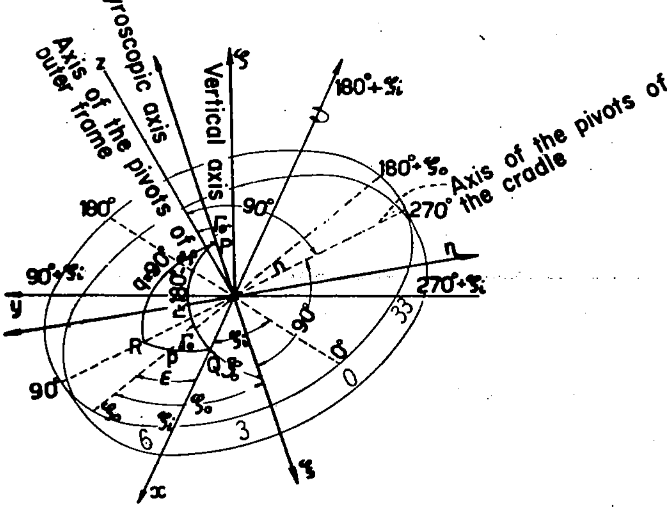

Fig. 5

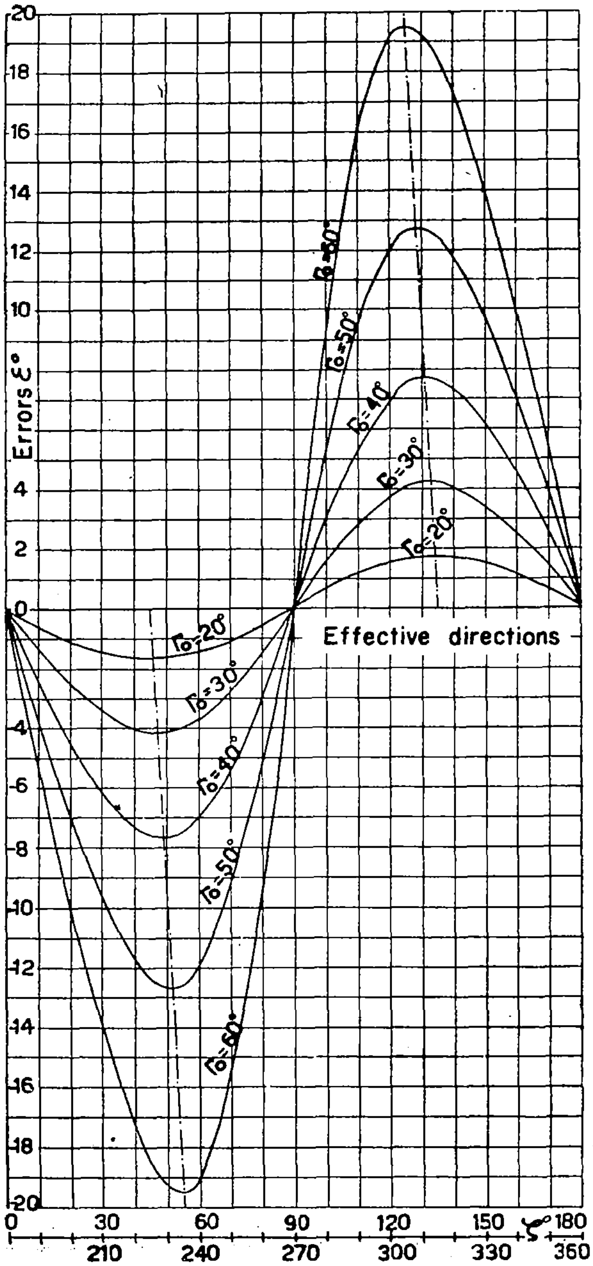
Errors $\mathcal{E}°$
20
18
16
14
12
10
4
2
0
-4
-6
-8
-10
-12
-14
-16
-18
-20
$\Gamma_0=60°$
$\Gamma_0=50°$
$\Gamma_0=40°$
$\Gamma_0=30°$
$\Gamma_0=20°$
Effective directions
0 30 60 90 120 150 180 $\zeta°$
210 240 270 300 330 360

Fig. 6

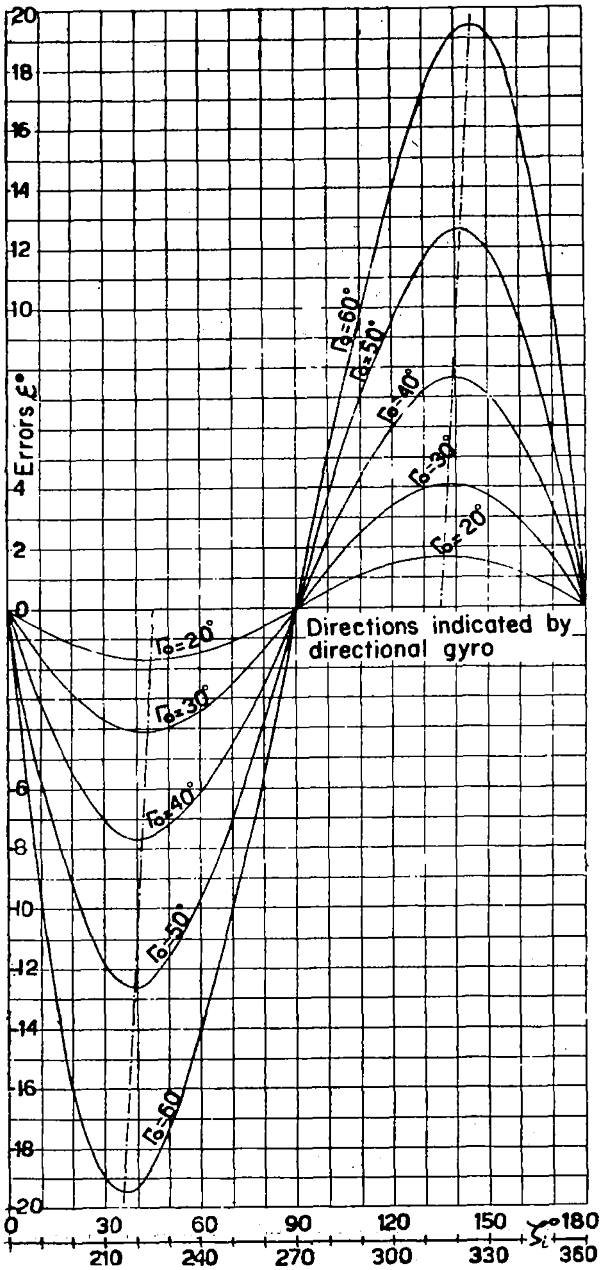
Errors $\mathcal{E}°$
20
18
16
14
12
10
4
2
0
-6
-8
-10
-12
-14
-16
-18
-20
$\Gamma_0=60°$
$\Gamma_0=50°$
$\Gamma_0=40°$
$\Gamma_0=30°$
$\Gamma_0=20°$
Directions indicated by directional gyro
0 30 60 90 120 150 180 $\zeta_i°$
210 240 270 300 330 360

Fig. 7