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TECHNICAL MEMORANDUMS

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No. 1031

CONTRIBUTION TO THE DESIGN OF PLYWOOD SHELLS

By S. Blumrich

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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CONTRIBUTION TO THE DESIGN OF PLYWOOD SHELLS[1]

By S. Blumrich

SUMMARY

The writer sets out to prove by calculation and experiment that by extensive utilization of the skin to carry axial load (reduction of stringer spacing) the stringer sections can be made small enough to afford a substantial saving in structural weight. This saving ranges from 5 to about 40 percent.

INTRODUCTION

One serious draw-back of modern shell constructions is their inability to take full advantage of the strength of the covering material under compression or shear. In shear the additive stresses caused by wrinkling become quickly decisive, in compression the exceeding of the buckling stress of the sheet causes the load to concentrate on the stringer and the skin to fail, of course, with increasing loading. The natural way to raise the critical skin loads by reducing the size of the panels, that is, the stringer spacing, leads - in metal designs - to uneconomical structures (multiplied riveting labor), with the result that the heavier type of construction: fewer stringers, thick skin is usually preferred over improved economy.

In wood designs this procedure is feasible as described hereinafter. The guiding principle is to have the skin, hence the plywood covering, buckle at the same time as the stringers. In this way the total section is completely evenly utilized, hence must result in a reduction of the structural weight per kilogram of load

[1] "Ein Beitrag zur Ausbildung von Sperrholzschalen." Luftfahrtforschung, vol. 18, no. 9, Sept. 20, 1941. pp. 331-337.

carried. This construction is hereafter designated with "strengthened plywood." Skin buckling is dependent upon the dimensions: length, width, and wall thickness; stringer buckling upon cross-sectional area, cross-sectional form and length.

I. MATHEMATICAL STUDY

1. Strengthened Plywood

a) Flat plate.- In order to assure concurrent buckling stresses in stringers and skin, the stringer spacing must be very small compared to the other quantities. The same holds, of course, for the dimensions of the stringer sections. With properly selected frame sections the buckling length of the stringers will not be restricted to the frame spacing, the frames themselves will be permitted to deflect. This affords a great variety in the choice of all dimensions for a given load.

Figure 1 is a diagrammatic view of such a strengthened plywood plate. The inferior thickness d of the frame as well as the large number of stringers prohibits their being let into the frame. The system skin-stringer is simply glued to the frame. (The glueing available surface is 2 ab.) Herewith one of the determinating quantities for the variation of the skin buckling load is removed; the panel length becomes infinite.

The panel boundary of the entire plate in direction of the width B may be established by strengthened diffusion sections or by fitting curved pieces. For buckling, the application of Wagner's formula[1]

$$P = \frac{20\ E}{B} \sqrt{J_v^*\ J_x^*} + \frac{10\ G}{B}\ T_v^*$$

is suggested. The second part on the right-hand side of the equation which allows for the torsional stiffness has been omitted. Table I gives the dimensions and cross-sectional data of the investigated plates. Here is, of

[1] Sheet-metal airplane construction.

TABLE I

a (mm)	l (mm)	t (mm)	Number	F (cm^2)	J_x (cm^4)	J_x* (cm^3)	i (cm)	J_v	J_v*	Buckling load (kg) calculation	test
		30	1	0.46	0.00868	0.00289	0.138			336	300
	100	50	2	.66	.00968	.00194	.121	0.2597	0.02597	250	250
		70	3	.86	.01028	.00147	.109			212	200
		30	4	0.46	0.00868	0.00289	0.138			256	200
4	200	50	5	.66	.00968	.00194	.121	0.3014	0.01507	191	225
		70	6	.86	.01028	.00147	.109			162	150
		30	7	0.46	0.00868	0.00289	0.138			216	400
	300	50	8	.66	.00968	.00194	.121	0.3219	0.01073	161	150
		70	9	.86	.01028	.00147	.109			136	150
		30	10	0.66	0.03095	0.01032	0.216			744	350
	100	50	11	.86	.03655	.00731	.206	0.3584	0.03584	572	400
		70	12	1.06	.03995	.00571	.194			489	300
		30	13	0.66	0.03095	0.01032	0.216			577	550
6	200	50	14	.86	.03655	.00731	.206	0.4254	0.02127	442	300
		70	15	1.06	.03995	.00571	.194			376	350
		30	16	0.66	0.03095	0.01032	0.216			488	500
	300	50	17	.86	.03655	.00731	.206	0.4549	0.01516	375	250
		70	18	1.06	.03995	.00571	.194			318	250

F and J values are valid for stringer and frame spring l, respectively.

course, only one specific skin buckling stress for each stringer spacing, the solution of which presents no unusual difficulties. The plate studies reported herein were primarily intended as an explanation of the fundamental principles involved and as a check on the mathematical treatment. The values J_v* and J_x* indicate the bending stiffness of the respective sections per centimeter of length.

In the determination of J_v* the dissimilarity of the elasticity values of the covering (transverse to the fiber) and pinewood was considered with

$$E_{Sph} = 0.7\ E_{pine}$$

(E = 120,000 kg/cm² for pine). J_x' was posed as variable across the panel width conformable to

$$J_x' = \frac{2}{B} \Sigma J_x \cos^2 \frac{\pi y}{B}$$

The buckling values obtained are then compared with the experimental buckling loads. The results are in close agreement.

The calculation of the buckling length by

$$L = 0.91\ B \sqrt[4]{\frac{J_x*}{J_v*}}$$

gives the values compiled in table II.

TABLE II.- BUCKLING LENGTHS
(cm)

Number	Theory	Test	Number	Theory	Test
1	21.3	----	10	27	30
2	19.2	21.5	11	25	25
3	18.1	----	12	23.4	25
4	24.4	25	13	31	24
5	22.2	20	14	28	25
6	20.7	20	15	27	--
7	26.6	20	16	34	18
8	24.0	25	17	31	25
9	22.5	--	18	29	21

The experimental values are also shown, but, being originally intended for information only, have no great claim to accuracy.

The derivation of a formula for the breaking load proceeds from the consideration of the strain condition. The symbols are shown in figure 2, while figure 3 gives the loading condition of a stringer and frame in the elastic zone which stresses the glued joint between frame and stringer in tension (fig. 4).

The stringer accordingly represents an elastically supported compression member. The support forces from the frames onto the stringers P_{Sp} are put equal to the stringer deflection over the buckling length L_o, that is, according to $\sin \frac{\pi x}{L_o}$. A hypothetical force P_{Sp} in the middle of the buckling length then affords

$$P'_{Sp} = P_{Sp} \sin \frac{\pi x}{L_o}$$

corresponding to

$$e' = e \sin \frac{\pi x}{L_o} \quad \text{(fig. 5)}$$

The maximum bending stress at stringer center (fig. 6) is

$$M = P\ e + \Sigma P'_{Sp}\ a - Q \frac{L_o}{2} \tag{1}$$

where $a = \frac{L_o}{2} - x$ and the term for P'_{Sp} is, for this purpose, more conveniently written in the form

$$P'_{Sp} = P_{Sp} \cos \frac{\pi x}{L_o}$$

Then

$$\Sigma P'_{Sp}\ a = \Sigma P_{Sp} \cos \frac{\pi a}{L_o} a$$

or

$$\Sigma P'_{Sp}\, a = P_{Sp}\, \Sigma \cos \frac{\pi a}{L_o}\, a \tag{2}$$

and

$$\Sigma P'_{Sp} = P_{Sp}\, \Sigma \cos \frac{\pi a}{L_o} = Q \tag{3}$$

whence follows:

$$M = P\, e + P_{Sp}\, \Sigma \cos \frac{\pi a}{L_o}\, a - P_{Sp}\, \frac{L_o}{2}\, \Sigma \cos \frac{\pi a}{L_o}$$

$$M = P\, e + P_{Sp} \left(\Sigma \cos \frac{\pi a}{L_o}\, a - \frac{L_o}{2}\, \Sigma \cos \frac{\pi a}{L_o} \right) \tag{4}$$

The bending stiffness of the frame governs P_{Sp}. With A as the force required for 1-centimeter-frame deflection, the equivalence of frame and stringer deflection results in

$$P_{Sp} = A\, e \tag{5}$$

In the case in point A must be the sum of the stringer forces P_{str} distributed over frame length B. For simplification a position of the forces symmetrical to $x = \frac{L_o}{2}$ is assumed which, in view of the multitude of stringers, can never lead to large errors. For a simple group of forces there is obtained, according to figure 7,

$$f = \frac{P'\, c}{24\, E\, J}\, (3B^2 - 4c^2)$$

With n = the number of panels and m denoting the respective place there is obtained

$$c = B\, \frac{m}{n}$$

Therefore

$$f = \frac{P'}{24\ E\ J_v}\ B\ \frac{m}{n}\left[3B^2 - 4B^2\left(\frac{m}{n}\right)^2\right]$$

$$f = \frac{P'\ B^3}{24\ E\ J_v}\ \frac{m}{n}\ \frac{3n^2 - 4m^2}{n^2}$$

or, with P' distributed over B corresponding to $\sin \frac{\pi c}{B}$:

$$\frac{c}{B} = \frac{m}{n}$$

$$f = \frac{P_{str}\ \sin \frac{\pi m}{n}\ B^3}{24\ E\ J_v}\ \frac{m}{n}\ \frac{3n^2 - 4m^2}{n^2}$$

where P_{str} similar to frame load P_{Sp} is the load at the point of maximum deflection, which serves as initial value. Under the effect of all forces P' there is then obtained

$$\Sigma f = \frac{P_{str}\ B^3}{24\ E\ J_v}\ \Sigma \sin \frac{\pi m}{n}\ \frac{m}{n}\ \frac{3n^2 - 4m^2}{n^2} \tag{6}$$

for $\Sigma f = 1$ cm, $P_{str} = A$ and with

$$C = \Sigma \sin \frac{\pi m}{n}\ \frac{m}{n}\ \frac{3n^2 - 4m^2}{n^2} \tag{7}$$

$$A_{Sp} = \frac{24\ E\ J_v}{C\ B^3} \tag{8}$$

Equation (8), written in equation (5), gives

$$P_{Sp} = \frac{24\ E\ J_v}{C\ B^3}\ e$$

hence the moment (equation (4)) is

$$M = P\,e + e\,\frac{24\;E\;J_v}{C\;B^3}\left(\Sigma\cos\frac{\pi a}{L_o}\,a - \frac{L_o}{2}\,\Sigma\cos\frac{\pi a}{L_o}\right)$$

as breaking condition in the stringer we get

$$\sigma_{br} = \sigma_D + \sigma_B = \frac{P}{F} + \frac{M}{W} \tag{9}$$

For simpler writing we put

$$K = \Sigma\cos\frac{\pi a}{L_o}\,a - \frac{L_o}{2}\,\Sigma\cos\frac{\pi a}{L_o} \tag{10}$$

We further can write $W = \frac{J_x}{s}$ with s indicating the centroidal distance from the inside fiber. Then the rearranged equation (9) gives:

$$P = \frac{F\;W}{W + F\;e}\left(\sigma_{br} - 24\;E\;\frac{e}{B^3}\;s\;\frac{K}{C}\;\frac{J_v}{J_x}\right) \tag{11}$$

The values F, W, J_v and J_x as well as the end load P apply per partial width t.

The separation of the stringers from the frame (tension failure in area 2 ab) has a pronounced effect on the breaking load, because it results in a sudden increase in deflection, hence of all stresses. So far no satisfactory mathematical solution has been found for it. In support of the deflection of the test plates we obtain with $e = 1.0$ centimeter, for instance, for the design with 6-millimeter stringers, the data given in table III.

b) Circular cylinders.- The subsequently described test confirms the assumption that the frame dimensions illustrated in figure 8 affords ample stiffness for preventing deformation of the frames. This simplifies the solution considerably, since the element, stringer plus skin, can be simply considered as compression member between the frames. To determine the cross-sectional inertia moment of the cylinder it is simply treated as a ring with mean wall thickness

$$\delta_m = \delta + \frac{a^2}{t}$$

For the buckling a stringer with the full effective width t is involved. For the cylinder of figure 8 the values are $J = 9500 \text{ cm}^4$, $W = 346 \text{ cm}^3$, for the skin-stringer element $i = 0.187$ cm, and at 150 mm frame spacing $\lambda = 80$. The result of the test is anticipated with buckling failure

$$\sigma = 173 \text{ kg cm}^{-2} \quad \text{calculation}$$

$$\sigma = 187 \text{ kg cm}^{-2} \quad \text{test}$$

The determination of σ is based on the buckling stress diagram of pinewood (fig. 16).

TABLE III

l (mm)	t (mm)	Ultimate load P (kg) theory	Ultimate load P (kg) test
100	30	75	73.5
	50	112	95.5
	70	141	113.0
200	30	67	74
	50	97	99
	70	121	125
300	30	56	78
	50	78	86
	70	97	115

2. Conventional Construction

a) Flat plates.- The prediction of the load carrying capacity of flat plates in conventional designs of wood presents no difficulty. Such a plate is shown in figure 9. The frame can be considered stiff enough to prevent its buckling, as subsequently proved. The static values of a stringer inclusive of an effective width of 3 centimeters in plywood are:

$$F = 1.02 \text{ cm}^2$$

$$J_x = 0.197 \text{ cm}^4$$

$$i = 0.44 \text{ cm}$$

At a 340 mm frame spacing it affords $\lambda = 78$ and, consequently, $\sigma_K = 175$ kg cm^{-2}. The subsequently described test showed $\sigma_K = 178$ kg cm^{-2}.

The determination of the effective width for both wood and metal causes at first an unnecessarily high expenditure of time. But it is soon apparent that - apart from exceptional cases - the obtained values are usually of the same order of magnitude and that accurately enough values are readily obtainable. Incident to the plywood portion counted here it should be noted that the E modulus, after which a diagonally covered panel is obtained, is figured at about 40 percent of that for pine.

b) Conventional cylinders (fig. 10).- Obviously the frames can again be considered as rigid support points. The data for the total cross section are

$$J = 11,760 \text{ cm}^4$$

$$W_{min} = 428 \text{ cm}^3$$

$$W_{12} = 496 \text{ cm}^3$$

The skin is fully accounted for. Since the effect of panel width on the buckling of curved plywood panels is not yet known, the result of the test made with this cylinder must be resorted to. (See section II, 2, b.) It affords at the instant of buckling a moment of 46,300 cm kg. Maintaining this amount as constant at a further loading, the stringers must take up the increase of the moment up to their failure. In correspondence with the previously introduced W_{min} the stress in the stringer at the "skin buckling moment" is

$$\sigma_{str} = 109 \text{ kg cm}^{-2}$$

The buckling stress of a stringer inclusive of 4 cm effective longitudinally fibered skin is 175 kg cm^{-2}. Thus the 66 kg cm^{-2} difference in stress should correspond to the still-to-be-applied moment up to failure. The moment of inertia of the stringers alone is

$$J_{str} = 2230 \text{ cm}^4$$

for the outermost stringer centroid we get $W = 82.5 \text{ cm}^3$, hence

$$\Delta M = 5450 \text{ cm kg}$$

Then the mathematical breaking moment is

$$M_{br} = 46,800 + 5450 = 52,250 \text{ cm kg}$$

which is equivalent to a transverse force of

$$P = 282 \text{ kg}$$

The experiment showed a force of $P = 295$ kg.

II. EXPERIMENTS

1. Strengthened Plywood

a) Flat plates.- It was essential to distribute the load as evenly as possible over the plate width. The stringers were therefore loaded separately and for the same reason two closely spaced frames were glued on the edge of each plate. The load was applied by a dynamometer across the levers on the application points (knife edges). The lateral edges of the plate were to be freely supported and permit flattening besides. This was accomplished by small wooden blocks.

The buckling load was determined with a ruler placed along the covering. By suitable illumination a curvature of the plate was rendered visible.

With the joining of frame and stringer by means of glueing of area 2 ab, as previously stated, the obviously decisive part of the S-shape elastics is that corresponding to the lower part of figure 4, for by greater stringer deflection the tension in the glue becomes ultimately so high that the joint separates, which is immediately followed, as a rule, by compression failure on the inside of the stringer as a result of the greater deflection. The failure usually starts at the middle stringers. The distribution of the areas of failure over the plate width has the characteristics of figure 11. With the exception of plate 9, the failure is reflected by buckling of the wood fibers on the inside of the curvature. Only the plate, just mentioned, disclosed tension failure in the plywood. There was no frame failure in any of the tests, but the frames stressed according to C showed, within 60 to 80 percent of the ultimate load, incipient buckling of the plywood strip of the frame (fig. 12).

The ultimate deflection of the stringers was not measured; it was estimated at 7 to 12 millimeters.

b) Circular cylinder.- The cylinder shown in figure 8 was solidly screwed to a horizontal bracket with end plate, while the transverse force for producing the bending moment was applied at the other end by dynamometer and steel strap. To prevent failure of the lower end plate, a 150-millimeter wide plywood strip, 2 millimeters thick was glued around the circumference, as shown in figure 13.

The load was applied evenly and progressively. According to figure 13, it did not succeed in making the buckling stresses of stringers and skin the same. This is due in part to the afore-mentioned lack of knowledge of the effect of panel width on the buckling stress, and in part to the attempt of obtaining - for reason of direct comparison with the cylinder of conventional design - equal end load in both tests, which likewise had some effect on the panel width.

At $P = 275$ kg the total deflection was recorded at 23 millimeters. Failure occurred at $P = 320$ kg, which, for the center between frames 14 and 15, corresponds to a moment of

$$M = 202 \times 320 = 64,600 \text{ cm kg}$$

With $W = 346 \text{ cm}^3$ the ultimate stress becomes

$$\sigma = 187 \text{ kg cm}^{-2}$$

which agrees very closely with the calculations.

2. Conventional Construction

a) Flat plates.- The experimental set-up is exactly as described under section II, 1, a. The theoretical assumption of adequate frame stiffness to permit treating the compression members as support was confirmed. The failure occurred ordinarily on the small side of the stringers, as compression failure of the fibers. The distribution of the breaks over the plate width follows the characteristics of figure 11. The original three-panel plate, subsequent to the occurrence of all failures in the edge panel, was shortened by the outer piece and loaded again. The practically constant breaking load indicates that the buckling is actually confined to the space between the rigid frames (table IV).

TABLE IV

Number	Design	Breaking load
1	3 buckling lengths	172
2		180
3		170
4	2 buckling lengths	168
5		200
6		196

Average: 181 kg; with $F = 1.02\ \mathrm{cm}^2$ the ultimate stress is $\sigma = 178\ \mathrm{kg\ cm}^{-2}$

b) Circular cylinder (fig. 10).- The experimental set-up is similar to that described in section II, 1, b. (See fig. 14.) At $P = 250$ kg the skin panels between frames 6 and 7 adjoining the maximum stressed stringers buckle. The section modulus at the panel center is

$$W_{12} = 496\ \mathrm{cm}^3$$

With the moment up to the center between the two frames reckoned as skin buckling stress we get

$$M = 187 \times 250 = 46{,}800\ \mathrm{cm\ kg}$$

as "skin buckling moment," and the corresponding skin buckling stress at

$$\sigma = 94 \text{ kg cm}^{-2}$$

The deflection, again measured at $P = 275$ kg, was 32 millimeters. The reason that this value is so much higher than that of the cylinder of strengthened plywood, in spite of the greater wall thickness, is due to the marked drop of the fictitious shear modulus of plywood with increasing shear stress.

The highest stressed stringer starts to buckle at $P = 280$ kg, followed immediately by incipient tearing of the skin at the area of maximum stringer curvature, in consequence of the sharp deflection. At $P = 295$ kg the stringer fails in compression, the crack in the skin increases rapidly (fig. 15).

III. WEIGHT COMPARISON

To make a comparison of the foregoing data in respect to structural weight, the reciprocal value of the stresses, that is, the number of square centimeters per kilogram of breaking load, is contrasted. The numerical values of table V are based upon the experimental breaking loads.

This result which of itself is already unfavorable for conventional designs would become even worse by increased stringer spacing, where with respect to weight each stringer would have to be figured with a greater width of skin.

TABLE V

WEIGHT COMPARISON $\left(\frac{cm^2}{kg}\right)$ OF PLATES OF CONVENTIONAL DESIGN WITH STRENGTHENED PLYWOOD

Strengthened plywood						Conventional design	
Number	$\left(\frac{cm^2}{kg}\right)$	Number	$\left(\frac{cm^2}{kg}\right)$	Number	$\left(\frac{cm^2}{kg}\right)$	Number	$\left(\frac{cm^2}{kg}\right)$ average
1	0.0094	7	0.0047	13	0.0089	1 to 6	0.0137
2	.0111	8	.0102	14	.0087		
3	.0129	9	.0116	15	.0085		
4	.0085	10	.0090	16	.0085		
5	.0097	11	.0090	17	.0100		
6	.0131	12	.0094	18	.0092		

The comparison of the cylinders is much simpler. The weighed weights of the two cylinders are:

G = 5.37 kg for 1957 mm length in strengthened plywood

G = 6.47 kg for 2004 mm length of conventional design. The weight ratio with allowance for the supported breaking moments is:

$$\frac{G_{\text{plywood}}}{G_{\text{conv. design}}} = \frac{5.37}{1957} \times \frac{2004}{6.47} \times \frac{55000}{64600} = \underline{0.723}$$

These two results, table V and the computed weight ratio, are ample proof that the design with strengthened plywood is lighter by a considerable percent than the conventional design. In addition, the frame weight of the flat plate had not been included in the comparison.

As to the manufacturing problem of strengthened plywood, it can be stated that the system stringer skin can be previously glued in very simple devices and then fitted to the frames (or ribs, etc.). The gripping strength is very good. The treatment of load application, tangentially or longitudinally, presents no fundamental difficulties, nor does the splicing of two such skin panels.

IV. CONCLUSIONS

It is proved by calculation and experiment that extended use of the skin to take up longitudinal forces (reduced stringer spacing) makes it possible to lower the stringer section so as to afford a substantial saving in structural weight. This saving ranges from about 5 to 40 percent.

V. APPENDIX

Because of the wide scatter of the strength and elasticity data of wood, it is a fairly thankless task to secure theoretical derivations of the buckling curves, as originally carried out by Ros and Brunner, from the compression flattening diagram for structural wood. No buckling curve has as yet been published for special kinds of wood, such as used in airplane design. Buckling tests had to be made therefore with such wood, the results of which are shown in figure 16. The curve then served as a basis for the buckling calculations of the present article.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

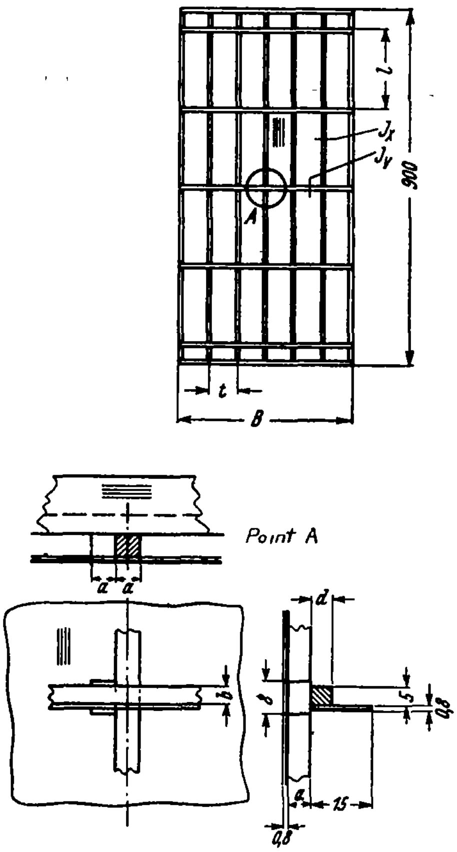

Figure 1.- Plate of strengthener plywood.

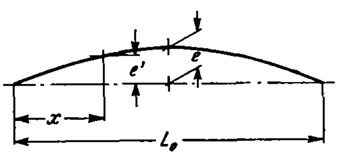

Figure 5.- Identification on a stringer.

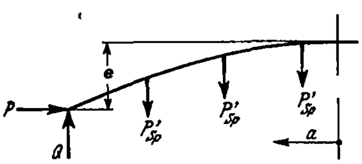

Figure 6.- Load pattern of a stringer for bending moment prediction.

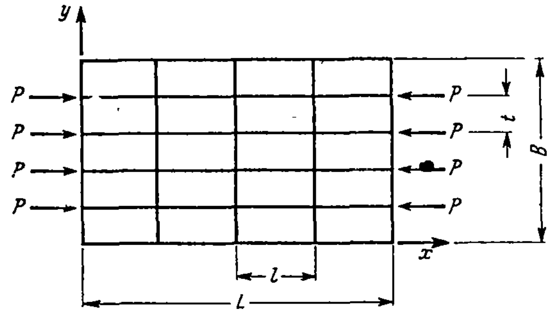

Figure 2.- Identification.

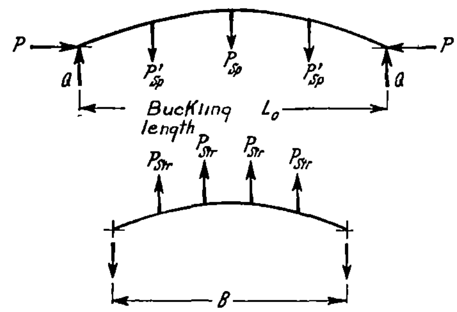

Figure 3.- Load pattern of buckled stringer and frame.

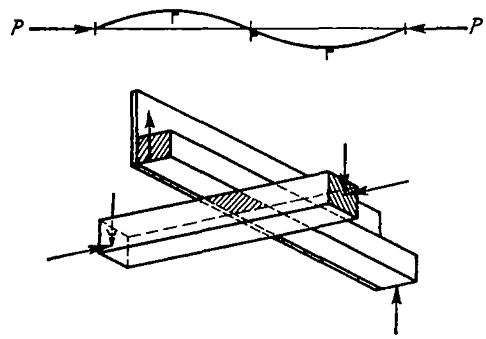

Figure 4.- Strain pattern of stringer. Load schedule at joint between frame and stringer.

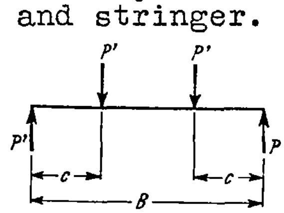

Figure 7.- Frame loading (simplified).

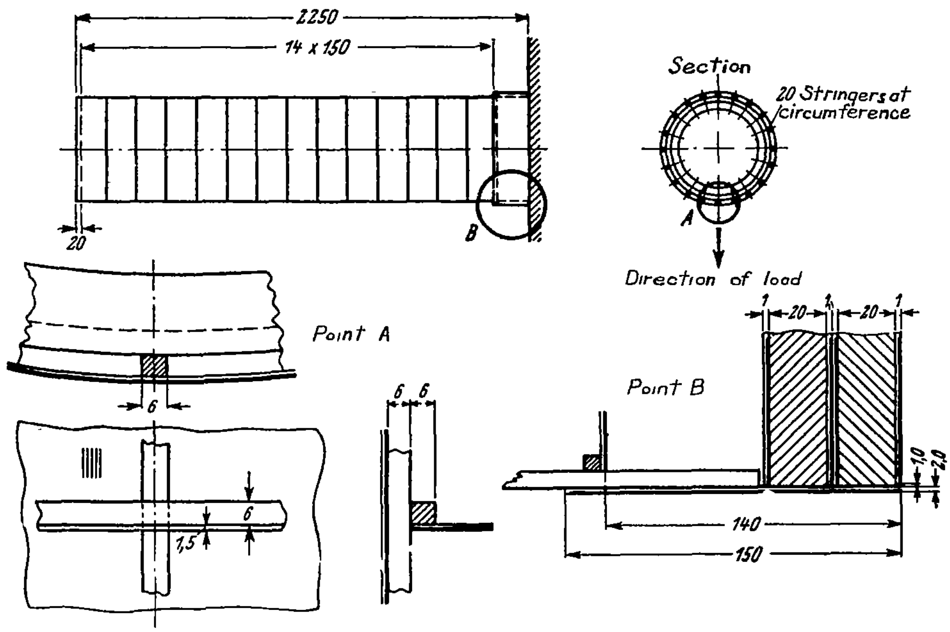

Figure 8.– Cylinder of strengthened plywood.

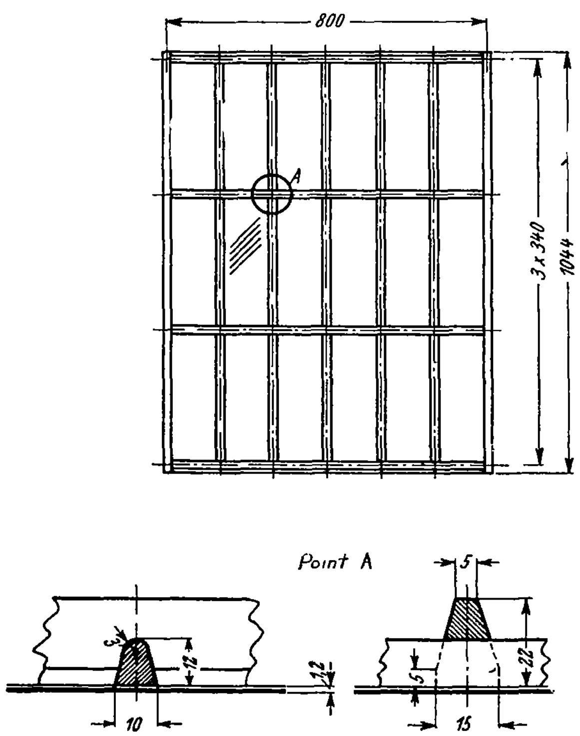

Figure 9.– Plate of conventional design.

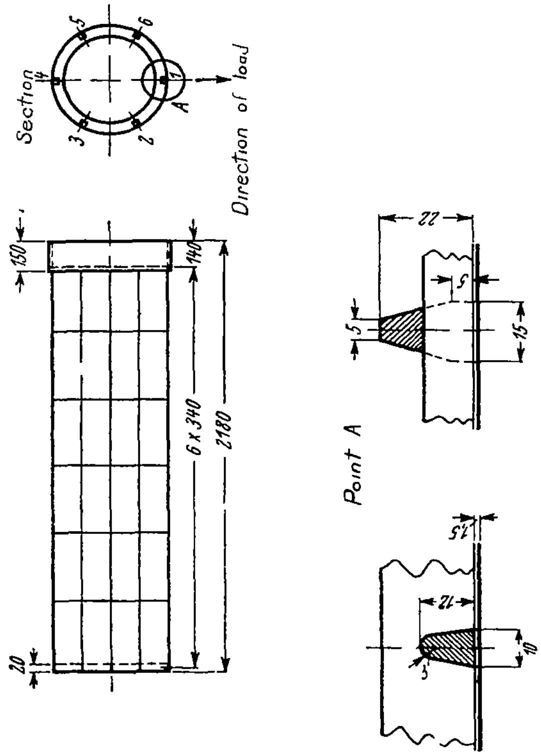

Figure 10.– Cylinder of conventional design.

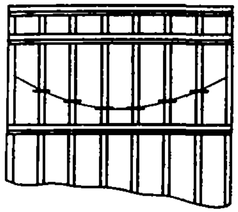

Figure 11.- Distribution of break areas over plate width.

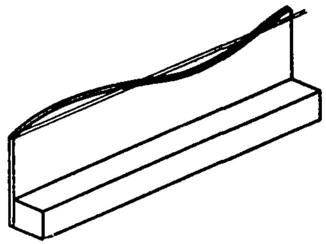

Figure 12.- Buckled plywood strip of a bent frame.

Figure 14.- Cylinder of conventional design at Q = 250 kg.

Figure 15.- Cylinder of conventional design at failure.

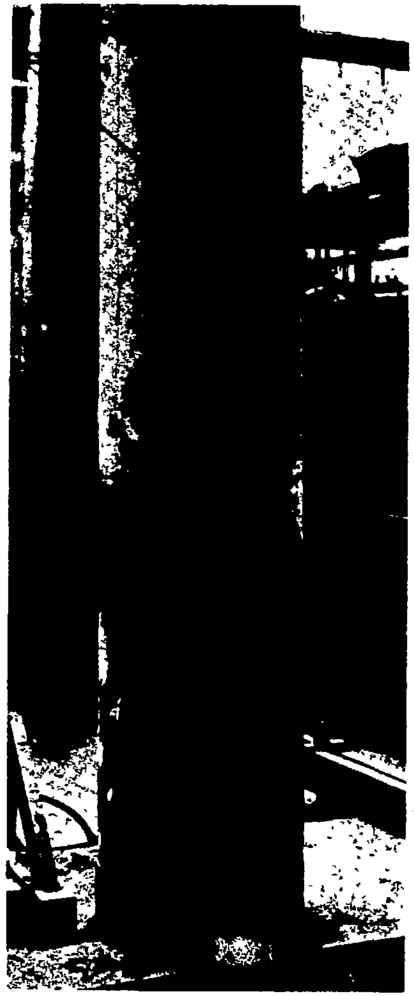

Figure 13.- Cylinder of strengthened plywood at Q = 250 kg.

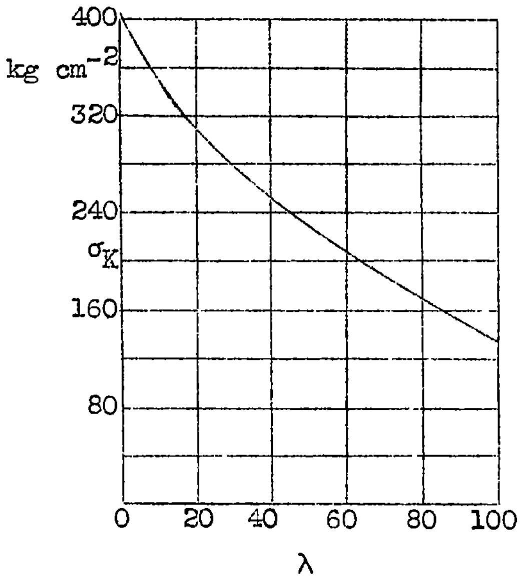

Figure 16.- Buckling stress of pine wood.