NEURAL NETWORK BASED MODELING AND ANALYSIS OF LP CONTROL SURFACE ALLOCATION

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ABSTRACT

This paper presents an approach to interpretive modeling of LP based control allocation in intelligent flight control. The emphasis is placed on a nonlinear interpretation of the LP allocation process as a static map to support analytical study of the resulting closed loop system, albeit in approximate form. The approach makes use of a bi-layer neural network to capture the essential functioning of the LP allocation process. It is further shown via Lyapunov based analysis that under certain relatively mild conditions the resulting closed loop system is stable. Some preliminary conclusions from a study at Ames are stated and directions for further research are given at the conclusion of the paper.

INTRODUCTION

The problem considered in this paper, i.e. development of an interpretive model and analysis of stability of LP based control surface allocation, emerges from an active research program at NASA Ames Research Center, on Intelligent Flight Control(IFC). In the past decade this program has been concerned with addressing two key issues in the design of flight control systems[1,2]:

- Relative shortcoming of existing flight control systems in dealing with in-flight failure of control surfaces/actuators as well as c.g. shifts or body damage

- High cost and relatively long duration of development and testing of flight control systems for each new aircraft.

These issues have led to the development of an architecture for intelligent flight control, which as shown in Figure 1, is centered on the use of a *dynamic inverse* based control strategy and is fur-ther augmented with a standard Proportional Integral (PI) controller. The architecture also makes provisions for adaptive neural network based control augmentation to enhance the performance of the system.

Control Allocation in IFC

A key element of the architecture shown in Figure 1 is the so called *control allocation* subsystem. This subsystem serves two fundamental purposes as follows:

- It reduces the complexity of the control design problem by allowing the design process to focus on the so called *virtual* control inputs to the aircraft, namely the triplet: aileron, elevator and rudder, irrespective of how the actual aircraft may implement these control inputs(e.g. multiple elevators or even evelons, or perhaps multiple rudders and so forth.)

- It allows for in-flight reconfiguration of the aircraft control surfaces to best meet the control demands. Such reconfiguration may occur continuously as the aircraft moves within its operating envelope. More importantly, however, reconfiguration of the flight control surfaces allows the flight control system to overcome failure or damage to one or more control surfaces by using the remaining control surfaces to fully or partially compensate for such failures.

To this end, the IFC team at NASA Ames, has considered a number of possible control allocation strategies that are further discussed below. It has been well understood, however, that regardless of which specific control allocation strategy is deployed in the architecture of Figure 1, two key issues must be further explored:

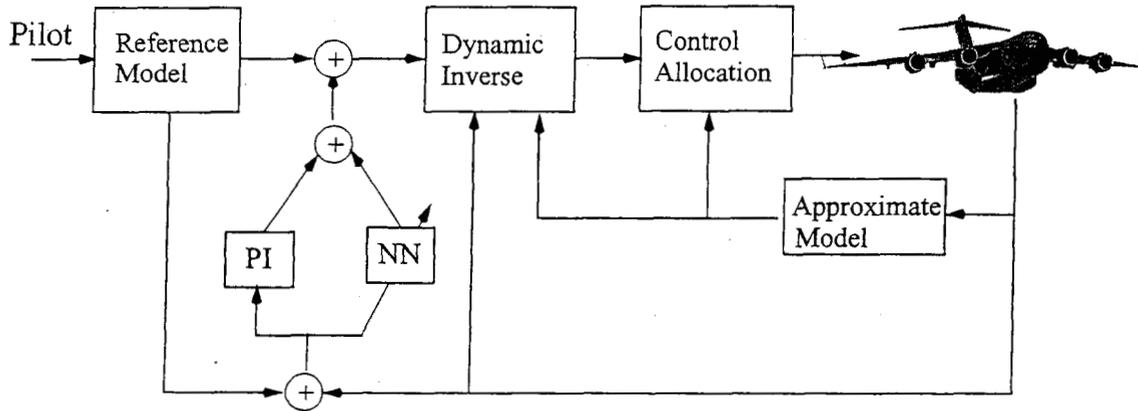- Interpretation of control allocation strategy in

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Figure 1 Architecture for Intelligent Flight Control (from K. Krishnakumar and K. Gundy-Burlet[2].)

terms that are clearly understood by pilots, flight researchers and others engaged in the development and evaluation of intelligent flight control systems

- Analysis of stability of the closed loop control system and evaluation of the impact of control allocation on the behavior of the flight control system

The remainder of this paper provides a preliminary assessment of the above issues. With this in mind, we start with the problem formulation

PROBLEM FORMULATION

In this report we consider a simple model of partial aircraft dynamics as follows[*]:

$$\dot{x} = Ax + A_1 x_1 + Bu \qquad (1)$$

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[*]. The development and notation closely follows[3].

where x is a vector representing the faster changing attitude rates, p, q, and r; x_1 represent the slower translational states; A, A_1, B represent the aerodynamic stability and control derivatives respectively, which may be nonlinearly dependent on x and x_1 but not on u; finally, u represents the actuator commands. We focus for the time being on the behavior of x. In the above model, any residual effects of linearization of the dynamics nonlinear in the control, u, is not included; neither is the cross coupling between the fast rotational states and the slow translation states considered. These effects give rise to *unmodeled* dynamics that is generally handled through (robust)closed loop control. While this issue is important, the main focus of the current discussion is on the impact of control surface allocation on the behavior of the system (1). For this purpose it can be assumed, for instance, that a *virtual* command vector, u_v, can be

generated based on a revised model of the aircraft as

$$\dot{x} = Ax + A_1 x_1 + B_v u_v \qquad (2)$$

where B_v is the corresponding control derivative matrix. A typical approach here it to perform (dynamic) model inversion as follows:

$$u_v = B_v^{-1}(A_m x + B_m r - Ax - A_1 x_1) \qquad (3)$$

which transforms the system into one with the desired behavior given in terms of A_m and B_m. In general, (dynamic) inversion is not perfect, as it is well known, and the resulting system may still incorporate residual effects that must be addressed through additional control augmentation. Moreover, note that the resulting controller can be viewed as a combination of state feedback control, with $B_v^{-1}(A - A_m)$ as the state feedback gain, and reference input feedforward, $B_v^{-1} B_m r$. This fact is depicted in Figure 2 and will later be utilized in the analysis of the system.
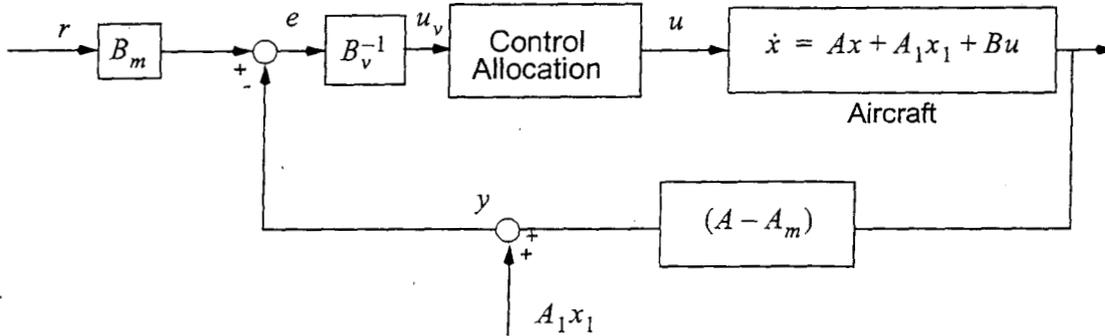

Figure 2 The structure of the closed loop control system

Control Allocation Problem

The main concern in this report is with the problem of translating the virtual control command u_v into the *actual* control vector, u, which could be used in conjunction with the original aircraft model (1). This issue can be stated formally as the problem of determining the optimal choice of u such that the objective

$$Bu = B_v u_v \qquad (4)$$

is met subject to constraints on the magnitude and rate of change of actuator commands, u. An equivalent version of this problem can be stated as

$$Bu = a_d \qquad (5)$$

where a_d is the desired acceleration vector. Either of the above can be solved in a number of ways as described in detail in [4]. These are recounted below:

1. *Direct allocation*
   Maximize ρ such that $Bu = \rho a_d$ if

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$\rho \le 1$; otherwise, $Bu = a_d$; subject to $u_{min} \le u \le u_{max}$. Note that actuator rates are not explicitly listed here but can be included in the solution strategy in incremental form.

2. *Error minimization*

Minimize $\|Bu - a_d\|$ subject to $u_{min} \le u \le u_{max}$

3. *Control minimization*

Minimize $\|u - u_p\|$ subject to $Bu = a_d$ and $u_{min} \le u \le u_{max}$; u_p is the preferred setting for u.

4. *Mixed error/control minimization*

Minimize $\|Bu - a_d\| + \varepsilon\|u - u_p\|$ subject to $u_{min} \le u \le u_{max}$.

It is known that any of the above, appropriately transformed, can be stated as a constrained linear optimization problem; i.e. a linear programming (LP) problem and solved used using some variant of the Simplex Method[4].

INTERPRETATION OF LP BASED CONTROL ALLO-CATION

One of the key issues raised in the introductory section of this report has been the problem of interpretation of the control allocation strategy. As it is evident from the brief discussion in the previous section, the variety of control allocation strategies discussed are generally stated in terms of optimization problems that produce the "best" possible solution given a certain cost function and subject to constraints on magnitudes (and rates) of actuator commands. However, as a rule, optimization algorithms do not provide an interpretive mechanism whereby one can readily assess what the solution

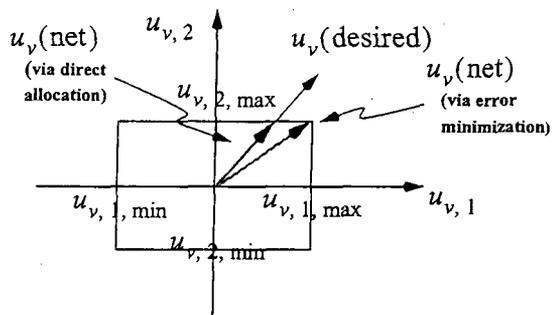means and how it is generated. This idea is depicted in Figure 3 in the particular cases of the

Figure 3 Schematic of the control allocation proces

direct allocation and error minimization. As evident from the figure, either of these strategies produces a solution that either falls inside the "constraint box" defined by the maximum available control authority in the given virtual direction or else leads to a solution that minimizes some error measure (directional accuracy or magnitude.) There is no evidence, however, from the functioning of these allocation strategies that would facilitate an interpretation that could aid the pilot or the designer determine what the significance of loss of control authority is and how that might vary from instant to instant.

Neural Network based Interpretation of Control Allocation

One possible means of addressing the interpretability issue is to determine a nonlinear functional relationship that, while remaining equivalent to the given control allocation strategy, can help provide some insight into its behavior. While in general this approach may not be feasible, the study reported here considered the question in its limited form as it pertains to the case of LP based allocation. In

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particular, the approach explored in this section is to employ a neural network based model of LP allocation strategy to capture the nonlinear relationship presumably embedded in this strategy. This approach in schematic form can be assumed to take the form shown on the left side of Figure 4.
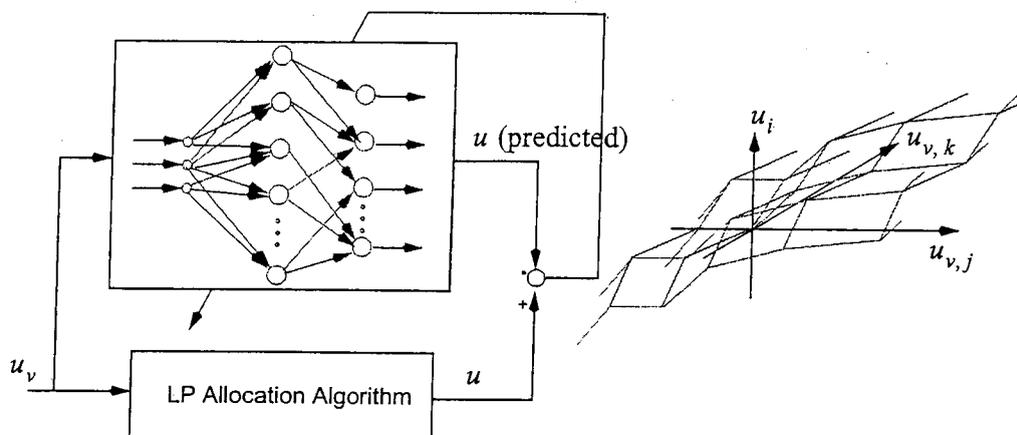

Figure 4 The neural network based framework for nonlinear representation of LP allocation

To this end, a series of "training data" representing the range of variations of the variables involved, i.e. virtual control inputs, were used to train a bi-layered neural feedforward neural network using the so called back-propagation training algorithm with 10 neurons in the first and 6 neurons in the second layers. As shown in Figure 5, the sample response of the left aileron compares well with the prediction of the relatively simple neural network referred above. Note that the intent here has not been to closely match the output of the LP allocation strategy, which can be done using further adaptation of the given neural network or using a large number of neurons at each layer. Rather, the intent has been to demonstrate that it is *in principle* possible to use a neural network to mimic the behavior of the LP allocation strategy. There do remain, to be sure, issues that must be addressed before one can claim that a neural network based model of LP allocation is truly viable in this
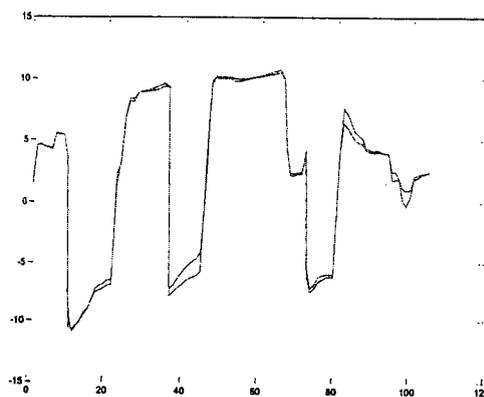

Figure 5 Neural network response (Left aileron response: Predicted versus actual)

respect. These issues are discussed at more length in the conclusion section of this report.

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IMPACT OF CONTROL ALLOCATION ON CLOSED LOOP PERFORMANCE

As discussed above, the behavior of LP can be represented as nonlinear mapping between the desired and the actual controls. This fact is exploited below to address the problem of stability of the resulting closed loop system. In particular, in what follows we assume that the given mapping can be stated as

$$LP_{nn}(u_v) \equiv W\sigma(Vu_v), \qquad (6)$$

in terms of a bi-layered neural network with a sigmoid type activation function in the first layer and a linear type activation function in the output layer. The weights are represented as V and W, respectively. With this in mind, we have

$$\dot{x} = Ax + A_1x_1 + BLP(u_v) \qquad (7)$$

where u_v is given by (3). Note that the ideal situation is represented by $LP(u_v) = B^+B_vu_v$, where B^+ is the pseudo-inverse of B. This would yield, starting from (7), and substituting for u_v from (3)

$$\begin{aligned}
\dot{x} &= Ax + A_1x_1 + BLP(u_v) \\
&= Ax + A_1x_1 + BB^+B_vu_v \\
&= Ax + A_1x_1 + B_vu_v \\
&= Ax + A_1x_1 + B_vB_v^{-1}(A_mx + B_mr - Ax - A_1x_1) \\
&= A_mx + B_mr
\end{aligned}$$
$$(8)$$

This last step, however, is not perfectly realized due to the nonlinear behavior of LP. Indeed it is the impact of this very issue that may potentially lead to instability in the system. To this end, we consider the system (7) and add and subtract the term, $B^+B_vu_v$, and simplify to obtain

$$\dot{x} = Ax + A_1x_1 + B_vu_v + B(LP(u_v) - B^+B_vu_v) \qquad (9)$$

Now given u_v in terms of (3) and using a similar development as in (8) the above resolves into

$$\dot{x} = A_mx + B_mr + B(LP(u_v) - B^+B_vu_v). \qquad (10)$$

The issue that remains is whether the above system remains stable in presence of the residual factor:

$$\delta(u_v) = B(LP(u_v) - B^+B_vu_v) \qquad (11)$$

A simple view of the situation here is that as long as $\delta(u_v)$ is bounded, say by some bound Δ and that A_m is Hurwitz, x remains close to $A_m^{-1}B_m$ at steady state. However, the fact that this residual factor is a function of u_v, which itself is a function of the states, renders the problem somewhat challenging. However, at least limited use can be made of some existing results in nonlinear systems theory which can be used to establish at least the limiting conditions under which the system can be guaranteed to be stable. For this purpose, let LP be given via a neural network approximation as in (7). We would then have

$$\begin{aligned}
\delta(u_v) &= W\sigma(Vu_v) - B^+B_vu_v \\
&= W\sigma(VB_v^{-1}(A_mx + B_mr - Ax - A_1x_1)) - \\
&\quad B^+B_vB_v^{-1}(A_mx + B_mr - Ax - A_1x_1)
\end{aligned}$$
$$(12)$$

and subsequently as

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$$\delta(u_\nu) = W\sigma(VB_\nu^{-1}(A_m x + B_m r - Ax - A_1 x_1)) - B^+(A_m x + B_m r - Ax - A_1 x_1)$$

$$(13)$$

For sufficiently large x the above resolves into

$$\|\delta(u_\nu)\| \leq \|B^+\| \|A_m - A\| \|x\| \qquad (14)$$

This fact is subsequently used in conjunction with the following theorem to determine the stability condition for the closed loop system.

Theorem 1. Consider the system

$$\dot{x} = Ax + g(x, t) \qquad (15)$$

where A is Hurwitz and $\|g(x, t)\| < \gamma \|x\|$ for $t \geq 0$ and all $x \in R^n$. Let $P = P^T > 0$ be the unique solution of

$$PA + A^T P = -Q \qquad (16)$$

where $Q = Q^T > 0$. Then the origin, $x = 0$ is globally asymptotically stable if

$$\gamma < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}. \qquad (17)$$

Proof. See Khalil[6].

Now given (14) the above stability condition resolves into

$$\|B^+\| \|A - A_m\| < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} \qquad (18)$$

For the particular case of the C17 aircraft,

$$A = \begin{bmatrix} -1.053 & 0.469 & 0 \\ -0.035 & -0.570 & 0 \\ 0 & 0 & -1.75 \end{bmatrix}, \qquad (19)$$

$$A_m = \mathrm{diag}(-1.5, -1, -2), \qquad (20)$$

and

$$B = \begin{bmatrix} 0.0101 & -0.0101 & 0.0179 & 0 \\ -0.0003 & 0.0003 & -0.0231 & 0 \\ -0.0031 & -0.003 & 0.0009 & -0.0552 \end{bmatrix}$$

$$\begin{bmatrix} 0.0243 & -0.0243 \\ 0.0031 & -0.0031 \\ -0.0026 & -0.0026 \end{bmatrix}$$

$$(21)$$

This leads to

$$\|B^+\| = 44.92, \qquad (22)$$

and

$$\|A - A_m\| = 0.772. \qquad (23)$$

Now let us choose $Q = I_3$, we thus have $P = \mathrm{diag}(0.33, 0.5, 0.25)$. Therefore the bound $\lambda_{\min}(Q)/2\lambda_{\max}(P) = 1$ and so the system may not be shown to be stable with the values given. However, it must be noted that $\|A - A_m\|$ can be made arbitrarily small so as to meet the above stability condition. This is an important fact, meaning that the designer has in principle the choice to make a trade off between the performance of the system and the desire for stability (and robustness.)

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CONCLUSIONS

The study reported above was conducted in collaboration with the Intelligent Flight Control(IFC) team at NASA Ames. It presents positive and generally optimistic results concerning the stability behavior of the IFC system depicted in Figure 1 with LP control allocation in place. In piloted simulation studies at Ames, bounded oscillations were observed for severe failure conditions. These have been rectified by using either pseudo control hedging or reference model adaptation techniques. In general, however, these studies confirmed the overall conclusions of the above analysis. These results are, nevertheless, still preliminary in nature and further issues must be addressed:

- *Refined Nonlinear Stability Analysis*; This is to reduce the conservatism of the approach discussed in this report. Various schemes might work, including but not limited to *absolute stability/hyperstability*[7] but this needs to be looked at more closely specially with rate limited actuators

- *Variations of the operating environment (stability/control derivatives.)* In principle a network of networks may be the best approach in terms of transparency but other options such as a single extended network may be possible

- *Consideration of rate limits. R*ate limits must be considered in the neural network based approach; the most obvious choice here is recurrent networks but other possibilities exist.

- *Exploration of other neural network architecture.* Clearly generalized basis function networks, support vector machines, are all candidates that can be explored as well in place of the simple network used in this report.

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