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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1424

ON THE THEORY OF ANISOTROPIC SHALLOW SHELLS

By S. A. Ambartsumyan

Translation

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1. INITIAL ASSUMPTIONS

We shall consider a thin-walled, sufficiently shallow and anisotropic shell whose material, at each point, has a plane of elastic symmetry parallel to the middle surface of the shell.[1]

For the coordinate surface of this shell, we take the middle surface in curvilinear orthogonal coordinates α and β, coinciding with the lines of curvature. Let $k_1 = k_1(\alpha,\beta)$, $k_2 = k_2(\alpha,\beta)$ be the principal curvatures of the coordinate surface, and $A = A(\alpha,\beta)$, $B = B(\alpha,\beta)$, be the coefficients of the first quadratic form.

In regard to the shell, we make the following simplifying assumptions:

(1) The hypothesis of Kirchhoff-Love (ref. 2) shows that the rectilinear elements of the shell normal to the middle surface maintain their initial length after deformation of the shell, and remain rectilinear and normal to this surface. The error of this hypothesis, shown in reference 3, has a value of the order of (δk) compared with unity, where δ is the constant thickness of the shell.

(2) The parameters $A(\alpha,\beta)$ and $B(\alpha,\beta)$ are regarded as constants in differentiation (ref. 4).

(3) Certain terms of secondary significance are neglected (ref. 5).

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*"K teorii anizotropnykh pologikh obolochek." Prik. Mat. i Mekh., vol. XII, 1948, pp. 75-80.

[1]A solution of the analogous problem for a plate has been given by S. G. Lekhnitskii (ref. 1).

2. EQUATIONS OF EQUILIBRIUM AND RELATIONS BETWEEN DEFORMATIONS AND STRESSES

The conditions of equilibrium of an element of a shell, for our initial assumptions, are expressed by the equations

$$\left.\begin{aligned}
&B\frac{\partial T_1}{\partial \alpha} + A\frac{\partial S}{\partial \beta} + ABX = 0\\
&A\frac{\partial T_2}{\partial \beta} + B\frac{\partial S}{\partial \alpha} + ABY = 0\\
&-(k_1T_1 + k_2T_2) + \frac{1}{AB}\left[\frac{\partial}{\partial \alpha}(BN_1) + \frac{\partial}{\partial \beta}(AN_2)\right] + Z = 0\\
&B\frac{\partial H}{\partial \alpha} - A\frac{\partial G_2}{\partial \beta} - ABN_2 = 0\\
&-A\frac{\partial H}{\partial \beta} + B\frac{\partial G_1}{\partial \alpha} + ABN_1 = 0\\
&S_1 + S_2 + k_1H_1 + k_2H_2 = 0
\end{aligned}\right\} \qquad (2.1)$$

The last relation, in virtue of the formulas expressing the forces and moments through deformation of the middle surface, is an identity.

For the deformations and the parameters of the changes in curvature, we have

$$\varepsilon_1 = \frac{1}{A}\frac{\partial u}{\partial \alpha} + k_1w \qquad \varepsilon_2 = \frac{1}{B}\frac{\partial v}{\partial \beta} + k_2w \qquad \omega = \frac{1}{B}\frac{\partial u}{\partial \beta} + \frac{1}{A}\frac{\partial v}{\partial \alpha} \qquad (2.2)$$

$$\varkappa_1 = -\frac{1}{A^2}\frac{\partial^2 w}{\partial \alpha^2} \qquad \varkappa_2 = -\frac{1}{B^2}\frac{\partial^2 w}{\partial \beta^2} \qquad \tau = -\frac{2}{AB}\frac{\partial^2 w}{\partial \alpha \partial \beta} \qquad (2.3)$$

where $u = u(\alpha,\beta)$, $v = v(\alpha,\beta)$ are the displacements in the middle surface along the coordinate lines, and $w = w(\alpha,\beta)$ is the normal displacement.

In equation (2.3) we neglected the components u and v in comparison with the component w. Hence, these relations do not differ from the corresponding expressions for plates. This interpretation of the change in curvature for the general case was originally given by V. S. Vlasov (ref. 5).

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Of the three differential relations given by A. L. Goldenveizer (ref. 6), for the case under consideration, only the last one is required:

$$k_2 x_1 + k_1 x_2 + \frac{1}{A^2}\frac{\partial^2 \varepsilon_2}{\partial \alpha^2} - \frac{1}{AB}\frac{\partial^2 \omega}{\partial \alpha \partial \beta} + \frac{1}{B^2}\frac{\partial^2 \varepsilon_1}{\partial \beta^2} = 0 \qquad (2.4)$$

The equations of the generalized Hooke's law in the chosen triorthogonal curvilinear system of coordinates are

$$\left.\begin{aligned}
\sigma_\alpha &= A_{11}e_\alpha + A_{12}e_\beta + A_{13}e_\gamma + A_{16}e_{\alpha\beta} \\
\sigma_\beta &= A_{12}e_\alpha + A_{22}e_\beta + A_{23}e_\gamma + A_{26}e_{\alpha\beta} \\
\sigma_\gamma &= A_{13}e_\alpha + A_{23}e_\beta + A_{33}e_\gamma + A_{36}e_{\alpha\beta} \\
\tau_{\beta\gamma} &= A_{44}e_{\beta\gamma} + A_{45}e_{\alpha\gamma} \\
\tau_{\alpha\gamma} &= A_{45}e_{\beta\gamma} + A_{55}e_{\alpha\gamma} \\
\tau_{\alpha\beta} &= A_{16}e_\alpha + A_{26}e_\beta + A_{36}e_\gamma + A_{66}e_{\alpha\beta}
\end{aligned}\right\} \qquad (2.5)$$

In the case under considerations for $\sigma_\gamma = 0$, we have

$$\left.\begin{aligned}
\sigma_\alpha &= B_{11}e_\alpha + B_{12}e_\beta + B_{16}e_{\alpha\beta} \\
\sigma_\beta &= B_{12}e_\alpha + B_{22}e_\beta + B_{26}e_{\alpha\beta} \\
\tau_{\alpha\beta} &= B_{16}e_\alpha + B_{26}e_\beta + B_{66}e_{\alpha\beta}
\end{aligned}\right\} \qquad (2.6)$$

where, following S. G. Lekhnitskii's theories (ref. 1), there is introduced the notation

$$B_{ik} = (A_{ik}A_{33} - A_{13}A_{k3})/A_{33} \qquad (i,k = 1,2,6)$$

These stresses produce the following internal generalized forces: tangential T, S, bending, and torsional moments G, H, which, on the two principal sections α = constant and β = constant, have the form

$$\left.\begin{aligned}
T_1 &= \delta(B_{11}\varepsilon_1 + B_{12}\varepsilon_2 + B_{16}\omega)\\
T_2 &= \delta(B_{22}\varepsilon_2 + B_{12}\varepsilon_1 + B_{26}\omega)\\
G_1 &= -\frac{\delta^3}{12}(B_{11}x_1 + B_{12}x_2 + B_{16}\tau)\\
G_2 &= -\frac{\delta^3}{12}(B_{22}x_2 + B_{12}x_1 + B_{26}\tau)\\
S_1 &= -S_2 = S = \delta(B_{66}\omega + B_{16}\varepsilon_1 + B_{26}\varepsilon_2)\\
H_1 &= -H_2 = H = -\frac{\delta^3}{12}(B_{66}\tau + B_{16}x_1 + B_{26}x_2)
\end{aligned}\right\} \qquad (2.7)$$

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Substituting the values G_1, G_2 in equation (2.1), and H from equation (2.7), we obtain

$$N_1 = -\frac{\delta^3}{12}C(B_{ik})w \qquad N_2 = -\frac{\delta^3}{12}D(B_{ik})w \qquad (2.8)$$

where

$$\left.\begin{aligned}
C(B_{ik}) &= B_{11}\frac{1}{A^3}\frac{\partial^3}{\partial\alpha^3} + 3B_{16}\frac{1}{A^2B}\frac{\partial^3}{\partial\alpha^2\partial\beta} + (B_{12}+2B_{66})\frac{1}{AB^2}\frac{\partial^3}{\partial\alpha\partial\beta^2} + B_{26}\frac{1}{B^3}\frac{\partial^3}{\partial\beta^3}\\
D(B_{ik}) &= B_{22}\frac{1}{B^3}\frac{\partial^3}{\partial\beta^3} + 3B_{26}\frac{1}{B^2A}\frac{\partial^3}{\partial\beta^2\partial\alpha} + (B_{12}+2B_{66})\frac{1}{BA^2}\frac{\partial^3}{\partial\beta\partial\alpha^2} + B_{16}\frac{1}{A^3}\frac{\partial^3}{\partial\alpha^3}
\end{aligned}\right\} \text{(a)}$$

3. FUNDAMENTAL DIFFERENTIAL EQUATIONS

For the unknowns, take $u(\alpha,\beta)$, $v(\alpha,\beta)$, and $w(\alpha,\beta)$. If ε_1, ε_2, ω, k_1, k_2, and τ from equation (2.2) and equation (2.3) are substituted in equation (2.7), T_1, T_2, S, G_1, G_2, and H can be determined as functions of u, v, and w.

Further, substituting the values of the internal forces in the equations of equilibrium, and considering equation (2.8), there is obtained a complete system of equations for the three principal unknown parameters, namely, u, v, and w. This system, following the work of V. Z. Vlasov (ref. 5), is presented in table I.

The equations (see 3.1, table I, connecting the unknowns u, v, w), and boundary conditions make it possible to investigate the problem of the equilibrium of thin-walled, shallow, and anisotropic shells by the method of displacements. The integration of this system is, however, connected with very great difficulties. Making use of the method proposed by V. Z. Vlasov (ref. 5), for isotropic and anisotropic shells, the problem can be reduced to a system of two simultaneous equations.

We shall assume that $X = Y = 0$, that is, if the case of a surface with a normal load is considered.

Setting

$$T_1 = \frac{1}{B^2}\frac{\partial^2\varphi}{\partial\beta^2} \qquad T_2 = \frac{1}{A^2}\frac{\partial^2\varphi}{\partial\alpha^2} \qquad S = -\frac{1}{AB}\frac{\partial^2\varphi}{\partial\alpha\partial\beta} \tag{3.2}$$

the first two equations of equilibrium are identically satisfied. Further, taking account of equations (2.3) and (3.2), we obtain, from equation (2.4) and the third equations of equations (2.1)

$$\begin{gathered} -\left(k_2\frac{1}{A^2}\frac{\partial^2 w}{\partial\alpha^2} + k_1\frac{1}{B^2}\frac{\partial^2 w}{\partial\beta^2}\right) + \frac{1}{\delta\Omega}L_1(B_{ik})\varphi = 0 \\ \left(k_1\frac{1}{B^2}\frac{\partial^2\varphi}{\partial\beta^2} + k_2\frac{1}{A^2}\frac{\partial^2\varphi}{\partial\alpha^2}\right) + \frac{\delta^3}{12}L(B_{ik})w - Z = 0 \end{gathered} \tag{3.3}$$

where

$$\begin{aligned} L_1(B_{ik}) = \frac{1}{B_{66}}\Bigg\{&(B_{11}B_{66} - B_{16}^2)\frac{1}{A^4}\frac{\partial^4}{\partial\alpha^4} + 2(B_{11}B_{26} - B_{12}B_{16})\frac{1}{A^3B}\frac{\partial^4}{\partial\alpha^3\partial\beta} + \\ &\left[(B_{11}B_{22} - B_{12}^2) - 2(B_{12}B_{66} - B_{16}B_{26})\right]\frac{1}{A^2B^2}\frac{\partial^4}{\partial\alpha^2\partial\beta^2} + \\ &2(B_{22}B_{16} - B_{12}B_{26})\frac{1}{AB^3}\frac{\partial^4}{\partial\alpha\partial\beta^3} + (B_{22}B_{66} - B_{26}^2)\frac{1}{B^4}\frac{\partial^4}{\partial\beta^4}\Bigg\} \end{aligned} \tag{3.4}$$

$$\Omega = \frac{1}{B_{66}^2}\left[(B_{11}B_{66} - B_{16}^2)(B_{22}B_{66} - B_{26}^2) - (B_{12}B_{66} - B_{16}B_{26})^2\right] \tag{3.5}$$

For the operators we introduce the notation

$$\nabla_r = k_1 \frac{1}{B^2} \frac{\partial^2}{\partial \beta^2} + k_2 \frac{1}{A^2} \frac{\partial^2}{\partial \alpha^2} \qquad \nabla_r^2 = \nabla_r \nabla_r \tag{3.6}$$

Equation (3.3), in this case, assume the form

$$\frac{1}{\delta \Omega} L_1(B_{ik})\varphi - \nabla_r w = 0 \qquad - \nabla_r \varphi - \frac{\delta^3}{12} L(B_{ik})w + Z = 0 \tag{3.7}$$

where $\varphi = \varphi(\alpha,\beta)$ is the stress function, analogous in the plane problem to the functions of Airy, and $w = w(\alpha,\beta)$ is the displacement function.

From equations (3.7), for $k_1 = k_2 = 0$, we obtain the well-known equations for the plane stress state of a plate $L_1(B_{ik})\varphi = 0$, and for the bending of an anisotropic plate, $L(B_{ik})w = 12Z/\delta^3$.

Thus, making use of the mixed method of V. Z. Vlasov (ref. 5), we obtain a more compact representation of the differential equations of the theory of anisotropic shells. The system (3.7) may be reduced to an equivalent single equation of the eighth order. We set

$$w = L_1(B_{ik})\Phi \qquad \varphi = \delta \Omega \nabla_r \Phi \tag{3.8}$$

From the second of equations (3.7), we obtain

$$L_1(B_{ik})L(B_{ik})\Phi + \frac{12\Omega}{\delta^2} \nabla_r^2 \Phi = \frac{12}{\delta^3} Z \tag{3.9}$$

We note that this equation is a generalization of the equation given by V. Z. Vlazov (ref. 7), for isotropic cylindrical shells, and can be obtained by another method from the system 3.1 (table I), analogous to the method by which B. G. Galerkin (ref. 9), obtained the equation of the isotropic cylindrical shell (ref. 8).

The internal forces, by equations (2.2), (2.3), (2.7), (2.8), and (3.8), are as follows:

$$T_1 = \delta \Omega \frac{1}{B^2} \frac{\partial^2}{\partial \beta^2} \nabla_r \Phi \qquad T_2 = \delta \Omega \frac{1}{A^2} \frac{\partial^2}{\partial \alpha^2} \nabla_r \Phi \qquad S = - \delta \Omega \frac{1}{AB} \frac{\partial^2}{\partial \alpha \partial \beta} \nabla_r \Phi \tag{3.10}$$

$$G_1 = \frac{\delta^3}{12}\left[B_{11}\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2} + B_{12}\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2} + B_{16}\frac{2}{AB}\frac{\partial^2}{\partial\alpha\partial\beta}\right]L_1(B_{ik})\Phi$$

$$G_2 = \frac{\delta^3}{12}\left[B_{22}\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2} + B_{12}\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2} + B_{26}\frac{2}{AB}\frac{\partial^2}{\partial\alpha\partial\beta}\right]L_1(B_{ik})\Phi \qquad (3.11)$$

$$H = -\frac{\delta^3}{12}\left[B_{66}\frac{2}{AB}\frac{\partial^2}{\partial\alpha\partial\beta} + B_{16}\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2} + B_{26}\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2}\right]L_1(B_{ik})\Phi$$

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$$N_1 = -\frac{\delta^3}{12}C(B_{ik})L_1(B_{ik})\Phi \qquad N_2 = -\frac{\delta^3}{12}D(B_{ik})L_1(B_{ik})\Phi \qquad (3.12)$$

For the displacement of a point of the middle surface, we have

$$w = L_1(B_{ik})\Phi \qquad (3.13)$$

$$u = -\frac{1}{B_{66}}\Bigg(\left[(B_{11}B_{66} - B_{16}^2)k_1 + (B_{12}B_{66} - B_{16}B_{26})k_2\right]\frac{1}{A^3}\frac{\partial^3\Phi}{\partial\alpha^3} +$$
$$\left[2(B_{11}B_{26} - B_{12}B_{16})k_1 - (B_{22}B_{16} - B_{12}B_{26})k_2\right]\frac{1}{A^2B}\frac{\partial^3\Phi}{\partial\alpha^2\partial\beta} +$$
$$\left\{\left[(B_{11}B_{22} - B_{12}^2) - (B_{12}B_{66} - B_{16}B_{26})\right]k_1 - (B_{22}B_{66} - B_{26}^2)k_2\right\}\frac{1}{AB^2}\frac{\partial^3\Phi}{\partial\alpha\partial\beta^2} +$$
$$(B_{22}B_{16} - B_{12}B_{26})k_1\frac{1}{B^3}\frac{\alpha^3\Phi}{\partial\beta^3}\Bigg) \qquad (3.14)$$

$$v = -\frac{1}{B_{66}}\Bigg(\left[(B_{22}B_{66} - B_{26}^2)k_2 + (B_{12}B_{66} - B_{16}B_{26})k_1\right]\frac{1}{B^3}\frac{\partial^3\Phi}{\partial\beta^3} +$$
$$\left[2(B_{22}B_{16} - B_{12}B_{26})k_2 - (B_{11}B_{26} - B_{12}B_{16})k_1\right]\frac{1}{B^2A}\frac{\partial^3\Phi}{\partial\beta^2\partial\alpha} +$$
$$\left\{\left[(B_{11}B_{22} - B_{12}^2) - (B_{12}B_{66} - B_{16}B_{26})\right]k_2 - (B_{11}B_{66} - B_{16}^2)k_1\right\}\frac{1}{BA^2}\frac{\partial^3\Phi}{\partial\beta\partial\alpha^2} +$$
$$(B_{11} + B_{26} - B_{12}B_{16})k_2\frac{1}{A^3}\frac{\partial^3\Phi}{\partial\alpha^3}\Bigg) \qquad (3.15)$$

4. LOCAL STABILITY AND VIBRATIONS

Repeating the considerations of V. Z. Vlasov (ref. 5) yields the equations of the local stability of an isotropic shallow shell in the form presented in table II.

Since, in this case, the components X and Y are proportional to the curvatures k_1 and k_2, they can be neglected. From equation (3.7), we then obtain

$$\frac{1}{\delta\Omega} L_1(B_{ik})\varphi - \nabla_r w = 0$$

$$\nabla_r\varphi + \frac{\delta^3}{12} L(B_{ik})w - \left[T_1^o \frac{1}{A^2}\frac{\partial w^2}{\partial\alpha^2} + 2S^o \frac{1}{AB}\frac{\partial^2 w}{\partial\beta\partial\alpha} + T_2^o \frac{1}{B^2}\frac{\partial^2 w}{\partial\beta^2}\right] = 0 \tag{4.2}$$

Thus, the problem likewise reduces to the solution of two simultaneous equations for the stress function φ, and the displacement function w. This system can be reduced to a single equation of the eighth order, for a single function. From equation (3.9) we have

$$L_1(B_{ik})L(B_{ik})\Phi + \frac{12\Omega}{\delta^2}\nabla_r^2\Phi -$$
$$\frac{12}{\delta^3}\left[T_1^o \frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2} + 2S^o \frac{1}{AB}\frac{\partial^2}{\partial\alpha\partial\beta} + T_2^o \frac{1}{B^2}\frac{\partial^2}{\partial\beta^2}\right] L_1(B_{ik})\Phi = 0 \tag{4.3}$$

From equations (4.2) or (4.3), the equations of vibration of anisotropic shallow shells can also easily be obtained by introducing in the computation, the inertia forces, and by setting $T_1^o = T_2^o = S^o = 0$.

Using γ to denote the specific weight of the shell, and g the acceleration of gravity, we obtain, from equations (7.8) or (7.9), respectively,

$$\frac{1}{\delta\Omega} L_1(B_{ik})\varphi - \nabla_r w = 0 \qquad \nabla_r\varphi + \frac{\delta^3}{12} L(B_{ik})w + \frac{\gamma\delta}{g}\frac{\partial^2 w}{\partial t^2} = 0 \tag{4.4}$$

$$L_1(B_{ik})L(B_{ik})\Phi + \frac{12}{\delta^2}\nabla_r^2\Phi + \frac{\gamma\delta}{g}\frac{\partial^2}{\partial t^2} L_1(B_{ik})\Phi = 0 \tag{4.5}$$

This problem, for shallow isotropic shells, was first solved in this form by V. Z. Vlasov (ref. 7). For $k_1 = k_2 = 0$, there is obtained, from the equations given, the fundamental equations of stability and vibration of anisotropic plates by S. G. Lekhnitskii (ref. 8).

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TABLE I.

$u(\alpha,\beta)$	$v(\alpha,\beta)$	$w(\alpha,\beta)$	(3.1)
$B_{11}\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2}+2B_{16}\frac{1}{AB}\frac{\partial^2}{\partial\alpha\partial\beta}+B_{66}\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2}$	$B_{16}\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2}+(B_{12}+B_{66})\frac{1}{AB}\frac{\partial^2}{\partial\alpha\partial\beta}+B_{26}\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2}$	$\frac{1}{A}(B_{11}k_1+B_{12}k_2)\frac{\partial}{\partial\alpha}+\frac{1}{B}(B_{16}k_1+B_{26}k_2)\frac{\partial}{\partial\beta}$	$\frac{X}{\delta}$
$B_{16}\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2}+(B_{12}+B_{66})\frac{1}{AB}\frac{\partial^2}{\partial\alpha\partial\beta}+B_{26}\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2}$	$B_{22}\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2}+2B_{26}\frac{1}{AB}\frac{\partial^2}{\partial\alpha\partial\beta}+B_{66}\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2}$	$\frac{1}{B}(B_{22}k_2+B_{12}k_1)\frac{\partial}{\partial\beta}+\frac{1}{A}(B_{26}k_2+B_{16}k_1)\frac{\partial}{\partial\alpha}$	$\frac{Y}{\delta}$
$\frac{1}{A}(B_{11}k_1+B_{12}k_2)\frac{\partial}{\partial\alpha}+\frac{1}{B}(B_{16}k_1+B_{26}k_2)\frac{\partial}{\partial\beta}$	$\frac{1}{B}(B_{22}k_2+B_{12}k_1)\frac{\partial}{\partial\beta}+\frac{1}{A}(B_{26}k_2+B_{16}k_1)\frac{\partial}{\partial\alpha}$	$(B_{11}k_1^2+2B_{12}k_1k_2+B_{22}k_2^2)+\frac{\delta^2}{12}L(B_{ik})$	$-\frac{Z}{\delta}$

$$L(B_{ik}) = B_{11}\frac{1}{A^4}\frac{\partial^4}{\partial\alpha^4} + 4B_{16}\frac{1}{A^3B}\frac{\partial^4}{\partial\alpha^3\partial\beta} + 2(B_{12}+2B_{66})\frac{1}{A^2B^2}\frac{\partial^4}{\partial\alpha^2\partial\beta^2} + 4B_{26}\frac{1}{AB^3}\frac{\partial^4}{\partial\alpha\partial\beta^3} + B_{22}\frac{1}{B^4}\frac{\partial^4}{\partial\beta^4}$$

Table II.

$u(\alpha,\beta)$	$v(\alpha,\beta)$	$w(\alpha,\beta)$	(4.1)
$B_{11}\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2} + 2B_{16}\frac{1}{AB}\frac{\partial^2}{\partial\alpha\partial\beta} + B_{66}\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2} - k_1^2T_1^0$	$B_{16}\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2} + (B_{12} + B_{66})\frac{1}{AB}\frac{\partial^2}{\partial\alpha\partial\beta} + B_{26}\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2} - k_1k_2S^0$	$\frac{1}{A}(B_{11}k_1 + B_{12}k_1)\frac{\partial}{\partial\alpha} + k_1T_1^0\frac{1}{A}\frac{\partial}{\partial\alpha} + \frac{1}{B}(B_{16}k_1 + B_{26}k_2)\frac{\partial}{\partial\beta} + k_1S^0\frac{1}{B}\frac{\partial}{\partial\beta}$	
$B_{16}\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2} + (B_{12} + B_{66})\frac{1}{AB}\frac{\partial^2}{\partial\alpha\partial\beta} + B_{26}\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2} - k_1k_2S^0$	$B_{22}\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2} + 2B_{26}\frac{1}{AB}\frac{\partial^2}{\partial\alpha\partial\beta} + B_{66}\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2} - k_2^2T_2^0$	$\frac{1}{B}\frac{\partial}{\partial\beta}(B_{22}k_2 + B_{12}k_1) + k_2T_2^0\frac{1}{B}\frac{\partial}{\partial\beta} + \frac{1}{A}\frac{\partial}{\partial\alpha}(B_{26}k_2 + B_{16}k_1) + k_2S^0\frac{1}{A}\frac{\partial}{\partial\alpha}$	
$\frac{1}{A}(B_{11}k_1 + B_{12}k_2)\frac{\partial}{\partial\alpha} + k_1T_1^0\frac{1}{A}\frac{\partial}{\partial\alpha} + \frac{1}{B}(B_{16}k_1 + B_{26}k_2)\frac{\partial}{\partial\beta} + k_1S^0\frac{1}{B}\frac{\partial}{\partial\beta}$	$\frac{1}{B}(B_{22}k_2 + B_{12}k_1)\frac{\partial}{\partial\beta} + k_2T_2^0\frac{1}{B}\frac{\partial}{\partial\beta} + \frac{1}{A}(B_{26}k_2 + B_{16}k_1)\frac{\partial}{\partial\alpha} + k_2S^0\frac{1}{A}\frac{\partial}{\partial\alpha}$	$\frac{\delta^2}{12}L(B_{ik}) + (B_{11}k_1^2 + 2B_{12}k_1k_2 + B_{22}k_2^2) - \left(T_1^0\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2} + 2S^0\frac{1}{AB}\frac{\partial^2}{\partial\alpha\partial\beta} + T_2^0\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2}\right)$	