A LATTICE BOLTZMANN METHOD FOR TURBOMACHINERY SIMULATIONS

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Outline

- Lattice Boltzmann Method
- Objectives
- Current LB model
- Simulation of cascades and result
- Parallel computing and result
- Conclusion remarks

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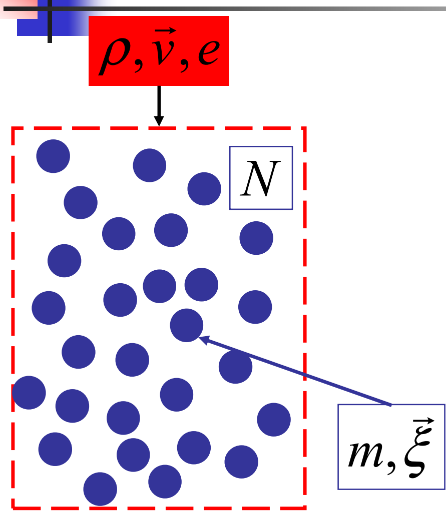
Introduction

- Lattice Boltzmann (LB) Method is a relatively new method for flow simulations.
- The start point of LB method is statistic mechanics and Boltzmann equation.
- The LB method tries to set up its model at molecular scale and simulate the flow at macroscopic scale
- LBM has been applied to mostly incompressible flows and simple geometry

Statistic Mechanics

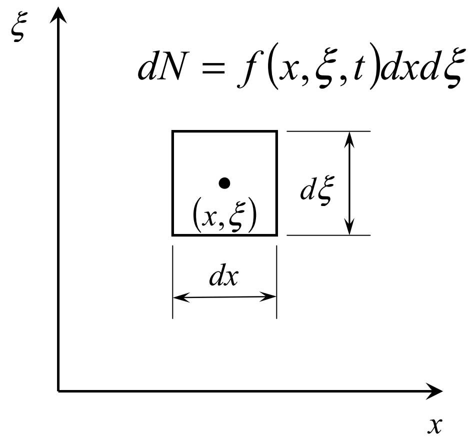


A small domain near point x

- Statistic mechanics views fluid as a collection of particles.
- The properties of the fluid are determined by the average properties of the particles in the collection

Boltzmann Equation
1) Distribution Function

$$dN = f(x,\xi,t)\,dx\,d\xi$$



Phase space for 1D problem

- **A phase space consists of both location and velocity of particles is introduced.**
- **The distribution function is defined as the density of the number of particles at point (x,ξ)**

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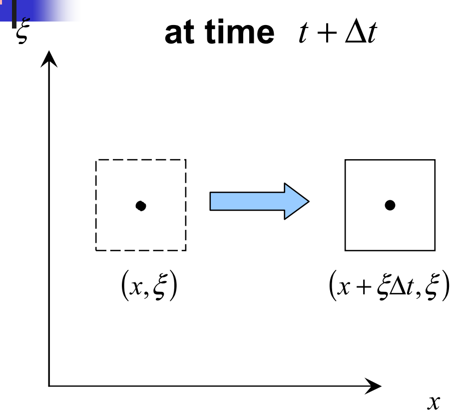
Boltzmann Equation
2) Time Evolution Of Distribution Function



- **Assume that there is no external force and no collision between the particles, the velocity of the particles will not change**

The two domain have the same number of particles

$$f(x+\xi\Delta t,\xi,t+\Delta t)\,dx\,d\xi = f(x,\xi,t)\,dx\,d\xi$$

$$f(x+\xi\Delta t,\xi,t+\Delta t) = f(x,\xi,t)$$

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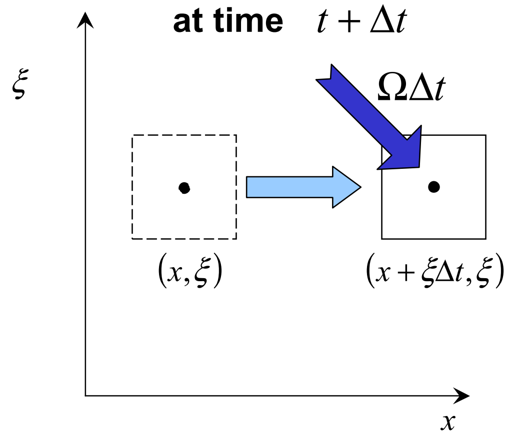
Boltzmann Equation

3) Collision Term



- Collisions between particles change their velocities, and make them move in and out of the domain
- A collision term describes the net increase of the density of the number of particles in the domain due to the collision

$$f(x+\xi\Delta t,\xi,t+\Delta t)=f(x,\xi,t)+\Omega\Delta t$$

Boltzmann Equation

4) BGK Collision Term

- One of the simplest collision models is the Bhatnagar, Gross and Krook (BGK) simplified collision model

$$\Omega=-\frac{1}{\tau}\left(f-f^{(eq)}\right)$$

- The BGK model is widely used in LB models

Boltzmann Equation

5) Boltzmann Equation

- **The Boltzmann equation in finite difference form:**

$$f\left(\vec{x}+\vec{\xi}\Delta t,\vec{\xi},t+\Delta t\right)=f\left(\vec{x},\vec{\xi},t\right)+\Omega\Delta t$$

- **By Taylor expansion, the above equation can be written in the differential form**

$$\frac{\partial f}{\partial t}+\vec{\xi}\cdot\vec{\nabla}f=\Omega$$

This is the Boltzmann Equation

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The Macroscopic Properties

$$Y\left(\vec{x},t\right)=\frac{\int\eta\left(\vec{\xi}\right)f\left(\vec{x},\vec{\xi},t\right)d^3\xi}{\int f\left(\vec{x},\vec{\xi},t\right)d^3\xi}$$

- **The Macroscopic properties are determined by the average value of properties of the particles**

density $$\rho=m\int f(\vec{x},\vec{\xi},t)d^3\xi$$

momentum $$\rho\vec{v}=m\int\vec{\xi}f\left(\vec{x},\vec{\xi},t\right)d^3\xi$$

thermal energy $$\rho\varepsilon=\frac{m}{2}\frac{D_f}{D}\int\left|\vec{\xi}-\vec{v}\right|^2 f\left(\vec{x},\vec{\xi},t\right)d^3\xi$$

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The Enskog-Chapman Expansion

- **It has been shown that the Euler equation and Navier-Stokes equation are the zeroth-order and first order approximations of the Boltzmann equation, respectively.,**
- **That is to say, the Boltzmann equation describes the fluid phenomena in a more accurate way than tradition fluid dynamics does**

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Lattice Boltzmann Method

- **Lattice Boltzmann Method can be reviewed as a numerical method to solve the Boltzmann equation**
- **In LB method, the phase space is discretized.**
- **In a LB model, the velocity of a particle can only be chosen from a velocity set, which has only a finite number of velocities**

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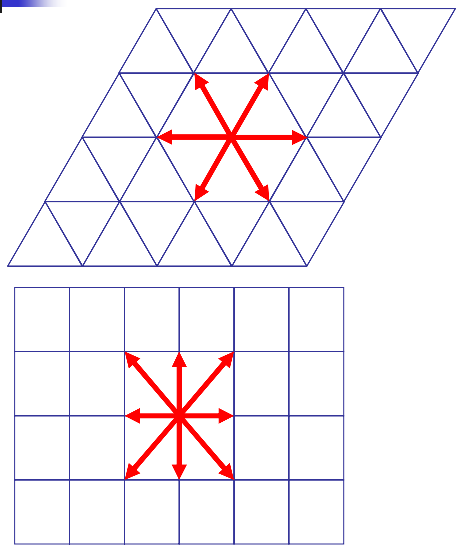
Lattice Boltzmann Method

1) Velocity Set and Grid



- **For the convenience of computation, the position space is discretized in such a way that the particles travel with one of the velocity in the velocity set will arrive at a correspondent node at next time step.**

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The Lattice Boltzmann Method

2) Macroscopic Properties

- **In the LB method, the macroscopic properties are evaluated through the weight summation**

$$Y = \int \eta(\vec{\xi}) f^{(eq)}(\vec{x}, \vec{\xi}, t) d\vec{\xi} = \sum_{\alpha} W_{\alpha} \eta(\vec{\xi}_{\alpha}) f^{(eq)}(\vec{x}, \vec{\xi}_{\alpha}, t)$$

$$\rho = m \sum_{\alpha} f_{\alpha}^{(eq)} \qquad \rho\vec{v} = m \sum_{\alpha} \vec{\xi}_{\alpha} f_{\alpha}^{(eq)} \qquad \rho\varepsilon = \frac{m}{2} \sum_{\alpha} \left| \vec{\xi}_{\alpha} - \vec{v} \right|^2 f_{\alpha}^{(eq)}$$

where $f_{\alpha}^{(eq)}(\vec{x}, \vec{\xi}_{\alpha}, t) = W_{\alpha} f^{(eq)}(\vec{x}, \vec{\xi}_{\alpha}, t)$

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Lattice Boltzmann Method

3) Time Evolution Equation

- Boltzmann equation in finite difference form with the BGK collision term

$$f\left(\vec{x}+\vec{\xi}_\alpha \Delta t, \vec{\xi}_\alpha, t+\Delta t\right)=f\left(\vec{x}, \vec{\xi}_\alpha, t\right)-\frac{\Delta t}{\tau}\left(f\left(\vec{x}, \vec{\xi}_\alpha, t\right)-f^{(eq)}\left(\vec{x}, \vec{\xi}_\alpha, t\right)\right)$$

where $\vec{\xi}_\alpha$ is one of velocities in the velocity set

- Set $\Delta t = \tau$

$$f\left(\vec{x}+\vec{\xi}_\alpha \Delta t, \vec{\xi}_\alpha, t+\Delta t\right)=f^{(eq)}\left(\vec{x}, \vec{\xi}_\alpha, t\right)$$

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The Lattice Boltzmann Method

4) The Calculation

$$Y(\vec{x}, t)=\sum_\alpha \eta\left(\vec{\xi}_\alpha\right) f_\alpha^{(eq)}\left(\vec{x}-\vec{\xi}_\alpha \Delta t, \vec{\xi}_\alpha, t-\Delta t\right)=\sum_\alpha Y_\alpha\left(\vec{x}-\vec{\xi}_\alpha \Delta t, t-\Delta t\right)$$

- The calculation of LB method can be viewed as following steps

  1) a node dividing its macroscopic quantities into parts

  2) the node sending parts of the macroscopic quantities to corresponding nodes

  3) a node receiving the parts of the macroscopic quantities sent by nearby nodes, adding them together and obtaining the macroscopic quantities for next time step

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Advantages of LB method

- Start from a clear and direct physical picture at molecular level;
- Algorithm is simple and straightforward;
- Natural parallel scheme;
- Easy to incorporate the physical phenomena at molecular level, possible of modeling fluid phenomena that can not be modeled with the traditional fluid mechanics.

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Motivation

- There are very few reports about the successful LB simulations of the real life problems
- Successful extension of LB method to the simulation of turbomachinery can show the maturity of LB method and the promise of this method in the simulation of real life problem
- The parallel nature of LB method make it a possible high performance solver for turbomachine simulations

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Challenges

1. **Most of LB models can only simulate the flow with a small Mach number**
2. **Most of past LB simulations are laboratory type simulations with simple computational domain and boundary**
3. **The proper LB model for simulation should be developed and necessary techniques should be developed**

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The Current Work

- **Develop a compressible LB model.**
- **Introduce a boundary condition that allows accurate turbine blade simulation**
- **Develop mesh treatment for the irregular computational domain**

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The Current Work

- Simulations carried out for three different cascades. It is the first time that turbomachines have been simulated by a LB model.

- The parallel performance of the LB model has been tested

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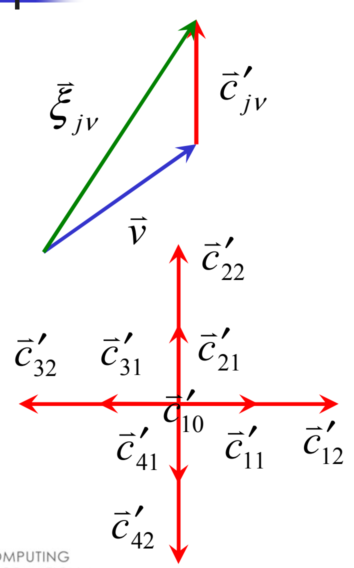
Current Model

1) The Velocity Set



- A velocity in the velocity set consists of three parts.

- First the macroscopic velocity has been included explicitly into the microscopic velocity.

- Second, there is a set of diffusion velocity

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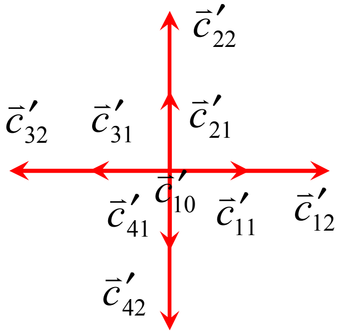
Current Model
2) The Diffusion Velocity



- There are 3 levels of diffusion velocity in current model
- The module of the diffusion velocities are determined by macroscopic quantities

$$c'_\nu = \begin{cases} 0 & for \;\; \nu = 0 \\ \mathrm{int}\left(\sqrt{D(\gamma-1)\rho e/(\rho - b_0 d_0)}\right) & for \;\; \nu = 1 \\ \mathrm{int}\left(\sqrt{D(\gamma-1)\rho e/(\rho - b_0 d_0)}\right)+1 & for \;\; \nu = 2 \end{cases}$$

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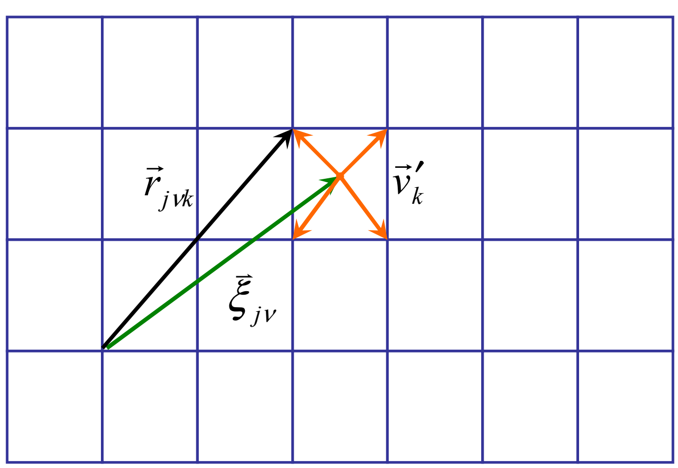
Current Model
3) The Velocity Set



- A third set of velocities $\vec{v}'_k$ is introduced to carry the particles to the nearest vertex nodes

$$\vec{r}_{j\nu k} = \vec{v} + \vec{c}'_{j\nu} + \vec{v}'_k$$

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Current Model
4) The Microscopic Quantities

- Mass, only one species is considered, is a constant

$$m = 1$$

- Velocity, consists of macroscopic velocity and a diffusion velocity

$$\vec{\xi}_{jv} = \vec{v} + \vec{c}\,'_{jv}$$

- Total energy is the same for all particles

$$\zeta = \frac{1}{2} v^2 + e$$

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Current Model
4) Modification of Microscopic Quantities

- For the purpose of recovering the correct Navier-Stokes equation, a correction term has been introduced

$$\chi_{jvk} = \frac{\rho}{\rho - b_0 d_0} \frac{D}{2c_v'^2} \left(\vec{c}\,'_{jv} \cdot \vec{v}\,'_k \right)$$

  - Mass $$m_{jvk} = 1 - \lambda_{jvk}$$
  - Velocity $$\vec{\xi}_{jvk} = \vec{v} + \vec{c}\,'_{jv} - \lambda_{jvk} \vec{v}$$
  - Total energy $$\zeta_{jvk} = (1 - \lambda_{jvk}) \left(\frac{1}{2} v^2 + e \right)$$

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Current Model

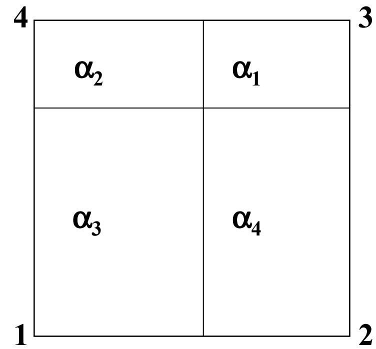
5) The Equilibrium Distribution Function

$$f_{j\nu k}^{(eq)} = f_{\nu k}^{(eq)}$$

$$f_{\nu k}^{(eq)} = \alpha_k d_\nu$$

$$d_1 = \frac{(\rho - b_0 d_0)c_2'^2 - D(\gamma - 1)\rho e}{b_1\left(c_2'^2 - c_1'^2\right)}$$

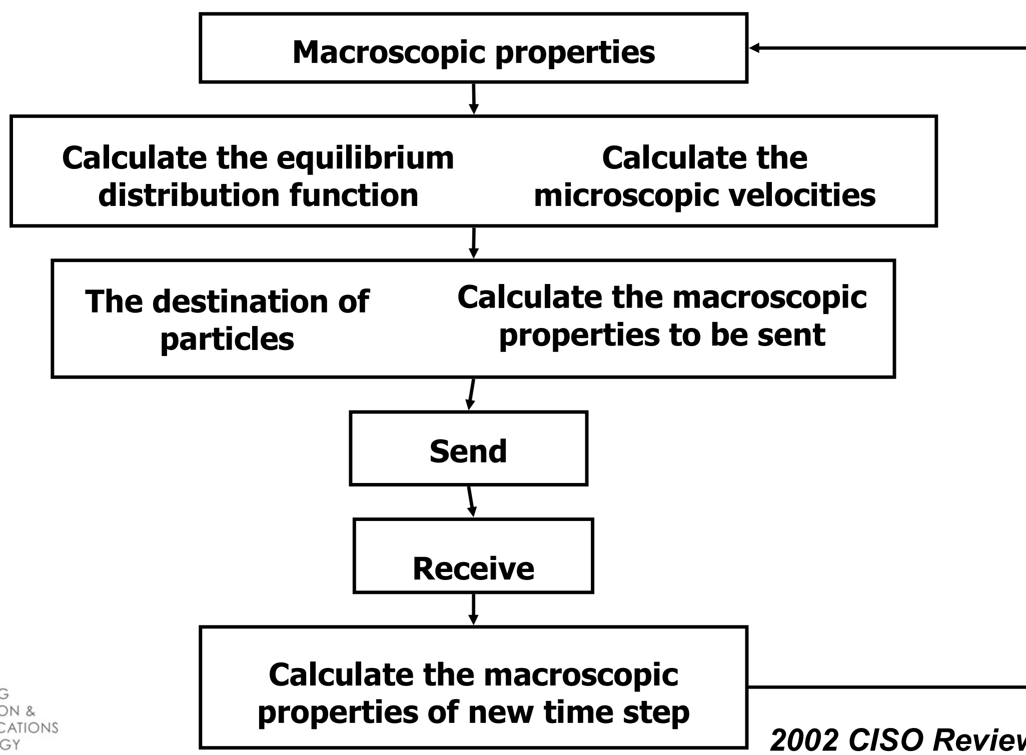
$$d_2 = \frac{D(\gamma - 1)\rho e - (\rho - b_0 d_0)c_1'^2}{b_2\left(c_2'^2 - c_1'^2\right)}$$



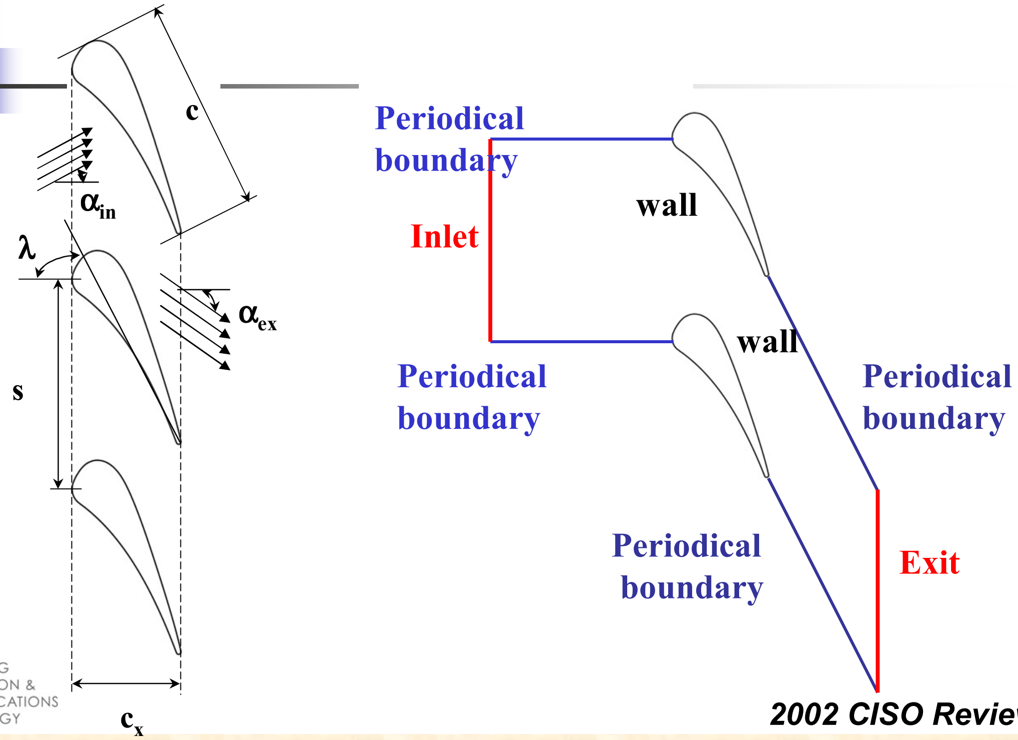
$$\alpha_1 = |u_3' v_3'| \qquad \alpha_2 = |u_4' v_4'|$$

$$\alpha_3 = |u_1' v_1'| \qquad \alpha_4 = |u_2' v_2'|$$

The Flow Chart Of Computation



Geometry of Cascade and Simulation Domain



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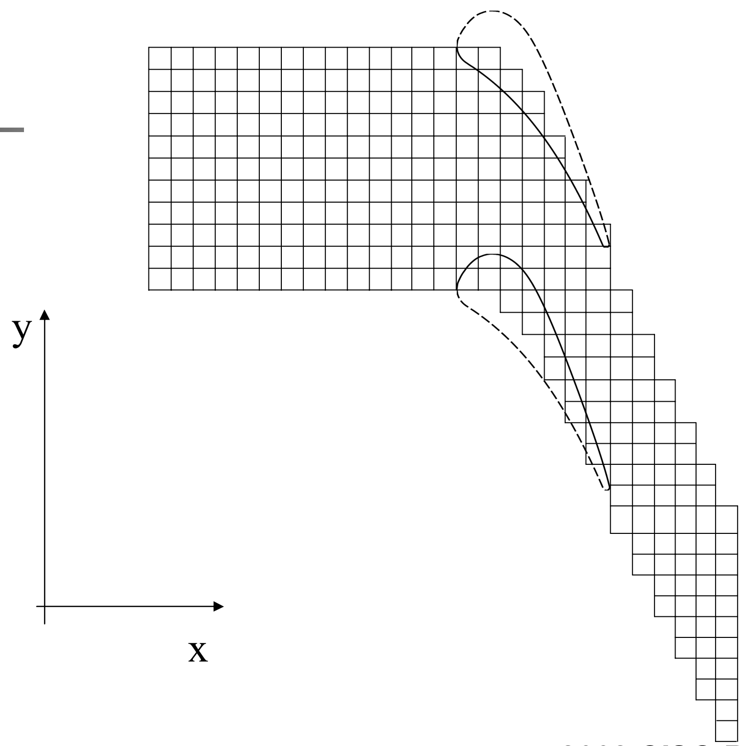
Mesh



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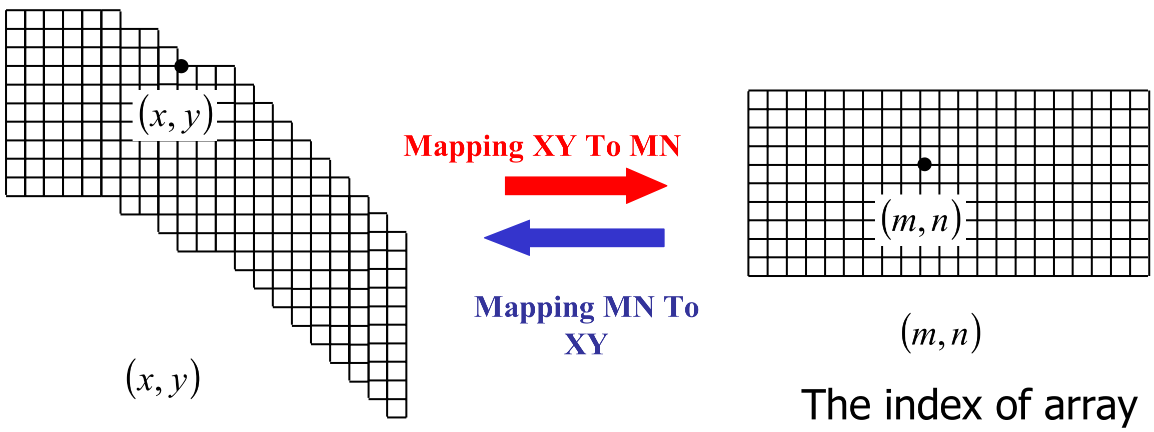
The Mapping Between Coordinates and Indexes



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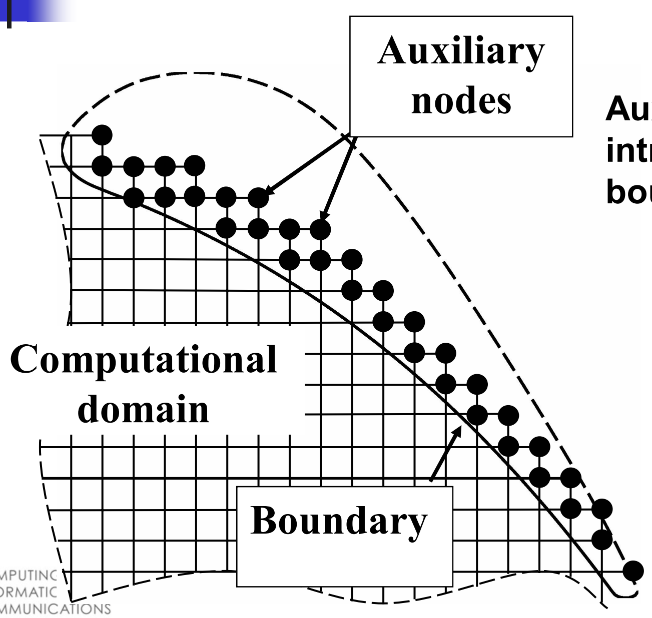
Implement of Boundary Condition

1) Wall Boundary - Auxiliary Nodes



Auxiliary nodes were introduced to implement wall boundary conditions

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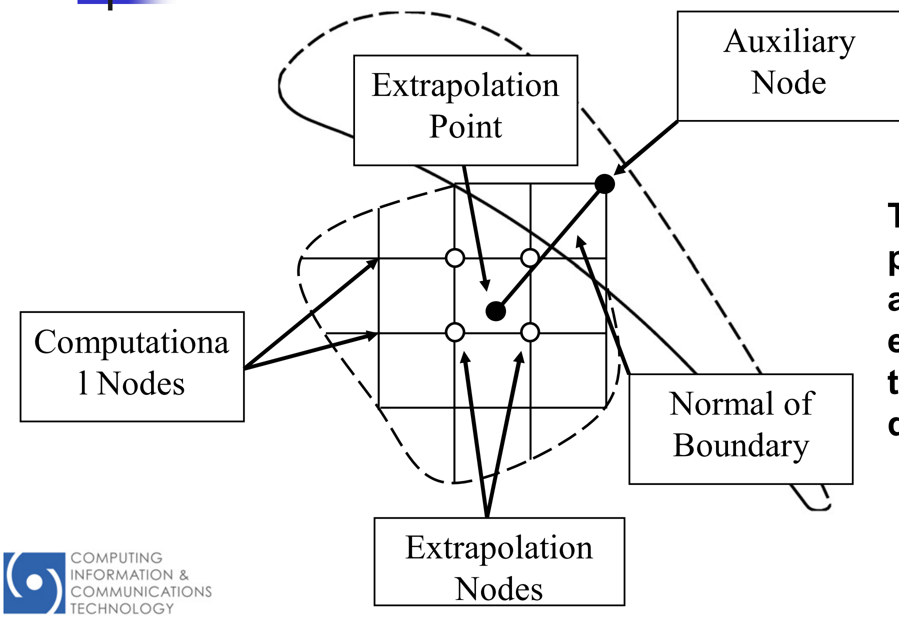
Implement of Boundary Condition

2) Wall Boundary - Extrapolation



The macroscopic properties of auxiliary nodes are extrapolated from the computational domain.

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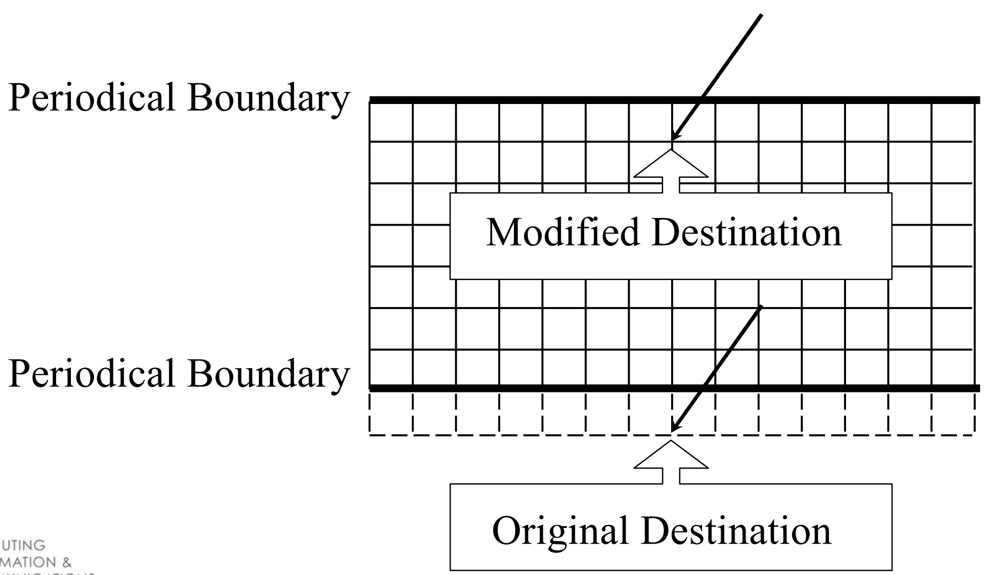
Implement of Boundary Condition

3) Periodical Boundary



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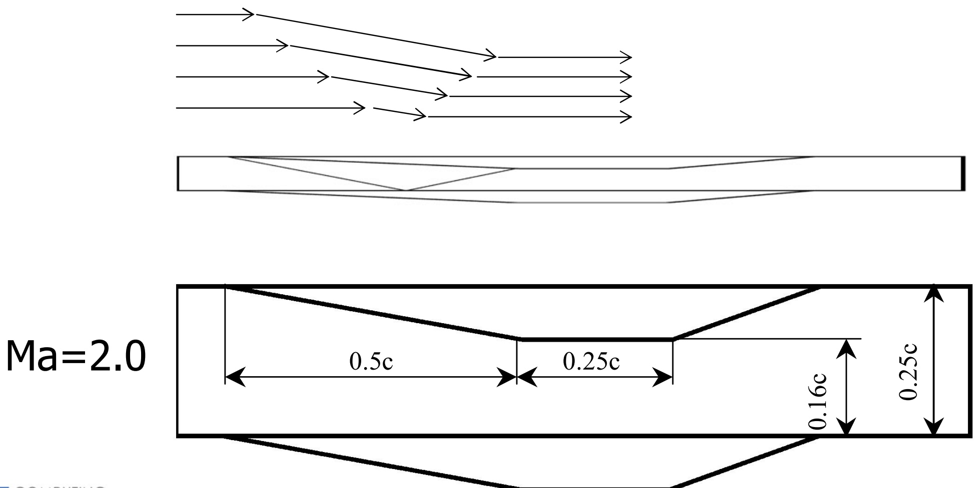
Wedge Cascade
Geometry



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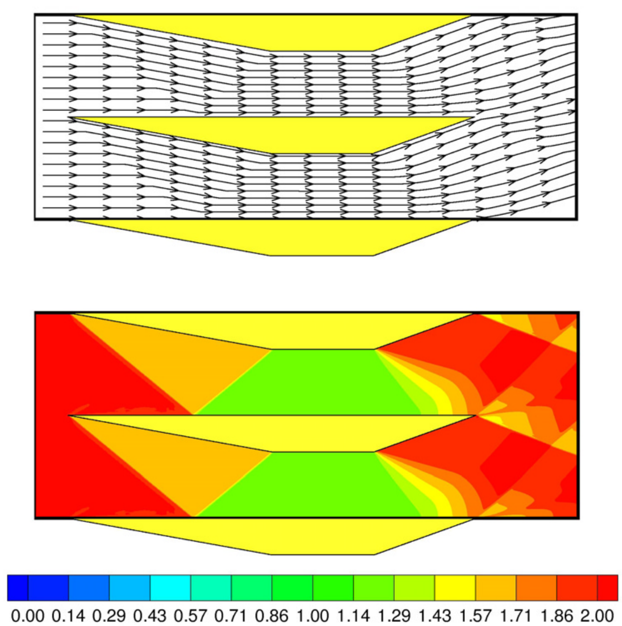
Wedge Cascade
Result: Stream Line and Ma Contour



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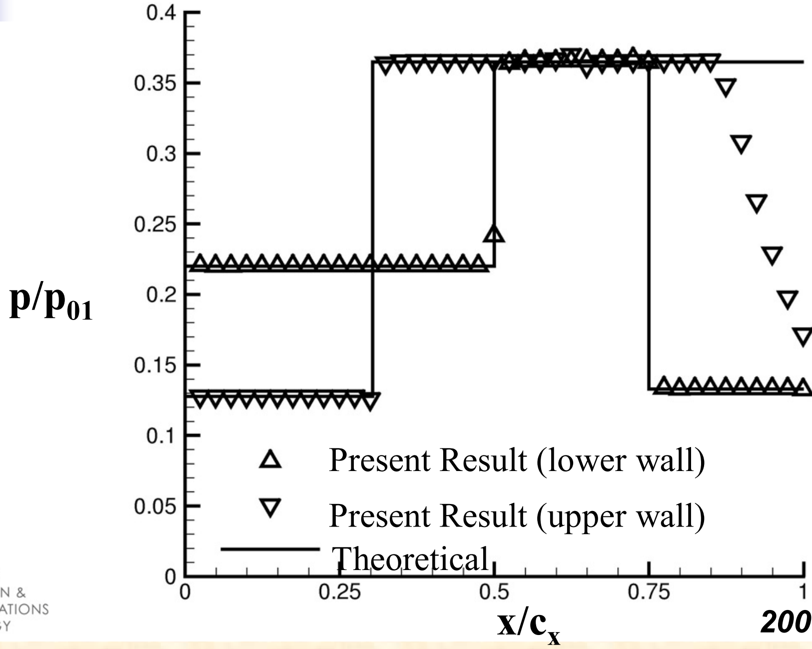
Wedge Cascade

Result: Static Pressure at Walls



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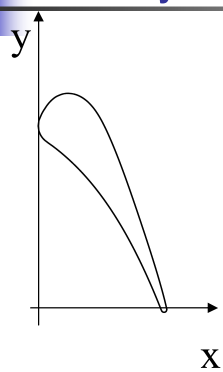
C3X Cascade

Geometry and Inlet and Outlet Parameters



Stagger Angle	**59.89°**
Chord	**14.49cm**
Spacing	**11.77cm**
Solidity	**1.23**
Axial Chord	**7.82cm**

Run Number	α_{in}	p_{t1} (Pa)	T_{t1} (K)	Ma_1	Re_1	p_2/p_{t1}
RUN 143	0	7755	811	0.17	0.63×10^6	0.50
RUN 144	0	7889	815	0.16	0.63×10^6	0.59

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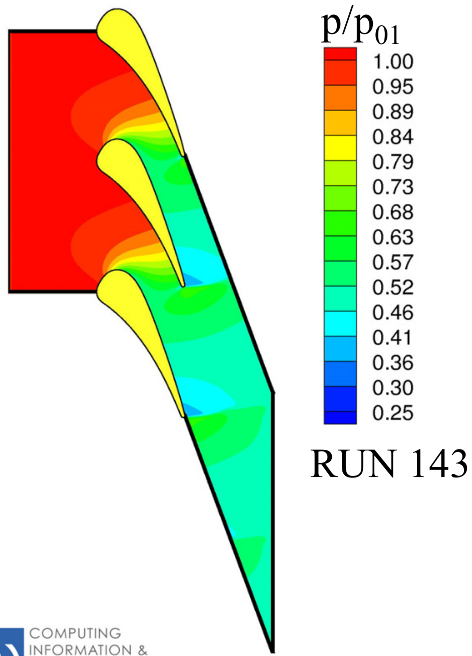
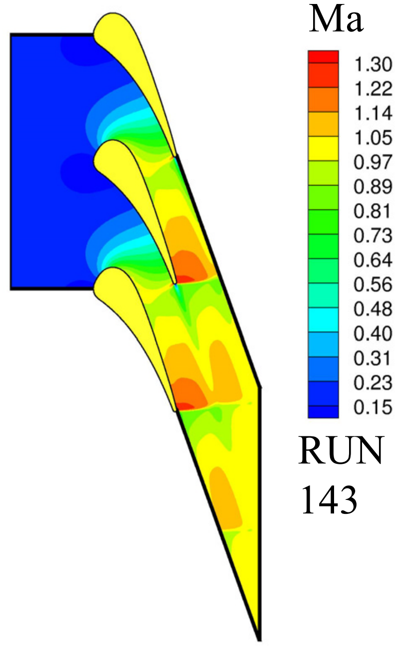
C3X Cascade RUN 143

esults





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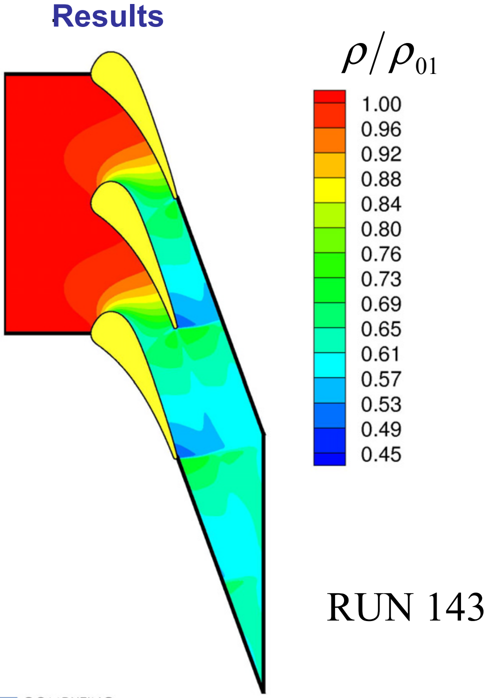
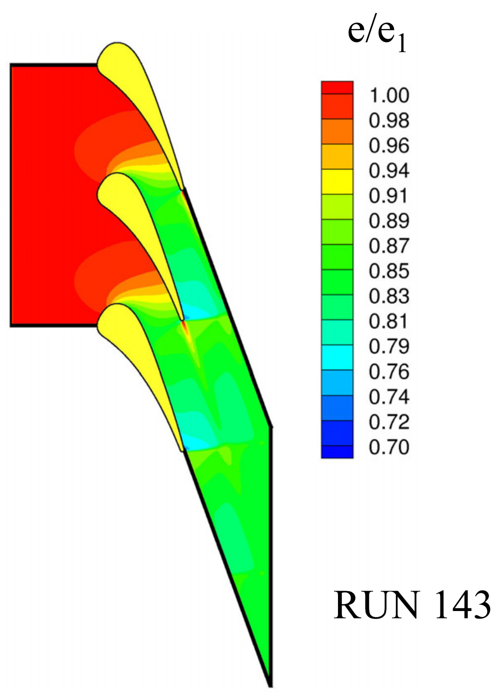
C3X Cascade RUN 143

Results





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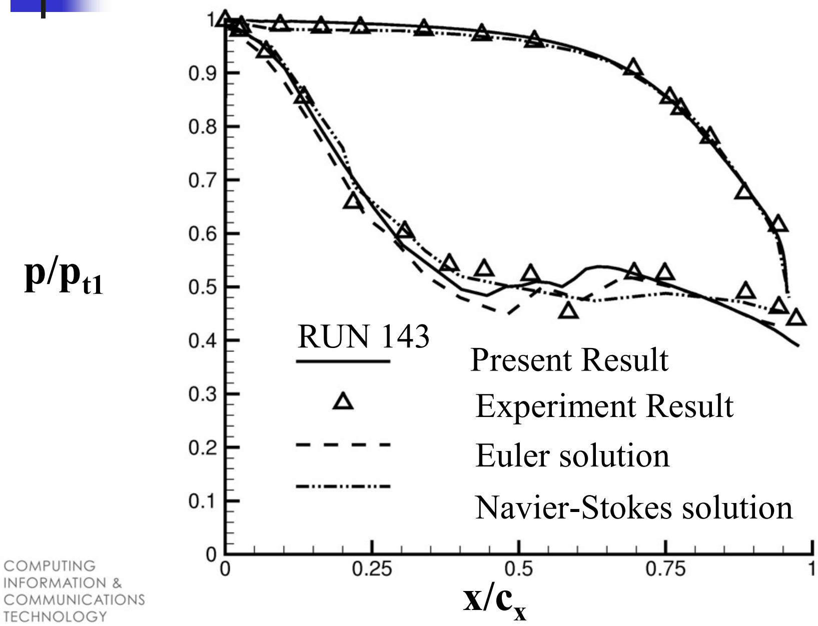
C3X Cascade RUN 143

Results



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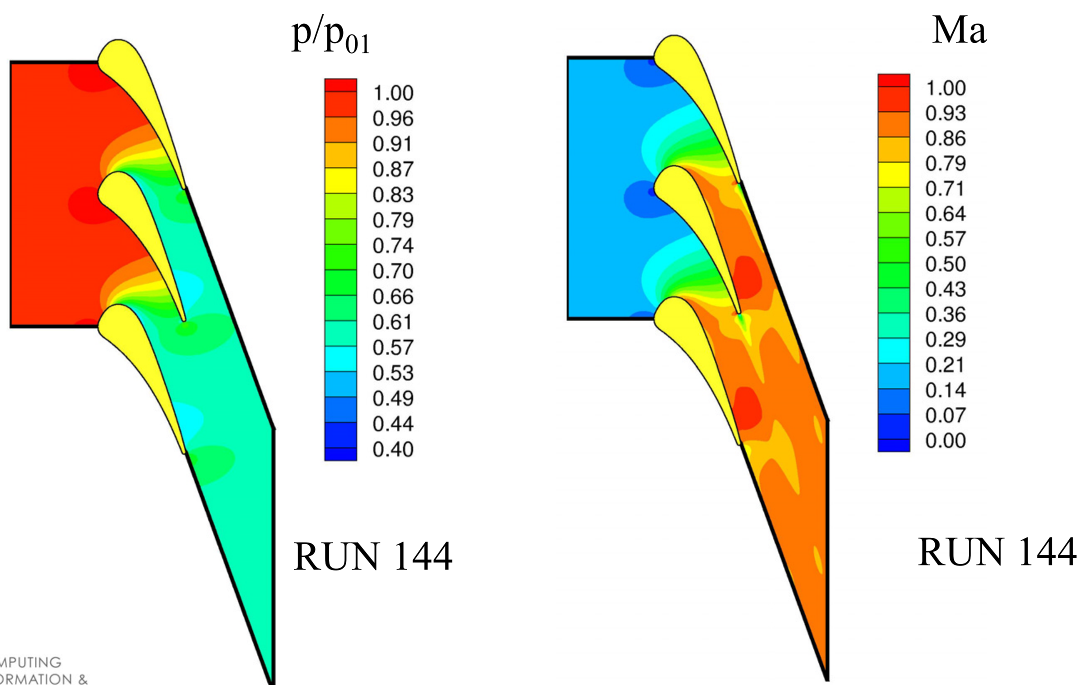
C3X Cascade RUN 144

Results



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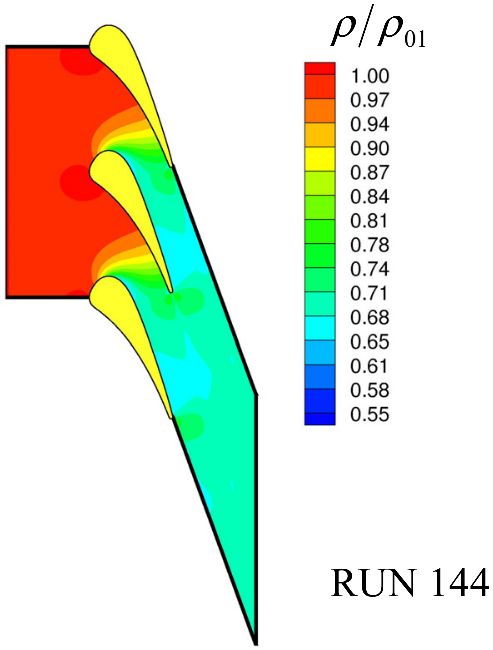
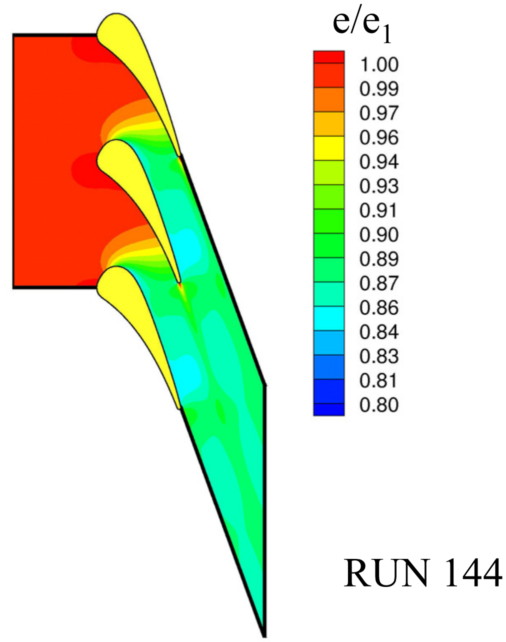
C3X Cascade RUN 144
Results





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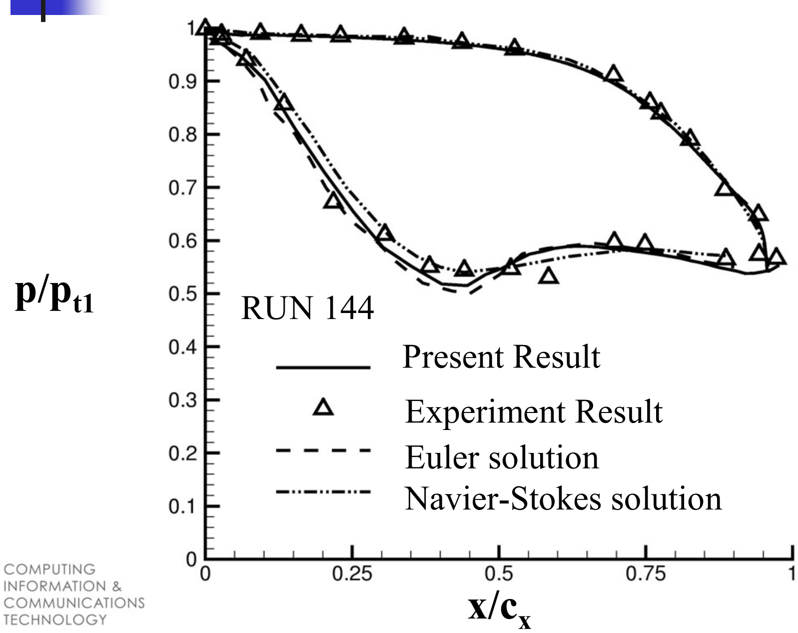
C3X Cascade RUN 144
Results



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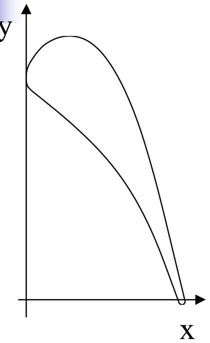
VKI Cascade

Geometry and Inlet and Outlet Parameters

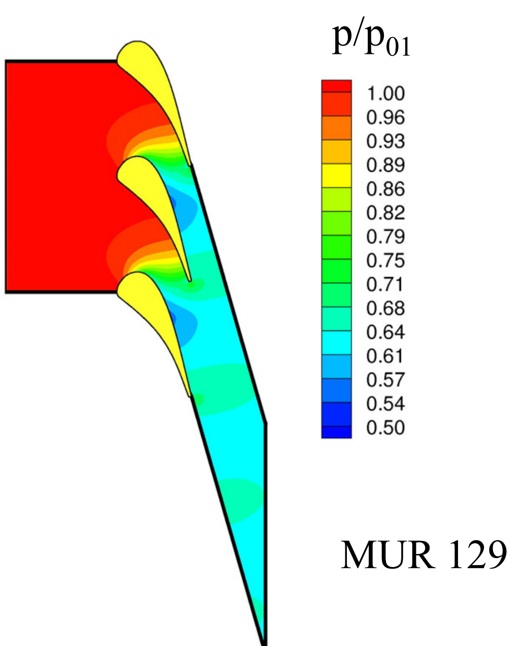


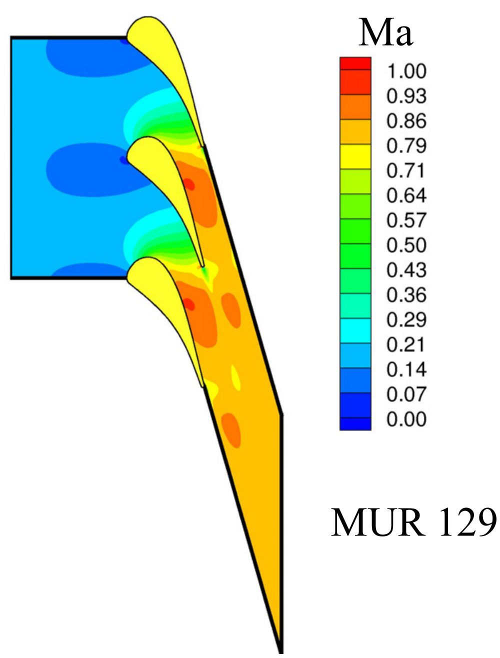
Stagger Angle	**55.0°**
Chord	**67.646 mm**
Spacing	**57.50 mm**
Solidity	**1.1765**
Axial Chord	**36.98 mm**

Run Number	α_{in}	p_{t1} (Pa)	T_{t1} (K)	Ma_1	Re_1	Ma_2
MUR 129	0	18200	409.20	0.15	0.27×10^6	0.84

VKI Cascade MUR 129

Results





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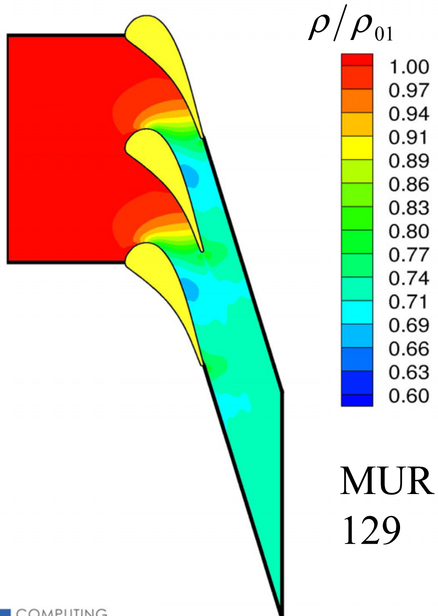
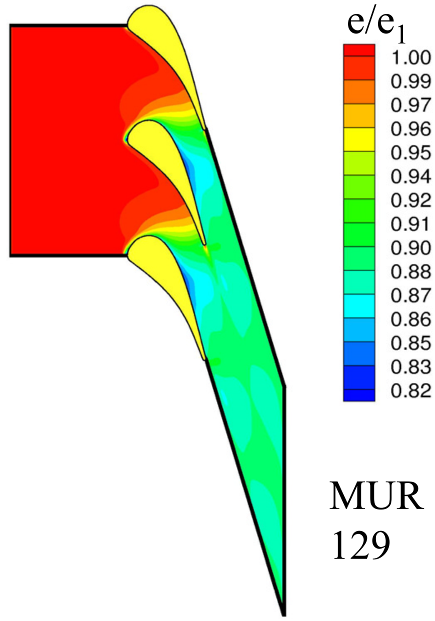
VKI Cascade MUR 129





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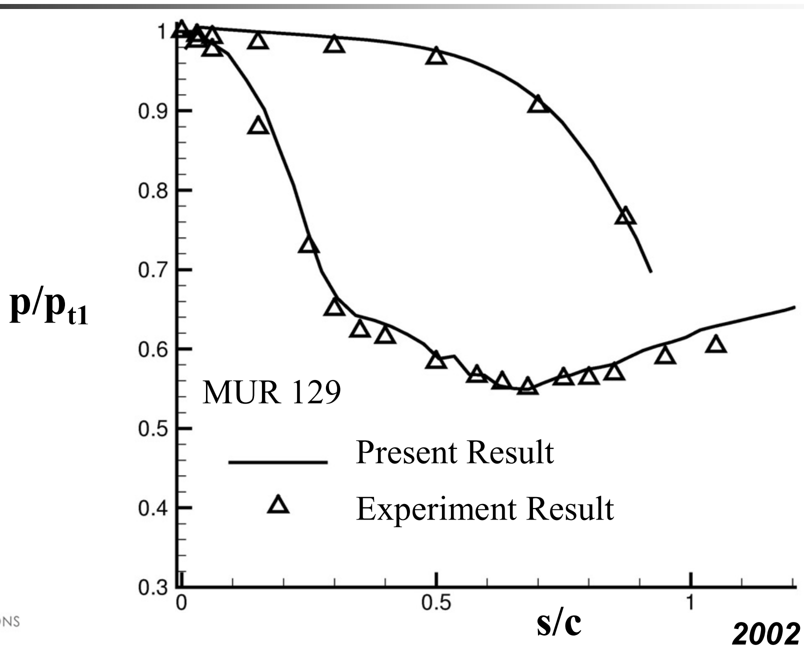
VKI Cascade MUR 129

Results



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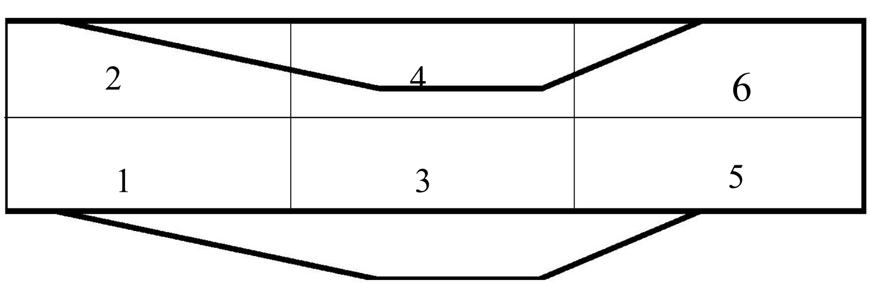
Parallel Computing

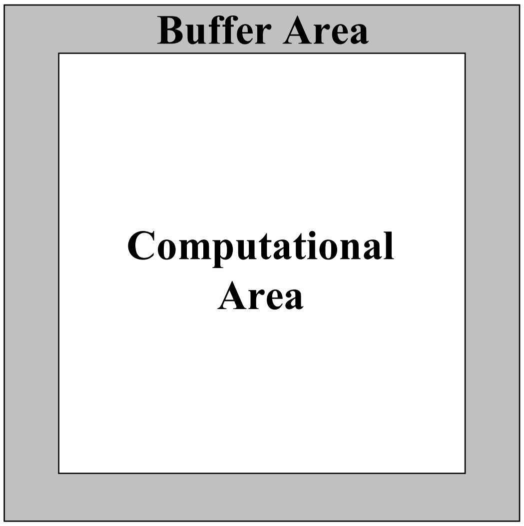
- **The LB method is a natural parallel method;**
- **The LB model is explicit in time;**
- **The area of dependent domain of a node is determined by the magnitude of velocity set, which is usually a small number.**

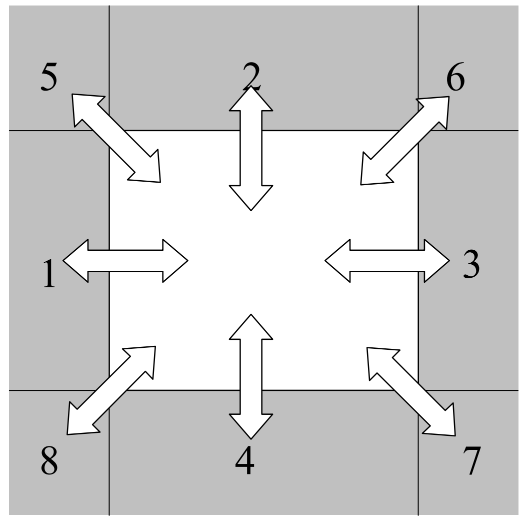
Parallel Computing
1) Division of Blocks

- **The simulation of wedge cascade was parallelized to test the parallel performance of current LB model**



Parallel Computing
2) Information Exchange





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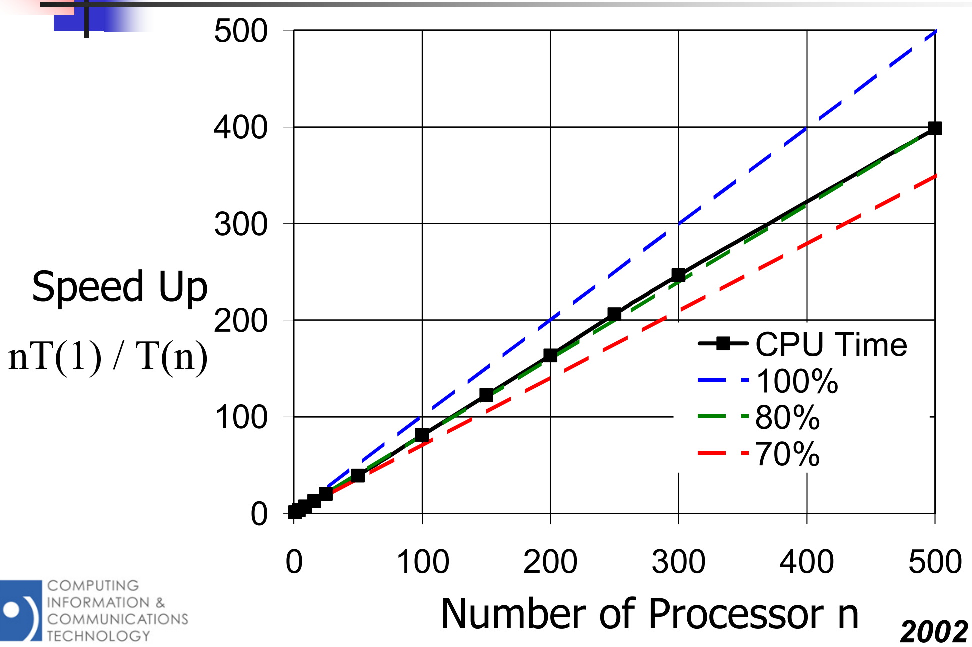
Parallel computing
3) Result



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Conclusion

1) A compressible LB model has been successfully developed for turbomachinery simulations.

2) Successful simulation of cascades has been carried out and it is the first successful turbomachinery simulation by a LB model.

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Conclusion

3) A treatment of boundary condition in LB method has been introduced to the current compressible LB model.

4) A new mesh treatment method has been devised in order to use regular mesh on a irregular geometry.

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Conclusion

5) **The parallel efficiency of the new compressible LB model is studied. A linear efficiency has been demonstrated.**

6) **The theoretical basis of the current model is analyzed in detailed.**

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