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Project Title: Preliminary Thermal Stress Analysis of a High-Pressure Cryogenic Storage Tank

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1. Abstract

The thermal stresses on a cryogenic storage tank strongly affect the condition of the tank and its ability to withstand operational stresses. These thermal stresses also affect the growth of any surface damage that might occur in the tank walls. These stresses are particularly of concern during the initial cooldown period for a new tank placed into service, and during any subsequent thermal cycles. A preliminary thermal stress analysis of a high-pressure cryogenic storage tank was performed. Stresses during normal operation were determined, as well as the transient temperature distribution. An elastic analysis was used to determine the thermal stresses in the inner wall based on the temperature data. The results of this elastic analysis indicate that the inner wall of the storage tank will experience thermal stresses of approximately 145,000 psi (1000 MPa). This stress level is well above the room-temperature yield strength of 304L stainless steel, which is about 25,000 psi (170 MPa). For this preliminary analysis, several important factors have not yet been considered. These factors include increased strength of 304L stainless steel at cryogenic temperatures, plastic material behavior, and increased strength due to strain hardening. In order to more accurately determine the thermal stresses and their affect on the tank material, further investigation is required, particularly in the area of material properties and their relationship to stress.

2. Introduction

A newly constructed cryogenic storage tank will experience a significant thermal gradient during its initial cooldown. This process frequently uses liquid nitrogen (LN2) to cool a storage vessel used to store liquid oxygen (LOX), with the LOX being pumped into the storage vessel after the tank is cooled sufficiently. Note that the saturation temperatures of these cryogenic fluids at atmospheric pressure are 77 K (139 R) for LN2 and 100 K (180 R) for LOX. The effects of these thermal stresses on the condition of the tank material are necessary to accurately determine the effects of the operational stresses placed on the tank during normal operations.

The tank is filled through an inlet located at the bottom of the tank. As the cryogenic fluid enters the tank at approximately atmospheric pressure, much of the fluid is boiled away as it encounters the hot inner wall of the tank. This combination of flowing, boiling cryogenic fluid produces a forced-convection boiling condition on the surface of the tank. This condition is characterized by extremely large values of the heat transfer coefficient.

Typical storage operation of the tank consists of the tank being nearly filled with LOX at approximately atmospheric pressure. The typical ullage (amount of unfilled space in the storage tank) is 10%. Additional LOX must be supplied to the tank to replace the amount vaporized due to the heat leak into the system. This storage operation comprises the majority of the cryogenic storage tank life. Under these circumstances, the temperature of the tank wall is approximately uniform at the saturation temperature of the stored fluid; for LOX at ambient pressure, the temperature is 100K (180 R).

High-pressure operation of the tank occurs when the cryogenic fluid is being supplied for a particular task, rocket engine component testing for example. Under these circumstances, another gas, typically nitrogen, is introduced at high pressure into the ullage space at the top of the tank, thereby pressurizing the tank and its contents.

This analysis will be performed for a tank whose diameter is 105 inches, wall thickness of 13 inches, and a maximum operating pressure of 8500 psi. The tank material is 304L stainless steel.

3. Analysis

The analysis of the stresses for the high-pressure cryogenic storage tank should include a determination of the stresses under normal high-pressure operation at cryogenic temperatures, thermal stresses under the initial transient chilldown operation as the tank is put into service, as well as the effect of repeated thermal cycles (warm up and chilldown). This analysis determines the spatial distribution of the operational stresses and a preliminary value of the thermal stress at the inner wall. Further analysis is required to more accurately model the thermal stresses and to estimate crack growth.

3.1 Operational Stresses Analysis

Although unrelated to the thermal stresses (since the cooldown and normal operation are separate and distinct events) an analysis of the stresses due to the pressurization of the tank was performed. The radial and tangential stresses were determined using equations for thick-walled vessels [1]. These equations are for thick-walled spherical shells, with an imposed internal pressure, p, and have been modified for the nomenclature used in this document. They do not include the effects of weight or support points. The tangential stresses ($\sigma 1$ and $\sigma 2$) shown in Equation 1 are equal due to the symmetry of the sphere. The radial stress ($\sigma 3$) is determined using Equation 2, while the shear stress (τ) is determined from Mohr's circle as in Equation 3. The radial location within the spherical shell is given by r, while r_i and r_o represent the inner and outer radii respectively.

$$\sigma_1(r) = \sigma_2(r) = \frac{p \cdot r_i^3}{2r^3}\left(\frac{r_o^3 + 2r^3}{r_o^3 - r_i^3}\right) \qquad \text{Eqn. 1}$$

$$\sigma_3(r) = \frac{-p \cdot r_i^3}{r^3}\left(\frac{r_o^3 - r^3}{r_o^3 - r_i^3}\right) \qquad \text{Eqn. 2}$$

$$\tau(r) = \frac{\sigma_1(r) - \sigma_3(r)}{2} \qquad \text{Eqn. 3}$$

A vessel is considered to be thick-walled if the ratio of the wall thickness to the inner radius is greater than about 0.1. For the tank described in this analysis, this ratio is about 0.25, definitely a thick-walled vessel. For this analysis, a pressure of 8500 psi was used, resulting in a maximum stress of –8500 psi (the negative sign indicates a compressive stress) in the radial direction at the inner wall of the tank. The maximum tangential stress was about 17,800 psi, while the maximum shearing stress was about 13,000 psi, both occurring at the inner wall. These results indicate that the imposed pressure produces a maximum stress approximately equal to the 17,500 psi allowable stress specified by the ASME Code, Section VIII for 304L stainless steel storage vessels at or below room temperature [2]. The spatial stress distribution is shown in Figure 1. Note that the tangential stresses are tensile stresses, while the radial stress is a compressive stress. Recall that these stresses are due to pressurization effects only; additional stresses due to the weight and support of the structure itself are not included. If desired these stresses could be determined and superimposed on the pressure stresses to find the total stress in the tank.

It is interesting to compare these results to the yield and ultimate strengths of 304L stainless. ASTM sets a yield strength of 25,000 psi (170 MPa) and an ultimate strength of 70,000 psi (485 MPa) as minimum requirements for 304L stainless steel, while Yuri, *et. al.* [3] have determined these strengths at LN2 saturation temperature (77 K/ 139 R) to be about 50,000 psi (350 MPa) and 225,000 (1550 MPa) psi respectively.

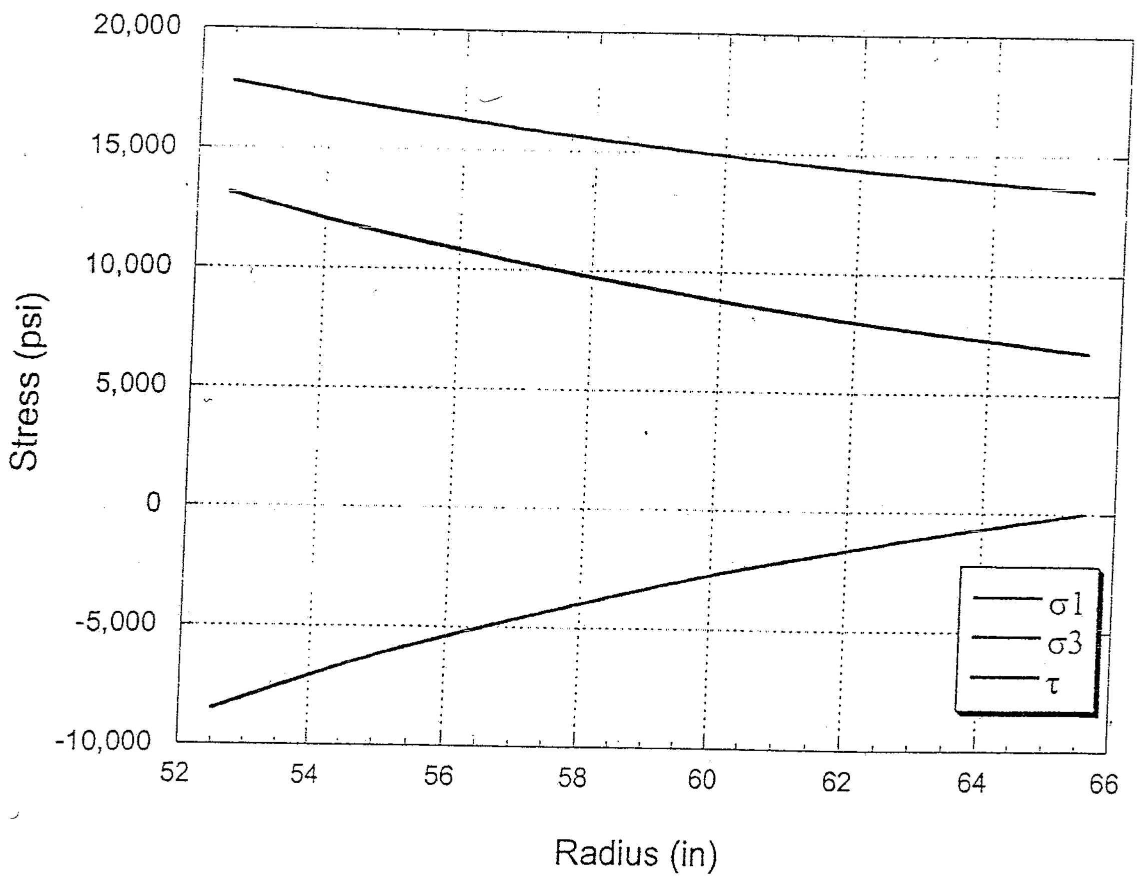

Figure 1. Stress distribution in thick-walled spherical vessel (high-pressure operation).

3.2 Thermal Analysis

As the liquid nitrogen is pumped into the storage tank from the pipe at the bottom of the tank to cool the tank, the liquid nitrogen entering the tank will boil as it touches the inner walls, which are initially at ambient temperature. Because the incoming nitrogen is both flowing and boiling, the heat transfer coefficient in this situation is quite high, on the order of several thousand BTU/(ft^2*hr*R). It is this high heat transfer coefficient, coupled with the relatively low thermal conductivity and the large spatial dimensions that result in the expected significant thermal stresses.

A thin-walled tank, or a tank made from a high thermal conductivity material, would not experience such dramatic thermal stresses. The temperature would be much more uniform throughout the material at any given time during the cooldown procedure, resulting in relatively little thermal stress, which is caused by the spatial thermal gradient.

Temperature Distribution

The temperature profile can be obtained by solving the heat conduction equation, a partial differential equation seen in Equation 4 for spherical coordinates. As can be seen, this is a three-dimensional, transient problem.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{q}=\rho c_P\frac{\partial T}{\partial t} \qquad \text{Eqn. 4}$$

Although the problem is a three-dimensional transient conduction problem, it can be simplified to a one-dimensional transient conduction problem in the radial direction. The reasons for the simplification are the relatively low thermal conductivity value for stainless steel and the relative magnitude of the spatial dimensions of the tank. The problem can be further simplified for this preliminary analysis by treating the thermophysical properties, such as thermal conductivity, as constants. Although acceptable for this analysis, this is not physically accurate, as the thermal conductivity varies with temperature over the range from ambient to cryogenic temperatures. The simplified version of the heat conduction equation (1D, transient, constant properties) is shown below in Equation 5, where r is the radial location, T is the temperature, t is time, and α is the thermal diffusivity.

$$\frac{2}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{Eqn. 5}$$

As with any partial differential equation, initial and boundary conditions are required to completely define the problem. Initially, the entire sphere is at ambient conditions, so a uniform temperature initial condition is appropriate, which is defined in Equation 6. The inner surface of the sphere is subjected to forced-convection boiling, which is located only in the region occupied by the liquid, so the boundary condition for the inner wall is actually transient, depending on the fill rate or the amount of ullage in the tank. For this preliminary analysis, a steady boundary condition representing the forced-convection boiling is used. This is justified, since the area of interest in the initial part of the cooldown is located at the inlet, which is at the bottom of the tank. The portion of the tank not covered with LN2 has a much smaller heat transfer coefficient, and is being cooled much more slowly, generating much smaller thermal stresses. Therefore, as soon as there is liquid in the tank, the steady boundary condition is locally appropriate. The outer wall of the sphere is encased in a vacuum jacket, which contains insulation. For this preliminary analysis, this boundary will be modeled as an insulated surface. This is acceptable for the modeling of the initial cooldown, but would need to be modified for a steady state thermal analysis of the tank to include the heat leak into the system. The convection and insulated boundary conditions are shown in Equations 7 and 8 respectively, where t is time, r is the local radial location, and r_i and r_o represent the inner and outer wall radii respectively. The heat transfer coefficient is given by h, and the thermal conductivity by k, both of which are found in Equation 7.

$$T(r, t=0) = T_i \qquad \text{Eqn. 6}$$

$$-k \left.\frac{\partial T}{\partial r}\right|_{r=r_i} = h\left[T(r_i, t) - T_\infty\right]_{r=r_i} \qquad \text{Eqn. 7}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=r_o} = 0_{r=r_o} \qquad \text{Eqn. 8}$$

The solution to this properly posed partial differential equation, which mathematically models the transient heat conduction in the sphere, can be approximated using finite difference techniques. Specifically, central differences were used for the spatial derivatives, while the explicit method was used for the time derivative. The resulting spatial temperature distribution is shown for various values of time (starting with t=0 and ending with t=6000 seconds) in Figure 2.

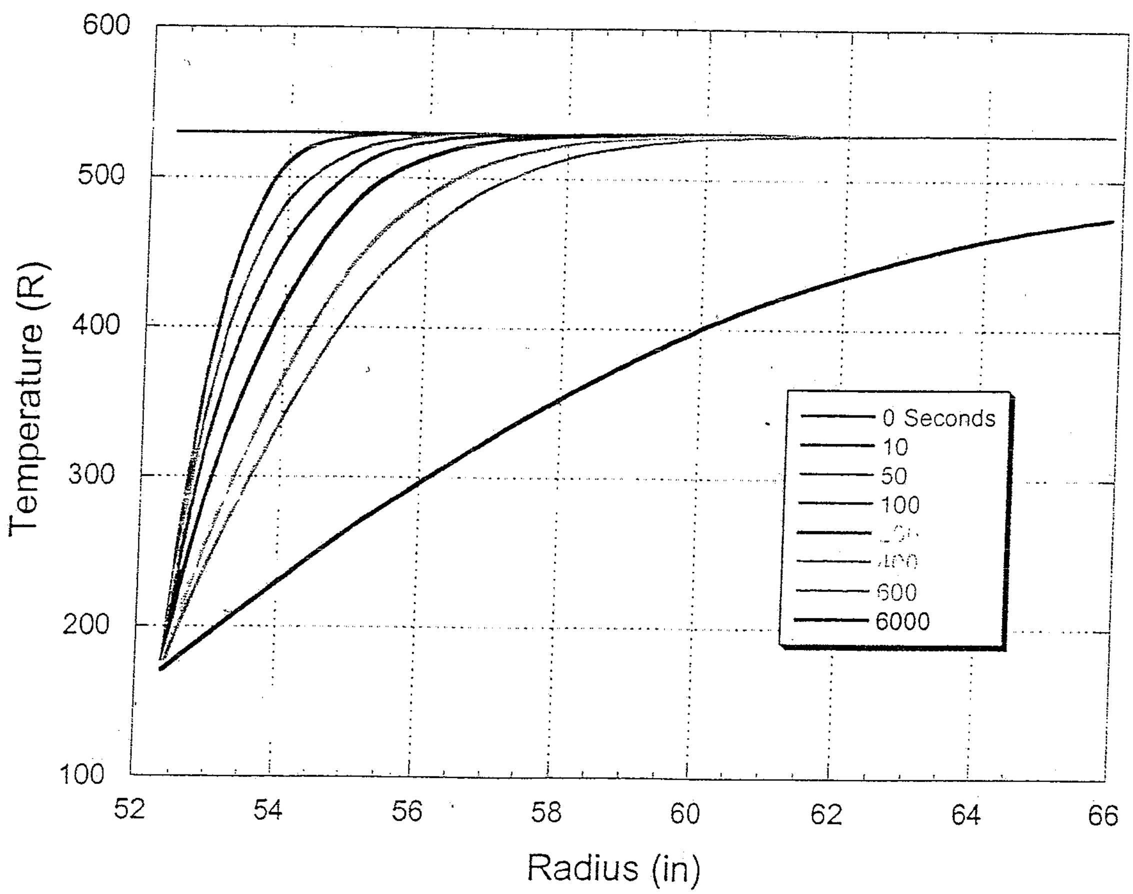

Figure 2. Temperature Distribution in Tank Wall for Selected Time Values.

These data show that the temperature gradient is quite steep at the beginning of the chilldown process, then tends to gradually decline in the first ten minutes. However, note that the inner wall temperature has essentially achieved the saturation temperature of the cryogenic fluid, in this case liquid oxygen at about 180 R (100 K), due to the large heat transfer coefficient (approximately 10,000 BTU/(ft^2*hr*R)). The chilldown process has not yet affected the temperature of the outer wall, even after 10 minutes (600 seconds). It is seen that after 100 minutes (6000 seconds) the temperature of the outer wall is now decreasing, having dropped approximately 50 degrees at this point in the process.

Thermal Stresses

Due to the thermal conditions, the inner wall of the spherical vessel attains the saturation temperature of the incoming cryogenic fluid (liquid nitrogen for the cool down operation) very quickly due to the high heat transfer coefficient, while the outer wall temperature does not change for several minutes, due primarily to the poor heat conduction (low thermal conductivity) of stainless steel and the dimensions of the vessel (thick-walled.) A preliminary analysis of the thermal stresses in the vessel can be performed using Hooke's law and the thermal strain equation. The elastic thermal stress is related to the thermal strain as follows,

$$\sigma_T = \frac{E\gamma}{1-\nu}(\Delta T) \qquad \text{Eqn. 9}$$

where E is Young's modulus or the modulus of elasticity, ν is Poisson's ratio, and γ is the coefficient of thermal expansion. (The subscript T denotes thermal stress.) The resulting thermal stress for this situation is approximately 145,000 psi, using the temperature difference for the inner wall of about 390 R (difference between ambient and LN2 saturation temperatures) for a liquid nitrogen chilldown. This stress is due to the thermal gradient in the sphere, where the inner wall is at the LN2 saturation temperature, while the outer wall remains at ambient temperature. The thermal stress on the inner wall is tensile, as the cooling tends to contract the material.

Typical values for the yield and ultimate strength of 304L stainless steel are 25,000 psi (170 MPa) and 70,000 psi (485 MPa), per the ASTM specifications for 304L. Based on the thermal stress result, it would seem that the tank would be expected to fail. However, as mentioned previously, the yield and ultimate strengths at cryogenic temperatures are much larger than the room temperature values. So even as the temperature gradient creates thermal stresses, the cold material has a much higher strength. Also recall that this is an elastic analysis and does not include the effects of plasticity; the strength of stainless steel at room temperatures increases dramatically due to strain hardening, with maximum values (yield and ultimate) of 160,000 and 185,000 psi.

3.3 Future Analyses

The analysis results will depend on the effects of these factors, strength as a function of both temperature and strain hardening. As alluded to previously, if the temperature of the inner wall material is 77 K, then there is a thermal stress of about 145,000 psi, but the yield and ultimate strengths of 304L at this temperature are about 50,000 and 225,000 psi. The material would be expected to yield, but would not be expected to reach the ultimate limit. Additionally, as the stress exceeds the yield strength, the material will experience strain hardening, increasing its yield strength as it yields. Therefore, as the material yields, its yield strength increases, until it reaches a point where it no longer yields, which should occur prior to reaching the ultimate strength (failure). Further investigation is required to test this supposition and to determine the extent of these effects.

As a material yields, the relationship between stress and strain typically determines the residual stress due to the plastic deformation. Consider that the material yields during the thermal cooldown. As the entire structure finally achieves a uniform temperature, the thermal stresses will return to zero, as there is no longer a thermal gradient to drive them. However, if plastic deformation has occurred, a portion of the material at the inner wall and into the sphere may be in a state of compressive stress due to the plastic deformation. Should this be the case, this residual stress would algebraically oppose the tensile stresses imposed by the internal pressure during high-pressure operation. Again, further investigation of the material behavior is required to test this supposition.

4. Conclusions

Before reasonably accurate conclusions can be made, further investigation is required to answer the questions regarding the material behavior during this initial cooldown process and subsequent thermal cycles. This research includes a search of the existing literature as well as experimentation in order to measure the important parameters and determine the effects of this initial cooldown. More appropriate analytical solutions are also needed as part of this further investigation. As this investigation provides better information about the stress state of the material, this information can be used to model crack growth in the tank wall.

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