TEMPERATURE GRADIENTS ON THE CELL WALL
IN THE
CRITICAL VISCOSITY EXPERIMENT

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Because of the diverging susceptibility $\partial\rho/\partial T$ near the liquid–vapor critical point, temperature gradients must be kept small to maintain adequate sample homogeneity. In our Science Requirements Document we paid particular attention to radial density gradients caused by equilibration of the xenon sample. Axial density gradients were addressed through the requirement that the cell's copper wall have a gradient less than 22 μK/m. This report re–examines the cell wall's temperature distribution in more detail by estimating all known significant contributions to temperature differences on the cell's wall.

Although temperature homogeneity requirements have previously been given in terms of temperature gradients, this report will instead use temperature differences to generalize the discussion to include nonlinear temperature distributions. In equilibrium, a temperature difference ΔT on the cell wall will induce a sample density difference $\Delta\rho$ according to

$$\frac{\Delta\rho}{\rho_c} = \left[\frac{T_c}{\rho_c}\frac{\partial\rho}{\partial T}\right]_P \frac{\Delta T}{T_c} \tag{1}$$

$$= \left[\frac{T_c}{P_c}\frac{\partial P}{\partial T}\right]_{\rho_c} \Gamma\epsilon^{-\gamma}\frac{\Delta T}{T_c} . \tag{2}$$

Here ρ_c, T_c, and P_c are the density, temperature, and pressure at the critical point and Γ and γ are the amplitude and exponent of the reduced susceptibility. The maximum temperature difference allowed at the planned closest reduced temperature of $\epsilon = 2\times10^{-6}$ is thus

$$\Delta T = \frac{\Delta\rho}{\rho_c}\left[\frac{T_c}{P_c}\frac{\partial P}{\partial T}\right]_{\rho_c}^{-1}\frac{\epsilon^{\gamma}T_c}{\Gamma} , \tag{3}$$

or

$$\Delta T = (0.003)[6.0]^{-1}\frac{\left(2\times10^{-6}\right)^{1.24}(290 \text{ K})}{(0.058)} = 0.21 \ \mu\text{K} .$$

(This is equivalent to the specification of 22 μK/m along the oscillating screen's radius of 0.010 m. At larger reduced temperatures this requirement is less stringent. Therefore, in the following sections, where we estimate possible sources of temperature differences, we address only the case where the thermostat temperature is nearest T_c and is ramping at

the slowest planned rate.

Thermistor dissipation

The gradient caused by the sample cell's thermistor power P is largest near the thermistor. The temperature field of the cell's wall can be estimated by imagining the cell to be slit along its length and unrolled into a plate of thickness d, length $2x_2$, and width $2\pi R_2$. Treating this plate as an infinite two–dimensional sheet gives the temperature at a distance r from the thermistor as approximately

$$\Delta T_{therm} = T(d) - T(r) \cong \frac{P}{2\pi d\lambda_{Cu}} \ln(r/d) , \tag{4}$$

where λ_{Cu} is the thermal conductivity of copper. Substituting the half length x_2 of the cell for r yields the maximum temperature difference of

$$\Delta T_{therm} = \frac{(5\times10^{-7} \text{ W})}{(2\pi)(0.006 \text{ m})(4\,00 \text{ W/(m·K)}} \ln(0.03 \text{ m}/0.006 \text{ m}) = 0.05 \text{ } \mu\text{K}.$$

Heat flow out of the copper body due to the cell's ramp rate

As the cell cools at the slowest ramp of $\dot{T} = 3\times10^{-8}$ K/s, the associated heat flow is about 6 μW, much more than the thermistor's power. If all of the heat is assumed to flow through two identical spacers located at the ends of the cell the temperature difference between the middle and the ends of the copper cell is

$$\Delta T_{body} = T(middle) - T(end) = \frac{x_2^2}{2}\frac{\dot{T}}{D_{T,Cu}} \tag{5}$$

where $D_{T,Cu}$ is the thermal diffusivity of copper. The result is

$$\Delta T_{body} = \frac{(0.030 \text{ m})^2(3\times10^{-8} \text{ K/s})}{(2)(1\,.\,16\times10^{-4} \text{ m}^2/\text{s})} = 0.12 \text{ } \mu\text{K} .$$

This estimate would be reduced by including the effects of air conduction and infrared radiation to the inner shell, and it would be increased by spacer asymmetry, considered below.

Symmetric conduction of the cell's two spacers, located at each end, can be encouraged but not guaranteed by identical construction. The relative contributions of the left and right spacers also depend on the details of the clamping of the spacers to the cell and to the inner shell. If we model this asymmetry as an air gap of length ΔR_{air} in series with the spacer length ΔR_{spacer}, then reasonable machining tolerances (0.005 inch) place an upper bound on the asymmetry $\Delta\kappa$ of

$$\Delta\kappa = \left[\frac{\lambda_{plastic}}{\lambda_{air}}-\right]\left[\frac{\Delta R_{air}}{\Delta R_{spacer}}-\right] . \tag{6}$$

$$\Delta\kappa = \left[\frac{0.2 \quad W/(m\cdot K)}{0.023 \quad W/(m\cdot K)}\right]\left[\frac{0.00013 \quad m}{0.013 \quad m}\right] = 0.09$$

In practice, the air gaps will be significantly reduced by slight deformation of the spacer when the the plastic and metal surfaces are clamped together.

Including spacer asymmetry in the model leads to the following dependence of the cell temperature on axial position x.

$$T(x,t) = \dot{T}\{[t + (2D_{T,Cu})^{-1}[(2x_2\Delta\kappa)x + x^2]\}\tag{7}$$

The maximum temperature difference is then

$$\Delta T_{body} = \frac{\dot{T}x_2^2}{2D_{T,Cu}}\,[1 + 2\Delta\kappa + \Delta\kappa^2]\,,\tag{8}$$

leading to the estimate

$$\Delta T_{body} = \frac{(3\times10^{-8}\ K/s)(0.03\ m)^2}{(2)(1.16\times10^{-4}\ m^2/s)}\,[1.19] = 0.14\ \mu K\,.$$

In the 1992 Science Requirements Document, the cell's two spacers were estimated to contribute 0.031 W/K out of a total of 0.065 W/K conductivity between the cell and the inner shell. Thus the estimate above should be reduced by a factor of about two to

$$\Delta T_{body} = 0.07\ \mu K\,.$$

Heat flow out the sapphire window due to the cell's ramp rate
The heat flow out of the sapphire window gives a radial temperature difference of

$$\Delta T_{window} = \frac{R_w^2}{4}\frac{\dot{T}}{D_{T,sapphire}}\,,\tag{9}$$

where R_w is the window's radius and $D_{T,sapphire}$ is the thermal diffusivity of sapphire, the window material. The result is

$$\Delta T_{window} = \frac{(0.013\ m)^2(3\times10^{-8}\ K/s)}{(4)(1.34\times10^{-5}\ m^2/s)} = 0.09\ \mu K\,,$$

Including the effects of radiation and air conduction would reduce this temperature difference.

Inner shell gradient
The temperature gradient on the inner shell is partially imposed on the cell. Here we estimate the gradient reduction ratio in two ways and combine the resulting values with an estimate of the inner shell's gradient to obtain the cell's gradient.

For the first estimate we assume that heat flows between the cell and the

4

surrounding inner shell through only the cell's spacers. The temperature difference on the cell is then approximately

$$\Delta T_{gradient} = T(right) - T(left) \cong \frac{dT_{in}}{dx} \frac{\kappa_{sp}}{\kappa_{cell}} 2x_2 , \qquad (10)$$

where $2x_2$ is the cell's length, dT_{in}/dx is the inner shell's temperature gradient, and κ_{sp} and κ_{cell} are the radial conductance of the spacer and the axial conductance of the cell respectively. The conductivity of the cell along its length is

$$\kappa_{cell} = \frac{A_{cell}\lambda_{Cu}}{2x_2} = \frac{(3/4)(\pi)(0.019 \text{ m}^2)(400 \text{ W/K/m})}{(2)(0.030 \text{ m})} = 5.7 \text{ W/K} ,$$

where A_{cell} is the cell's metallic cross section area. The spacer conductivity was already estimated in the Science Requirements Document at 0.031 W/K.

For the second estimate we assume heat flows through only the radial air gap between the cell and the inner shell.

$$\Delta T_{gradient} = T(right) - T(left) \cong \frac{dT_{in}}{dx} \left[\frac{x_2}{d_{cell}^2} \right]^2 x_2 , \qquad (11)$$

where $d_{cell} = 0.74$ m is a length characterizing the cell's conductance to the surrounding inner shell (defined by Eq.(20), p.17 of the Science Requirements Document).

Now we estimate the inner shell's temperature gradient, due mainly to the heaters located at the ends. Assuming the right heater produces 10% more heat than the left heater, the inner shell's gradient is

$$\frac{dT_{in}}{dx} \cong (0.10) \frac{x_{2,in}}{d_{in}^2} (T_{in} - T_{mid}) , \qquad (12)$$

where $x_{2,in}$ is the half–length of the inner shell, d_{in} is a length characterizing the inner shell's conductance to the surrounding middle shell, and $(T_{in}-T_{mid})$ is the regulated temperature offset between the inner and middle shells. Eq.(12) yields

$$\frac{dT_{in}}{dx} = (0.10)\frac{(0.070 \text{ m})}{(0.82 \text{ m})^2} (0.030 \text{ K}) = 3.1 \times 10^{-4} \text{ K/m} .$$

Using the above results in Eq.(10) gives

$$\Delta T_{gradient} \cong (3 \times 10^{-4} \text{ K/m}) \frac{(0.031 \text{ W/m})}{(5.7 \text{ W/m})} (0.06 \text{ m}) = 0.01 \text{ } \mu K ,$$

and for Eq.(11) gives

$$\Delta T_{gradient} \cong (3{\times}10^{-4} \ K/m) \left[\frac{(0.030 \ m)}{(0.74 \ m)} \right]^2 (0.03 \ m) = 0.01 \ \mu K \ ,$$

Both assumptions lead to small estimates of the temperature difference imposed on the cell by the inner shell.

Asymmetrical connections to the cell

Asymmetric thermal conduction of the plastic spacers between the cell and the inner shell was considered above in connection with gradients induced by the cell's ramp rate. Although spacer asymmetry does not play an important role in connection with the temperature difference imposed by the inner shell's gradient, it is important for heat flows originating on the cell.

There is an additional asymmetry due to the electrical connections to the cell. Each coaxial cable contains a central #30 wire and a #30 drain wire with aluminized plastic film. The total cross section from the three coaxial cables and the thermometer and heater connections is approximately equal to ten #30 copper wires, giving a thermal conductivity along the 0.03 m length of

$$\kappa_{wires} \cong 7{\times}10^{-4} \ W/K \ ,$$

about 1% of the total conductivity to the cell. This is negligible in comparison with the spacer asymmetry already considered.

In the Science Requirements Document radiation was estimated to account for about one quarter of the total conductance between the inner shell and the cell. If we assume the unrealistic case where radiation is emitted from only one end then ΔT_{body} will be roughly doubled. More realistically, the surface with the highest emissivity is the tape wrapped around the cell's middle to secure the electrical leads. The resulting effect of this emissivity asymmetry on the cell's temperature distribution will be small compared with the effects already considered. As was the case with spacer asymmetry, emissivity asymmetry is more important in connection with heat flows originating on the cell than with the temperature gradient of the surrounding inner shell.

Summary

The temperature differences calculated above are

Allowed maximum	ΔT	0.21 μK
Thermistor power	ΔT_{therm}	0.05 μK
Cooling of body	ΔT_{body}	0.07 μK
Cooling of window	T_{window}	0.09 μK
Imposed by inner shell	$\Delta T_{gradient}$	0.01 μK

The cell has two "hot spots", one at the center of the window and one near the thermistor. The second spot, the sum of ΔT_{therm} and ΔT_{body}, is hotter. Therefore the maximum expected temperature difference is

$$\Delta T_{total} = \Delta T_{therm} + \Delta T_{body} + \Delta T_{gradient}$$

$$= 0.05 \; \mu K + 0.07 \; \mu K + 0.01 \; \mu K$$

$$= 0.12 \; \mu K \; .$$

Though acceptable, the estimated magnitude ΔT_{total} is rather close to the allowed maximum of 0.21 μK. The most important contributions are due to the thermistor power and the ramp rate $\dot{T}$. The first contribution could be approximately halved by dividing the thermistor power between two thermistors imbedded on opposite sides of the cell body. The second contribution could be halved by halving the ramp rate, although any such decrease must be weighed against the available mission time.

      The cell design will be modified to include a metal cover plate for the sapphire window to prevent radiative heat transfer directly to the interior of the cell.