**2002**

**NASA FACULTY FELLOWSHIP PROGRAM**

**MARSHALL SPACE FLIGHT CENTER**
**THE UNIVERSITY OF ALABAMA**

# SUBSURFACE STRESS FIELDS IN SINGLE CRYSTAL (ANISOTROPIC) CONTACTS


| | |
|---|---|
| Prepared By: | Nagaraj K. Arakere |
| Academic Rank: | Associate Professor |
| Institution and Department: | University of Florida<br>Mechanical & Aerospace Engineering Department |
| NASA/MSFC Directorate: | Engineering |
| MSFC Colleague: | Gregory R. Swanson |

## Introduction

Single crystal superalloy turbine blades used in high pressure turbomachinery are subject to conditions of high temperature, triaxial steady and fatigue stresses, fretting stresses in the blade attachment and damper contact locations, and exposure to high-pressure hydrogen. The blades are also subjected to extreme variations in temperature during start-up and shutdown transients. The most prevalent HCF failure modes observed in these blades during operation include crystallographic crack initiation/propagation on octahedral planes, and noncrystallographic initiation with crystallographic growth. Numerous cases of crack initiation and crack propagation at the blade leading edge tip, blade attachment regions, and damper contact locations have been documented. Understanding crack initiation/propagation under mixed-mode loading conditions is critical for establishing a systematic procedure for evaluating HCF life of single crystal turbine blades.

Techniques for evaluating two and three dimensional subsurface stress fields in anisotropic contacts are presented in this report. Figure 1 shows typical damper contact locations in a turbine blade. The subsurface stress results are used for evaluating contact fatigue life at damper contacts and dovetail attachment regions in single crystal nickel-base superalloy turbine blades.

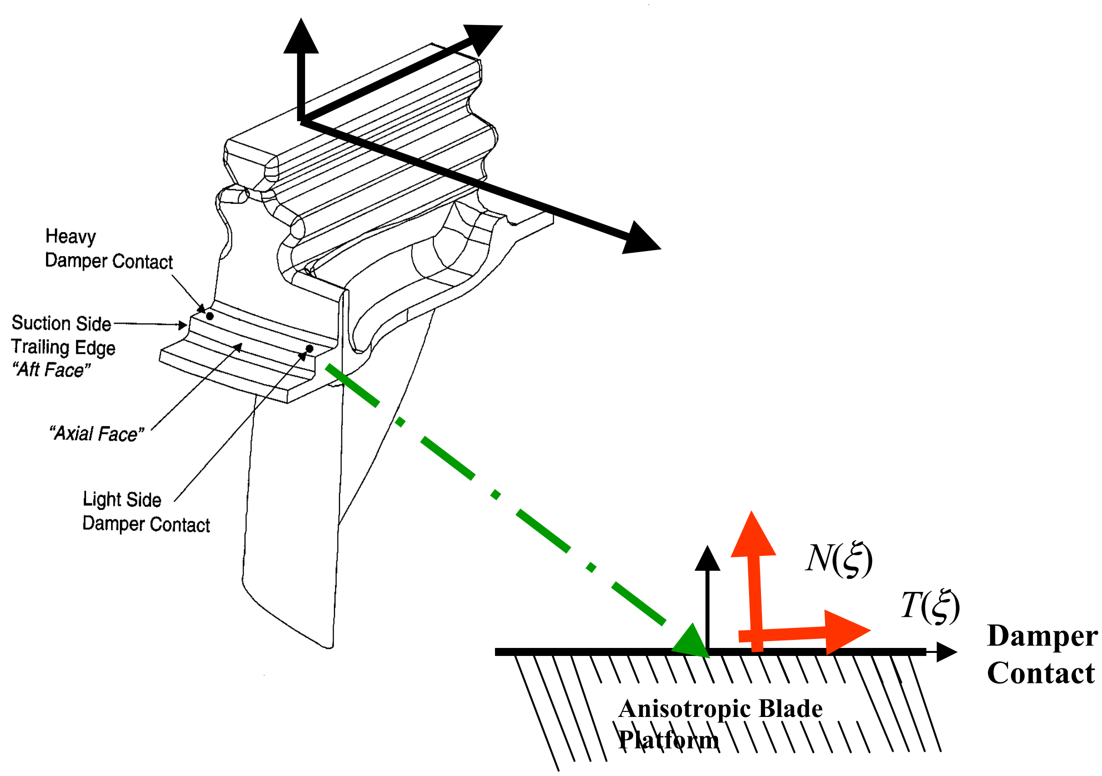


Figure 1: Damper contact locations on a typical turbine blade

## Two-Dimensional Stress Distribution in an Anisotropic Elastic Half-Space

The damper contact regions shown in Figure 1 will be modeled as an elastic anisotropic half-space. This approximation is reasonable since Hertzian type contact stresses are confined to very small volumes in the vicinity of the contact. An analytical procedure will be presented for evaluating the subsurface stresses in the elastic half-space using a complex potential method outlined by Lekhnitskii [1]. Figure 2 shows the elastic half-space subjected to normal traction $N(\xi)$ and tangential traction $T(\xi)$ over the region –a to +a on the x-axis. The traction forces are independent of z, and functions of x and y only. The stresses are also functions of x and y only. Under the assumptions of generalized plane strain, the subsurface stresses due to the applied traction forces can be determined as outlined below.

The stress functions are given by

$$\phi_1'(z)+\phi_2'(z)+\lambda_3\phi_3'(z) = -\frac{1}{2\pi i}\int_{-a}^{+a}\frac{N(\xi)}{\xi - z}d\xi \tag{1}$$

$$\mu_1\phi_1'(z)+\mu_2\phi_2'(z)+\mu_3\lambda_3\phi_3'(z) = -\frac{1}{2\pi i}\int_{-a}^{+a}\frac{T(\xi)}{\xi - z}d\xi \tag{2}$$

$$\lambda_1\phi_1'(z)+\lambda_2\phi_2'(z)+\phi_3'(z) = 0 \tag{3}$$

The $\mu_i$ are the roots of the cylindrical characteristic equation, given by Eq. (4), and $z = x + \mu y$.

$$l_4(\mu).l_2(\mu)-l_3^2(\mu)= 0 \tag{4}$$

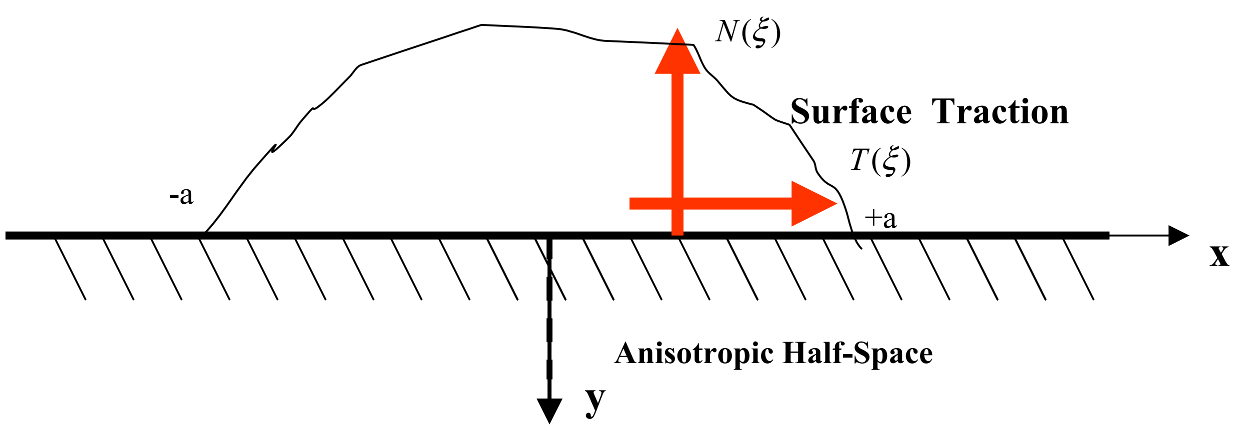


Figure 2: Anisotropic elastic half-space under generalized plane deformation subjected to normal and tangential traction forces.

$$l_2(\mu) = \beta_{55}\mu^2 - 2\beta_{45}\mu + \beta_{55}; \qquad l_3(\mu) = \beta_{15}\mu^3 - (\beta_{14} + \beta_{56})\mu^2 + (\beta_{25} + \beta_{46})\mu - \beta_{24}$$

$$l_4(\mu) = \beta_{11}\mu^4 - 2\beta_{16}\mu^3 + (2\beta_{12} + \beta_{66})\mu^2 - 2\beta_{26}\mu + \beta_{22}; \qquad \beta_{ij} = a_{ij} - \frac{a_{i3}.a_{j3}}{a_{33}} \tag{5}$$

$$\lambda_1 = -\frac{l_3(\mu_1)}{l_2(\mu_1)}, \quad \lambda_2 = -\frac{l_3(\mu_2)}{l_2(\mu_2)}, \quad \lambda_3 = -\frac{l_3(\mu_3)}{l_4(\mu_3)}$$

The matrix $a_{ij}$ relates the strains to the stresses. The $a_{ij}$ are functions of the crystal orientation. The stresses are then given by:

$$\begin{aligned}
&\sigma_x = 2\,\mathrm{Re}\left[\mu_1^2\phi_1'(z) + \mu_2^2\phi_2'(z) + \mu_3^2\lambda_3\phi_3'(z)\right]; \qquad \sigma_y = 2\,\mathrm{Re}\left[\phi_1'(z) + \phi_2'(z) + \lambda_3\phi_3'(z)\right] \\
&\tau_{xy} = -2\,\mathrm{Re}\left[\mu_1\phi_1'(z) + \mu_2\phi_2'(z) + \mu_3\lambda_3\phi_3'(z)\right]; \qquad \tau_{xz} = 2\,\mathrm{Re}\left[\mu_1\lambda_1\phi_1'(z) + \mu_2\lambda_2\phi_2'(z) + \mu_3\phi_3'(z)\right] \\
&\tau_{yz} = -2\,\mathrm{Re}\left[\lambda_1\phi_1'(z) + \lambda_2\phi_2'(z) + \phi_3'(z)\right] \\
&\sigma_z = -\frac{1}{a_{33}}\left[a_{13}\sigma_x + a_{23}\sigma_y + a_{34}\tau_{yz} + a_{35}\tau_{xz} + a_{36}\tau_{xy}\right]; \qquad z_i = x + \mu_i y
\end{aligned} \tag{6}$$

The normal traction force $N(\xi)$ is given by the Hertzian cylindrical contact force as $N(\xi) = p_o\sqrt{1-\xi^2/a^2}$ and $T(\xi) = \mu_f p_o\sqrt{1-\xi^2/a^2}$ , where $p_o$ is the peak pressure and $\mu_f$ the coefficient of friction. The stress solution has been programmed in Mathcad and subsurface stresses computed for various crystal orientations. Figure 3 shows a representative half-space $\sigma_y$ stress distribution for a = 0.01 inch, po = 260 ksi, and for the (x, y, z) axes parallel to the edges of the FCC crystal, i.e. x = <100>, y = <010> and z = <001>. The analytical solution shows good agreement with the finite element numerical solution.

Subsurface fretting fatigue cracks at the damper contacts caused by oscillatory tangential stresses, $T(\xi)$, is of much practical interest. Figure 4 shows the variation in resolved shear stress

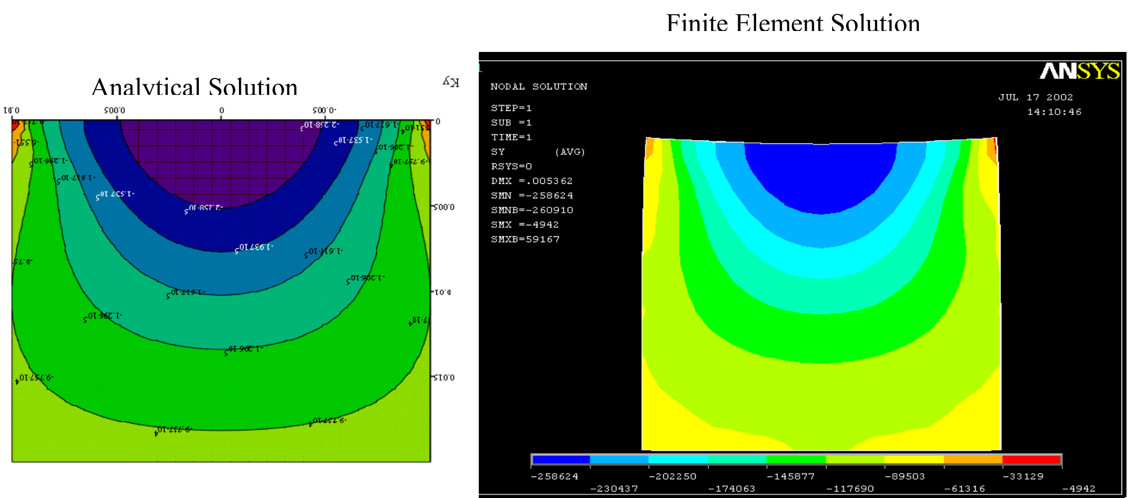


Figure 3: Stress ($\sigma_y$) contours using analytical solution and finite element (Ansys) solution

amplitude on the primary octahedral planes, as a function of secondary crystallographic orientation. It is seen that the highest resolved shear stress amplitude, $\Delta\tau_{rss}$, varies 32 percent as the secondary orientation varies from 0 to 90 deg. The secondary orientation of the blades is not controlled during the casting process and hence its variation can result in blade-to-blade variation in fatigue stresses ($\Delta\tau_{rss}$) at constant load, which is clearly undesirable from a design standpoint.

## Conclusions

The two-dimensional analytical solution presented is an efficient and accurate method for obtaining contact stresses in anisotropic half-spaces. Finite element solution of contact problems is very tedious and requires highly refined meshes to obtain accurate stress solutions. Variation in secondary orientation between blades can result in large variation in fatigue life at damper contact locations.

Three-dimensional contact stress solutions for anisotropic half-spaces has also been obtained for elliptical contacts. Stress solutions have been obtained by linear superposition of a point-load solution outlined in Lekhnitskii [1]. The details of this solution could not be included in this report because of lack of space. Complete details of the 2D and 3D solutions will be documented in a future NASA Technical Publication.

## Acknowledgements

The author would like to thank Gregory R. Swanson of the Marshall Space Flight Center's Engineering Directorate (ED22) for the summer program and also for supporting graduate student Erik Knudsen. Many thanks to Gilda Battista and Greg Duke at Sverdrup for numerous fruitful discussions, and for providing the finite element solutions to the contact problem.

## References

[1] Lekhnitskii, S. G., "Theory of Elasticity of an Anisotropic Elastic Body," Holden Day, Inc. Publishers, San Francisco, 1963.

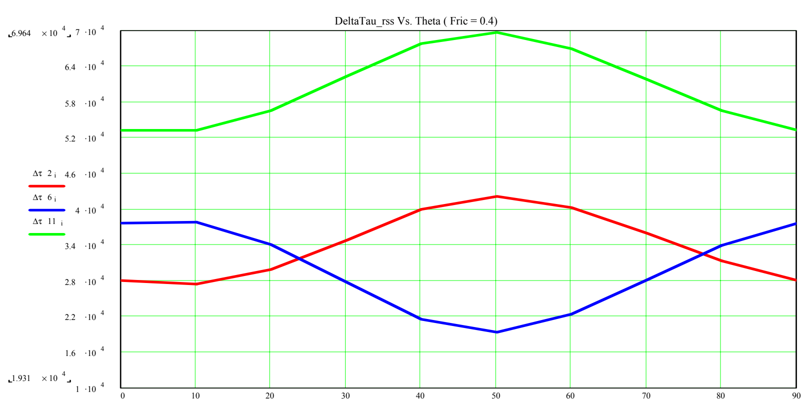


Figure 4: Effect of secondary orientation on resolved shear stress amplitude ($\Delta\tau_{rss}$)