2002

NASA FACULTY FELLOWSHIP PROGRAM

MARSHALL SPACE FLIGHT CENTER
THE UNIVERSITY OF ALABAMA

MATHEMATICAL INVESTIGATION OF GAMMA RAY AND NEUTRON ABSORPTION GRID PATTERNS FOR HOMELAND DEFENSE RELATED FOURIER IMAGING SYSTEMS

Prepared By:	Dona Boccio
Academic Rank:	Professor
Institution and Department:	Queensborough Community College of the City University of New York
	Department of Mathematics and Computer Science
NASA/MSFC Directorate:	Flight Projects
MSFC Colleague:	Jonathan Campbell

Introduction

Terrorist suitcase nuclear devices typically using converted Soviet tactical nuclear warheads contain several kilograms of plutonium. This quantity of plutonium emits a significant number of gamma rays and neutrons as it undergoes radioactive decay.

These gamma rays and neutrons normally penetrate ordinary matter to a significant distance. Unfortunately this penetrating quality of the radiation makes imaging with classical optics impractical. However, this radiation signature emitted by the nuclear source may be sufficient to be imaged from low-flying aerial platforms carrying Fourier imaging systems.

The Fourier imaging system uses a pair of co-aligned absorption grids to measure a selected range of spatial frequencies from an object. These grids typically measure the spatial frequency in only one direction at a time. A grid pair that looks in all directions simultaneously would be an improvement over existing technology.

A number of grid pairs governed by various parameters were investigated to solve this problem. By examining numerous configurations, it became apparent that an appropriate spiral pattern could be made to work.

A set of equations was found to describe a grid pattern that produces straight fringes. Straight fringes represent a Fourier transform of a point source at infinity. An inverse Fourier transform of this fringe pattern would provide an accurate image (location and intensity) of a point source.

Methodology

Several types of spirals were investigated. The most basic type of spiral, the Spiral of Archimedes, has equation $r = a\theta$ for constant a. This is a special case of the Archimedean Spiral, $r = a \theta^{b}$ for constants a and b. Fermat's Spiral, $r = \theta^{1/2}$, is a special case of the Archimedean Spiral, with a = 1 and b = ½. Other types of spirals investigated were logarithmic spirals with $r = ae^{b\theta}$ for constants a and b, and $r = \ln \theta$.

In order to design a grid, a double spiral would need to be utilized. To create a double spiral, the equation and its negative were used. Figure 1a shows two spirals, $r = \theta^{1/2}$ and $r = -\theta^{1/2}$. In Figure 1b, the area between the two spirals is filled in to indicate the material of the grid.
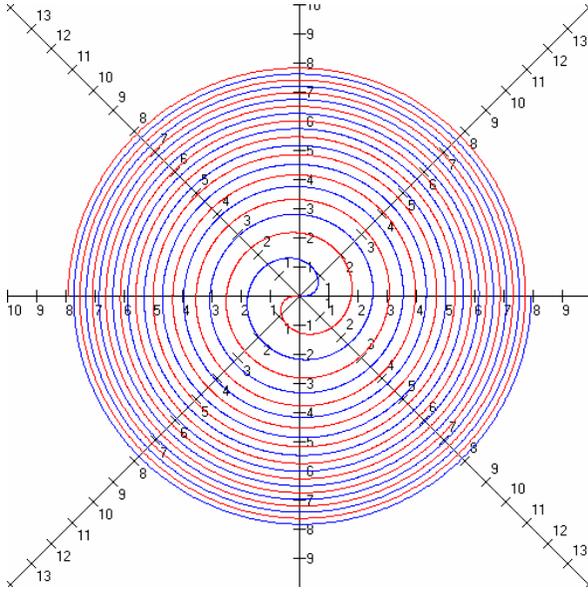
Figure 1a: Double Spiral   Figure 1b: Double Spiral
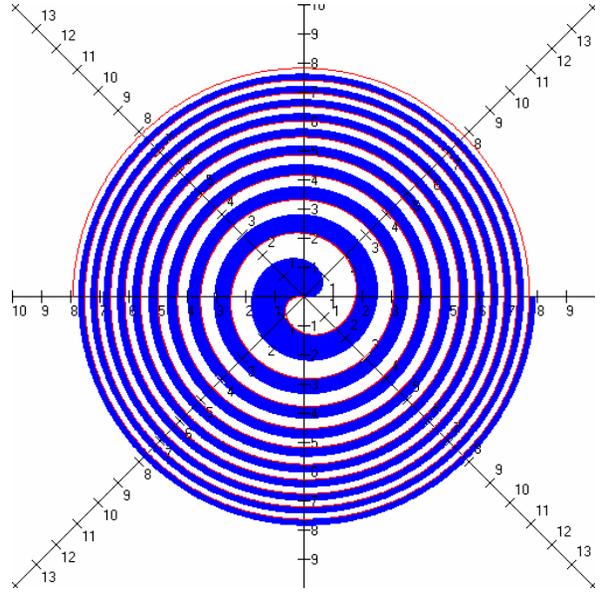
In order to show the pair of absorption grids to be used in the Fourier imaging system, it was necessary to represent a translation of the original curves. The equations were first converted from polar to rectangular coordinates using $x = r \cos \theta$ and $y = r \sin \theta$. $r = \theta^{1/2}$ becomes

$$x = t^{1/2} \cos t, \quad y = t^{1/2} \sin t \tag{1}$$

and $r = -\theta^{1/2}$ becomes

$$x = -t^{1/2} \cos t, \quad y = -t^{1/2} \sin t \tag{2}$$

To produce a horizontal translation, $\pi/2$ was added in the x – direction, giving

$$x = \pi/2 + t^{1/2} \cos t, \quad y = t^{1/2} \sin t \tag{3}$$

$$x = \pi/2 - t^{1/2} \cos t, \quad y = - t^{1/2} \sin t \tag{4}$$

Graphs of the resulting equations are shown in Figure 2.
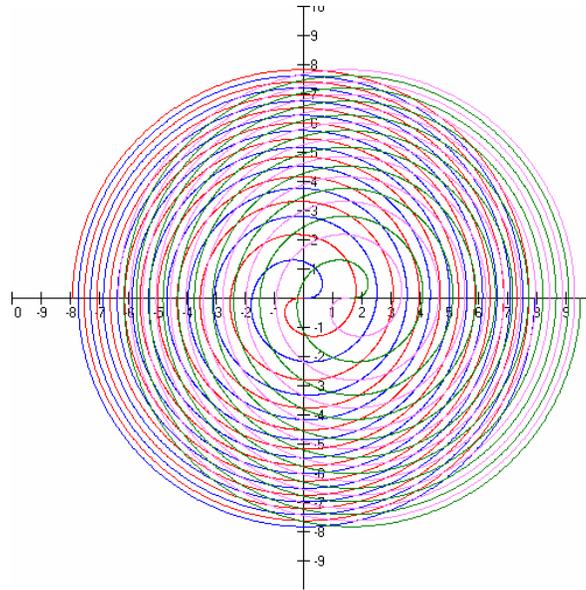
Figure 2: Equations (1), (2), (3), and (4)

In order to visualize the fringe pattern, the area between the spirals was again filled in (see Figure 3).
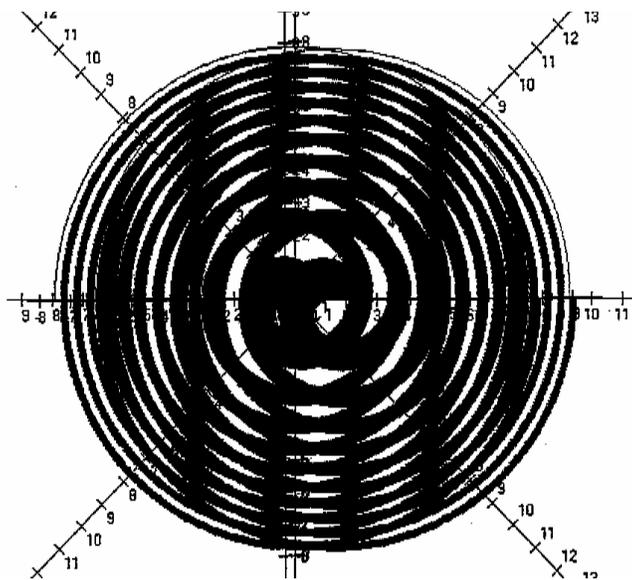
Figure 3: Equations (1), (2), (3), and (4)

Conclusion

A set of equations was found to describe a grid pattern that produces straight fringes. Straight fringes represent a Fourier transform of a point source at infinity. An inverse Fourier transform of this fringe pattern would provide an accurate image (location and intensity) of a point source. For an extended object, the resulting fringe pattern may be inversely Fourier transformed to provide an image. This image may be overlaid on a map to allow a precise location of the nuclear source.

References

[1] Lawrence, J. D. (1972), *A Catalog o f Special Plane Curves*, Dover Publications, pp. 184-190.