

**A Numerical Gamma-Ray Burst Simulation Using
Three-Dimensional Relativistic Hydrodynamics: The Transition
from Spherical to Jet-like Expansion**

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1. Introduction

Gamma-ray bursts are the most powerful explosions in the Universe. (For reviews see Piran 1999 and Mészáros 2002.) A crucial advance in understanding gamma-ray bursts began with the discovery of “afterglows”, starting with *BeppoSax* observations in the soft X-ray band of GRB 970228 (Costa et al. 1997; Wijers, Rees, & Mészáros 1997). (For a review of GRB afterglows see van Paradijs, Kouveliotou, & Wijers 2000.) If GRBs were isotropic, then the measured redshifts would imply total explosion energies of $\sim 10^{52} - 10^{54}$ ergs (Frail et al. 2001). Theoretical work on relativistic jet expansion, however, shows that one expects a steepening in the decay light curve if one is looking down the axis of a jet as the flow decelerates from a bulk Lorentz factor $\gamma^{-1} < \theta$ to $\gamma^{-1} > \theta$, where θ is the jet beaming angle (e.g., Rhoads 1997, Sari, Piran, & Halpern 1999, Panaitescu & Mészáros 1999, Panaitescu & Kumar 2001ab, 2002). One does in fact see such steepenings in the light curves (e.g., Stanek et al. 1999; Harrison et al. 1999). Prior to the time when $\gamma^{-1} \simeq \theta$ the expansion is effectively “spherical” from the observer’s viewpoint because the relativistic beaming is narrower than the jet itself. In other words, if the GRB emission were coming from one spot on a large, relativistically expanding sphere, aimed directly at the observer, the observer would not see any emission from the other parts of the sphere. After the time when $\gamma^{-1} \simeq \theta$ the observer can “see” the entire jet, and a faster rate of decline in the luminosity is predicted. A separate issue that we will address in this work is the sideways or lateral expansion of the jet as the increasing solid angle of the jet enables a larger fraction of the circumstellar medium (CSM) surrounding the progenitor star to be intercepted and provide decelerating gas. Several groups have claimed that this leads to a faster (exponential) decrease in γ , which acts as an additional agent to diminish the amplitude of the relativistic beaming.

The concept of a “break” in the afterglow light curves occurring when $\gamma^{-1} \simeq \theta$ has

the jet, and (iii) a deceleration phase during which the accumulated mass forces a rapid decrease in γ . Previous studies using analytical methods divided the evolution into regimes defined by some ordering of distance, velocity, or energy scales (see Table 3 of Piran 1999 and Figure 7 of Mészáros 2002). The main findings that seem to be common to all studies are that the break in the decaying light curve $d \log L(t)/d \log t$ occurs roughly when the deceleration has decreased γ to roughly the reciprocal of the jet beaming angle, and that the subsequent decrease in γ is roughly exponential with distance. Also, Panaitescu & Mészáros (1999) present axisymmetric calculations to study the combined effects of the transition from $\gamma^{-1} < \theta$ to $\gamma^{-1} > \theta$ and the lateral jet expansion.

In other subdisciplines of astronomy the use of relativistic hydrodynamics codes has been standard for some time. For example, workers studying extragalactic jets have used such codes to continuously inject a collimated supersonic beam into a surrounding medium, usually under the assumption of pressure equilibrium. (see, e.g., Norman et al. 1982 for a thorough discussion). In the context of GRBs, work has been done using 2D and 3D relativistic hydro codes to consider the evolution of the GRB jet as it propagates through the envelope of the progenitor star, up to the point where it breaks out of the stellar surface and produces the prompt GRB emission (Zhang, Woosley, & MacFadyen 2003, Zhang, Woosley, & Heger 2003). In this work we consider the evolution covering the afterglow time (i.e, after the period considered by Zhang et al. 2003ab). We utilize a three dimensional relativistic hydrodynamical code to study the propagation of an initially ultrarelativistic blob into a dense CSM. We study the spatial spreading of the blob both along the direction of propagation and orthogonal to it, as well as the evolution of γ in space and time. We also calculate afterglow light curves, taking a simple prescription in which the local emissivity scales with the local pressure.

DB02 present a suite of test results for 1, 2, and 3 dimensions. We have reproduced these tests, and now show the results of a 3D spherical expansion that results in higher γ values than the tests discussed in DB02. We utilize a cubical grid of $100 \times 100 \times 100$ points, with each side having length unity.² Initial conditions are that everywhere $v^j = 0$, $\rho = 2$, and $p = 3 \times 10^5$ within a radius 0.1 of the center of the cube, and $p = 1$ outside. This is the 3D analog of the 1D “piston” problem. (An identical test with less extreme initial conditions is shown and discussed by Hughes, Miller, & Duncan 2002, see their Fig. 3.) The enormous over-pressure launches a relativistic blast wave at $t = 0$. Figure 1 shows the evolution of ρ , v_x , p , and γ for a slice taken along the x -axis. The density inside the sphere becomes small for late times. To avoid numerical instabilities in this region we utilize the smoother LLF fluxes for small densities ($\rho < 0.0225$), and the HLL fluxes elsewhere. We show the initial configuration plus 10 time slices from a run with 360 total time steps, taking a Courant number of 0.05. After the run begins one can see the development of a strong spherical outflow from the center of the cube. The Lorentz γ factor has increased to $\sim 20 - 30$ in the outer parts of the expanding shell by the end of the run. The run was halted before the expansion reached the edge of the grid. For the last three time steps, a kink develops in the v_x profile at the center of the sphere because of the prolonged spherical outflow from localized point. Figure 2 shows the corresponding evolution for pure LLF fluxes. All variables now show a smoother evolution and $\gamma \rightarrow 20$ at large radii for late times – somewhat less than for the HLL fluxes. Conservation of rest-mass energy $M \equiv \sum_i(\rho_i \gamma_i)$ and energy $E \equiv \sum_i(e_i \gamma_i^2 + p_i(\gamma_i^2 - 1))$ (where the specific energy $e_i = \rho_i + p_i/(\Gamma - 1)$)

²DB02 quote a CPU run time of ~ 2 minutes per Runge-Kutta integration for a 3D problem with 100^3 nodes, utilizing the third order reconstruction for the L and R values of the primitives, and using a 1 GHz Linux PC; our run time of ~ 70 s is consistent with the 1.7 GHz Linux PC used in this work.

relaxes to one in which $\rho_{\text{blob}}/\rho_{\text{CSM}} \simeq 10$, therefore the effective density contrast for the early run is ~ 200 . This is less than expected astrophysically, but required in our computations in order to see significant deceleration of the blob by the time it reaches the end of the grid $x = 5$. In other words, we must telescope the evolution into the finite dimensions of our grid. From conservation of momentum, the condition for significant deceleration is that the total swept-up mass-energy roughly equals that in the initial blob.

Experimentation using HLL fluxes shows that strong internal shocks almost immediately create large-amplitude sawtooth ρ variations within the blob, leading to noisy results. Therefore we utilize the smoother LLF fluxes in this work. We follow the evolution of the blob in terms of both its motion in x and its spreading in y and z . Taking a Courant number of 0.25 necessitates ~ 2000 time steps for the blob to reach the end of the grid, traveling at $v_x \simeq 1$. This is a simple consequence of the box length and grid spacing $\Delta x = 0.01$, from which it follows that the time to traverse the box is $2000 \times 0.25 \times 0.01 = 5$ units traveling at $v \simeq 1$. A density contrast of $\sim 10^2$ is sufficient to see the desired deceleration from $\gamma^{-1} < \theta$ to $\gamma^{-1} > \theta$ during a run.

In order to avoid undue complexity in these experiments, that currently are purely hydrodynamic and do not yet contain proper prescriptions for emission from bremsstrahlung and synchrotron processes, we calculate a simple measure of the emissivity by taking the local emission to scale as p , which would be expected roughly for optically thin synchrotron emission characteristic of frequencies significantly above the self-absorption frequency. We avoid the issues of synchrotron self-absorption (Granot, Piran, & Sari 1999b, 2000; Panaitescu & Kumar 2000; Sari & Esin 2001), and of whether the evolution is adiabatic or radiative (Panaitescu & Mészáros 1998b; Sari, Piran, & Narayan 1998) – our calculations are adiabatic. We calculate effective “light curves” for observers at various viewing angles between 0° and 15° from the center of the jet. The amplification of the photon energy flux is

behind this shocked region. Although the initial pressure is small, after the bow shock is fully developed we have $p_{\text{shock}}/\rho_{\text{shock}} \simeq 0.3$ throughout the subsequent evolution. The evolution proceeds in a roughly self-similar manner and the forward/reverse shock system is stable, in accord with analytic estimates (Wang, Loeb, & Waxman 2002).

Figures 4 and 5 show a time series of contour plots in γ and ρ that follow the blob propagation and radial spreading. The full 500 grid points along the x direction are shown, and each frame represents 240 time steps (i.e., $\Delta t = 0.6$). The leading contours show the density enhancement associated with the shock. The ρ contours indicate the values 0.2, 0.3, 0.4, and 0.5, while the (trailing) γ contours, indicate values of 1.25, 2.5, 5, 10, 15, and 20. The contours are formed by taking a cut at $z = 0$ through the $x - y$ plane. The initial velocity vector of the blob points toward the top of the plot, with $\gamma = 25$. One sees a strong bow shock associated with the maximum in pressure at the point where the blob encounters the CSM. The lack of a bow shock in the γ contours suggests that little or no material in advance of the blob is accelerated to a significant bulk Lorentz factor. As the evolution progresses the deceleration causes the higher γ contours to disappear, and those that remain become increasingly distorted. The bilateral symmetry of the contours evident in Figures 4 and 5 attests to the power of the third order differencing scheme given in DB02, insofar as the 3D model has no enforced symmetry.

Figure 6 shows the evolution of the total rest-mass energy M (dotted) and total energy E (dashed) within the grid, over the course of the conical run. The values of M and E have been normalized to their initial values. The curves that extend significantly below an abscissa value of unity indicate the values summed over the grid. At the six faces of the computational box we continually reset the values of all variables to their initial values, and keep track of the differences between those values and the initial ones. As the run progresses and more high velocity gas reaches the edge of the computational domain and is extracted,

different γ cuts follow the values $\gamma_{\text{cut}} = 1.001, 1.01, 1.1, 1.5, 2, 4, 6,$ and 8 . The higher γ cuts $\gamma_{\text{cut}} \gtrsim 2$ reveal that a negligible fraction of CSM matter gets accelerated to significant values. This is because the curved bow shock shunts material laterally in front of the advancing blob, rather than accelerating it up to a significant fraction of the blob's bulk Lorentz factor $\sim 10 - 20$. The rest-mass energy curves are relatively constant in time up to $t \simeq 2$. At roughly this point the total accumulated CSM gas becomes comparable to that in the initial blob. If the blob did not spread laterally, this time would occur when the blob had plowed through a density $\sim \gamma_{\text{initial}}(\rho_{\text{blob}}/\rho_{\text{CSM}})$, where $\gamma_{\text{initial}} = 20$ and $\rho_{\text{blob}}/\rho_{\text{CSM}} \simeq 10$. This means that the blob would have shed of order its initial momentum by the time it had traveled ~ 200 times its initial length $\sim \delta x = 0.1$, or ~ 20 units. In practice, the initially imposed tangential velocity v_T leads to the lateral expansion that effectively increases the cross section for interaction of the blob. The negligible decrease in $\langle \gamma v_x \rangle$ for the $v_T(0) = 0$ run demonstrates the importance of this effect.

Figure 8 shows the lateral expansion of the blob for the conical run. The blob edge is computed in each time step by first finding the local maximum in either ρ , γ , or $\rho\gamma$ along the x -axis, and then stepping laterally to the position at which the background (CSM) value of the relevant quantity has been increased by 10% due to expansion of the blob. The transient lateral expansion $v_{\text{edge}} \simeq 0.3 - 0.4c$ lasting until $x \simeq 1$ is unphysical insofar as it occurs during the period of adjustment to the initial density and velocity profiles. The value of the later spreading rate $v_{\text{edge}} \simeq 0.1c$ is basically dictated by the initial v_T value given to the gas.

Figure 9 shows the evolution of $\langle \gamma v_x \rangle$ and θ as the blob propagates. The solid curve in the top panel indicates the same weighted value of $\langle \gamma v_x \rangle$ as shown in Fig. 7. After the transient physical conditions associated with the initial state have vanished, one sees a period of deceleration associated with the increasing drag force. The dashed curves indicate

4. Discussion

Utilizing 3D relativistic hydrodynamical calculations, we have examined the evolution of an expanding relativistic blob of gas intended to be representative of a jet associated with ejecta from an extremely energetic event such as a hypernova, that produces a gamma-ray burst (Aloy et al. 2000; Tan, Matzner, & McKee 2001; MacFadyen, Woosley, & Heger 2001, Zhang, Woosley, & Heger 2003, Zhang, Woosley, & MacFadyen 2003). Since these are the first such calculations applied to the blob during the time in which the afterglow radiation is produced, we have purposely kept them simple in an effort to concentrate on the most fundamental aspects of the physics. We restrict our attention to the transition from spherical to jetlike expansion that occurs during the time that the Lorentz factor becomes less than the reciprocal of the jet spreading angle.

We have not yet attached specific numbers to our results. From the SRHD equations, one sees that the relevant quantities are the ratios of pressure to density, and of distance to time. If we specify either one of these two sets of numbers, the other one is also determined. The column giving the observed afterglow break time t_j in Table 1 of Frail et al. (2001) indicates $t_j \simeq 2$ d as being representative. For an observer directly on the velocity vector of the blob, the time T between the GRB and afterglow

$$T = \int \delta t = \int_{t_{\text{GRB}}}^{t_{\text{afterglow}}} \left(\frac{\delta x}{v(x)} - \frac{\delta x}{c} \right) = \frac{1}{c} \int \delta x \left[\frac{1}{\sqrt{1 - \gamma^{-2}}} - 1 \right] \simeq \frac{1}{2c} \int \delta x \gamma(x)^{-2},$$

where the dominant contribution to the integral comes from later times. Thus the light travel time of 1 day is multiplied by $\sim 2\gamma_{\text{afterglow}}^2 \simeq 2 \times 10^2$, assuming the spherical-to-jetlike transition giving the break in the afterglow light curve happens at $\gamma \simeq 10$. For the conical run, the break in the light curve occurs at $x \simeq 2.5$. If we designate this point as corresponding to a time 2 d, then $x = 2.5$ translates to $ct_j \simeq 5.2 \times 10^{15}(200)$ cm $\simeq 10^{18}$

precursor and continues to propagate, the medium through which it travels should be dominated by the density profile left from the precursor's stellar wind (Chevalier & Li 1999, 2000, Li & Chevalier 2001). If the density varies as r^{-2} away from the star, the mass loss of the wind $\dot{M}_{\text{wind}} \simeq 10^{-5} M_{\odot} \text{ yr}^{-1}$, and the wind velocity $v_{\text{wind}} \simeq 10^3 \text{ km s}^{-1}$, the circumstellar density at the point where the afterglow radiation is emitted $\sim 10^{18} \text{ cm}$ will be $n_{\text{CSM}} \sim \dot{M}_{\text{wind}} / (4\pi r^2 v_{\text{wind}} m_p) \simeq 0.3 \text{ cm}^{-3}$. In the calculation of Zhang et al (2003), the density inside the jet is $\sim 10^{17} \text{ cm}^{-3}$ at $\sim 10^{12} \text{ cm}$, and $p \simeq \rho$. If the spreading angle of the jet remains roughly constant from $\sim 10^{12} \text{ cm}$ to $\sim 10^{18} \text{ cm}$ (Lithwick & Sari 2001), then the density inside the jet at $\sim 10^{18} \text{ cm}$ should be lower by $\sim (r_2/r_1)^2 \simeq 10^{12}$, or about $\sim 10^5 \text{ cm}^{-3}$. At this point p inside the jet should be negligible compared to ρ . The density contrast between the blob and CSM is greater than what we have assumed in the results shown previously. A separate run taking an initial density contrast of 10^6 shows the same basic effects as the previous runs, however, namely a lateral expansion rate of $\sim 0.1c$ and the non-accretion of CSM gas.

Our computations lie within the "deceleration phase" discussed by Kobayashi et al. (1999, see also Kobayashi & Sari 2000, 2001). We find a change in the form of the luminosity decrease corresponding to the transition between spherical and jetlike expansion. The determination of the average spreading angle $\langle \theta \rangle$ is nontrivial because it depends on how one does the averaging, and how much of the diffuse, sideways-expanding jet material is included in the computation. In Figure 9 we presented cuts for gas possessing $\gamma > \gamma_{\text{cut}} = 4, 5, \text{ and } 6$ as representative of material in the flow that partakes most strongly in producing the observed radiation. We do not see a dramatic increase in $\langle \theta \rangle$ during the deceleration phase; the $\langle \theta \rangle$ value basically reflects the ballistic motion of material following its initial v_T value. Also, because the deceleration in our problem is forced not by the accumulation of gas from the CSM but rather the drag force of the CSM on the blob, we do not see an exponential decrease in γ with distance during the deceleration phase, but rather a decrease

During the latter stage, the swept-up mass increases exponentially in time. Panaitescu & Mészáros (1999) calculate light curves for observers at varying angles from the jet axis, and calculate separately the effects of including and excluding the lateral jet expansion. They find that the maxima in the light curves occur substantially later in runs which do not take into account the jet broadening (see their Fig. 4). In the analytical models of both Rhoads (1999) and Panaitescu & Mészáros (1999), the physics of mass accumulation from the CSM is an integral component of the formalism; all mass within the solid angle of the expanding shell is assumed to accrete. Workers have applied the results of Rhoads (1999) and Sari, Piran, & Halpern (1999) to the afterglow evolution, however the results of Zhang et al. (2003) cast doubt on the validity of this exercise, because at the time corresponding to the afterglow emission one anticipates that $p \ll \rho$ and the lateral spreading rate would not be governed by the internal sound speed but rather the ballistic motions of the ejecta comprising the blob as they leave the vicinity of the progenitor star. The blob has a large internal thermal energy $p/\rho \simeq 10$ as it emerges from the progenitor star, and even by the time the expanding ejecta have become optically thin to their own emission from internal shocks (producing the GRB), one still expects $p/\rho \simeq 1$. By the (much later) time of the afterglow emission, however, the blob would have cooled to the point that $p/\rho \ll 1$, which provides the impetus for our initial condition $p_{\text{blob}}/\rho_{\text{blob}} = 10^{-4}$. Following the arguments of Rhoads (1999) and Sari, Piran, & Halpern (1999), if the lateral spreading rate were mandated by the internal sound speed, then in our calculations it should be $\sim \sqrt{p/\rho} = 0.01c$, whereas we find it to be ~ 10 times larger. The spreading rate basically reflects our initial v_T value. Thus the physics of the lateral expansion is different than in Rhoads (1999) and Sari, Piran, & Halpern (1999). How might this result be influenced by systematic effects present in our model? One obvious potential shortcoming is the absence of cooling. In this work we have assumed an adiabatic gas, whereas in reality one might envision the presence of cooling within the shock. This might then reduce the ability of

where U_0 is the initial value of U and $a = (K/3)\rho_{\text{CSM}}\sigma/M_{\text{blob}}$. From the fitting to $\langle U \rangle$ presented in Figure 7, we infer that $K \simeq 2.4$. The specific numerical value for K is probably influenced by our numerical resolution. A comparison of the curves labelled “C” and “ $f(x)$ ” in Fig. 7 shows that the functional decrease in $\langle U \rangle$ with x is reasonably described by a cubic, as expected if the cross sectional area increases quadratically with x , or equivalently t . Note that we assumed $v_x = 1$ in this exercise, which is a good approximation for the evolution of interest.

5. Conclusion

The calculations we present are the first 3D relativistic hydrodynamical calculations of GRB jet evolution pertinent to the afterglow phase that do not enforce any special symmetry (e.g., spherical or axial). We find that (i) the CSM gas does not accrete onto the advancing blob, but rather is shunted aside by the bow shock, (ii) the decay light curve steepens roughly when one first “sees” the edge of the jet $\gamma^{-1} \approx \theta$, with this effect being strongest for “face-on” observers (confirming previous studies), and (iii) the rate of decrease of the x -component of momentum $\langle \gamma v_x \rangle$ is well-characterized by a simple model in which the cross sectional area of the blob increases quadratically with laboratory time (or distance). The primary impetus for the built-in assumption of accretion of matter in previous studies was the influential work of BM76 in which spherical relativistic expansion was considered. Accretion of gas onto the relativistically expanding shell is obviously justified for spherical expansion, but subsequent GRB workers applied the results to the case of the GRB jet, in which a thin wedge of material propagates through a low density medium. In such a situation the natural tendency of material in front of the jet is to be pushed aside and to form a “channel flow” around the jet, rather than to accrete. A separate issue is that workers used the results of Rhoads (1999) and Sari et al. (1999) that

ϕ , averaged along the axis of the jet, seems questionable. In addition, one would still be missing important physical effects, such as the non-accretion of the forward CSM material.

Two obvious refinements, currently being carried out, are to (i) treat the problem on a Lagrangian grid in which the mesh points follow the blob and are adaptively inserted in regions with strong gradients, so as to be able to explore regimes in which the density contrast between the blob and CSM is much larger, and (ii) include provisions for realistic bremsstrahlung and synchrotron physics, in order to produce light curves that can be compared directly with observations so as to test different aspects of the theory and thereby constrain the allowed parameter space.

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- Kobayashi, S., & Sari, R. 2001, *ApJ*, 551, 934
- Kumar, P., & Granot, J. 2003, *ApJ*, 591, 1075
- Kumar, P., & Panaitescu, A. 2000a, *ApJ*, 541, L9
- Kumar, P., & Panaitescu, A. 2000b, *ApJ*, 541, L51
- Lax, P. D., & Liu, X.-D. 1998, *SIAM J. Sci. Comput.*, 19, 319
- Levinson, A., Ofek, E. O., Waxman, E., & Gal-Yam, A. 2002, *ApJ*, 576, 923
- Li, Z.-Y., & Chevalier, R. A., 2001, *ApJ*, 551, 940
- Lithwick, Y., & Sari, R. 2001, *ApJ*, 555, 540
- Londrillo, P., & Del Zanna, L. 2000, *ApJ*, 530, 508
- MacFadyen, A. I., Woosley, S. E., & Heger, A. 2001, *ApJ*, 550, 410
- Mészáros, P., & Rees, M. J. 1997, *ApJ*, 476, 232
- Mészáros, P. 2002, *ARAA*, 40, 137
- Norman, M. L., Winkler, K.-H. A., Smarr, L., & Smith, M. D. 1982, *A&A*, 113, 285
- Panaitescu, A., & Kumar, P. 2000, 543, 66
- Panaitescu, A., & Kumar, P. 2001a, 554, 667
- Panaitescu, A., & Kumar, P. 2001b, 560, L49
- Panaitescu, A., & Kumar, P. 2002, 571, 779
- Panaitescu, A., & Mészáros, P. 1998a, *ApJ*, 492, 683
- Panaitescu, A., & Mészáros, P. 1998b, *ApJ*, 501, 772
- Panaitescu, A., & Mészáros, P. 1999, *ApJ*, 526, 707
- Panaitescu, A., Mészáros, P., & Rees, M. J. 1998, *ApJ*, 503, 314
- Piran, T. 1999, *Physics Reports*, 314, 575

FIGURE CAPTIONS

Figure 1. The evolution of a spherical relativistic expansion due to a large over-pressure inside a sphere of radius 0.1, adopting HLL fluxes. The evolution encompasses 360 time steps, taking a Courant number of 0.05. Shown are the initial conditions plus ten equally spaced time steps taken from a slice along the x -axis depicting the evolution of (i) pressure p (*top panel*), (ii) density ρ (*second panel*), (iii) v_x/c (*third panel*), and (iv) Lorentz factor γ (*bottom panel*). The small numbers beside each curve indicate the evolution. One sees an expansion of the central over-pressurized sphere into the surrounding medium. The strong relativistic outflow peaks at $\gamma \simeq 20 - 30$ near the end of the evolution. The general trend in which $\gamma \propto r$ within the expansion can be derived from fundamental principles and is well known from classical solutions. At late times the strong decrease in ρ at the center of the sphere, which is a singularity in this test, begins to cause numerical instability. We utilize the LLF fluxes for $\rho < 0.0225$ and the HLL fluxes for $\rho \geq 0.0225$.

Figure 2. The evolution of a spherical relativistic expansion due to a large over-pressure inside a sphere of radius 0.1, adopting pure LLF fluxes. Quantities shown are the same as in Fig. 1. The evolution is nearly identical to that in Fig. 1, except the variations in the physical variables are smoother, and $\gamma \simeq 20$ at larger radii by the end of the run – somewhat smaller than in Fig. 1.

Figure 3. The evolution of a relativistic blob launched as a small cone traveling along the $+x$ -axis, initially confined to between $x = 0.1375$ and $x = 0.2$ and a maximum radius 0.025. The small numbers beside each curve indicate the time step. The panels are the same as shown in Fig. 1. Snapshots represent conditions along a slice through the center of the rectangular grid, taken every 120 time steps ($\Delta t = 0.2$). For this trial and all that follow we utilize a Courant number of 0.25 and pure LFF fluxes. The initial bulb Lorentz factor γ was set to 25, and the initial full width flaring angle to 0.07 radian as measured by

to the conical run given by $21 - 0.15(x + 0.45)^3$. Although the rest mass energy curves for $\gamma_{\text{cut}} = 1.001$ and 1.01 show a sharp increase (up until $t \sim 1.7$ when material starts to leave the edges of the grid), the near constancy of the $\gamma_{\text{cut}} = 2$ curve and the slight decline for higher γ_{cut} curves indicate that there is negligible acceleration of high- γ material. Also, the functional form of the decrease in $\langle \gamma v_x \rangle$ is similar for all runs in which $\theta = 4^\circ$ initially.

Figure 8. The location of the edge of the jet, determined by first finding the location along the direction of propagation of the maximum in either γ (*top curve*), $\rho\gamma$ (*middle curve*), or ρ (*bottom curve*), then stepping laterally to the point at which the background value has increased by 10% of its original value. (The precise value of this constant does not affect the results.) The initial maximum radius of the blob is 0.05. After $x \sim 1.8$ the two lower curves coincide.

Figure 9. The evolution of the weighted mean of the x - component of momentum and a measure of the spreading angle $\langle \theta \rangle$, taken to be the ratio of tangential to axial speeds, in the observer's frame. Shown are (i) $\langle \gamma v_x \rangle$ (solid line) and the reciprocal of $\langle \theta \rangle$, measured in radians (dashed lines), where the lower limiting γ value used in the averaging for $\langle \theta \rangle$ is taken to be either 4, 5, or 6 (*top panel*). (ii) the $\langle \theta \rangle$ values whose reciprocal values are indicated in the first panel (*middle panel*), and (iii) the number of cells entering into the averages for the three γ_{cut} values in the first two panels (*bottom panel*). At late times the deceleration makes the $\langle \theta \rangle$ calculations problematic because the number of high γ cells drops. This progression in the loss of high γ cells can be seen at $t \gtrsim 4$ in the bottom panel.

Figure 10. Light curves constructed by summing the quantity $p_i(1 + \beta_i \cos \phi)/(1 - \beta_i \cos \phi)^2$ over the grid. We take this global sum as being a measure of the luminosity seen by an observer looking down the jet. The light curves are built up during the course of the run by summing the emission from 100 slices moving toward the observer at c that straddle









