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FILM CONDENSATION WITH AND WITHOUT BODY FORCE

IN BOUNDARY-LAYER FLOW OF VAPOR OVER

A FLAT PLATE

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SUMMARY


    Laminar film condensation under the simultaneous influence of gas-
liquid interface shear and body force (g force) is analyzed over a flat
plate. Important parameters governing condensation and heat transfer of
pure vapor are determined. Mixtures of condensable vapor and nonconden-
sable gas are also analyzed. The conditions under which the body force
has a significant influence on condensation are determined.

INTRODUCTION


    Condensation is one of the basic processes which are important in
the operation of a power plant. Most of the condensation processes which
take place on the earth depend on the strength of the gravitational field.
If condensation is to be carried out effectively in a space vehicle, away
from a strong influence of gravity, another force should be provided
which would continuously remove the condensate from the condensing
surfaces. One of the natural existing forces available for the condensate
removal in forced flows is the shear force at the gas-liquid interface.

    Nusselt, in his classical analysis of film condensation on a vertical
plate under the influence of the gravitational force (ref. 1), neglected
the shear stress which exists at the gas-liquid interface. However, the
fact that this interface shear stress can have an appreciable effect on
the condensation process has been shown in reference 2. When condensation
is taking place under the condition of a forced flow and a reduced body
force, the interface shear stress may become the controlling force of the
condensation process and a better understanding of gas-liquid interaction
phenomena becomes important.

    The purpose of the present paper is to study: (1) the interaction
phenomena between the gas-phase boundary layer and the layer of the con-
densate which control the condensing rate and the heat transfer, and
(2) the added effect of the body force (g force) on the condensing rate
and the heat transfer. For the stated purpose, we shall choose a simple

shape, the flat plate, and shall analyze the film condensation under the influence of the interface shear stress and the body force.

This report is divided into three parts. Part I contains a description of the problem and the method of analysis. Part II supplies the mathematical details. Part III contains a discussion and summary of results. It is suggested that casual readers may omit part II.

## LIST OF SYMBOLS

$c_p$     specific heat at constant pressure

$D$      coefficient of binary diffusion

$F(0)$   Blasius stream function for gas phase at the interface, defined by equation (45)

$F''(0)$  interface shear stress defined by equation (48)

$f$      dimensionless stream function for liquid film, defined by equation (26)

$Fr$     Froude number based on $L$, $\dfrac{u_{v,\infty}^2}{gL}$

$Fr_x$   Froude number based on $x$, $\dfrac{u_{v,\infty}^2}{gx}$

$g$      constant body force per unit mass

$h^o$    heat of condensation

$k$      thermal conductivity

$L$      reference length

$m$      mass fraction of condensable vapor in the gas-vapor mixture

$Nu_x$   Nusselt number based on $x$, $\dfrac{q_w x}{(T_o - T_w)k}$

$Pr$     Prandtl number, $\dfrac{\mu c_p}{k}$

$q_w$    heat-transfer rate to plate per unit area

$Re$     Reynolds number based on $L$, $\dfrac{u_{v,\infty}L}{\nu}$

$Re_x$   Reynolds number based on $x$, $\dfrac{u_{v,\infty}x}{\nu}$

Sc      Schmidt number, $\dfrac{\nu}{D}$

T      absolute temperature

u      x   component of velocity

U      $\dfrac{u}{u_{V,\infty}}$

v      y   component of velocity

V      $\dfrac{v}{u_{V,\infty}}$

x      distance along the interface

X      $\dfrac{x}{L}$

y      distance normal to interface (see fig. 1)

Y      $\dfrac{y}{L}$

$\Gamma$      $\dfrac{1}{Pr} \dfrac{c_p(T_o - T_w)}{h^o}$

$\Delta$      modified thickness of liquid film, defined by equation (23)

$\delta$      liquid-film thickness

$\eta$      similarity variable for liquid film, defined by equation (22)

$\eta_B$      Blasius similarity variable for gas boundary layer, defined by equation (43)

$\theta$      $\dfrac{T - T_w}{T_o - T_w}$

$\mu$      dynamic viscosity

$\nu$      kinematic viscosity

$\xi$      streamwise variable defined by equation (21)

$\rho$      density

$\tau_o$      interface shear stress

$\Psi$      stream function for liquid film, defined by equations (24) and (25)

## Subscripts

v       condition of gas phase

w       condition at the plate

o       condition at gas-liquid interface

o,1,2   order of perturbation defined by equations (37) through (39)

∞       free gas stream

10      value of the parameter which would vary $\dfrac{v_o}{u_{v,\infty}}\sqrt{Re_x}$ or $\dfrac{Nu_x}{\sqrt{Re_x}}$ by 10

        percent from those for $g = 0$

## Superscript

'      total differentiation with respect to the variable concerned

Symbols without subscripts refer to liquid unless otherwise specified.

## I. GENERAL PROBLEM AND METHOD OF SOLUTION

### Description of Problem

We consider that a mixture of a condensable vapor and a noncondensable gas is flowing over a flat plate whose uniform temperature is below the condensation temperature of the vapor. We further consider that the condensate wets the surface and that the entire system is subjected to an accelerative body force in the direction of flow and parallel to the plate. When a steady state is achieved, the following phenomena will be observed on the plate. A layer of condensed liquid will be flowing along the surface of the flat plate, dragged by the gas-phase boundary layer over it and also influenced by the body force (see fig. 1). Condensation will continuously take place at the gas-liquid interface, and the heat of condensation will be transferred to the plate across the liquid layer. The present problem is then to predict theoretically the condensing rate, the heat transfer, and the liquid film thickness for the steady state.

In order to make the analysis tractable, it is assumed that the flow of the gas and the liquid is stable and laminar, and that the condensate has constant properties.

Method of Analysis


The thickness of the liquid layer is generally very small and, in the present analysis, it is considered to be in the order of a boundary-layer thickness. There are two layers of fluids which must be studied simultaneously. They are the liquid layer (liquid film) and the gas-phase boundary layer over the liquid. Solutions are first obtained for the velocity and the temperature distributions through the liquid film in terms of the yet unknown temperature[1] and the shear stress at the gas-liquid interface. The interface temperature, shear stress, and mass transfer are then respectively matched between the solution of the liquid film and the existing solution of the gas-phase boundary layer to obtain the final condensing rate and heat transfer.

Liquid layer.- Let us first consider the liquid film. The governing equations for the liquid film are the usual laminar boundary-layer equations with a body force term. The equations to be solved are essentially the same as the equations used in reference 3 to study the effect of vehicle deceleration on melting surfaces. The equations are also similar to the equations used in reference 4 to analyze the effect of the buoyancy force in a boundary-layer flow. The basic differences between the analyses of the references and the present analysis lie, as far as the solution of the liquid film is concerned, in the boundary conditions and, particularly, in the positions at which they must be applied. In the analyses of the references, the boundary conditions are applied at the two predefined boundaries which are at finite boundary and at infinity. In the present analysis, the boundary conditions must be applied at the gas-liquid interface and at a position which is a liquid-film thickness away from the interface. The thickness of the film is not a predefined quantity and it may be obtained only as a result of the solutions. The thickness, moreover, varies along the plate.

In the present study, the solutions of the governing equations are obtained by expanding the dependent variables and the modified film thickness in terms of the reciprocal of Froude number, $1/\mathrm{Fr}_x$. The solutions thus obtained with the help of a digital computer show the effect of the body force as a perturbation to the solution with no body force when a finite number of the terms of the series is retained. The solutions presented here are, therefore, applicable only when $1/\mathrm{Fr}_x \ll 1$. A solution could be similarly derived which would show the effect of the interface shear stress as a perturbation to the solution with no interface drag. The present study, however, is mainly concerned with the relatively small body forces which exist in space flight and, therefore, the former solution only was considered.

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[1]The interface temperature is unknown only when the gas phase is a mixture of a condensable vapor and a noncondensable gas.

The major parameters $Nu_x/\sqrt{Re_x}$, $(v_o/u_{v,\infty})\sqrt{Re_x}$, and $(\delta/x)\sqrt{Re_x}$ are derived from the solutions of the governing equations as functions of the parameters $\sqrt{\rho_v\mu_v/\rho\mu}\ F''(0)$, $(1/Pr)[c_p(T_o-T_w)/h^o]$, Froude number, and Prandtl number. Then for the given values of Prandtl and Froude numbers and the parameter, $(1/Pr)[c_p(T_o-T_w)/h^o]$, the proper values of $Nu_x/\sqrt{Re_x}$, $(v_o/u_{v,\infty})\sqrt{Re_x}$, and $(\delta/x)\sqrt{Re_x}$ for a particular problem are obtainable as soon as $F''(0)$ is found. The interface shear stress represented by $F''(0)$ depends on the momentum, mass, and energy interactions between the gas-phase boundary layer and the liquid film. The gas-phase boundary layer and the interaction phenomena will be discussed in the following section and the method of determining $F''(0)$ will be explained.

Gas-phase boundary layer and interface interaction.- Consider now the gas-phase boundary layer over the liquid film. The velocity at the interface is much smaller than the free-stream gas velocity, but not zero. In the analysis of the gas-phase boundary layer, however, the velocity at the interface is assumed to be zero in order that the results of the existing solutions of the boundary-layer equations may be utilized. There is a net mass transfer at the interface toward the liquid film because of the condensation process. The flow of gas near the interface, therefore, is essentially a boundary-layer flow with suction. It is known that suction in a boundary-layer flow increases the drag drastically. The increased interface shear stress due to the condensation process itself will be shown to increase the condensing rate. The increased condensing rate, in turn, means again increased interface shear stress. It is seen then that the gas-phase boundary layer and the liquid film complement each other in increasing the interface shear stress represented by $F''(0)$. Eventually, an equilibrium value of $F''(0)$ will be reached for which both the gas-phase and liquid-phase requirements are satisfied. It is obvious, however, that the equilibrium value of the interface stress thus reached could be considerably greater than that with no mass transfer. The equilibrium value of $F''(0)$ is obtained in the following manner. The ratio of the suction to the stress functions, $F(0)/F''(0)$, is first obtained as a function of $F''(0)$ for the gas-phase boundary layer from the existing solutions of the boundary-layer equations such as are found in references 5 and 6. Next, from the solutions of the governing equations, the same ratio, $F(0)/F''(0)$, for the liquid film is derived also as a function of $F''(0)$. The two functions $F(0)/F''(0)$ for the gas phase and liquid phase are then compared and the matching values of $F(0)$ and $F''(0)$ are obtained.

## II. DETAILS OF ANALYSIS

The outline of the method of analysis has been given in part I of the report. In the present part, the mathematical details will be given.

## Governing Equations

Under the assumptions of the thin, stable, and laminar liquid film with constant properties and low speeds, the following equations describe the behavior of the liquid film. The equations are essentially the normalized boundary-layer equations, derivations of which are found in reference 7.

Continuity

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

Momentum

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{1}{Re} \frac{\partial^2 U}{\partial Y^2} + \frac{1}{Fr} \tag{2}$$

Energy

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{RePr} \frac{\partial^2 \theta}{\partial Y^2} \tag{3}$$

where

$$U = \frac{u}{u_{V,\infty}}, \qquad V = \frac{v}{u_{V,\infty}} \tag{4}$$

$$\theta = \frac{T - T_W}{T_O - T_W} \tag{5}$$

$$X = \frac{x}{L}, \qquad Y = \frac{y}{L} \tag{6}$$

The boundary conditions are as follows:

For $Y = 0$ (at the interface)

$$\frac{\partial U}{\partial Y} = -\frac{L}{u_{V,\infty}\mu} \tau_O(X) \tag{7}$$

$$\frac{\partial \theta}{\partial Y} = -\frac{\rho u_{V,\infty} VLh^O}{(T_O - T_W)k} \tag{8}$$

$$\theta = 1 \tag{9}$$

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For $Y = \delta/L$ (at the plate)

$$U = 0 \tag{10}$$

$$V = 0 \tag{11}$$

$$\theta = 0 \tag{12}$$

Boundary condition (7) is a statement of the shear stress at the interface, and will be evaluated by matching with that of the gas phase. Boundary condition (8) is derived from the consideration that the heat released at the interface by the condensing vapor is carried away into the film by conduction. Boundary condition (8) is strictly true only when the gas-phase boundary layer is isothermal. For practical purposes, however, it applies to the nonisothermal vapors also and this point will be explained following the gas-phase boundary-layer equations.

For the incompressible[2] gas-phase boundary layer, we have the following well-known equations:

Continuity

$$\frac{\partial U_V}{\partial X} + \frac{\partial V_V}{\partial Y} = 0 \tag{13}$$

Momentum

$$U_V \frac{\partial U_V}{\partial X} + V_V \frac{\partial U_V}{\partial Y} = \frac{1}{Re_V} \frac{\partial^2 U_V}{\partial Y^2} \tag{14}$$

Diffusion

$$U_V \frac{\partial m}{\partial X} + V_V \frac{\partial m}{\partial Y} = \frac{1}{Sc_V Re_V} \frac{\partial^2 m}{\partial Y^2} \tag{15}$$

The boundary conditions are as follows:

For $Y = 0$

$$U_V = 0 \qquad \text{(assumed approximation)} \tag{16}$$

$$V_V = \frac{\rho}{\rho_V} V \tag{17}$$

$$m = m_O \qquad \begin{array}{l}\text{(saturation concentration} \\ \text{corresponding to } T_O)\end{array} \tag{18}$$

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[2]An analysis very similar to that presented here can be obtained for the compressible boundary layer if it can be assumed that $\mu_V/\mu_{V,\infty} \propto \rho_{V,\infty}/\rho_V$.

For  Y = -∞

$$U_V = 1 \tag{19}$$

$$m = m_\infty \tag{20}$$

When the vapor is superheated a temperature gradient exists across the boundary layer, and the gas-phase energy equation is also needed to analyze the energy transfer associated with the superheat. The energy associated with the superheat, however, is very small compared to the latent heat of condensation for most cases. Superheat, therefore, is neglected in the present analysis of heat transfer and thus the energy equation for the gas phase is not needed.

## Solution of Equations

Our main concern is the solution of the governing equations for the liquid film since, as will be seen later, the solution of the equations for the gas-phase boundary layer can be obtained from the literature. We shall, therefore, begin the analysis with equations (1), (2), and (3).

Experience in boundary-layer analysis tells us that a similarity solution of equations (1), (2), and (3) could be obtained when $1/Fr_x$ is either zero or infinity. The present study is mainly concerned with the problem when $1/Fr_x$ is rather small. We shall, therefore, handle the effect of $1/Fr_x$ in the form of a perturbation to the similarity solution for $1/Fr_x = 0$. It was stated in part I that one of the complexities facing us in solving the governing equations is that one set of the boundary conditions must be applied along $\delta(x)$ which is not known a priori. In order to handle this complexity and also to arrive at the particular series form of the solutions sought, the original equations (1), (2), and (3) and the boundary conditions (7) through (12) are first transformed to more convenient forms as follows:

We define a set of dimensionless variables as

$$\xi = \frac{1}{Fr_x} \tag{21}$$

$$\eta = \frac{y}{\delta(x)} = \frac{Y}{\Delta(\xi)}\sqrt{\frac{Re}{Fr}} \tag{22}$$

Equation (22) defines the variable $\eta$ in such a way that the boundary conditions on all the subsequent transformed equations may be applied at $\eta = 0$ and $\eta = 1$ only. It can be seen from equations (21) and (22) that the variable $\Delta(\xi)$ is related to the liquid-film thickness $\delta(x)$ as

$$\Delta(\xi) = \frac{\delta(x)}{L}\sqrt{\frac{Re}{Fr}} \tag{23}$$

The continuity equation (1) is satisfied in the usual manner by defining a stream function $\Psi(X,Y)$ by the equations

$$U = \frac{\partial \Psi}{\partial Y} \tag{24}$$

$$V = -\frac{\partial \Psi}{\partial X} \tag{25}$$

Now we let

$$\Psi(X,Y) = \sqrt{\frac{Fr}{Re}}\,\Delta(\xi)f(\xi,\eta) \tag{26}$$

Then equations (24), (25), and (26) yield

$$U = \frac{\partial f}{\partial \eta} \tag{27}$$

and

$$V = -\sqrt{\frac{1}{ReFr}}\left(f\Delta' + \Delta\frac{\partial f}{\partial \xi} - \Delta'\eta\frac{\partial f}{\partial \eta}\right) \tag{28}$$

The continuity and the conservation equations (1), (2), (3) are now transformed to the following set of equations:

$$\Delta^2\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \xi\,\partial \eta} - \frac{\partial f}{\partial \xi}\frac{\partial^2 f}{\partial \eta^2} - 1\right) - \Delta\Delta'f\frac{\partial^2 f}{\partial \eta^2} = \frac{\partial^3 f}{\partial \eta^3} \tag{29}$$

$$\Delta^2\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial \xi} - \Delta\left(f\Delta' + \Delta\frac{\partial f}{\partial \xi}\right)\frac{\partial \theta}{\partial \eta} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial \eta^2} \tag{30}$$

The boundary conditions (7) through (12) become:

For $\eta = 0$

$$\frac{\partial^2 f}{\partial \eta^2} = -\frac{L\Delta}{u_{V,\infty}\mu}\sqrt{\frac{Fr}{Re}}\,\tau_0(X) \tag{31}$$

$$\frac{\partial \theta}{\partial \eta} = -L\Delta \sqrt{\frac{Fr}{Re}} \frac{\rho v h^o}{k(T_o - T_w)} \tag{32}$$

$$\theta = 1 \tag{33}$$

and for $\eta = 1$

$$\frac{\partial f}{\partial \eta} = 0 \tag{34}$$

$$f = 0 , \qquad \frac{\partial f}{\partial \xi} = 0 \tag{35}$$

$$\theta = 0 \tag{36}$$

The above transformed equations are solved by first expanding the variables $f$, $\theta$, and $\Delta$ in terms of $\xi$ as

$$f(\xi,\eta) = f_o(\eta) + \xi f_1(\eta) + \xi^2 f_2(\eta) + \ldots \tag{37}$$

$$\theta(\xi,\eta) = \theta_o(\eta) + \xi \theta_1(\eta) + \xi^2 \theta_2(\eta) + \ldots \tag{38}$$

and

$$\Delta(\xi) = \xi^{1/2}\Delta_o + \xi^{3/2}\Delta_1 + \xi^{5/2}\Delta_2 + \ldots \tag{39}$$

When the series (37), (38), and (39) are incorporated into equations (29) and (30) and when the terms with the equal powers of $\xi$ are collected, there result the equations

$$\left. \begin{array}{l} f_o''' + \dfrac{\Delta_o^2}{2} f_o f_o'' = 0 \\[4mm] \dfrac{1}{Pr} \theta_o'' + \dfrac{\Delta_o^2}{2} f_o \theta_o' = 0 \end{array} \right\} \tag{40}$$

$$\left. \begin{array}{l} f_1''' - \Delta_o^2 \left( f_o' f_1' - \dfrac{3}{2} f_1 f_o'' - \dfrac{1}{2} f_1'' f_o - 1 \right) + 2\Delta_o \Delta_1 f_o f_o'' = 0 \\[4mm] \dfrac{1}{Pr} \theta_1'' + \Delta_o^2 \left( \dfrac{1}{2} f_o \theta_1' - f_o' \theta_1 + \dfrac{3}{2} f_1 \theta_o' \right) + 2\Delta_o \Delta_1 f_o \theta_o' = 0 \end{array} \right\} \tag{41}$$

Now consider the expansion of the boundary conditions. First, let us consider the boundary conditions (31) and (32). Before we can expand these boundary conditions, we must know something about the functions $\tau_0(X)$ and $V_0(X)$. For this purpose, we shall briefly investigate the nature of the solution of the gas-phase boundary-layer equations (13) and (14).

The continuity and the momentum equations (13) and (14) are transformed to the following Blasius equations (see ref. 7) with the assumption that the dimensionless stream functions $F$ depend on the variable $\eta_B$ only; that is, we have a similarity solution:

$$F'''(\eta_B) + \frac{1}{2} F(\eta_B)F''(\eta_B) = 0 \tag{42}$$

where

$$\eta_B = y \sqrt{\frac{u_{V,\infty}}{\nu_V x}} \tag{43}$$

and

$$F'(\eta_B) = \frac{u_V}{u_{V,\infty}} \tag{44}$$

The boundary conditions (16), (17), and (19) become:

For $\eta_B = 0$

$$F = -2v_{V,o} \sqrt{\frac{x}{u_{V,\infty}\nu_V}} \qquad . \tag{45}$$

$$F' = 0 \tag{46}$$

and for $\eta_B = -\infty$

$$F' = 1 \tag{47}$$

It is seen from equation (42) and the boundary conditions (45) through (47) that $F$ indeed depends only on $\eta_B$ if $V_{V,o}$ is proportional to $1/\sqrt{x}$ and therefore $F(0)$ is constant. In reference 5 the solution of equation (42) is obtained for many different values of $F(0)$.[3] Also an

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[3]There exist the following relationships between the present function $F$ and the corresponding stream function used in reference 5.

$$F(0) = -[F(0) \text{ of ref. 5}]$$

$$F''(0) = -\frac{1}{4} [F''(0) \text{ of ref. 5}]$$

approximate solution of the equation can be obtained from reference 6. The interface shear stress $\tau_0(x)$ can be derived from the solutions as

$$\tau_0(x) = - \sqrt{\rho_V \mu_V} \frac{u_{V,\infty}^{3/2}}{x^{1/2}} F''(0) \tag{48}$$

From equations (45) and (48) we see that when the condensing rate and $\tau_0(x)$ vary as $1/x^{1/2}$ we have similarity in the gas-phase boundary layer and the solution can be obtained from reference 5.

Now let us examine the liquid film to determine whether the condensing rate and the interface shear stress vary with $1/x^{1/2}$ and therefore whether the solution of the liquid film is compatible with that of the gas phase. When $\xi = 0$ ($g = 0$), an examination of the appropriate governing equations (40) and the boundary conditions (31) through (36) shows that the condensing rate $V_0$ and the shear stress along $\eta = 0$ vary with $1/\sqrt{x}$ and, therefore, they can be made to match with the solution of equation (42). When $\xi > 0$, on the other hand, the distribution of the condensing rate and the shear stress deviate from that corresponding to $1/\sqrt{x}$. However, for the small values of $\xi$ considered in the present analysis, the deviation will be accordingly small and the effect of the deviation on the final results for $\xi > 0$ is neglected. The boundary conditions (31) and (32) are now expanded to the following form.

$$f_0''(0) + \xi f_1''(0) + \xi^2 f_2''(0) + \ldots = F''(0) \sqrt{\frac{\rho_V \mu_V}{\rho \mu}} (\Delta_0 + \xi \Delta_1 + \xi^2 \Delta_2 + \ldots) \tag{49}$$

$$\theta_0'(0) + \xi \theta_1'(0) + \ldots$$

$$= \frac{\text{Pr } h^o}{c_p(T_0 - T_w)} \left\{ \frac{1}{2} \Delta_0^2 f_0(0) + \xi \left[ \frac{3}{2} \Delta_0^2 f_1(0) + 2 \Delta_0 \Delta_1 f_0(0) \right] + \ldots \right\} \tag{50}$$

Expansion of the rest of the boundary condition is rather straightforward and the boundary conditions (31) through (36) become for equations (40) and (41), respectively:

For $\eta = 0$

$$\left. \begin{aligned} f_0'' &= \sqrt{\frac{\rho_V \mu_V}{\rho \mu}} F'' \Delta_0 \\ \frac{\Delta_0^2}{2} \frac{f_0}{\theta_0'} &= \frac{1}{\text{Pr}} \frac{c_p(T_0 - T_w)}{h^o} \\ \theta_0 &= 1 \end{aligned} \right\} \tag{51}$$

$$f_1'' = \sqrt{\frac{\rho_V \mu_V}{\rho \mu}} \, F'' \Delta_1$$

$$\frac{\frac{3}{2} \Delta_0{}^2 f_1 + 2\Delta_0 \Delta_1 f_0}{\theta_1'} = \frac{1}{Pr} \frac{c_p(T_0 - T_w)}{h^o}$$

$$\theta_1 = 0$$

$$\left. \right\} \quad (52)$$

and for $\eta = 1$

$$f_0 = f_0' = 0$$

$$\theta_0 = 0$$

$$\left. \right\} \quad (53)$$

$$f_1 = f_1' = 0$$

$$\theta_1 = 0$$

$$\left. \right\} \quad (54)$$

A study of equations (40) and (41) and boundary conditions (51) through (54) shows that the solutions of the equations depend on the three parameters, $Pr$, $\sqrt{\rho_V \mu_V / \rho \mu} \, F''(0)$, and $(1/Pr)[c_p(T_0 - T_w)/h^o]$. Solutions of the equations are obtained by the use of an IBM 704 digital computer for several combinations of the parameters. Each system of equations, equations (40) and (41), contains three unknowns whereas it has only two equations. The unknowns are the functions $f$, $\theta$, and the constant $\Delta$. We have, however, one extra boundary condition for each of the systems. In order to solve the systems, the constant $\Delta$ is first assumed and the system is integrated. The compatibility of the result is then checked for the extra boundary condition. The foregoing procedure is repeated until the iterative scheme on $\Delta$ produces a solution which satisfies all of the six boundary conditions. A typical result is shown in figures 2 and 3. Figures 2 and 3 are plots of the solutions of equations (40) and (41), respectively, for the liquid Prandtl number of unity, $\sqrt{\rho_V \mu_V / \rho \mu} \, F''(0)$ of -0.01, and $(1/Pr)[c_p(T_0 - T_w)/h^o]$ of 0.05. Figure 2 shows the velocity and temperature profiles for the case of zero body force. Figure 3 shows the first-order effect of the body force on the profiles.


Heat Transfer, Condensing Rate, and Film Thickness


For a given set of the parameters, $Pr$, $\sqrt{\rho_V \mu_V / \rho \mu} \, F''(0)$, $(1/Pr)[c_p(T_0 - T_w)/h^o]$, and $1/Fr_x$, the heat transfer, condensing rate, and film thickness can be obtained, respectively, from the following equations.

$$\frac{Nu_x}{\sqrt{Re_x}} = -\left(\frac{\theta_o{}'}{\Delta_o} + \frac{1}{Fr_x}\frac{\theta_1{}' - \frac{\Delta_1}{\Delta_o}\theta_o{}'}{\Delta_o} + \ldots\right) \tag{55}$$

$$\frac{v_o}{u_{v,\infty}}\sqrt{Re_x} = -\frac{1}{2}\left[\Delta_o f_o + \frac{3}{Fr_x}(\Delta_o f_1 + \Delta_1 f_o) + \ldots\right] \tag{56}$$

$$\frac{\delta}{x}\sqrt{Re_x} = \Delta_o + \frac{1}{Fr_x}\Delta_1 + \ldots \tag{57}$$

Some of the typical results are shown in figures 4 through 6. Figure 4 shows the dimensionless heat transfer and condensing rate, $Nu_x/\sqrt{Re_x}$ and $v_o/u_{v,\infty}\sqrt{Re_x}$, as functions of the interface shear-stress parameter $\sqrt{\rho_v\mu_v/\rho\mu}\,F''(0)$ for the case of zero body force. There are two sets of curves in the figure. They are for $(1/Pr)[c_p(T_o-T_w)/h^o]$ of 0.005 and 0.05, respectively. Figure 5 shows the dimensionless film thickness as a function of the same variables as figure 4. Figure 6 shows, for the two values of $(1/Pr)[c_p(T_o-T_w)/h^o]$, the value of $1/Fr_x$ which would cause $Nu_x/\sqrt{Re_x}$ and $v_o/u_{v,\infty}\sqrt{Re_x}$ to deviate by 10 percent from those for the case of $1/Fr_x = 0$. Figure 6, therefore, shows the effect of the body force on the heat transfer and the condensing rate as a function of $\sqrt{\rho_v\mu_v/\rho\mu}\,F''(0)$. The parameters $\sqrt{\rho_v\mu_v/\rho\mu}\,F''(0)$ and $(1/Pr)[c_p(T_o-T_w)/h^o]$ now must be matched to the appropriate parameters of the gas-phase boundary layer in order that the final solution of the problem may be obtained in terms of the free-stream gas characteristics. The method of obtaining the equilibrium value of $F''(0)$ will now be explained for the case of the gas phase comprised of a pure vapor. In this case, the entire resistance to heat transfer is essentially offered by the liquid film alone. We derive from equation (45) the expression

$$\frac{F(0)}{F''(0)} = -\frac{2\frac{v_o}{u_{v,\infty}}\sqrt{Re_x}}{\sqrt{\frac{\rho_v\mu_v}{\rho\mu}}\,F''(0)} \tag{58}$$

The left side of the equation, $F(0)/F''(0)$, for the gas-phase boundary layer is obtained from data given in references 5 and 6 and is expressed as a function of $F''(0)$ in figure 7. Next the right side of equation (58) is obtained from figures, such as 4 through 6, for a given value of $Pr$, $(1/Pr)[c_p(T_o-T_w)/h^o]$,[4] and $1/Fr_x$. The values thus obtained are then plotted in figure 7, against $F''(0)$ for a given value of $\sqrt{\rho_v\mu_v/\rho\mu}$. The intersections, such as point a, shown then give the desired equilibrium value of $F''(0)$ for the particular problem. Once the equilibrium value

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[4]The interface temperature $T_o$ is the saturation temperature of the vapor corresponding to the pressure.

of $F''(0)$ is known, parameters such as $Nu_x/\sqrt{Re_x}$ and $v_0/u_{v,\infty}\sqrt{Re_x}$ can be immediately obtained from figures 4 through 6.

Next consider the case in which the gas phase is a mixture of a condensable vapor and a noncondensable gas. For this case, resistance to heat and mass transfer is offered both by the gas-phase boundary layer and the liquid film. It can be seen from reference 6 that, for Schmidt numbers sufficiently close to 1, the diffusion process through the gas-phase boundary layer requires that the following relationship must be satisfied in addition to equation (58) in the present problem.

$$\frac{F(0)}{F''(0)} = 2\,\frac{m_\infty - m_0}{1 - m_0} \tag{59}$$

The additional equation (59) can be satisfied because now the saturation temperature $T_0$ is an unknown quantity and is a function of $m_0$.

Derivation of the equilibrium value of $F''(0)$ for this case is considerably more complicated. For this case, $(1/Pr)[c_p(T_0-T_w)/h^0]$ is not a priori a known factor because $T_0$ is not known. One may start an iteration analysis by assuming a suitable value of $T_0$, and therefore corresponding values of $m_0$ and $(1/Pr)[c_p(T_0-T_w)/h^0]$. A point such as a is then obtained in figure 7 in the same manner as for the case with the pure vapor. The value of $F(0)/F''(0)$ corresponding to a point a is next compared to the right side of equation (59). If equation (59) is not satisfied, then another value of $T_0$ is assumed and the foregoing procedure is repeated. The iteration process is continued until equation (59) is satisfied. Proper values of $Nu_x/\sqrt{Re_x}$, $v_0/u_{v,\infty}\sqrt{Re_x}$, etc., could then be obtained for the case of the gas phase comprised of both condensable vapor and noncondensable gas. Results of this nature will not be presented, but can be obtained for any specific case by the procedure outlined.

## III. DISCUSSION AND RESULTS

### Discussion

It was shown in part II that the heat transfer, condensing rate, and other phenomena depend on the four parameters, $Pr$, $\sqrt{\rho_v\mu_v/\rho\mu}\,F''(0)$, $(1/Pr)[c_p(T_0-T_w)/h^0]$, and $1/Fr_x$. The analysis of reference 8 showed that, in the process of film condensation due to gravity alone, the solution of the problem did not explicitly depend on $Pr$ except for extremely small values of $Pr$. For the present problem, the effect of the Prandtl number on the heat transfer and the condensing rate is shown in figure 8 for $\sqrt{\rho_v\mu_v/\rho\mu}\,F''(0) = -0.001$, $(1/Pr)[c_p(T_0-T_w)/h^0] = 0.05$ and $1/Fr_x = 0$. It is seen that for the conditions represented in figure 8, the heat transfer and condensing rate do not explicitly depend on the Prandtl

number appreciably. Though the effect of Pr was checked here only for the particular values of the parameters, $\sqrt{\rho_V \mu_V / \rho \mu}\, F''(0)$, $(1/Pr)[c_p(T_O-T_W)/h^O]$, and $1/Fr_X$, it may be reasonable to believe, in the light of reference 8, that the solution of the present problem does not depend on Pr explicitly except for extreme values of the Prandtl number. The solution, therefore, mainly depends on the three parameters, $\sqrt{\rho_V \mu_V / \rho \mu}\, F''(0)$, $(1/Pr)[c_p(T_O-T_W)/h^O]$, and $1/Fr_X$, and the effect of the Prandtl number appears through the parameter $(1/Pr)[c_p(T_O-T_W)/h^O]$ only.

Figure 9 shows the heat transfer and condensing rate as a function of $\sqrt{\rho_V \mu_V / \rho \mu}$ for the two different values of $(1/Pr)[c_p(T_O-T_W)/h^O]$ of 0.005 and 0.05, respectively. The figure is for $1/Fr_X = 0$ and for the condensation of a pure vapor. It is seen that the condensing rate increases slowly with $\sqrt{\rho_V \mu_V / \rho \mu}$ and increases rapidly with $(1/Pr)[c_p(T_O-T_W)/h^O]$. The fact that it increases rapidly with the parameter $(1/Pr)[c_p(T_O-T_W)/h^O]$ is expected from the nature of the parameter. The comparative insensitiveness of the condensing rate with respect to the parameter $\sqrt{\rho_V \mu_V / \rho \mu}$ may be rather surprising at first glance. Figure 4 shows that the condensing rate increases quite rapidly with $\sqrt{\rho_V \mu_V / \rho \mu}\, F''(0)$. Therefore, if the gas-phase boundary layer were unaffected by the condensing process, $F''(0)$ would be constant and the condensing rate would increase with $\sqrt{\rho_V \mu_V / \rho \mu}$ as rapidly as shown in figure 4. The gas-phase boundary layer, however, is influenced substantially by the suction velocity at the interface which is created by the condensation process itself as was pointed out in part I. Assume that the density of the vapor is increased, which implies that the parameter $\sqrt{\rho_V \mu_V / \rho \mu}$ is increased. The condensing rate, according to figure 4, should increase provided $F''(0)$ remains unchanged. The increased vapor density, however, also tends to decrease the suction velocity of vapor at the interface and, therefore, tends to decrease $F''(0)$. The net result is that the condensing rate increases with $\sqrt{\rho_V \mu_V / \rho \mu}$ but not nearly as rapidly as shown by figure 4.

Next, consider the heat transfer. The parameter $Nu_X/\sqrt{Re_X}$ for $(1/Pr)[c_p(T_O-T_W)/h^O]$ of 0.005 is seen in figure 9 to be considerably higher than that for $(1/Pr)[c_p(T_O-T_W)/h^O]$ of 0.05 in the upper part of the range of $\sqrt{\rho_V \mu_V / \rho \mu}$ considered. This, however, does not necessarily mean that the heat transfer increases as $(1/Pr)[c_p(T_O-T_W/h^O]$ decreases. Besides the magnitude of $(1/Pr)[c_p(T_O-T_W)/h^O]$, the heat transfer depends also on the reason for the variation of the parameter $(1/Pr)[c_p(T_O-T_W)/h^O]$. This may be seen more clearly from the following example. Heat transfer can be obtained from $Nu_X/\sqrt{Re_X}$ by the following equation

$$q_W = \frac{Nu_X}{\sqrt{Re_X}} \sqrt{\frac{u_{V,\infty}}{\nu x}}\, k(T_O - T_W) \qquad (60)$$

Consider that, for $\sqrt{\rho_v \mu_v / \rho \mu} = 0.03$, the parameter $(1/Pr)[c_p(T_o-T_w)/h^o]$ is decreased from 0.05 to 0.005. Figure 9 shows that $Nu_x/\sqrt{Re_x}$ is then increased by the factor of 1.53. Now, let us assume that the decrease in $(1/Pr)[c_p(T_o-T_w)/h^o]$ is caused by the decrease in $(T_o-T_w)$. Then equation (60) shows that the heat transfer is decreased by the factor 0.153. Next, let us assume that the decrease in $(1/Pr)[c_p(T_o-T_w)/h^o]$ is caused by an increase in $h^o$. Then it is seen in equation (60) that the heat transfer increases by the factor 1.53. Therefore the heat transfer may either increase or decrease with the parameter $(1/Pr)[c_p(T_o-T_w)/h^o]$ depending on the reason for the variation of the parameter. The physical explanations for these phenomena can be made as follows. Figure 10 shows that the liquid-film thickness is reduced as the parameter $(1/Pr)[c_p(T_o-T_w)/h^o]$ is decreased. Now first consider the case in which $(1/Pr)[c_p(T_o-T_w)/h^o]$ is decreased because of a reduction in $(T_o-T_w)$. The liquid film is now thinner and offers less resistance to heat transfer to the wall. However, the heat-transfer potential $(T_o-T_w)$ is also decreased. The decrease in the heat-transfer potential is such that the net effect on heat transfer is to decrease it even with the reduced film thickness. Next, consider the case in which the parameter, $(1/Pr)[c_p(T_o-T_w)/h^o]$, is decreased because of an increase in $h^o$. Now the only change, as far as the heat transfer is concerned, is the thinning of the liquid film. The heat transfer to the wall is therefore increased. It is also interesting to note that $Nu_x/\sqrt{Re_x}$ increases with $\sqrt{\rho_v \mu_v / \rho \mu}$ much faster when $(1/Pr)[c_p(T_o-T_w)/h^o]$ is 0.005 than when it is 0.05. The reason for this can be seen in figure 10. As $\sqrt{\rho_v \mu_v / \rho \mu}$ is increased, the film thickness decreases much more rapidly when $(1/Pr)[c_p(T_o-T_w)/h^o]$ is 0.005 than when it is 0.05.

Before leaving the case of $1/Fr_x = 0$ it should be noted that the complementing interaction between the gas phase and the liquid film (explained in part I of the report) has a drastic effect in increasing the condensing rate and the heat transfer. The following numerical example may bring out this point a little more clearly. Consider pure steam condensing at atmospheric pressure. The value of the parameter $\sqrt{\rho_v \mu_v / \rho \mu}$ is about 0.005. When $T_o-T_w \approx 90^o$ F, the value of the parameter $(1/Pr)[c_p(T_o-T_w)/h^o]$ is about 0.05. If it is erroneously assumed that the condensation does not affect the gas-phase boundary layer, the corresponding $F''(0)$ is -0.33206 according to reference 5. Figure 4 then yields $Nu_x/\sqrt{Re_x} \approx 0.19$ and $v_o/u_\infty \sqrt{Re_x} \approx 0.0094$. When the actual interaction is considered, on the other hand, $F''(0)$ is increased substantially because of the effect of the condensation on the gas-phase boundary layer. The resulting values of $Nu_x/\sqrt{Re_x}$ and $v_o/u_{v,\infty} \sqrt{Re_x}$ are obtained from figure 9 as 0.53 and 0.0268, respectively. The effect, of course, is not as large when $(1/Pr)[c_p(T_o-T_w)/h^o]$ is 0.005.

The effect of the body force on the condensing rate and heat transfer is seen in figure 11. In the figure, the values of $1/Fr_x$ which would increase the condensing rate and heat transfer by 10 percent from the values for $1/Fr_x = 0$ are plotted against the parameter $\sqrt{\rho_V \mu_V / \rho \mu}$ for $(1/Pr)[c_p(T_o-T_w)/h^o]$ of 0.05 and 0.005, respectively. It is seen in the figure that, for the values of $\sqrt{\rho_V \mu_V / \rho \mu}$ considered, the values of $(1/Fr_x)_{1o}$ for the condensing rate at $(1/Pr)[c_p(T_o-T_w)/h^o]$ of 0.05 are one to two orders of magnitude greater than those at $(1/Pr)[c_p(T_o-T_w)/h^o]$ of 0.005. The values of $(1/Fr_x)_{1o}$ for the Nusselt number are 1/2 to one order of magnitude greater when $(1/Pr)[c_p(T_o-T_w)/h^o]$ is 0.05 than when it is 0.005. Thus for the higher values of $(1/Pr)[c_p(T_o-T_w)/h^o]$ it requires a much larger body force to affect the heat transfer and the condensing rate a given relative amount than it does at the lower values. The relative effect of the body force on the condensing rate and heat transfer decreases as $\sqrt{\rho_V \mu_V / \rho \mu}$ is increased at the lower value of $(1/Pr)[c_p(T_o-T_w)/h^o]$ whereas it becomes rather insensitive to $\sqrt{\rho_V \mu_V / \rho \mu}$ at the higher values of $(1/Pr)[c_p(T_o-T_w)/h^o]$.

The typical velocity and temperature profiles for the liquid film are given in figures 12 and 13. It is seen that the velocity and temperature profiles are almost linear when $1/Fr_x = 0$. As the body force is added, however, the velocity profiles have a parabolic form and, at the same time, the liquid film becomes thinner.

## Summary of Results

The laminar film condensation on a flat plate under the influence of the gas-liquid interface shear and the body force (g force) has been analyzed.

It is shown that the major governing parameters in the condensation of a pure vapor are the ratio of the product of the density and viscosity of the gas to that of the liquid, $(\rho_V \mu_V / \rho \mu)^{1/2}$, the ratio of the temperature potential to the heat of condensation, $(1/Pr)[c_p(T_o-T_w)/h^o]$, and the Froude number, $u_{V,\infty}^2/gx$. When the gas phase is a mixture of a condensable vapor and a noncondensable gas, the gas-phase diffusion potential also becomes an important parameter.

The heat transfer, the condensing rate, and the liquid-film thickness are calculated for the case of a condensing pure vapor. The results show that the condensing rate increases slowly with the density-viscosity ratio, $\rho_V \mu_V / \rho \mu$, and rapidly with the ratio of temperature potential to heat of condensation, $(1/Pr)[c_p(T_o-T_w)/h^o]$. The Nusselt number also increases with the density-viscosity ratio but the rate of the increase becomes smaller as the ratio of driving temperature potential to heat of condensation increases.

It is found that as the condensation proceeds, it creates a suction velocity at the interface for the gas-phase boundary layer which results in an increased interface shear stress. The increased shear increases the condensation rate by increasing the liquid velocity. The increased condensing rate, in turn, increases the interface shear stress imposed by the gas-phase boundary layer. Because of this complementary interaction between the gas and liquid, it is found that the film condensation in a boundary-layer flow occurs at a high rate.

The ratio of temperature potential to heat of condensation is found to have a large influence on the magnitude of the body force which can be supported by the liquid film before the characteristics of the film are altered substantially.

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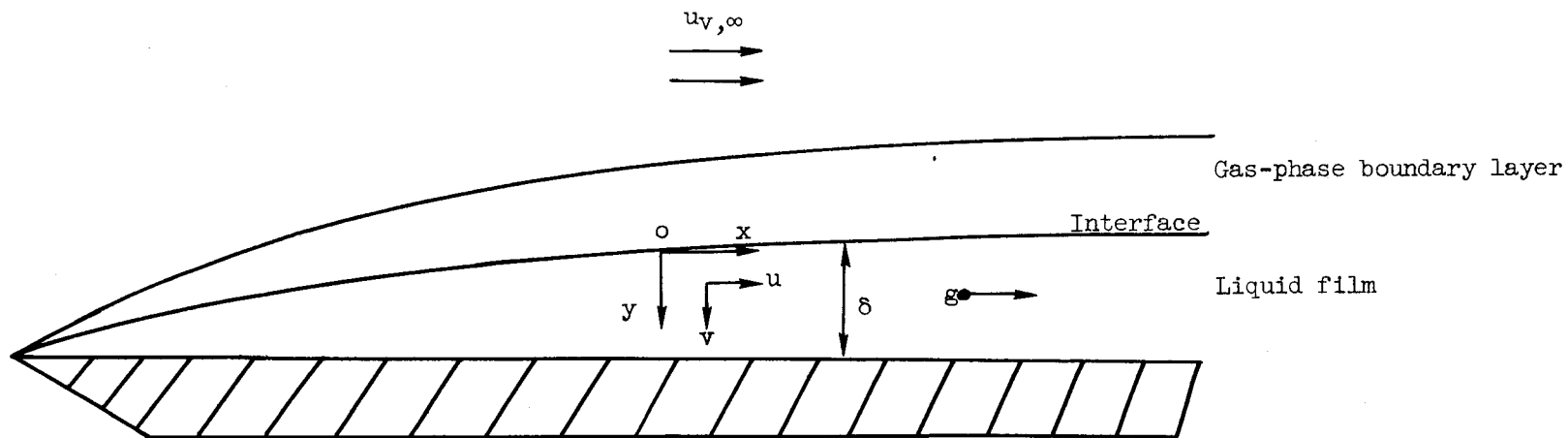
Figure 1.- Sketch of flow model.
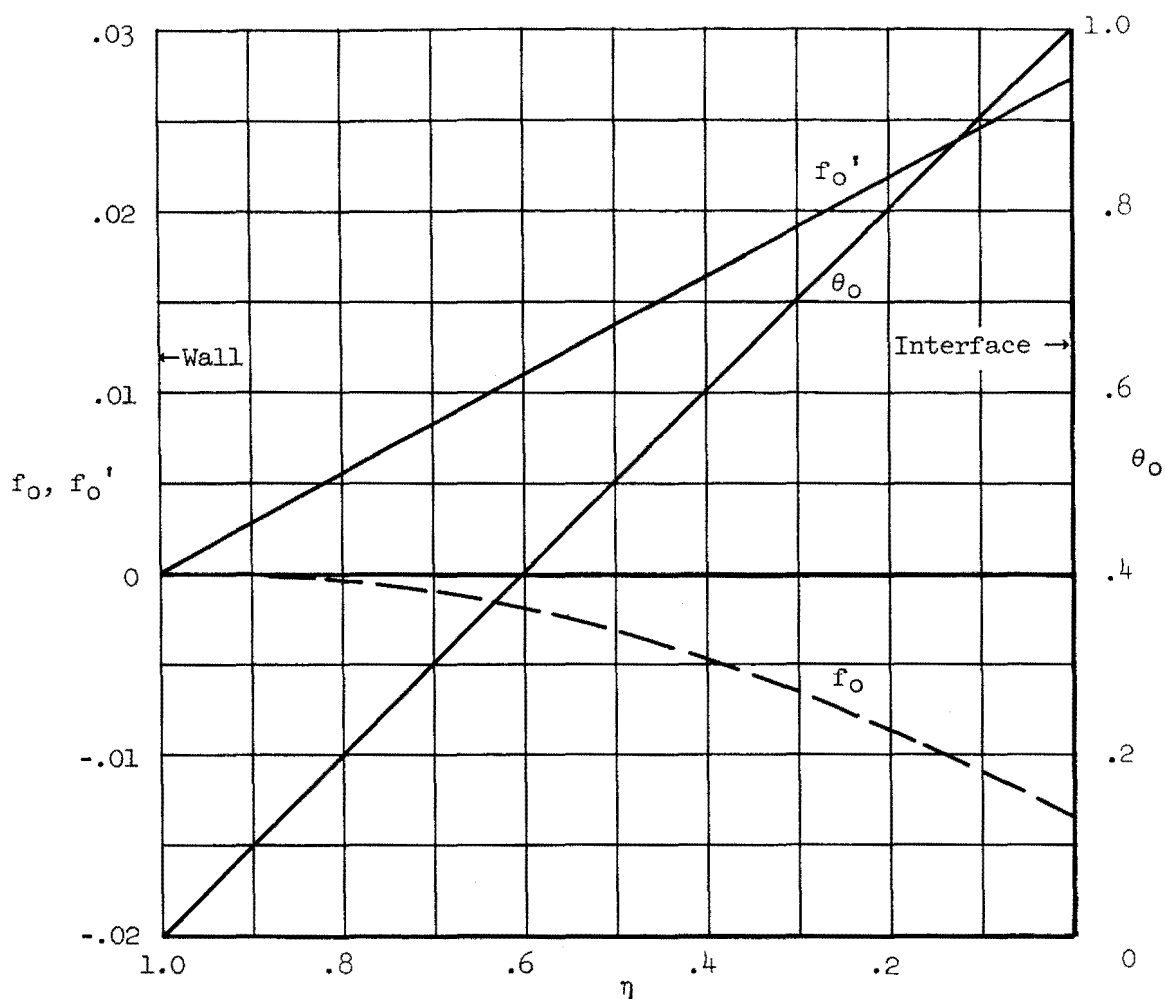
Figure 2.- Typical solutions of the zeroth order perturbation equations, equation (40), for the stream function, velocity profile, and temperature profile; Pr = 1.0; $\Gamma$ = 0.05; $\sqrt{\rho_V \mu_V / \rho \mu} \ F''(0)$ = -0.01.
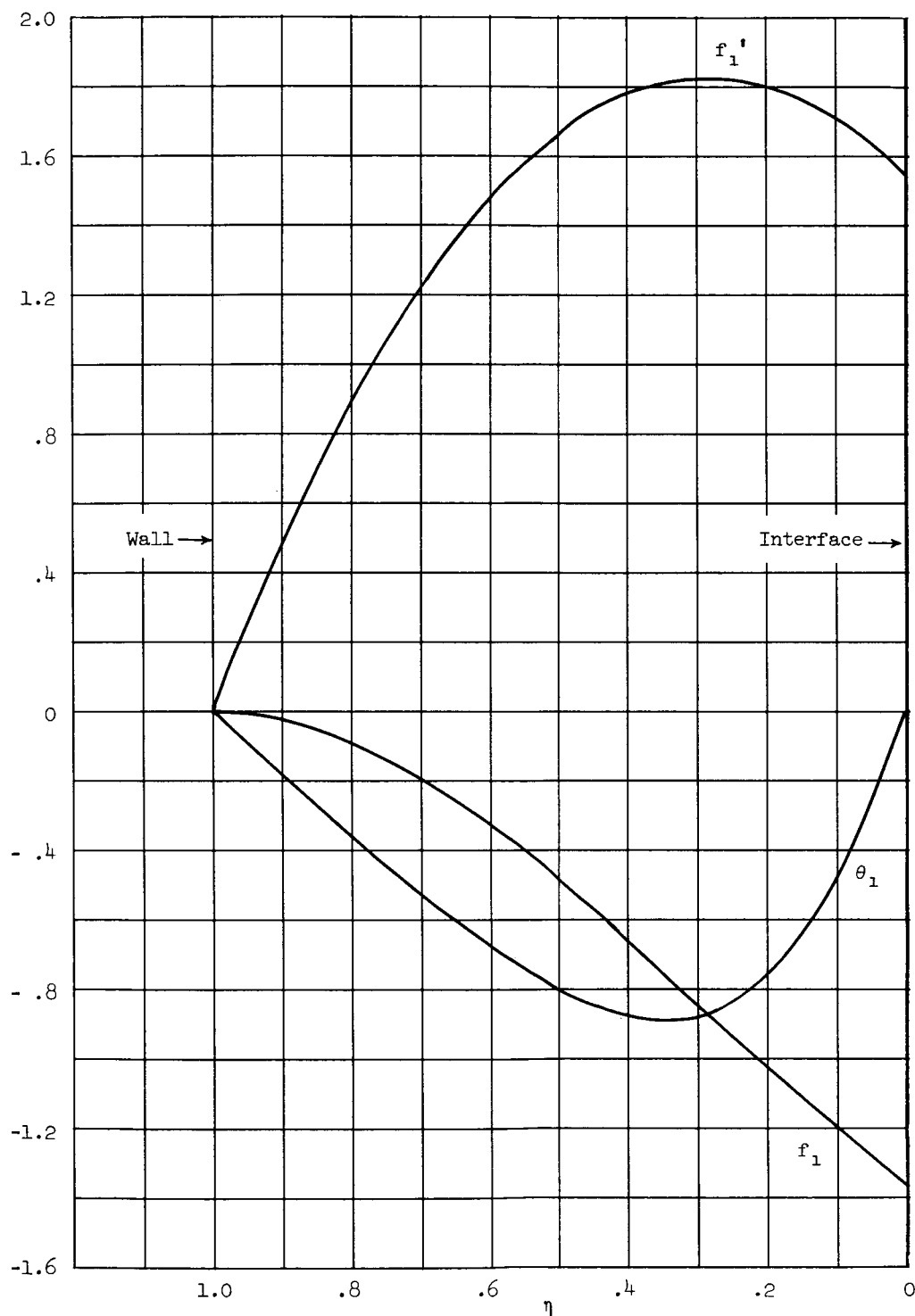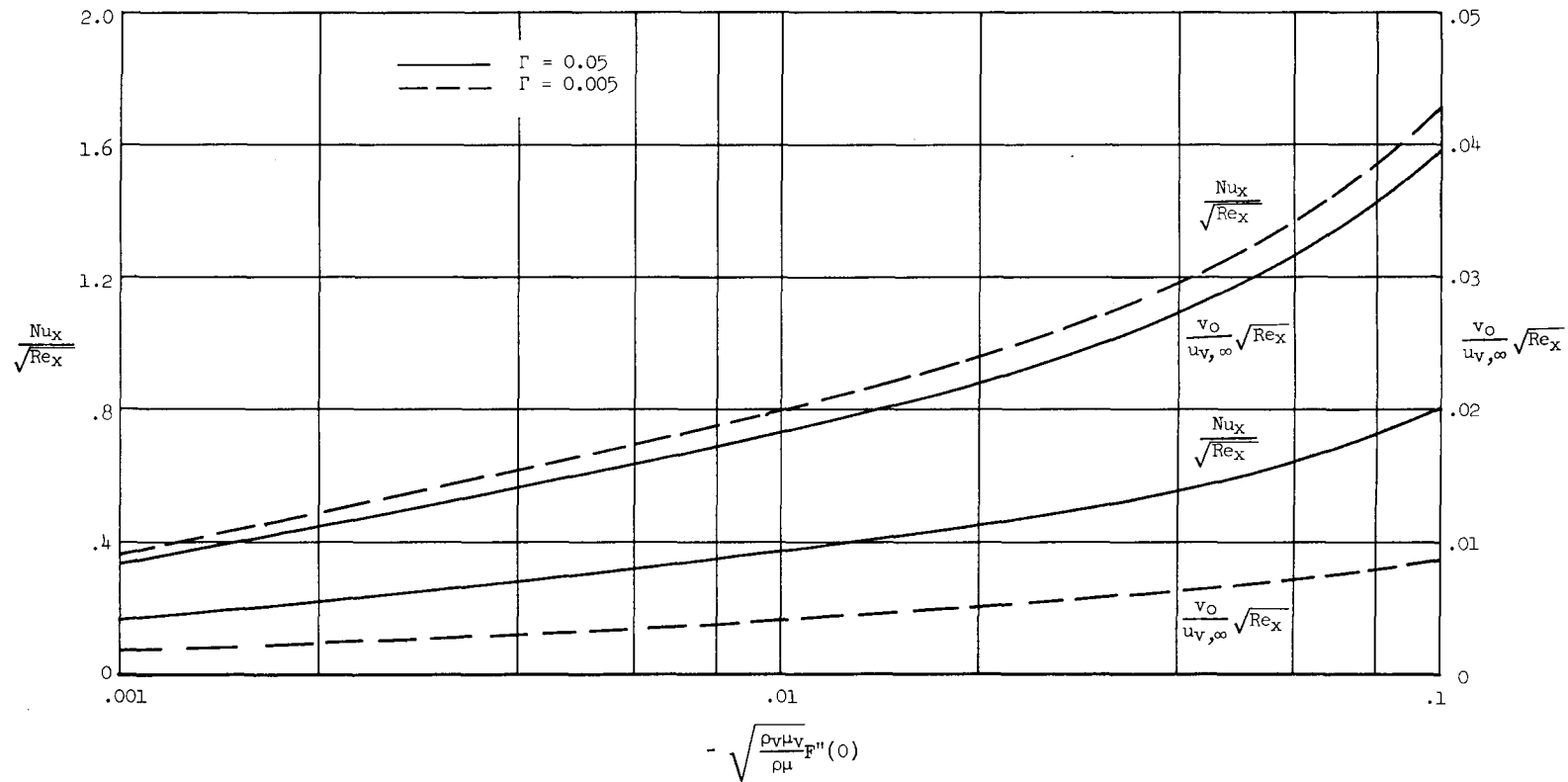
Figure 3.- Typical solutions of the first-order perturbation equations, equation (41), for the stream function, velocity profile, and temperature profile; $Pr = 1.0$; $\Gamma = 0.05$; $\sqrt{\rho_V \mu_V / \rho \mu} \ F''(0) = -0.01$.

Figure 4.- Variation of condensing rate and heat transfer with interface shear-stress parameter; $Pr = 1.0$; $1/Fr_X = 0$.
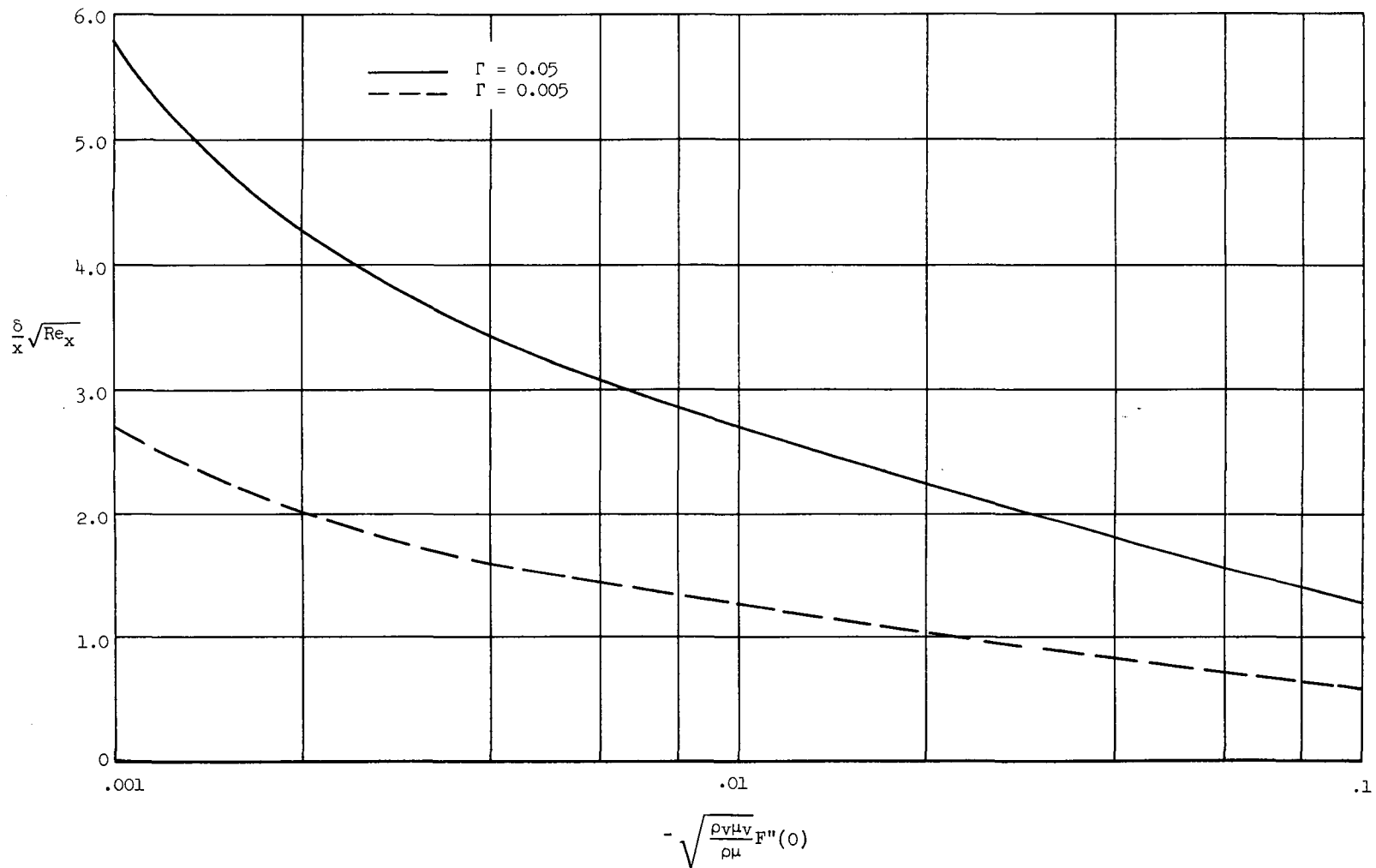
Figure 5.- Variation of liquid film thickness with interface shear-stress parameter; Pr = 1.0; $1/\text{Fr}_x = 0$.
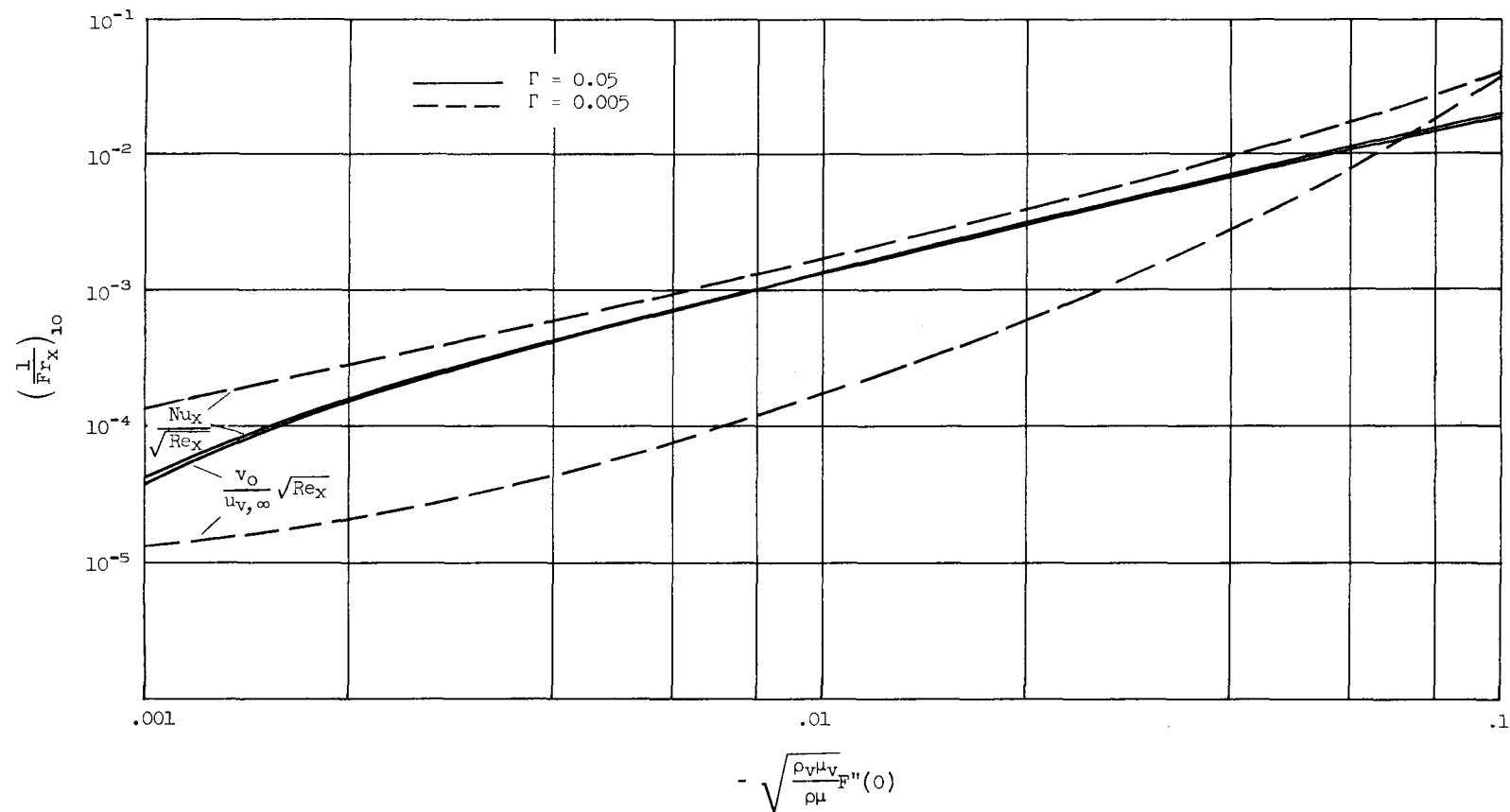
Figure 6.- Variation of Froude number which will vary the condensing rate and heat transfer by 10 percent from those for the case of no body force with respect to interface shear-stress parameter; Pr = 1.0.
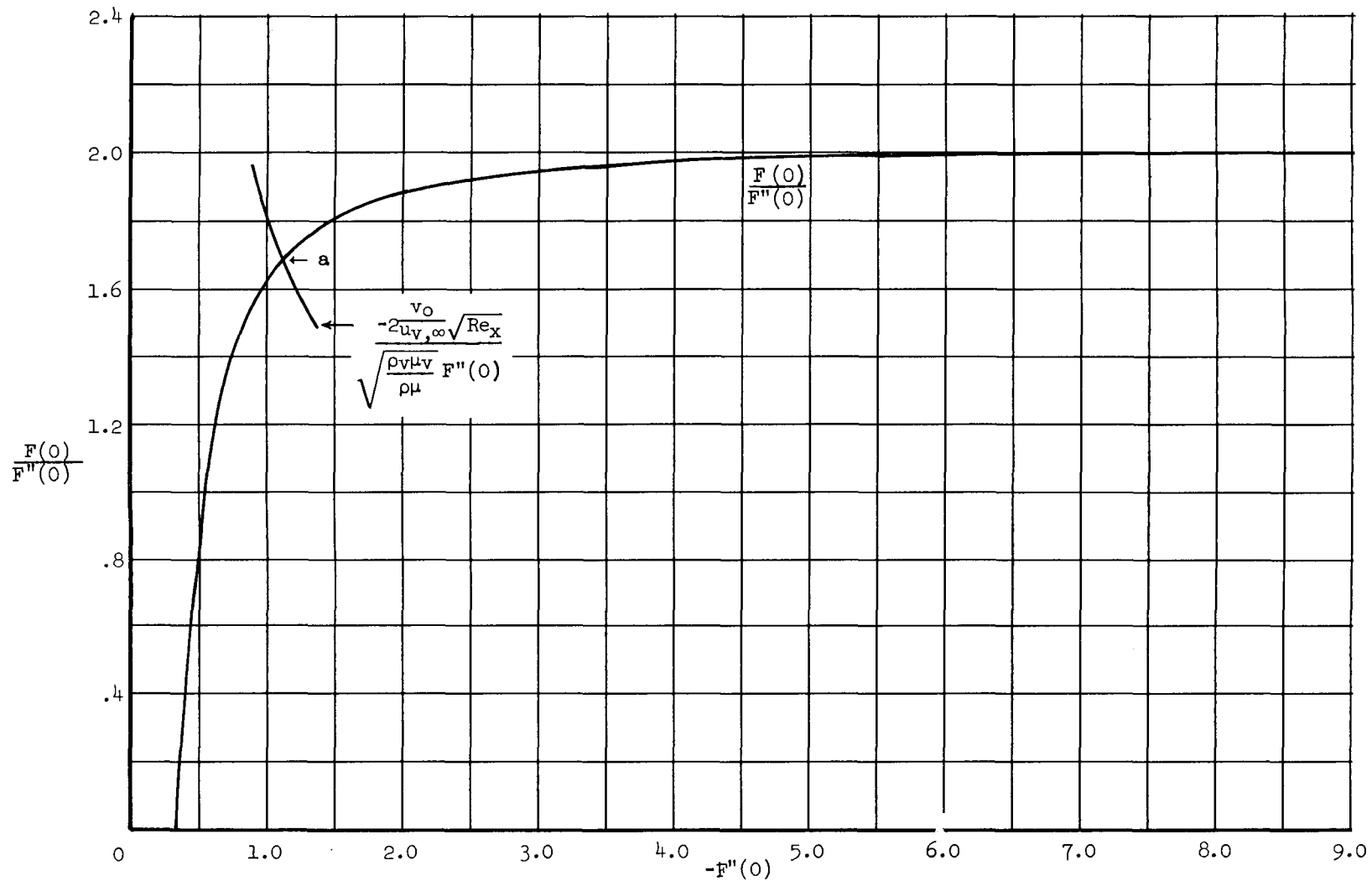
Figure 7.- Variation of the ratio of condensing rate to interface shear with respect to the interface shear stress.
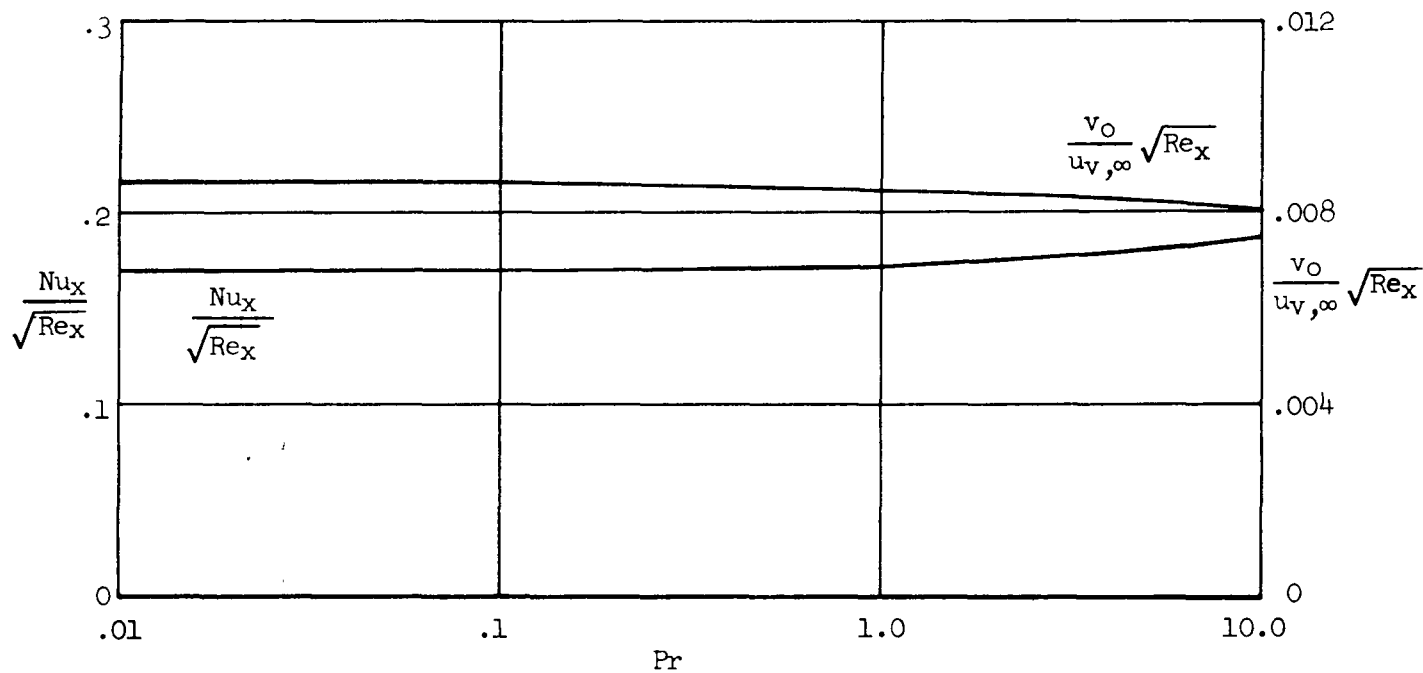
Figure 8.- Effect of Prandtl number on condensing rate and heat transfer; $1/Fr_x = 0$; $\Gamma = 0.05$; $\sqrt{\rho_V \mu_V / \rho \mu}\ F''(0) = -0.001$.
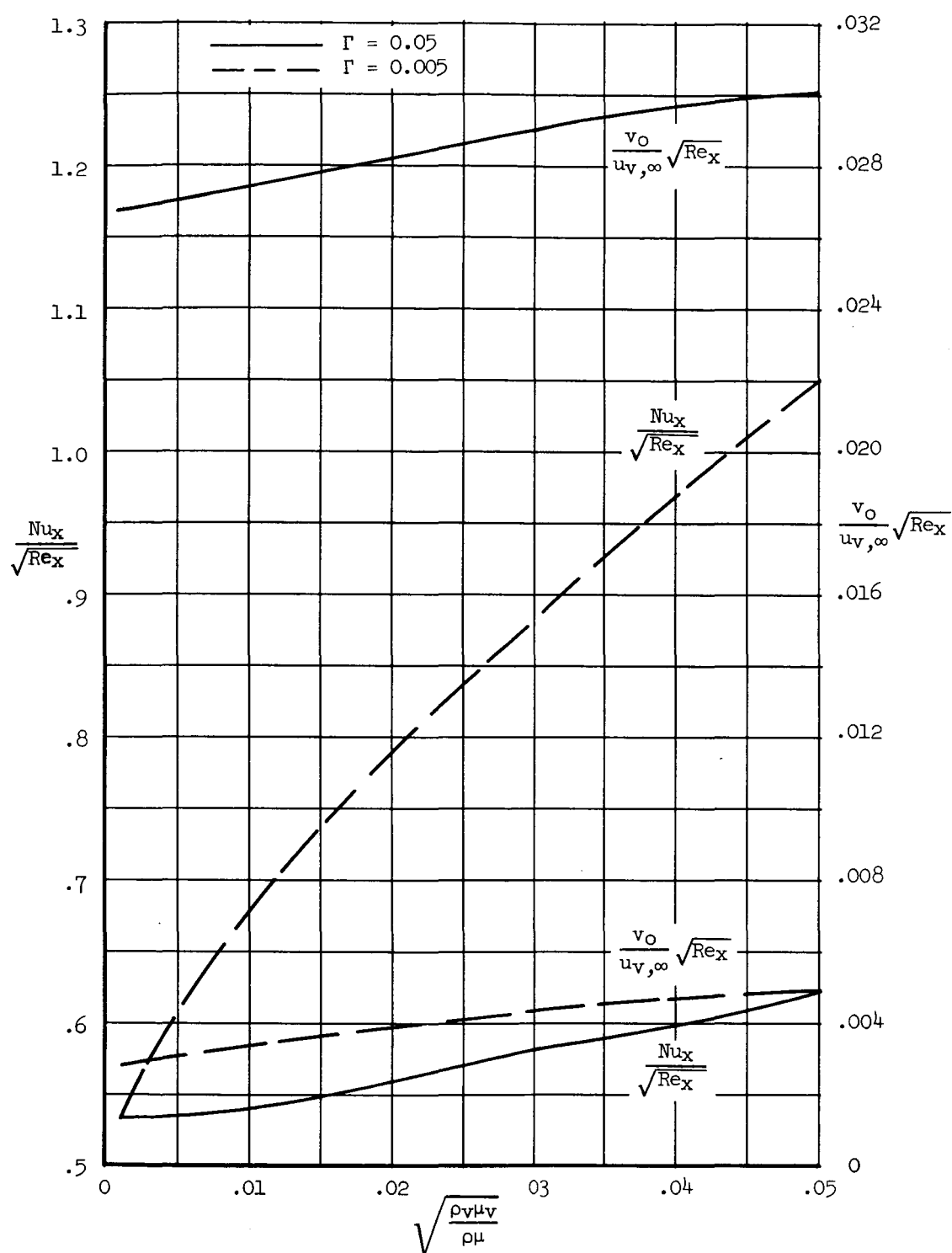
Figure 9.- Variation of condensing rate and heat transfer with $\rho\mu$ ratio; Pr = 1.0; $1/Fr_X$ = 0.
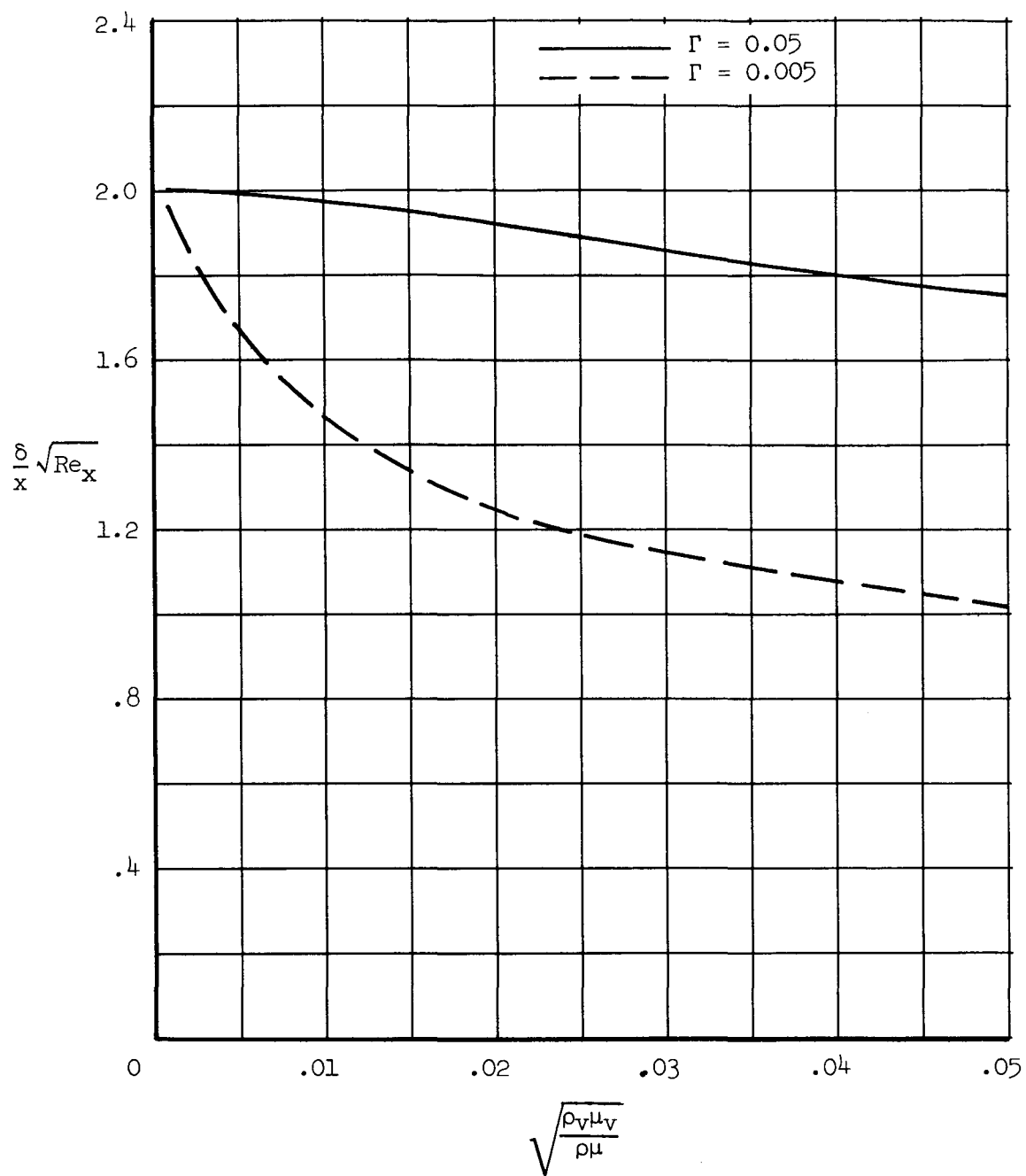
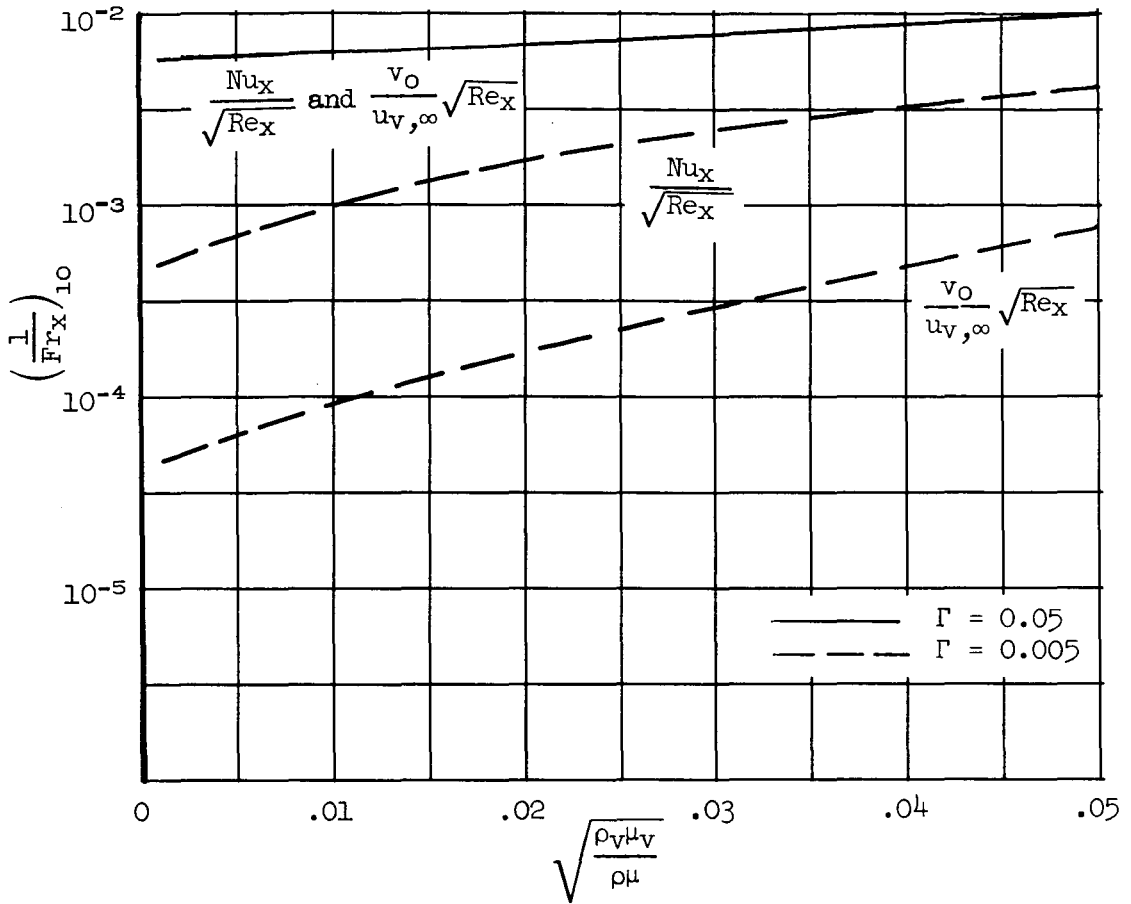Figure 10.- Variation of liquid film thickness with $\rho\mu$ ratio; Pr = 1.0; $1/Fr_x = 0$.

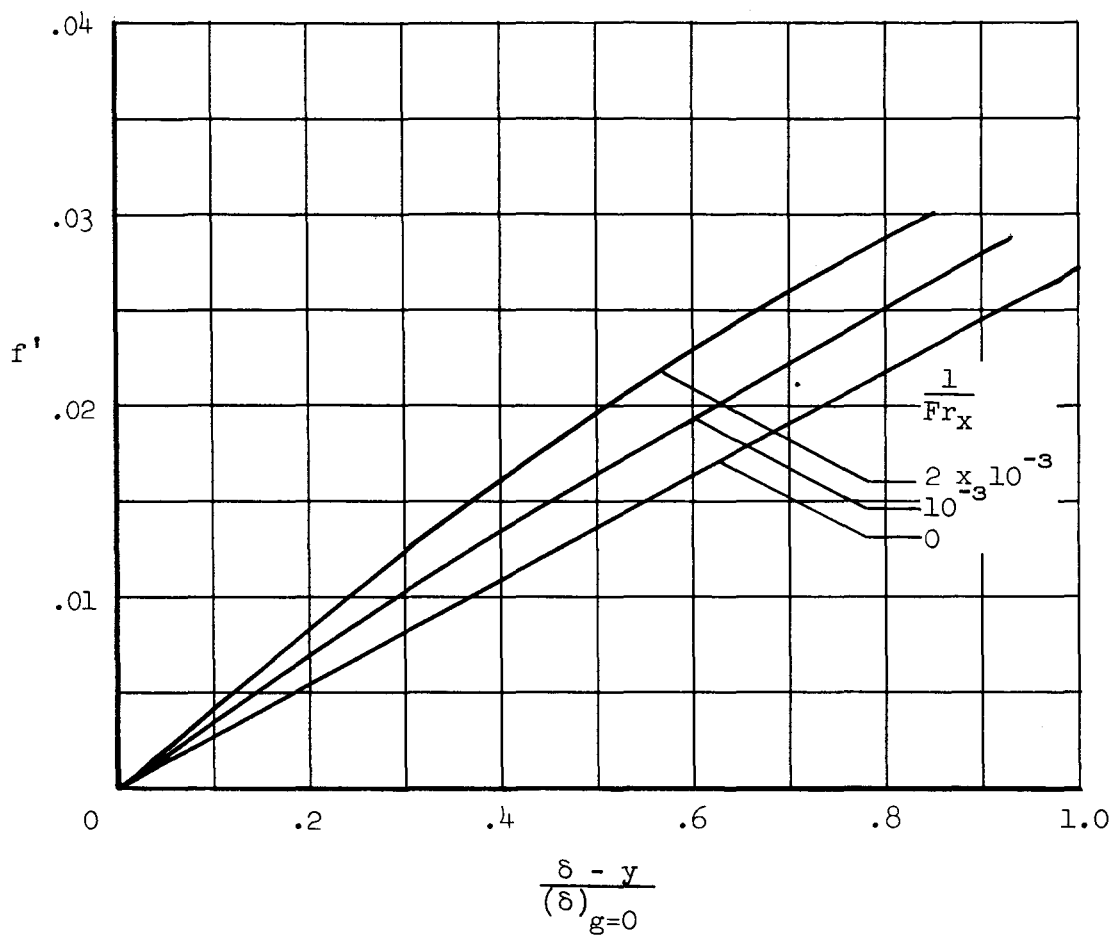Figure 11.- Relative effect of body force on condensing rate and heat transfer; Pr = 1.0.

Figure 12.- Typical velocity profile for liquid film; Pr = 1.0; $\Gamma$ = 0.05; $\sqrt{\rho_v \mu_v / \rho \mu}$ $F''(0)$ = -0.01.
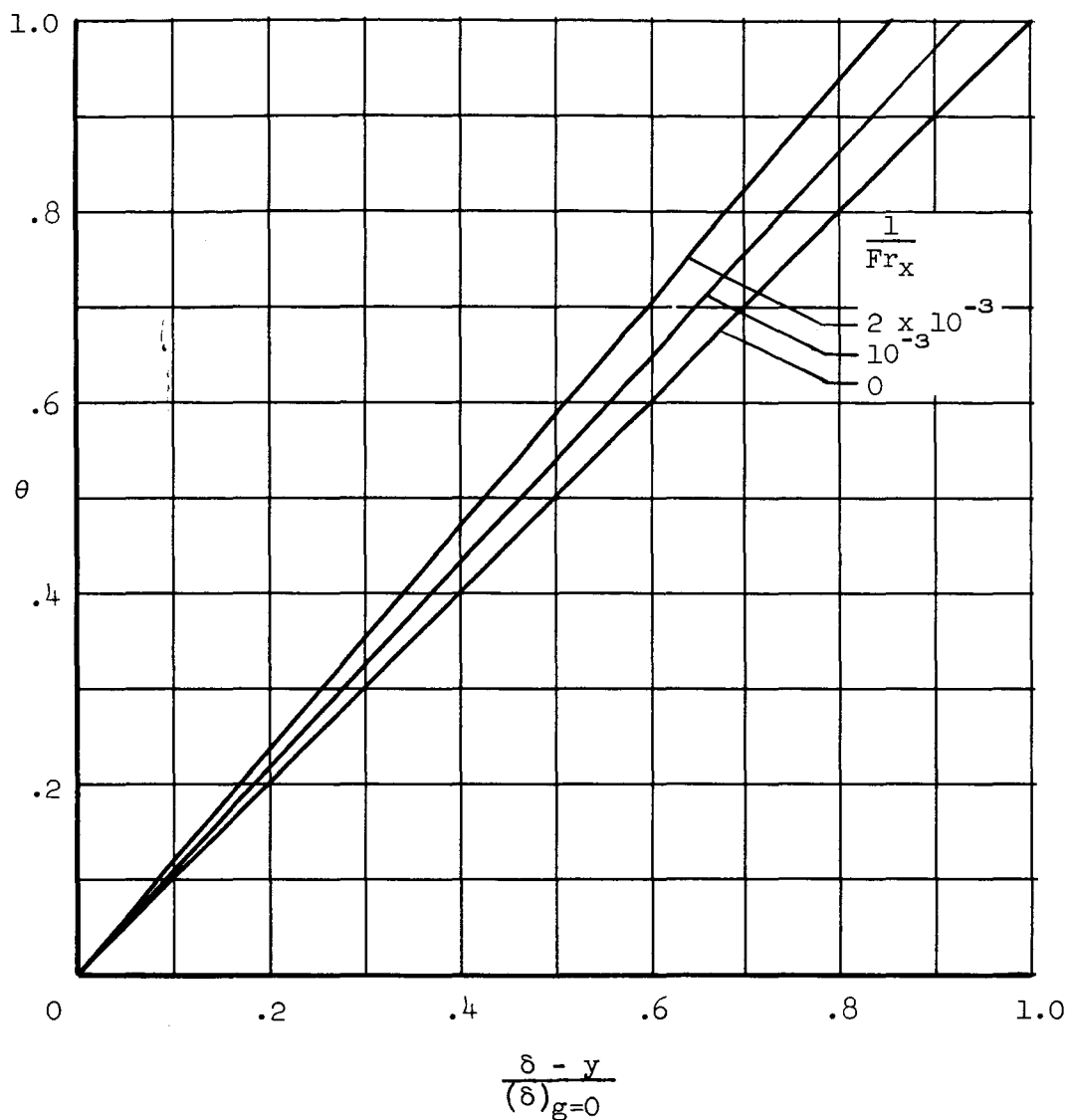
Figure 13.- Typical temperature profile for liquid film; $Pr = 1.0$; $\Gamma = 0.05$; $\sqrt{\rho_V \mu_V / \rho \mu}\ F''(0) = -0.01$.