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SATELLITE ATTITUDE CONTROL USING A COMBINATION OF INERTIA WHEELS AND A BAR MAGNET

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SUMMARY

The use of inertia wheels to control the attitude of a satellite has currently aroused much interest. The stability of such a system has been studied in this investigation. A single-degree-of-freedom analysis indicates that a response with suitable dynamic characteristics and precise control can be achieved by commanding the angular velocity of the inertia wheel with an error signal that is the sum of the attitude error, the attitude rate, and the integral of the attitude error. A digital computer was used to study the three-degree-of-freedom response to step displacements, and the results indicate that the cross-coupling effects of inertia coupling and precession coupling had no effect on system stability. A study was also made of the use of a bar magnet to supplement the inertia wheels by providing a means of removing any momentum introduced into the system by disturbances such as aerodynamic torques. A study of a case with large aerodynamic torques, with a typical orbit, indicated that the magnet was a suitable device for supplying the essential trimming force.

Single-degree-of-freedom bench tests generally verified the dynamic response predicted by the analytical study. It was possible to control the test table to within ±9 arc-seconds of the reference direction, even though the hardware components that were used in these tests were not specifically designed for the control system.

INTRODUCTION

The use of inertia wheels to supply control torques for a satellite in orbit has been suggested previously by other investigators (for example, see refs. 1, 2, and 3). It has been recognized that, for a satellite which has little or no stability or damping, inertia wheels can provide an adequate means of changing the attitude of the satellite. To a certain extent, inertia wheels can also maintain the attitude of a satellite in the presence of disturbances. However, since the inertia wheels can only absorb the momentum produced by disturbances, a supplemental device that can provide an external torque and thereby remove momentum from the inertia wheels is needed. A magnet reacting to the magnetic field of the earth is an example of such a device.

This paper presents the results of a study that has been made of the use of inertia wheels in combination with a permanent bar magnet to supply control torques for a satellite. Such a combination would be applicable to cases in which an adequate supply of electrical power would be always available, but the period of operation would be for such an extended time that any loss of mass through the use of jet devices, even at a slow rate, would be undesirable. The general requirements for the design were to provide precise control, with reasonable response time, and to keep the angular velocities of the wheels close to zero or at least to prevent them from exceeding some practical limit. It is not within the scope of this paper to determine particular specifications for these requirements. One of the purposes of this investigation was to determine the cross-coupling and nonlinear effects of the control system on precision and response time. An analytical study of the three-degree-of-freedom response to step displacements and a study of the problem of a satellite that must always point towards the sun while subject to small aerodynamic disturbances have been made. No other disturbance torques were considered in this analytical study. Also, single-degree-of-freedom bench tests using hardware mock-ups have been conducted.

SYMBOLS

G_1,G_2,G_3,G_4 gimbal angles, radians

g_1,g_2,g_3,g_4 gimbal axes

I_R moment of inertia of control wheel, slug-ft^2

I_X,I_Y,I_Z moment of inertia of satellite about X-, Y-, and Z-axis, respectively, slug-ft^2

K_1 angular error gain, $\frac{\text{radians/sec}}{\text{radian}}$

K_2 rate gain, $\frac{\text{radians/sec}}{\text{radians/sec}}$

K_3 integral gain, $\frac{\text{radians/sec}}{\text{radians-sec}}$

M	moment, ft-lb
r,q,p	yawing, pitching, and rolling angular velocities, radians/sec
s	Laplace operator, per sec
T	torque, ft-lb
t	time, sec
V	velocity, ft/sec
X,Y,Z	satellite body axes
α	angle of attack, radians
β	angle of sideslip, radians
γ	angle between magnet and magnetic field (same as G_4 in three-degree-of-freedom cases), radians
δ	angle between plane of orbit and plane of magnetic equator, radians
λ	angle defining position of satellite in orbit, radians
ρ	mass density, slugs/cu ft
τ	time constant, sec
ψ,θ,ϕ	yaw, pitch, and roll attitude angles, radians
Ω	dihedral angle between plane of orbit and ecliptic plane, radians
ω	angular velocity of control wheel, radians/sec

Subscripts:

c	command
MF	magnetic field
o	initial
S	satellite

X,Y,Z body axes

X",Y",Z" magnetic axes

ϵ error

Dots over symbols indicate differentiation with respect to time.

ANALYTICAL STUDY

Single Degree of Freedom

By accelerating a wheel in one direction, a satellite can be caused to accelerate in the opposite direction. Thus, the attitude of the satellite can be changed. The single-degree-of-freedom equation of motion for a satellite with no stability or damping and with an inertia wheel control is

$$I_X\ddot{\theta} = -I_R\dot{\omega}$$

In like manner, the momentum produced by a disturbance can be absorbed in the control wheel. Thus, the attitude of the satellite can be maintained at a prescribed position while it is subject to disturbances.

The means of achieving these operations are now discussed. A block diagram of the assumed control system is shown in figure 1. In this investigation it is assumed that an error signal causes the inertia wheel to rotate at some proportional angular velocity. This manner of operation can be obtained by incorporating a rate feedback signal (tachometer signal) in the motor control or by selecting a motor which produces an angular velocity proportional to the input signal. Other modes of operation which produce a motor position or angular acceleration proportional to the error signal are also possible but were not considered.

The error signal was assumed to be the sum of an angular error of the satellite, the angular velocity of the satellite, and the integral of the angular error of the satellite. The command equation for the angular velocity of the inertia wheel is, then,

$$\omega_c = K_1\theta_\epsilon + K_2\dot{\theta}_\epsilon + K_3\int\theta_\epsilon\,dt \tag{1}$$

It was also assumed that there would exist a lag between the command for an angular velocity and the attainment of that angular velocity. It was assumed that this lag would be represented by a first-order term with a time constant τ so that the equation for the angular velocity of the wheel could be

$$\omega = \frac{\omega_c}{1 + \tau s}$$

The equation of motion including a constant disturbance moment M_D in Laplace operator form is, then,

$$I_X s^2 \theta = -I_R s\left(K_1\theta_\epsilon + K_2 s\theta_\epsilon + K_3 \frac{\theta_\epsilon}{s}\right)\frac{1}{1 + \tau s} + M_D \tag{2}$$

If

$$\theta_\epsilon = \theta - \theta_c$$

then, the transfer function of the system is

$$\theta = \frac{I_R K_1 s + I_R K_2 s^2 + I_R K_3}{I_X \tau s^3 + \left(I_X + I_R K_2\right)s^2 + I_R K_1 s + I_R K_3}\theta_c$$

$$+ \frac{1 + \tau s}{I_X \tau s^3 + \left(I_X + I_R K_2\right)s^2 + I_R K_1 s + I_R K_3} M_D \tag{3}$$

In order to illustrate the dynamic characteristics of this single-degree-of-freedom system, some examples are given in the following table: (These examples assume a 3,000-pound satellite with a moment of inertia of 1,000 slug-feet^2 and a 5-pound control wheel with a moment of inertia of 0.04 slug-foot^2. Several different combinations of system control gains are presented to illustrate the dependence of system characteristics on the system parameters.)

Case	K_1, radians/sec / radians	K_2, radians/sec / radians/sec	K_3, radians/sec / radians-sec	τ, sec	Characteristics		
					Oscillatory root		Other roots
					Period, sec	Damping ratio	
1	15,000	0	0	0	--	----	s = 0.6, 0
2	15,000	0	0	5	17	0.27	s = 0
3	15,000	0	0	10	25	.20	s = 0
4	15,000	30,000	0	5	16	.60	s = 0
5	15,000	30,000	750	5	20	.60	s = 0.062

In case 1, in which an angular error commands a rotation of the control wheel with no lag, the characteristics of the system are a first-order root with a time constant of 1.66 seconds and a zero root. The significance of the zero root is that the system will stabilize at some angle other than the desired attitude if there is some momentum introduced into the system. A study of the disturbance term in the transfer function indicates that this bias error is a function of the integral of the disturbance. A steady torque on the satellite would cause an increasing error. If the torque were removed, the bias error would remain at the angle existing at that time, and the control wheel would continue turning at a steady angular velocity. The error would be proportional to the momentum imparted to the system by the disturbance and inversely proportional to the error gain. There is no theoretical limit to the size of the error gain and, therefore, the steady-state error could be kept small by the use of a large gain.

The addition of the lag term for the control-wheel rotation changes the first-order root to an oscillatory mode of motion. The larger the lag time constant is, the lower the damping ratio of motion. An infinite lag would, in effect, cause an error signal to command an acceleration of the motor instead of a velocity and would result in an undamped, or harmonic, motion. With some types of direct-current motors, eliminating the rate feedback control (tachometer signal) would produce the same acceleration, rather than velocity, response of the motor. Thus, various combinations of motor lag and rate feedback result in system characteristics somewhere between a first-order response and harmonic motion.

It is probably desirable to have a damping ratio higher than those given in cases 2 and 3. Such an increase could be achieved, if it were not possible to decrease the motor lag, by adding a rate signal to the error signal as is done in case 4.

The addition of the integral term to the error signal, as in case 5, eliminates the zero root and thus insures that the satellite would be driven to the desired attitude even though some disturbance torques were applied. With this type of control, the angular error of the satellite would be reduced to zero, but the control wheel would be required to continue turning with an angular velocity that would represent the momentum produced by the disturbance.

Three Degrees of Freedom

Use of inertia wheels alone.- Consider a three-degree-of-freedom situation in which three inertia wheels would be used for control, one on each mutually perpendicular axis. Each of the wheels would be subject to rotations about axes other than their spin axis and would therefore produce precession torques as well as the desired control torque. For example, the Y and Z precession torques of the X-axis wheel would be as follows:

$$T_Y = -I_R \omega_X r$$

$$T_Z = I_R \omega_X q$$

The three-degree-of-freedom equations of motion describing the moments produced by the satellite and the control system are, then,

$$I_X \dot{p} + \left(I_Z - I_Y\right) qr = -I_R \dot{\omega}_X + I_R \omega_Y r - I_R \omega_Z q \tag{4}$$

$$I_Y \dot{q} + \left(I_X - I_Z\right) rp = -I_R \dot{\omega}_Y + I_R \omega_Z p - I_R \omega_X r \tag{5}$$

$$I_Z \dot{r} + \left(I_Y - I_X\right) qp = -I_R \dot{\omega}_Z + I_R \omega_X q - I_R \omega_Y p \tag{6}$$

It is interesting to note that, if the momentum of each axis of the satellite is equal and opposite to the momentum of the inertia wheel mounted on that axis, the inertia-coupling terms of the left-hand side of equations (4) to (6) exactly cancel the precession-torque terms of the right-hand side, and each of the motions of the satellite is the same as though it were occurring in a single-degree-of-freedom system.

The three-degree-of-freedom response to step displacements, including the nonlinear inertia-coupling and precession-coupling terms, was solved by using a digital computer. The results are presented in figures 2 to 4. For the case shown in figure 2, the following command and control equations were assumed:

$$\omega_{X,c} = 15{,}000\phi + 30{,}000p \qquad \text{(limited to } \pm 2{,}100 \text{ radians/sec)}$$

$$\omega_{Y,c} = 15{,}000\theta + 30{,}000q \qquad \text{(limited to } \pm 2{,}100 \text{ radians/sec)}$$

$$\omega_{Z,c} = 15{,}000\psi + 30{,}000r \qquad \text{(limited to } \pm 2{,}100 \text{ radians/sec)}$$

$$\omega_X = \frac{\omega_{X,c}}{1 + 5s}$$

$$\omega_Y = \frac{\omega_{Y,c}}{1 + 5s}$$

$$\omega_Z = \frac{\omega_{Z,c}}{1 + 5s}$$

A simplifying assumption that the attitude angles of the satellite would be equal to the integral of the attitude rates was made as follows:

$$\theta = \theta_o + \int q\, dt$$

$$\psi = \psi_o + \int r\, dt$$

$$\phi = \phi_o + \int p\, dt$$

This assumption causes an error in the time history of the attitude angles, with the error being proportional to the size of the angles. This error results because the higher order terms present in the exact relationship between the Euler angles and body-axes rates are neglected. However, for the small angles used in these examples, there should be no effect on the stability or performance characteristics of the system. Other assumptions are as follows:

$$I_X = 800 \text{ slug-feet}^2$$

$$I_Y = 1{,}200 \text{ slug-feet}^2$$

$$I_Z = 400 \text{ slug-feet}^2$$

$$I_R = 0.04 \text{ slug-foot}^2$$

$$\theta_o = 0.35 \text{ radian}$$

$$\phi_o = 0.35 \text{ radian}$$

$$\psi_o = 0$$

$$p_o = q_o = r_o = 0$$

As is shown in figure 2, the pitch and roll wheels accelerate rapidly at first; this acceleration causes pitch and roll motions in the satellite. The satellite is brought to within a few arc-seconds of the desired attitude in 60 seconds. The yaw attitude is not disturbed during these motions. In figures 2 to 4 the ordinate scale is enlarged in the middle of the run to show more clearly the final portion of the run.

Another case in which it was assumed that the pitch wheel had an inital amount of momentum, or rotation $\left(\omega_{Y,o} = 500 \text{ radians/sec}\right)$, is shown in figure 3. In this case the momentum stored in the wheel causes a combination of inertia-coupling and precession-coupling torques to occur which result in final steady rotations of the roll and yaw wheels, and steady-state errors of 0.03, 0.01, and 0.003 radian in pitch, yaw, and roll, respectively.

The satellite can be driven to zero error by the addition of an integral of the angular error term to the error signal, so that, for example,

$$\omega_{Y,c} = 15{,}000\theta + 30{,}000q + 750\int \theta \, dt$$

A case with the same initial conditions as those given in the foregoing case is shown in figure 4. The transient response is more oscillatory in this example because of the addition of the integral term, but the satellite is driven to within 20 arc-seconds of the desired attitude in 120 seconds. The steady-state rotations of the control wheels are approximately the same as in the previous case. Thus, the use of the sum of the attitude error, the attitude rate, and the integral of the attitude error provides a precise control with good dynamic response.

Addition of a magnet.- If some disturbance was continuously applied to the satellite, it would cause the inertia wheels to reach their maximum angular velocities and make them useless for control. A study has been made to determine whether a bar magnet could produce a trimming torque that would cancel the disturbance torques, such as those due to aerodynamic moments, which will be encountered in orbit and prevent saturation of the control wheels. The torque produced by a magnet is given by the following formula, where mks units are used:

$$T = \frac{B}{\mu_o} I'Al \sin \gamma \tag{7}$$

where

B	magnetic-field flux density, webers/m^2
μ_o	permeability of free space, 12.7×10^{-7} weber/amp-m
I'	magnetic moment per unit volume, webers/m^2
A	cross-sectional area of magnet, m^2
l	length of magnet, m
γ	angle of magnet with respect to magnetic field, radians

The variations in magnitude and direction of the magnetic field of the earth are given in references 4 and 5. At the equator this field strength is 0.31×10^{-4} weber per square meter. A suitable magnetic material, such as Alnico V, has a residual intensity of 1.2 webers per square meter. Inserting these values into equation (7) gives the torque in newton-meters for a given volume of magnet. Expressed in engineering units, the maximum torque produced by a magnet at a 300-mile altitude near the equator where the field strength is 0.25×10^{-4} weber per square meter is approximately 0.001 foot-pound per pound of magnet.

One of the problems associated with the use of a magnet is that of properly orienting it with respect to the earth's magnetic field. The direction of the magnetic field varies with respect to the gimbal mount of the magnet either when the attitude of the satellite changes or when the position of the satellite in its orbit changes so as to change the latitude of the satellite with respect to the magnetic equator. A magnet can produce a torque vector only in a plane which is perpendicular to the magnetic lines of force.

A method of obtaining useful control torques with a magnet, in spite of these restrictions, is described now. The discussion is of a conceptual arrangement which is easy to describe. Methods of simplifying the actual construction of the mechanism are described in subsequent sections. The mechanism consists of a relatively large magnet mounted in a servo gimbal arrangement, which would be used to create torques, and a small magnet, which would be used to measure the orientation of the magnetic field with respect to the body axes of the satellite. A sketch of the arrangement is shown in figure 5. Because the outer two gimbals of the large magnet mount are driven to the same angle as the two gimbals of the small magnet, the third gimbal axis is pointed in the direction of the magnetic lines of force. The third and fourth gimbals are oriented as follows. Since the purpose of the magnet is to remove residual momentum from the inertia wheels, the angular velocity of these wheels is used to command the third and fourth gimbal angles. Assume that the three inertia wheels, which are mounted on the fixed axes in the body of the satellite, have some angular velocity. These angular-velocity vectors can be resolved to the axis system defined by the magnetic field. The magnetic axes are displaced from the body axes by the angles ψ_{MF} and θ_{MF}. The X"-axis is alined with the magnetic field. Then,

$$\omega_{X''} = \omega_X \cos \psi_{MF} \cos \theta_{MF} + \omega_Y \sin \psi_{MF} \cos \theta_{MF} - \omega_Z \sin \theta_{MF} \qquad (8)$$

$$\omega_{Y''} = -\omega_X \sin \psi_{MF} + \omega_Y \cos \psi_{MF} \qquad (9)$$

$$\omega_{Z''} = \omega_X \cos \psi_{MF} \sin \theta_{MF} + \omega_Y \sin \psi_{MF} \sin \theta_{MF} + \omega_Z \cos \theta_{MF} \qquad (10)$$

The third gimbal can be rotated so that the fourth gimbal axis points in the direction of the resultant vector $\overline{\omega_{Y''}} + \overline{\omega_{Z''}}$ by the command

$$G_3 = \tan^{-1} \frac{\omega_{Z''}}{\omega_{Y''}} \qquad (11)$$

If the magnet is mounted on the fourth gimbal axis and is rotated on this axis, it produces a torque vector which is opposite to the vector

$\overline{\omega_{Y''}} + \overline{\omega_{Z''}}$. This torque causes the vector $\overline{\omega_{Y''}} + \overline{\omega_{Z''}}$ to vanish. With this arrangement the magnet can be used to its fullest advantage all of the time. However, the fact that the vector $\overline{\omega_{Y''}} + \overline{\omega_{Z''}}$ is made to disappear does not necessarily mean that all of the inertia-wheel angular velocities are stopped or even reduced.

Consider the two-dimensional situation shown in figure 6. If the magnetic field is displaced from the body X-axis by the angle ψ_{MF} and there exists an angular velocity of the X-axis wheel, the magnet would be commanded to produce a torque which would be directed in the Y"-direction. This torque would have components in the X and Y directions, so that ω_X would be reduced, but some ω_Y would be caused to come into existence. When the Y"-components of ω_X and ω_Y become equal, and they would of necessity be of opposite sign, $\omega_{Y''}$ would become zero and G_4 would be commanded to produce zero torque. Thus, the system would not necessarily reduce all of the angular velocities of the wheels to zero, but the amount of cross coupling due to the magnet is limited and a net reduction in momentum would be realized.

The four-gimbal arrangement previously described is convenient for explaining the system, but it is not necessary to build the mechanism in this manner. The operations just described can be achieved with only the first two gimbals. If the angles G_3 and G_4 are resolved so as to obtain their components in the direction of the g_1 and g_2 axes and these components are added to the angles measured by the small magnet, the resulting orientation of the magnet would be exactly the same as obtained by using the four gimbals with the exception of the rotation of the magnet about its long axis, which is of no significance. The equations for G_1 and G_2 are as follows:

$$G_1 = \psi_{MF} - G_3 \sin \theta_{MF} + G_4 \sin G_3 \cos \theta_{MF}$$

$$G_2 = \left(\theta_{MF} + G_4 \cos G_3\right)\cos\left(-G_3 \sin \theta_{MF} + G_4 \sin G_3 \cos \theta_{MF}\right)$$

$$+ \left(G_3 \cos \theta_{MF} - G_4 \sin G_3 \sin \theta_{MF}\right)\sin\left(-G_3 \sin \theta_{MF} + G_4 \sin G_3 \cos \theta_{MF}\right)$$

In order to treat the problem analytically using the magnet it is necessary to formulate equations for the body-axis torques of the magnet. These equations are

$$T_X = T \sin G_3 \sin \theta_{MF} \cos \psi_{MF} - T \cos G_3 \sin \psi_{MF} \qquad (12)$$

$$T_Y = T \cos G_3 \cos \psi_{MF} + T \sin G_3 \sin \theta_{MF} \sin \psi_{MF} \qquad (13)$$

$$T_Z = T \sin G_3 \cos \theta_{MF} \qquad (14)$$

The response of the system, using both the inertia wheels and a magnet for control, to the same initial displacement as used before is shown in figure 7. It was assumed that the magnetic field was at a yaw angle of 0.35 radian and a pitch angle of zero radian to the initial body axes. The angles between the magnetic field and the body axes are equal to the sum of the body-axis Euler angles from the initial position and the angle of the magnetic field from the initial position; thus,

$$G_1 = -\psi_S + \psi_{MF}$$

$$G_2 = -\theta_S + \theta_{MF}$$

In this example, G_4 was assumed to be a discontinuous function so that it was 90° whenever the sum of the absolute values of $\omega_{Y''}$ and $\omega_{Z''}$ is greater than 0.02 radian per second; that is,

$$\left.\begin{aligned} G_4 &= 90^\circ \qquad \left(\text{when} \quad |\omega_{Y''}| + |\omega_{Z''}| > 0.02\right) \\ G_4 &= 0^\circ \qquad \left(\text{when} \quad |\omega_{Y''}| + |\omega_{Z''}| < 0.02\right) \end{aligned}\right\} \qquad (15)$$

It was assumed that a 25-pound magnet, which could produce 0.025 foot-pound of torque, was used. Therefore, when $G_4 = 90^\circ$,

$$T = 0.025 \sin G_4 = 0.025$$

and when $G_4 = 0^\circ$,

$$T = 0.025 \sin G_4 = 0$$

The initial response of the system, as shown in figure 7, is approximately the same as was obtained previously with the inertia wheel alone (fig. 4). However, ω_Z is now being steadily reduced. This reduction in ω_Z would continue at the same rate until it reached zero. Also, ω_Y is being reduced and ω_X is being increased. These changes would continue until ω_Y and ω_X reached values proportional to the sine and cosine of the yaw angle of the magnetic field.

Attitude Control of a Space Station

During Several Orbits

In order to determine the feasibility of the inertia-wheel and magnet control system for stabilizing an orbiting space station, the ability of the magnet to prevent saturation of the inertia wheels during a number of orbital revolutions must be studied. The station is assumed to derive some energy from a solar collector and, therefore, the requirement must be met that the station point at the sun continuously with fairly good accuracy. It was assumed that even when the station was in the shade of the earth, it would still point in the direction of the sun. It was also assumed that the station would be in a 300- or 400-mile-high orbit. At these altitudes, the station would be subject to appreciable aerodynamic torques. The assumed configuration of the station is shown in figure 8. The main aerodynamic forces would be on the sun-collecting parabola, and with the assumed configuration, in which the parabola is located as far from the center of gravity of the station as is likely, maximum torques would be experienced. If the station were in a circular orbit, these aerodynamic torques would be nearly symmetrical - that is, plus for half the orbit and minus for the other half - so that the net momentum imparted to the station during one complete orbit would be zero. However, if the orbit were elliptical, the variation in dynamic pressure experienced during the orbit would cause the aerodynamic torque to be unsymmetrical, and the total amount of momentum imparted to one of the axes of the satellite would increase with each orbit. A typical variation of the aerodynamic torques is shown in figure 8. These torques are derived in appendix A. It was assumed that the altitude of the orbit varied from 300 miles to 400 miles, with the 400-mile altitude occurring on the sunny side of the orbit; it was also assumed that there was a dihedral angle of 30° between the plane of the orbit and the line to the sun. It is shown in figure 8 that the net amount of momentum imparted to the Z-axis is negative for this situation.

If inertia wheels alone were used for control in this problem, the Z-axis wheel would become velocity saturated in two or three orbits. It is reasonable to consider the use of a magnet to prevent such an occurrence. For example, a 30-pound magnet would produce approximately 0.030 foot-pound of torque in the orbit considered herein, which is of the same order of magnitude as the maximum disturbance torque. The question, then, is will the magnet spend enough time in a favorable position with respect to the earth's field so that it can eliminate all of the momentum from the system? The inertia wheels would act as accumulators of momentum so that the magnet would not have to oppose the aerodynamic torques at the instant that they are in force.

The effectiveness of a magnet will depend on the variation of the direction of the magnetic field. The earth's field could be represented with sufficient accuracy by a dipole field (ref. 4). The variation of the direction and the relative magnitude of such a field are shown in the upper part of figure 9. It can be seen that a variation in latitude from the magnetic equator to the pole results in a 180° change in direction of the field and that the magnitude of the strength of the field is doubled. Also, since the magnetic field is inclined to the geographic axis of the earth by approximately 12°, a total variation in the inclination of any given satellite orbit to the magnetic equator is 24°. This variation will have a period of one day. One other factor that must be considered is the dihedral angle between the plane of the orbit and the line to the sun which dictates the angle that the X body axis of the satellite must make with respect to the plane of the orbit. The effects of these factors on the gimbal angles of the freely mounted magnet are considered in appendix B, and some examples of variations are presented. The numbers presented are not exact values but involve some small approximations.

The particular problem considered assumes an orbit inclination of 15° and a dihedral angle of 30°. For purposes of determining the magnet gimbal angles the orbit was assumed to be circular at an altitude of 350 miles. The problem was assumed to start with the orbit that was inclined to the magnetic equator by 3°. During this first orbit the magnet would have the least effect on the Z-axis of the satellite. The problem was continued for a period representing one-half day.

The control-system gains were assumed to be the same as before; the inertia of the wheels was assumed to be 0.1 slug-foot^2 and the magnet was assumed to weigh 30 pounds and to produce 0.03 foot-pound of torque. The discontinuous control was used for the magnet. The inertias of the satellite were assumed to be 50,000, 50,000, and 5,000 slug-feet^2 for the Y-, Z-, and X-axis, respectively. For this satellite—inertia-wheel combination, the characteristics of the single-degree-of-freedom linear response of the motion of the long axis are a period of 160 seconds with a damping ratio of 0.3 and a first-order root with a time constant of 5 seconds.

The three-degree-of-freedom response is shown in figure 10. The variation of the angular velocity of the inertia wheels for seven orbits is shown in the lower plot. The dashed curves shown for the first orbit are the variations in ω_Y and ω_Z that would occur if the magnet were not included in the control system. In this case the angular velocity of the Z-axis wheel would increase 400 radians per second each orbit and, therefore, would soon reach a value representative of a mechanical limit. During the first orbit the magnet is the least effective on the Z-axis and, therefore, has little effect on ω_Z. During the second orbit, when the inclination of the orbit to the magnetic equator is increased, the magnet is able to increase the amount of momentum it removes from the X-axis, and there is very little increase in ω_Z during this orbit. On succeeding orbits more momentum is removed from the Z-axis than is put in. The seven orbits shown represent only one-half of the daily cycle that will occur, but it is concluded that the 30-pound magnet that was assumed in this problem is adequate to keep the inertia wheels from becoming saturated.

The upper plot of figure 10 shows the pitch-angle variations that occur. Because of the discontinuous signal in the magnet control, a hunting cycle oscillation occurs whenever either of the resolved vectors ω_Y'' or ω_Z'' approaches zero and, therefore, the vectors change sign in response to small changes in satellite attitude. The period of these oscillations is approximately 100 to 150 seconds. The other attitude angles ψ and ϕ show similar variations. The maximum deviation from the desired attitude was always within ±0.001 radian (3.5 arc-minutes).

The operation just described for the magnet is very ideal in that the need for torque from the magnet on each axis is considered simultaneously and weighed according to the ability of the magnet to supply these torques. A computer would be needed to implement these control laws. However, the mechanisms for deriving operating signals for the magnet can be greatly simplified if some of these features can be deleted. If, from a foreknowledge of the orbit and the motions of the satellite, it can be predicted that the direction of the magnetic field with respect to the body axis of the satellite will move from one axis to another in a short time, then the magnet could be operated according to the need for torque of each axis separately and in succession. The succession would be controlled by the movement of the direction of the magnetic field as measured by the free magnet, and torque would be supplied to that axis for which the magnet was most effective. A slip-ring arrangement on the gimbals of the free magnet could be used to command the succession of control from one axis to another, and a system of relays could be used to control the gimbal angle G_3.

Such a simple control law was applied to the problem of the seven orbits previously described. The control was as follows:

The value of G_4 was assumed to be 90° all of the time.

A sample of the control for G_3 is as follows:

If θ_{MF} is between -45° and 45° and if ω_Z is +, then G_3 is 90°; if ω_Z is -, then G_3 is -90°.

If θ_{MF} is between 45° and 135° and if ψ_{MF} is between 45° and -45°,
and if ω_Y is +, then G_3 is 0°;
if ω_Y is -, then G_3 is 180°.

A complete statement of the G_3 control is given in table I. The results of this application are shown in figure 11 and are quite similar to those shown in figure 10. With the simplified control, slightly larger values of ω_Z are encountered. The on-off nature of the control again results in a limit cycle oscillation, but it is not as apparent as in the previous case.

It should be noted that the inertia of the magnet was not included in the equation of motion. If it had been included, some small additional motions would appear in the time history of the attitude of the satellite. Another problem that must be given consideration is the interference that may exist between the two magnets. It will be necessary, in the actual system, to separate the two magnets so that the large magnet will have only a negligible effect on the small magnet. For example, in the case used in this paper, it would be necessary to locate the small magnet 32 feet from the 30-pound magnet so that the strength of the magnetic field of the 30-pound magnet would be one-tenth the strength of the field of the earth at the location of the small magnet. If it is not possible to achieve this separation, the small magnet could be eliminated and the large magnet could be used to determine the direction of the magnetic field. At proper intervals of time, the large magnet could be freed and the necessary information could be obtained from the position assumed by the first and second gimbals. This information could then be stored and used for the next operating interval.

BENCH TESTS

Bench tests were conducted with a single-degree-of-freedom space stabilization system made up of surplus autopilot components mounted on a waterborne platform. Control torque for maintaining the attitude of the platform was furnished by the acceleration reaction of an inertial flywheel, and the torque for eliminating residual flywheel momentum was obtained from a permanent bar magnet mounted on a servomotor. The initial configuration of the space control simulator used a directional gyro for an error detector and had a self-contained power supply. Due to steady-state drift, the directional gyro was replaced by a photoelectric sensor. In addition, the self-contained battery-inverter power supply was replaced by small, lightly suspended lead-in wires. This modification eliminated frequent battery changes and furnished unwavering power required for long running times, with a negligible response penalty. A sketch of the modified configuration is shown in figure 12.

Description of Apparatus

Sensor.- The sensor for the final configuration of the space control simulator was made up of a pair of phototransistors. The wide-angle capabilities of the control system to return the platform to trim in the presence of large step displacements or rate inputs had been investigated previously with the directional gyro, and the photocell sensor was designed for high resolution about zero. Figure 12 shows the sensor guide horn containing the photocells. The stationary arc-light target rays were maintained parallel by a diverging lens off the platform and a mating converging lens on the platform and thence were reflected by a mirror into the photocell guide horn. When the platform yawed, a 0.1-inch-diameter light spot swept across the photocells. This optical arrangement prevented any sweep of the light rays across the cells due to translational movement of the platform and, consequently, any error seen by the photoelectric detector was an angular error. When the platform was in trim, the photocells produced equal but opposite voltages for zero error.

Amplifier.- A standard alternating-current amplifier from a displacement-type autopilot was used to control the flywheel drive motor. The voltage input to the amplifier was the servo-loop error made up of the difference between the input voltage from the photoelectric sensor and the compatible rate-feedback voltage from the tachometer windings of the flywheel servomotor.

Flywheel servomotor.- The servomotor which turned the reaction flywheel was a conventional motor-tachometer combination. The motor feedback commanded a flywheel rate. The motor had a 7.5-second time constant. At large step-displacement errors of up to 1 radian in simulator attitude, the inertial wheel produced a restoring torque that could accelerate the platform up to 0.04 radian per second in 7.5 seconds, quickly stop it at trim, and bring it to rest. If some initial motion, within the 0.04 radian-per-second capability of the wheel, existed in the platform before the inertial wheel undertook to correct for an error, this motion was represented by an equal amount of residual angular momentum in the wheel after the platform was brought to the desired attitude, and the wheel continued to spin unless some external trimming force was applied. If a minute undirectional force was applied to the platform for a long period of time, the inertia wheel would maintain the platform at a very small error by accelerating in the same direction as the force until the wheel reached its limiting speed where it could no longer produce torque; at this point the platform responded to the disturbing force.

Magnet circuit.- The 2.55-pound permanent bar magnet was mounted on a servomotor. The signal from the error sensor commanded the magnet circuit as well as the flywheel circuit. A selsyn position followup was used as the stabilizing feedback on the magnet circuit and, consequently, the bar magnet was commanded to a position proportional to the angular error of the platform. The magnet was set at magnetic north with the flywheel circuit trimmed so that the two control circuits were compatible at high resolution. When the control system received a signal from the sensor, the magnet sought a position in the same direction that the flywheel was accelerating; when the magnet was 90° to the lines of the earth's magnetic force, it produced 0.00176 foot-pound of external moment, even though the plane of magnetic flux had a 70° angle incident to the surface of the earth. The characteristic of the table-magnet combination was an undamped oscillation with a period of 2 minutes. A simplified block diagram of the space control system is presented in figure 13.

System moments of inertia.- The initial control system had a self-contained power supply. This power supply was replaced by small external lead-in wires. Tests using a calibrated torsion spring, with and without the lead-in wires attached, showed no appreciable difference in the response of the waterborne tank when oscillating at the representative 2-minute period. The moment of inertia of the final version of the test table was 5 slug-feet^2. The flywheel-servomotor combined moment of inertia was 0.0001 slug-foot^2. Thus, the flywheel-table moment-of-inertia ratio was 2×10^{-5}.

Instrumentation.- Remote instrumentation for the bench tests was composed of a system of mirrors. A direct-current arc light was beamed to a small mirror mounted on the platform, thence folded through three external mirrors to a length of 48 feet, and projected on a scale calibrated to read 180 arc-seconds per inch. Readings of 18 arc-seconds, or a limit-cycle double amplitude of 0.1 inch, could be made quite readily.

Results

A typical response time history of the control system is shown in figure 14. Figure 14(a) presents a response of the original system, with a directional gyro for a sensor, in the presence of a 1-radian step displacement. The response of the table using the flywheel alone is compared with that using the flywheel-magnet combination. The continuing effect of the magnet, after the flywheel had reached limiting speed, is shown as the platform rate approaches the design value of 0.06 radian per second. Figure 14(b) shows a high-resolution run with the photoelectric sensor. The run was started with some momentum in the table. The input for this run was a 0.018-radian disturbance accomplished by shielding the photocells from the light source momentarily as the platform was displaced by the weight of a sheet of writing paper held gently against a peripheral component on the platform. The inertia wheels bring the table to an error of 0.005 radian, which corresponds to the angular velocity of the wheel divided by the error gain. The angular velocity of the wheel represents the initial momentum of the table, which is now stored in the wheel. The magnet then reduces the stored momentum, and the error is further reduced until a limit cycle of ±0.000045 radian (±9 arc-seconds) with a period of 2 minutes is reached.

CONCLUSIONS

An analytical study of the use of a combination of inertia wheels and a magnet to control the attitude of a satellite indicates that the following conclusions may be stated:

1. An error signal that consists of the sum of the attitude error, the attitude rate, and the integral of the attitude error commanding the angular velocity of the wheel can give good dynamic response and precise control.

2. The three-degree-of-freedom cross-coupling moments had no serious effects on the stability of the system.

3. The magnet can remove the angular momentum of the system resulting from large aerodynamic moments in a case with a typical orbit. The maximum angular deviations in this case were within ±3.5 arc-minutes.

Single-degree-of-freedom bench tests generally verified the dynamic response predicted by the analytical study. It was possible to control the bench-test configuration to within ±9 arc-seconds of the reference direction, even though the hardware components that were used in these tests were not specifically designed for the control system.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., September 2, 1960.

APPENDIX A

AERODYNAMIC MOMENT ON SATELLITE

If the satellite is assumed to be attitude-stabilized in a space reference system, the aerodynamic moments on the satellite can be calculated in the following manner. The orbital velocity for several points around the orbit was determined first. In this particular example the 300-mile altitude was assumed to occur at position 1 and the 400-mile altitude at position 7. (See fig. 15.) Then, by using the assumed value for the inclination of the orbital plane with the ecliptic plane Ω and an angle which describes the position of the satellite in its orbit λ, the body-axis velocities can be determined from the following relations:

$$V_Y = V \cos \lambda$$

$$V_X' = V \sin \lambda$$

$$V_X = V_X' \cos \Omega$$

$$V_Z = V_X' \sin \Omega$$

The angle of attack and the angle of sideslip can now be determined by use of the following formulas:

$$\alpha = \sin^{-1} \frac{V_Z}{V}$$

$$\beta = \sin^{-1} \frac{V_Y}{V}$$

Then the total angle of incidence is the vector sum of α and β. The variation of the moment factor for the assumed shape of this satellite with angle of incidence, based on Newtonian flow theory, is shown in figure 15. This information, together with the assumed variation in density shown in figure 15, can be used to determine the total aerodynamic moment on the satellite at various points in the orbit. The velocity vectors V_Y and V_Z determine the direction of this moment vector in the YZ-plane of the satellite, as is shown in figure 15, and from this relationship the body-axis moments M_Y and M_Z can be determined. These moments were programed in the problem as a function of time.

APPENDIX B

VARIATION OF GIMBAL ANGLES DURING ORBIT

The Euler angles, which describe the direction of the magnetic field with respect to the body axes of the satellite and which are equivalent to the gimbal angles G_1 and G_2, depend on the longitude and latitude of the satellite with respect to the magnetic equator. The longitude and latitude are defined by the inclination of the orbit to the magnetic equator δ and an angle which defines the position of the satellite in its orbit λ. The inclination of the orbit to the geographic equator was assumed to be 15°. The magnetic equator is inclined to the geographic equator by an angle of 12°. Because of these relationships, the inclination of the orbit to the magnetic equator varies from 3° to 27° in one-half day. It is assumed that the problem starts when the orbit is inclined to the magnetic equator by an angle of 3°. The variation of the inclination of the orbit to the magnetic equator with time would be a sine function of the angle of rotation of the earth, but in this example a linear variation was used. Since $7\frac{1}{2}$ orbits occur in one-half day, this inclination increases by 3.2° each orbit. To simplify the calculations, it was assumed that this inclination remained constant during each orbit and made a step increase for the succeeding orbit. During the first orbit the satellite will be at zero latitude at positions 10 and 4. (See fig. 16.) The variation in latitude and longitude from these points, called 1' and 7', can be calculated by the following equations:

$$\sin(\text{latitude}) = \sin \delta \sin \lambda \tag{B1}$$

$$\sin(\text{longitude}) = \tan(\text{latitude})\cos \delta \tag{B2}$$

The variation in longitude is equal to the variation in the yaw gimbal angle G_1. It was assumed that the yaw gimbal angle would be zero when the magnetic-field direction was in the XZ-plane of the satellite and, therefore, would be zero at position 1. Thus, by shifting both the zero and the position number of the longitude variation calculated by use of equations (B1) and (B2), the variation in yaw gimbal angle with position, and therefore with time, could be obtained.

The dip angle of the magnetic field is a function of latitude. By taking the latitude calculated by use of equation (B1), by using values from the curve of dip angle plotted against magnetic latitude shown in figure 9, and by taking the proper shift in position number, the variation in dip angle with time can be obtained. The resulting dip-angle variation for the first orbit is shown in figure 16. Since the satellite is oriented parallel to the ecliptic plane, to obtain the pitch gimbal angle the projection of the angle between the ecliptic plane and the magnetic equator must be added to the magnetic dip angle. The approximate expression for this addition is $(\Omega - \delta)\cos G_1$. A sample calculation of G_1 and G_2 for the first orbit is presented in table II.

During the second orbit the inclination δ is 6.2°. Also, the line of intersection of the orbit plane with the magnetic equator shifts. This shift occurs at a rate that would place the intersection line at position 7 during the fourth orbit and back to position 11 during the eighth orbit. It was assumed that a 22.2° shift occurs between successive orbits. The situation for the second orbit is shown in figure 16. The longitude and latitude from position 1' are now calculated for the new inclination angle, and the procedure just described is repeated to obtain the gimbal angles.

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TABLE I.- CONTROL FOR G_3

θ_{MF}, deg	ψ_{MF}, deg	ω_X	ω_Y	ω_Z	G_3, deg
-45 to 45	0 to 360			+	90
-45 to 45	0 to 360			-	-90
135 to 225	0 to 360			+	-90
135 to 225	0 to 360			-	90
45 to 135	-45 to 45		+		0
45 to 135	-45 to 45		-		180
45 to 135	135 to 225		+		180
45 to 135	135 to 225		-		0
45 to 135	45 to 135	+			180
45 to 135	45 to 135	-			0
45 to 135	225 to 315	+			0
45 to 135	225 to 315	-			180
225 to 315	-45 to 45		+		0
225 to 315	-45 to 45		-		180
225 to 315	135 to 225		+		180
225 to 315	135 to 225		-		0
225 to 315	45 to 135	+			180
225 to 315	45 to 135	-			0
225 to 315	225 to 315	+			0
225 to 315	225 to 315	-			180

TABLE II.- DETERMINATION OF G_1 AND G_2 FOR THE FIRST ORBIT

Position	Magnetic longitude, deg	Magnetic latitude, deg	Dip angle, deg	Position	G_1, deg	Dip angle, deg	$(\Omega - \delta)\cos G_1$, deg	G_2, deg
1'	0	0	90	1	0	80	-27	53
2'	30	1.5	87	2	30	83	-23	60
3'	60	2.6	83	3	60	87	-13	74
4'	90	3.0	80	4	90	90	0	90
5'	120	2.6	83	5	120	93	13	106
6'	150	1.5	87	6	150	97	23	120
7'	180	0	90	7	180	100	27	127
8'	210	-1.5	93	8	210	97	23	120
9'	240	-2.6	97	9	240	93	13	106
10'	270	-3.0	100	10	270	90	0	90
11'	300	-2.6	97	11	300	87	-13	74
12'	330	-1.5	93	12	330	83	-23	60
1'	0	0	90	1	0	80	-27	53

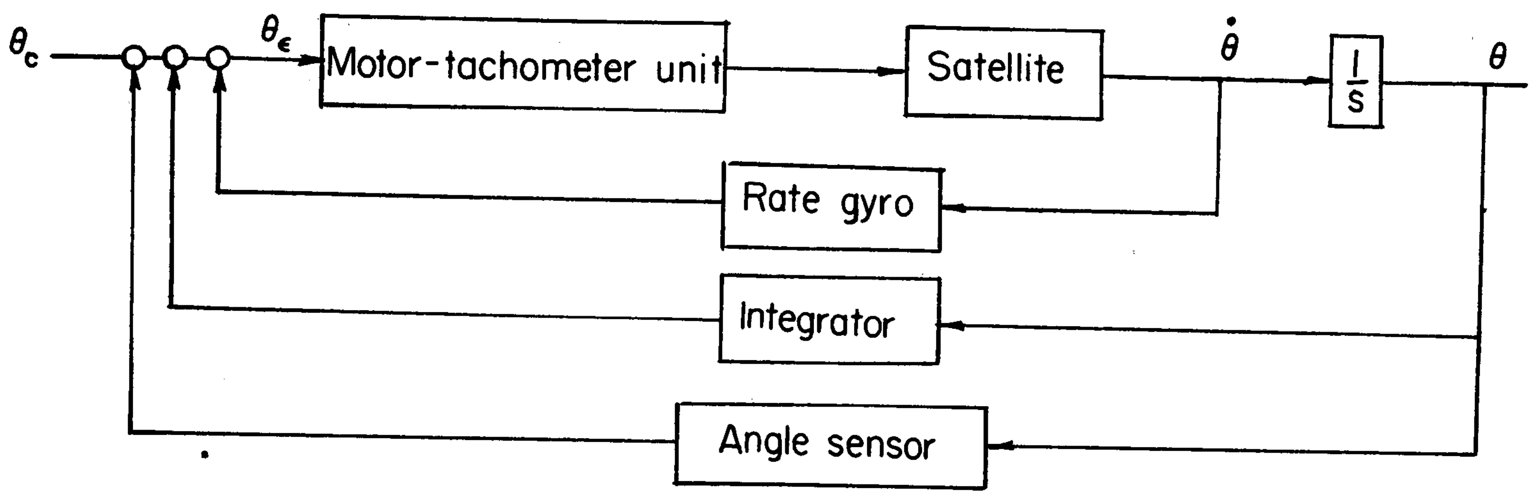


Figure 1.- Block diagram of inertia-wheel control system.

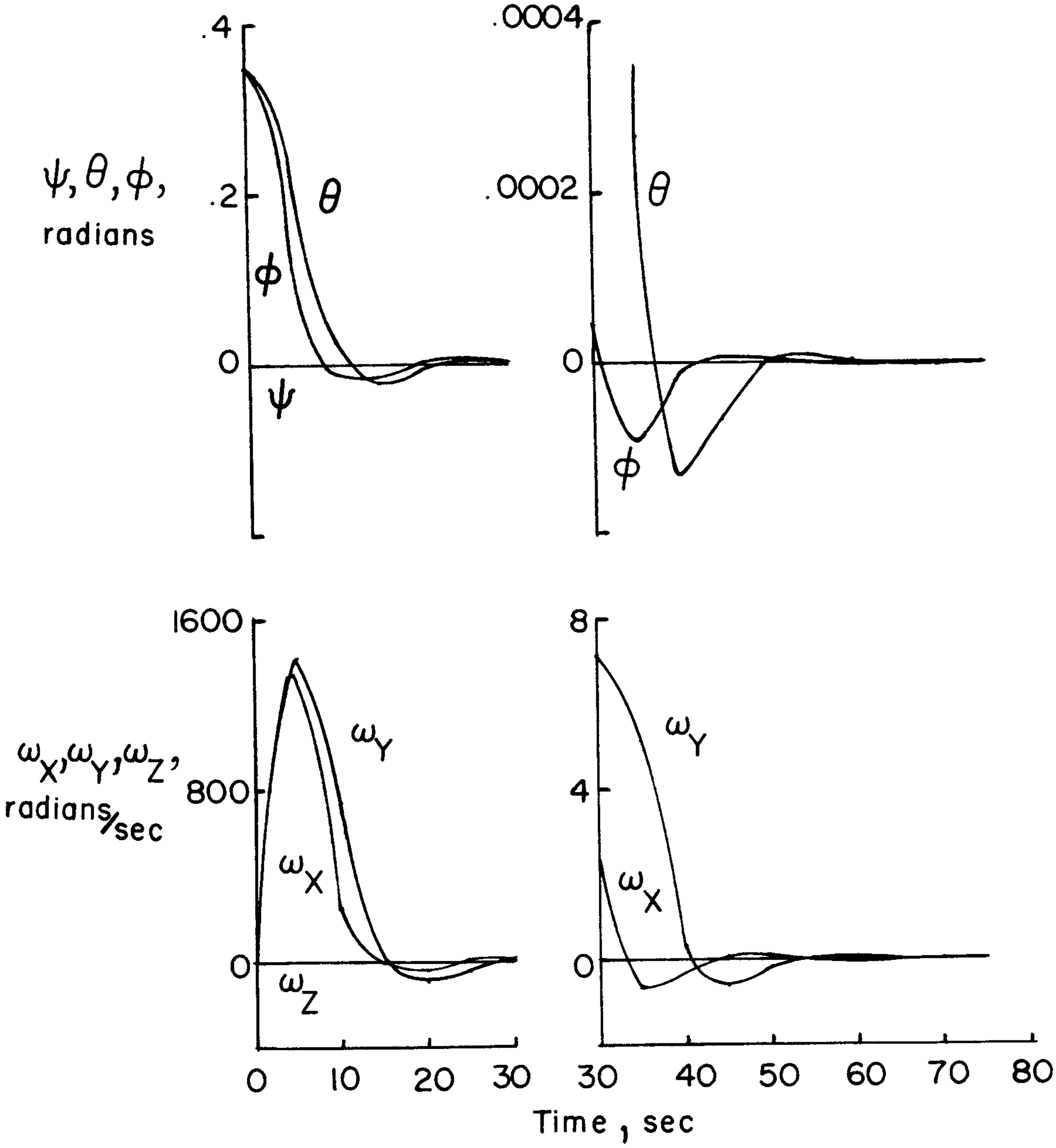


Figure 2.- Three-degree-of-freedom response to step displacement. Control by inertia wheels alone; I_X = 800; I_Y = 1,200; I_Z = 400; ω_c = 15,000(angle) + 30,000(angular rate).

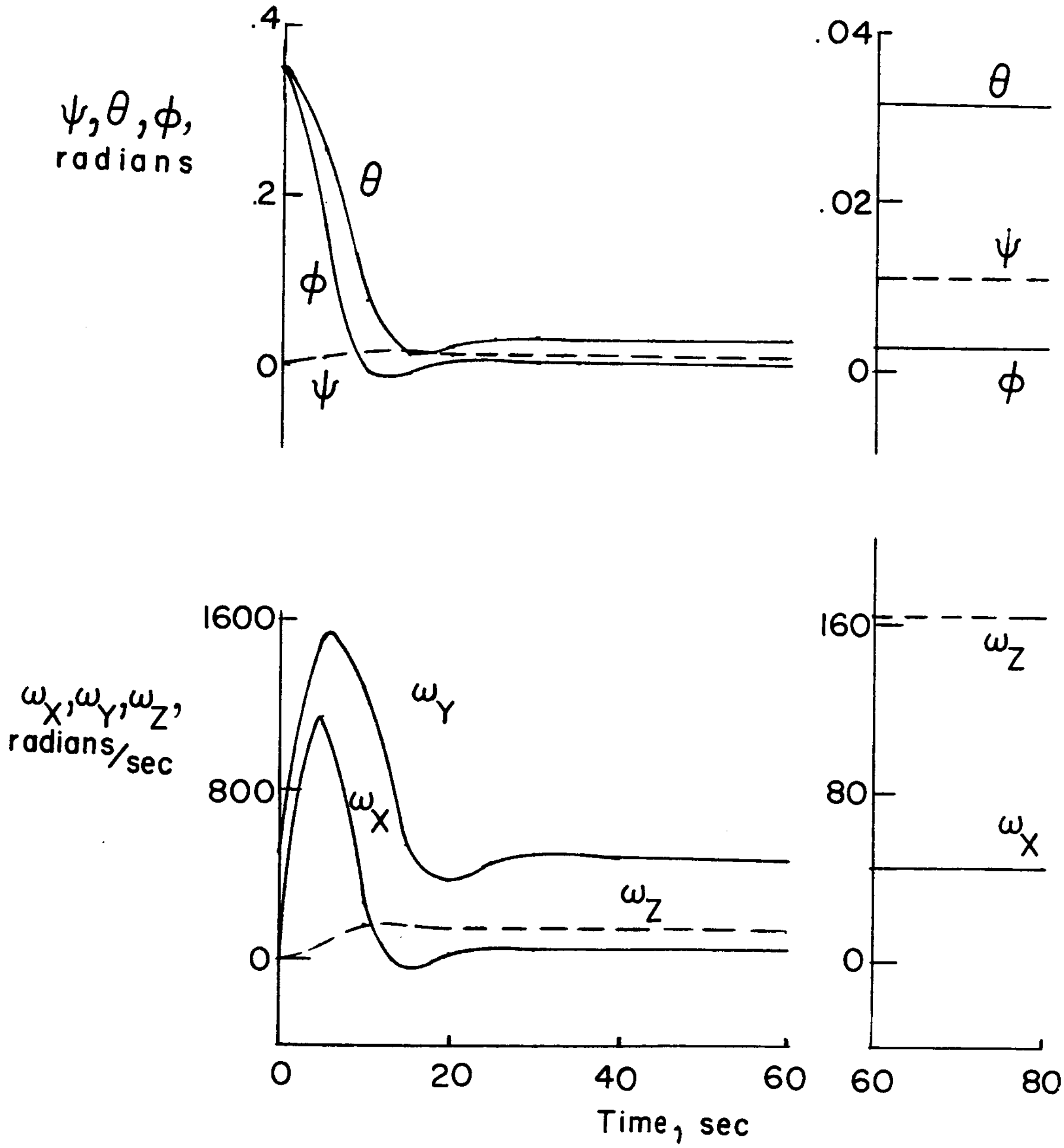


Figure 3.- Three-degree-of-freedom response to step displacement. Control by inertia wheels alone; I_X = 800; I_Y = 1,200; I_Z = 400; $\omega_{Y,o}$ = 500; ω_c = 15,000(angle) + 30,000(angular rate).

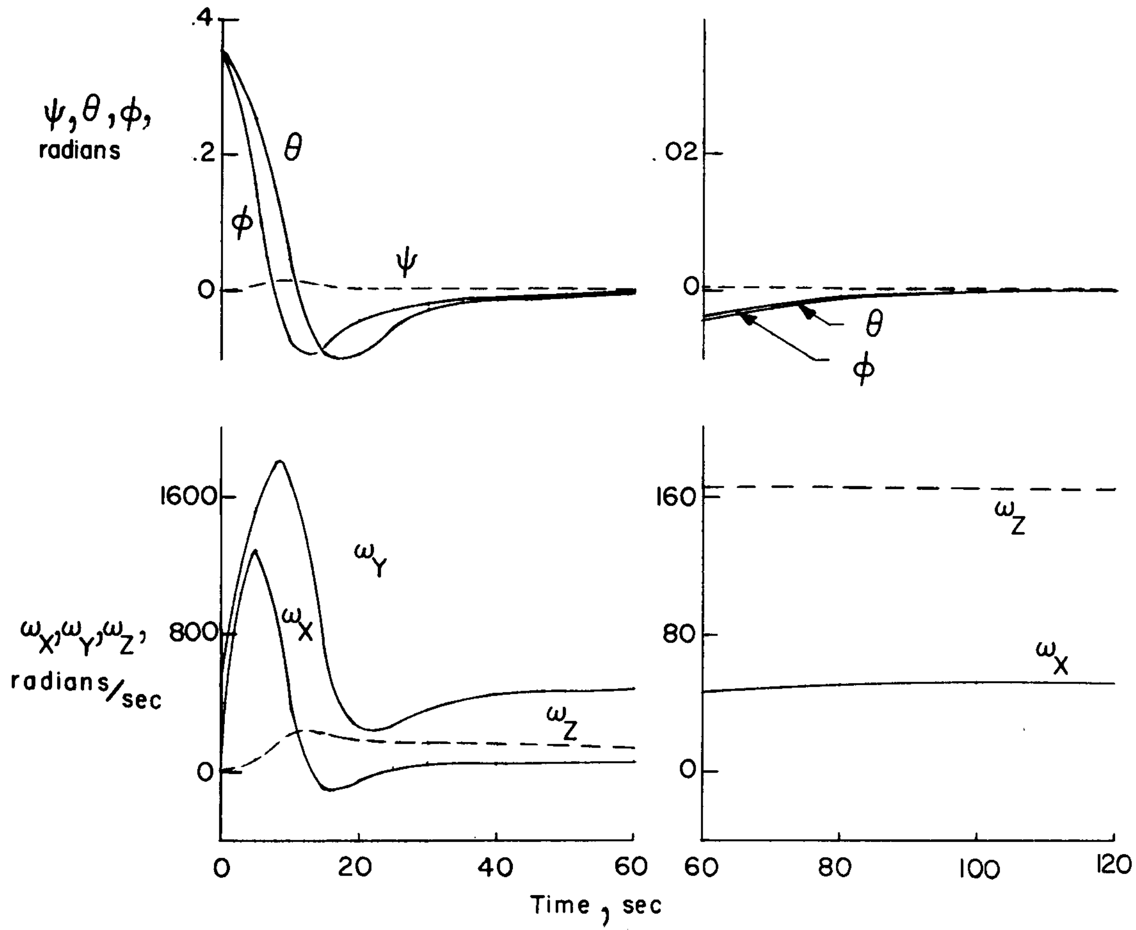


Figure 4.- Three-degree-of-freedom response to step displacement. Control by inertia wheel alone; $I_X = 800$; $I_Y = 1,200$; $I_Z = 400$; $\omega_{Y,o} = 500$;

$$\omega_c = 15,000(\text{angle}) + 30,000(\text{angular rate}) + 750 \int (\text{angle})dt.$$

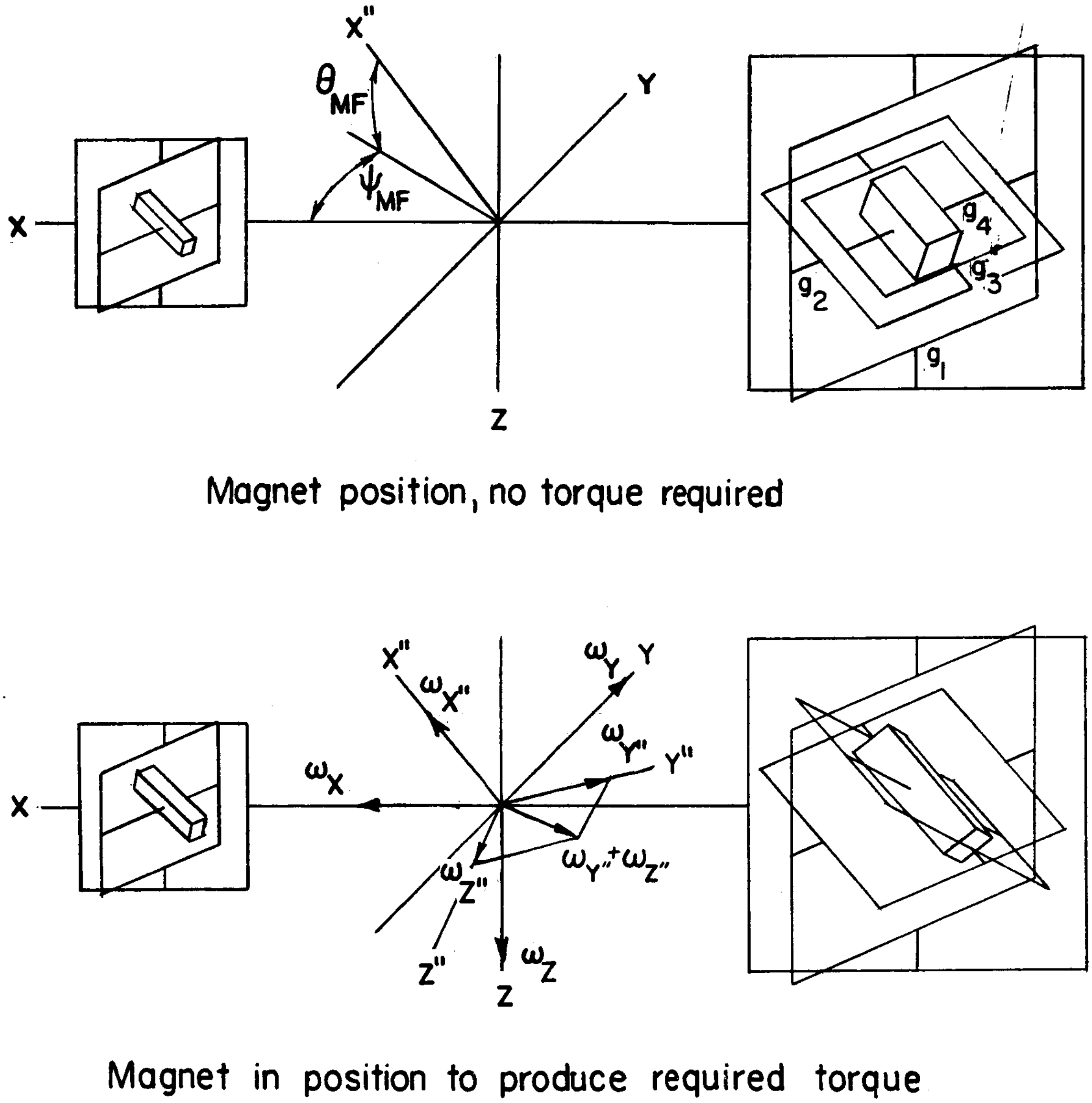
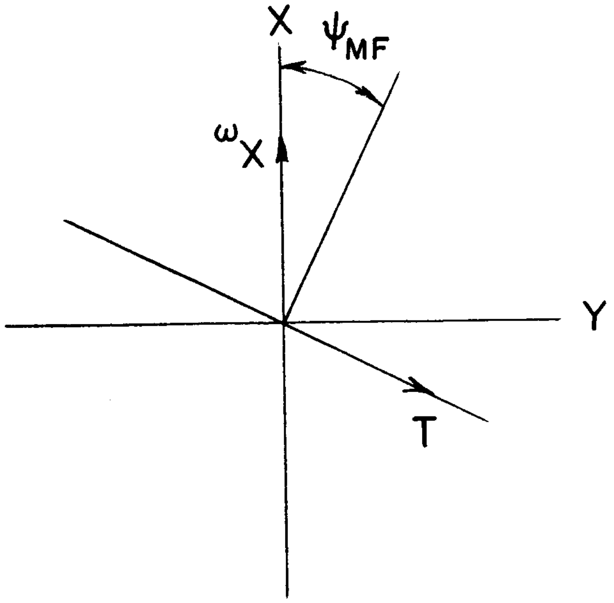
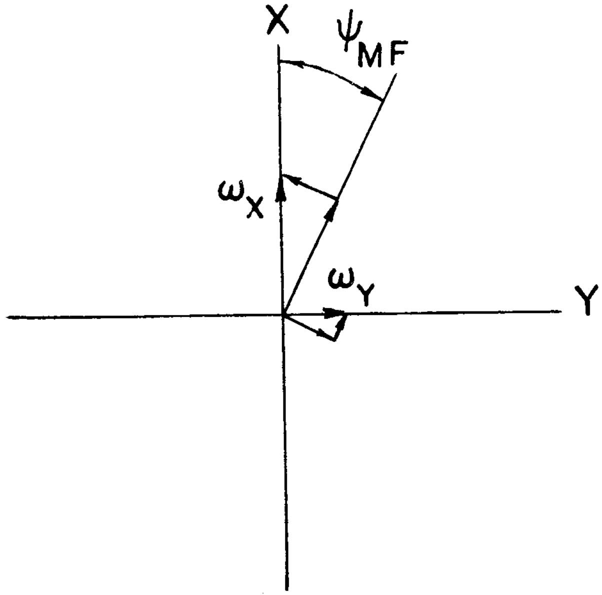


Figure 5.- Sketch of magnet gimbal system.



Beginning of run



Conclusion

Figure 6.- Two-degree-of-freedom illustrative example of effect of magnet.

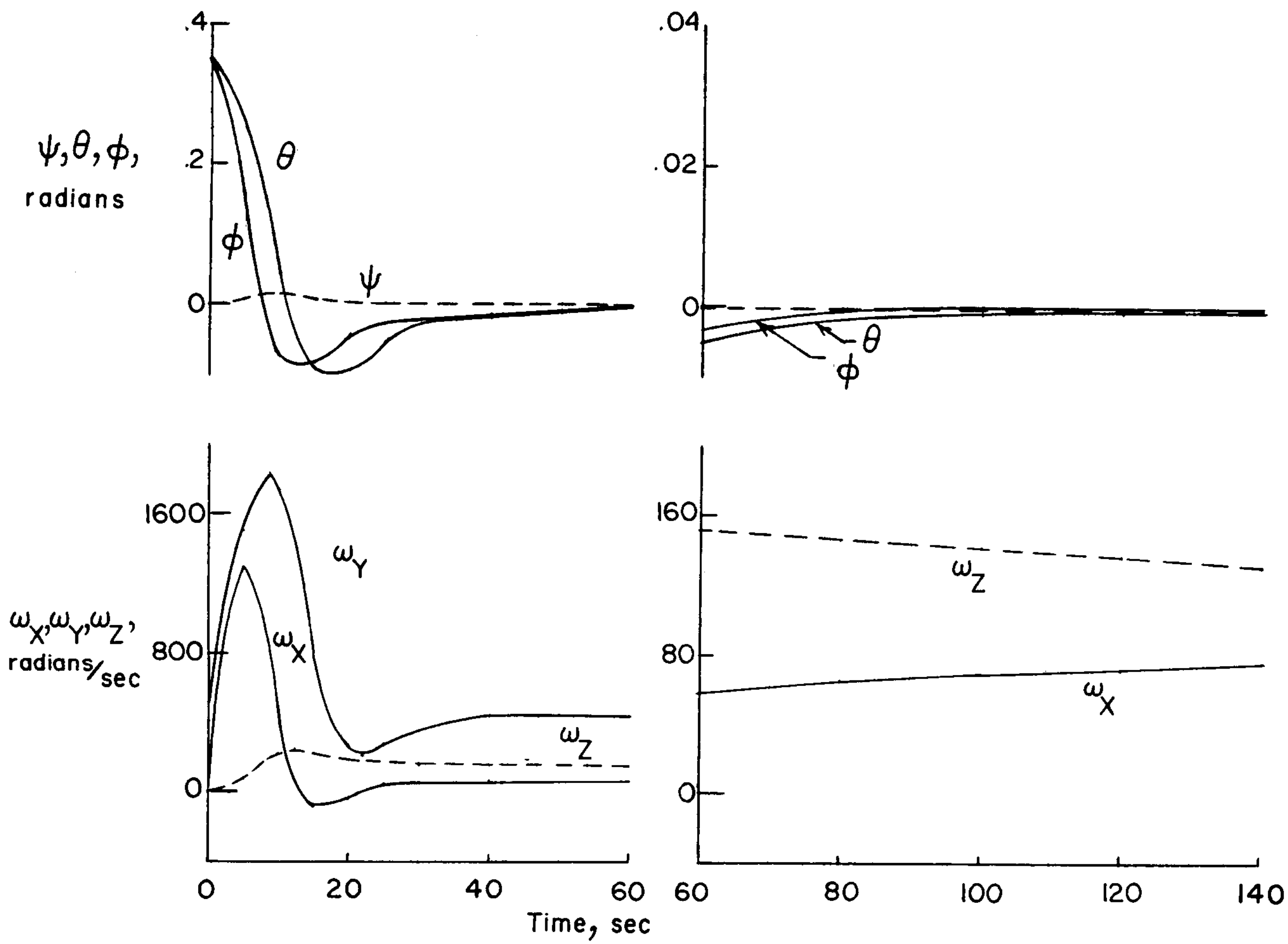


Figure 7.- Three-degree-of-freedom response to step displacement. Control by inertia wheels and magnet; $I_X = 800$; $I_Y = 1,200$; $I_Z = 400$; $\omega_{Y,o} = 500$;

$$\omega_c = 15,000(\text{angle}) + 30,000(\text{angular rate}) + 750 \int (\text{angle})dt.$$

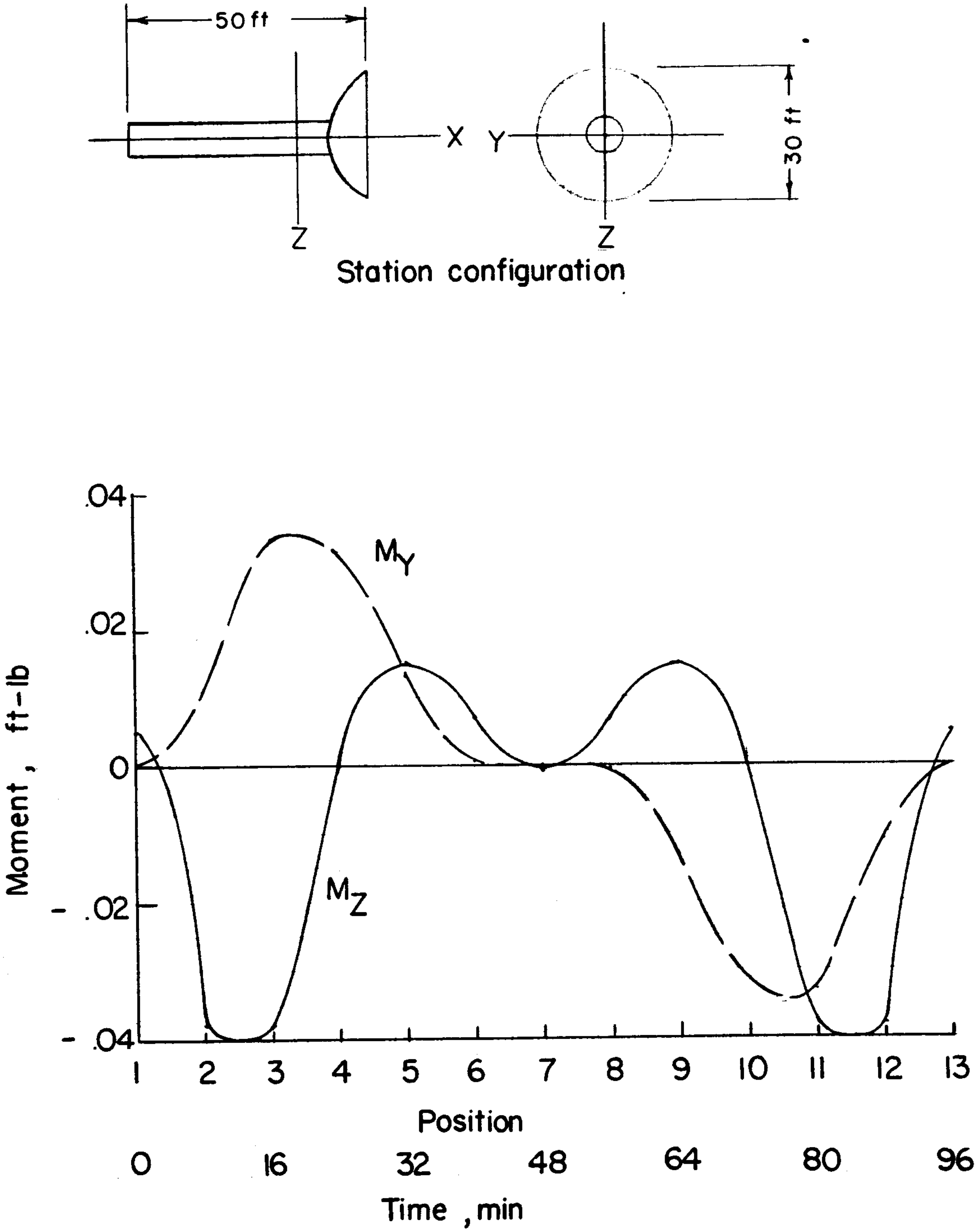
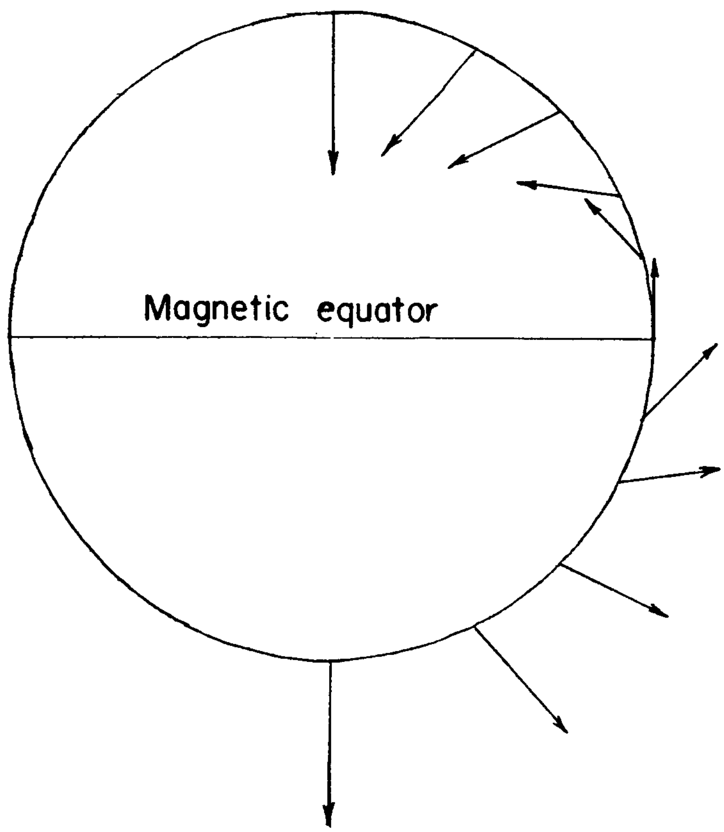


Figure 8.- Space-station configuration and aerodynamic moments for elliptical orbit.



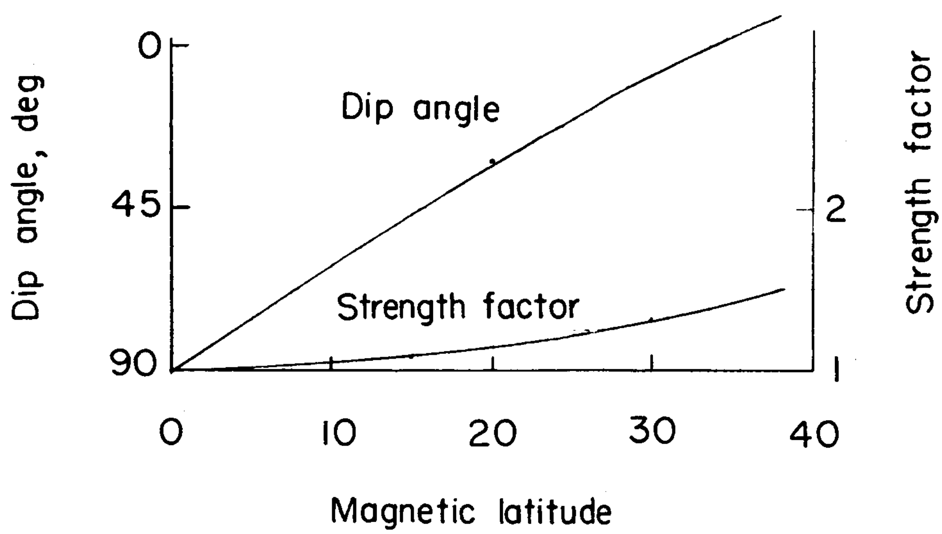


Figure 9.- Direction and relative magnitude of magnetic field.

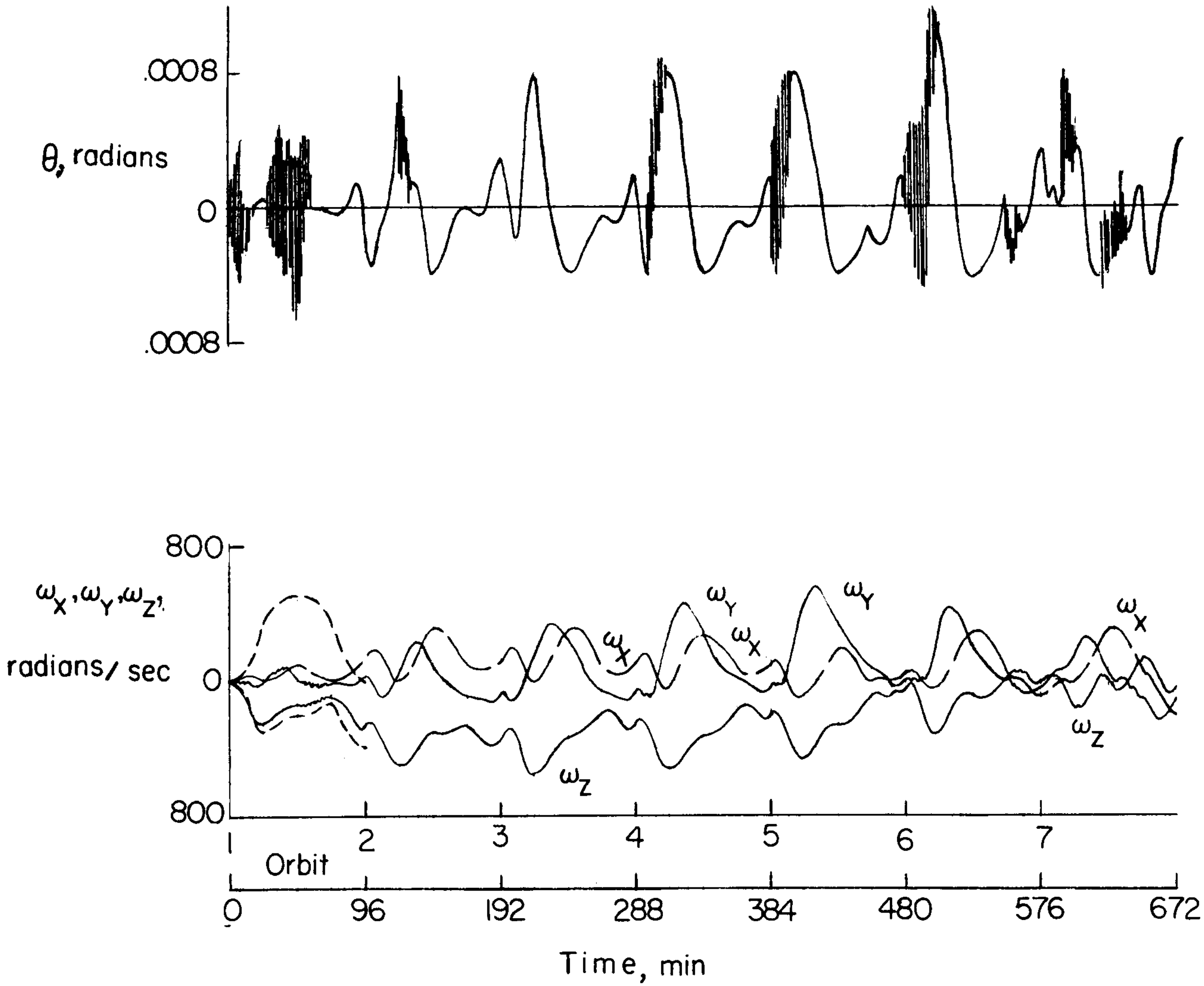


Figure 10.- Three-degree-of-freedom response of satellite to aerodynamic moments for several orbits. Control by inertia wheels and magnet.

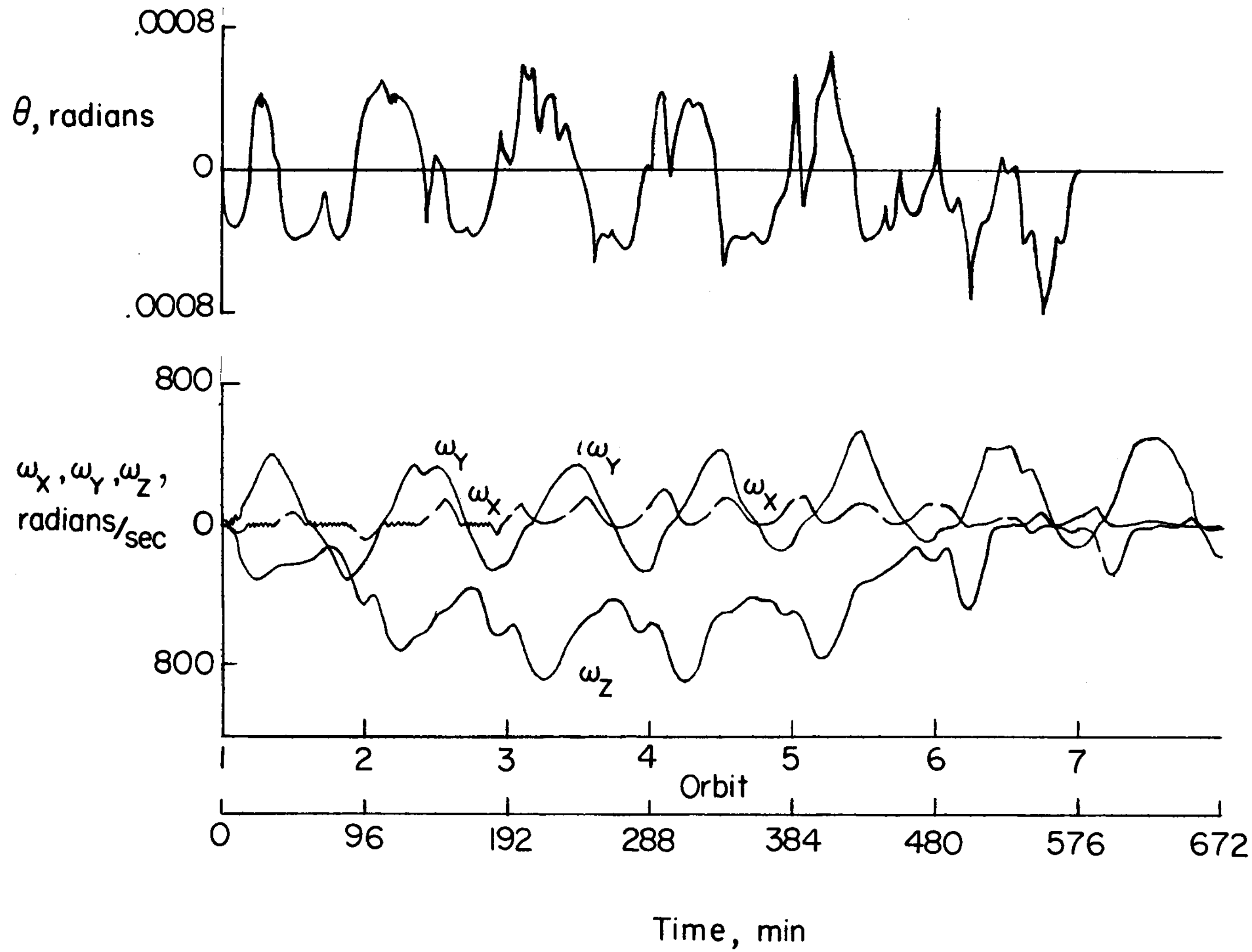


Figure 11.- Three-degree-of-freedom response of satellite to aerodynamic moments for several orbits. Control by inertia wheels and magnet, using the simplified control for the magnet.

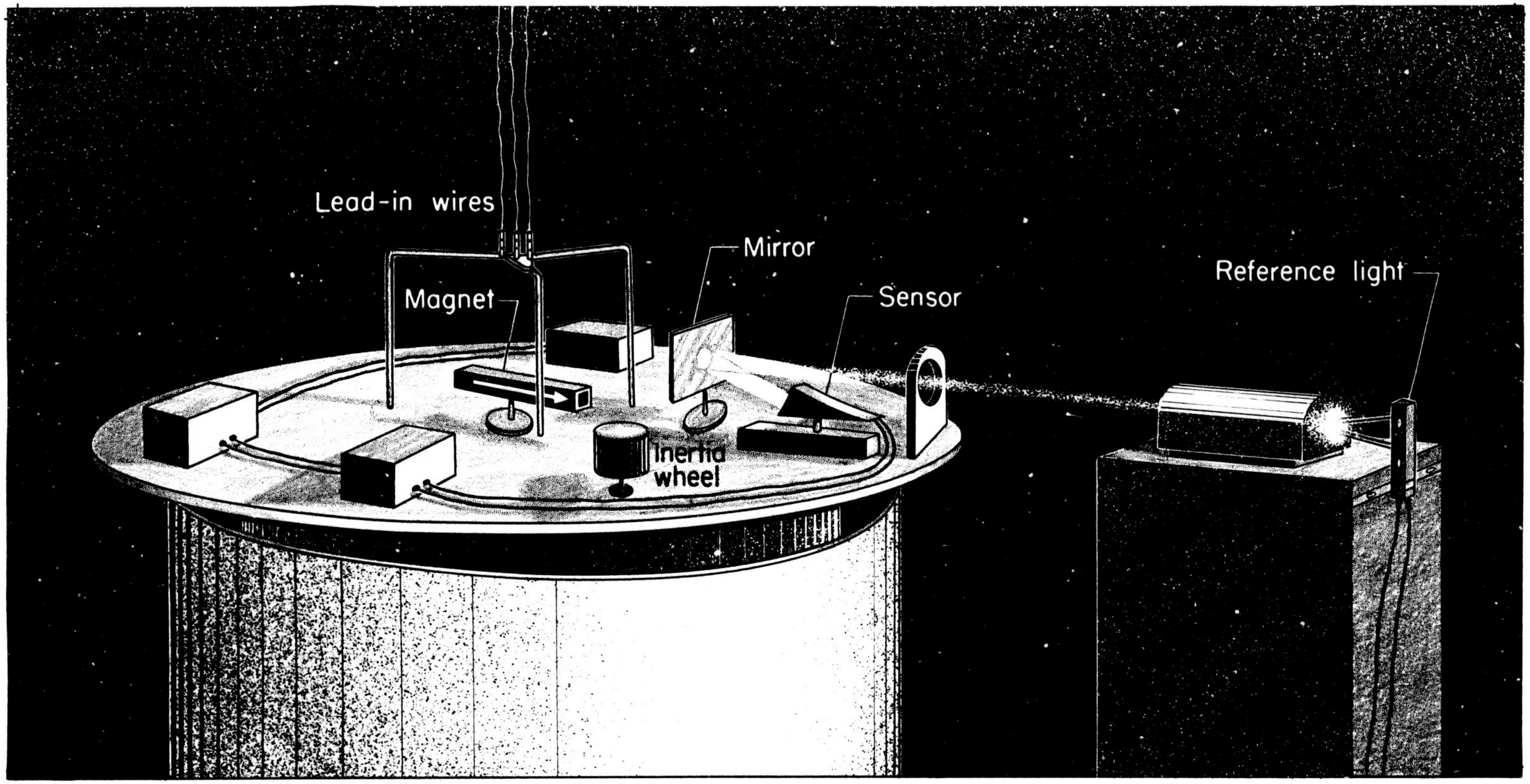


Figure 12.- Sketch of single-degree-of-freedom table.

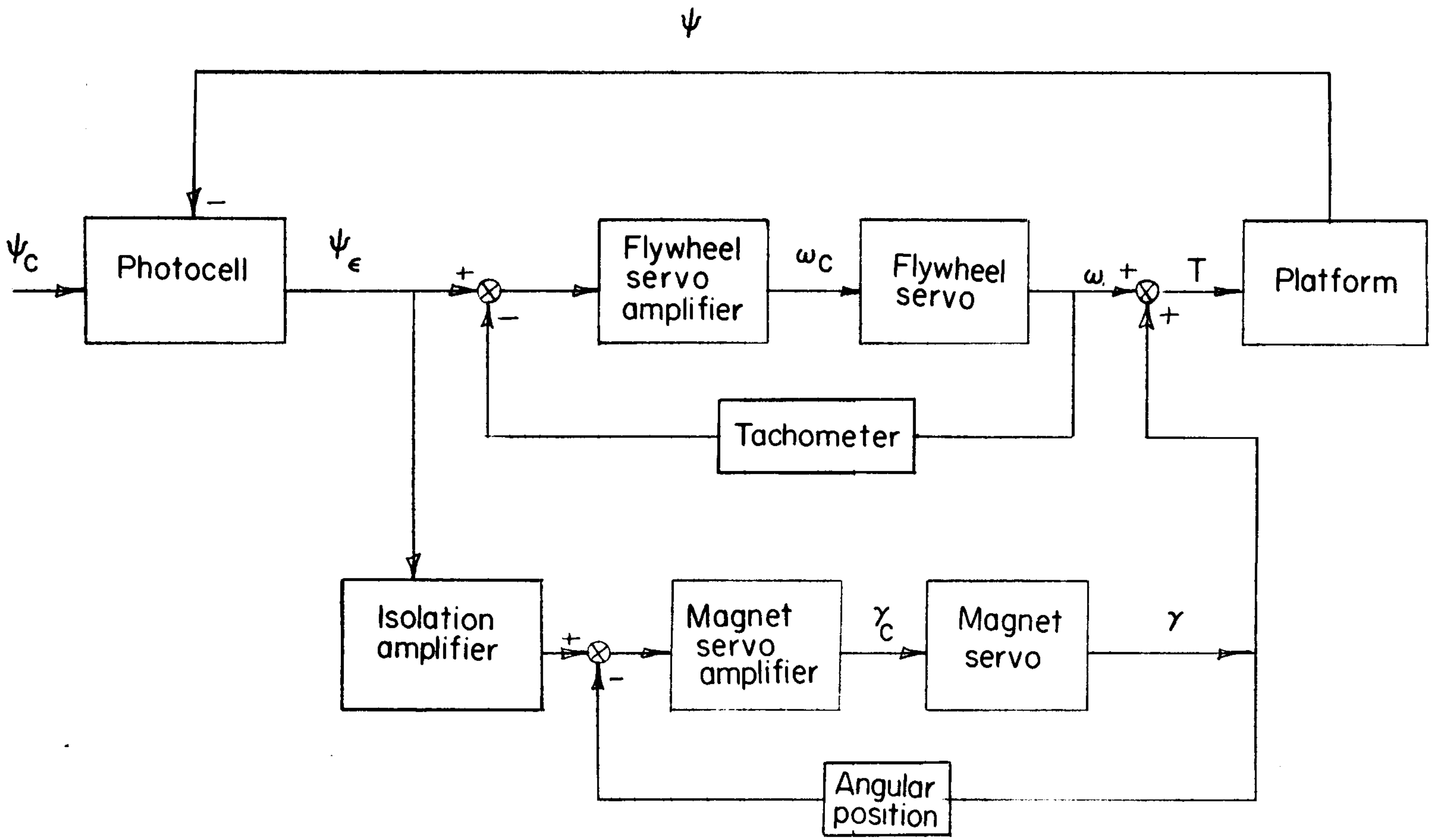
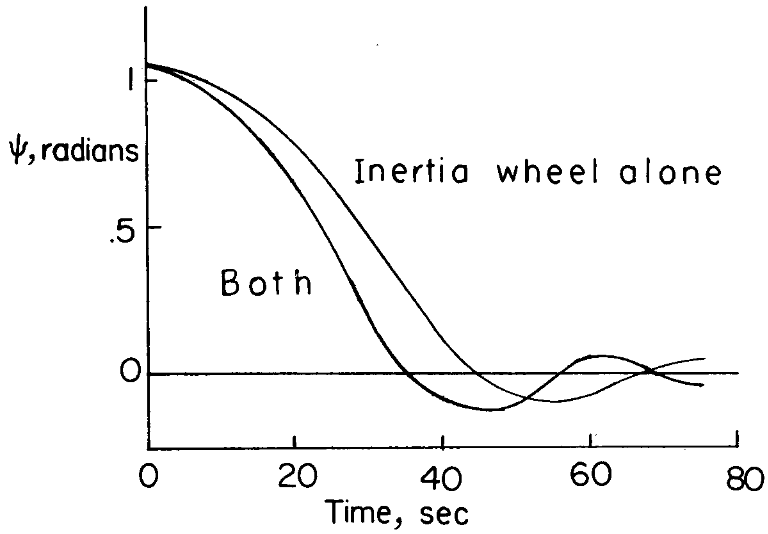
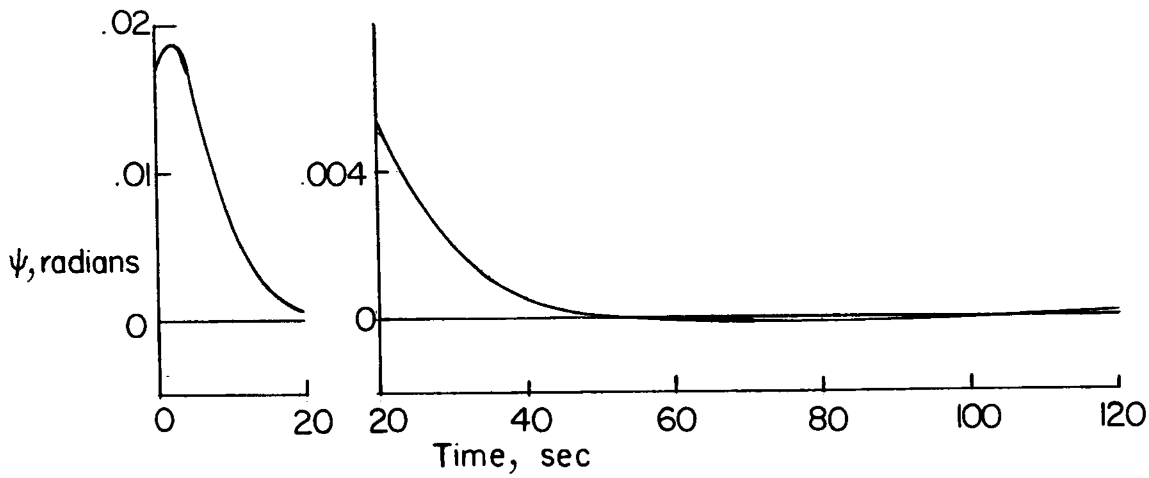


Figure 13.- Simplified block diagram of control system used in bench tests.



(a) Using directional-gyro error sensor.



(b) Using photocell error sensor (magnet and flywheel).

Figure 14.- Response of single-degree-of-freedom bench-test systems.

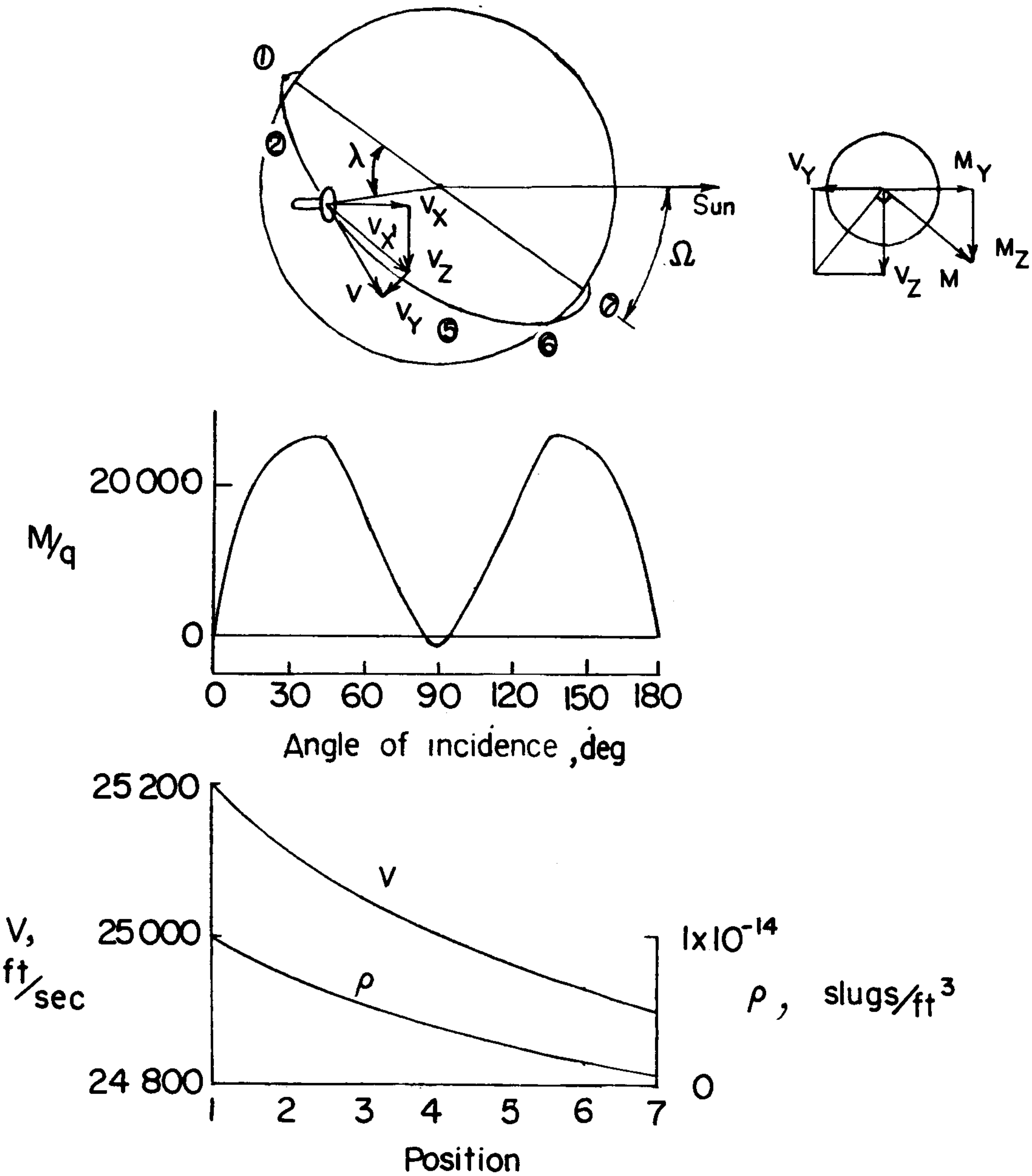
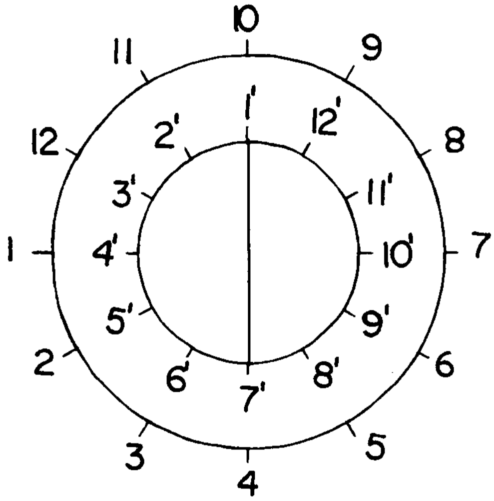
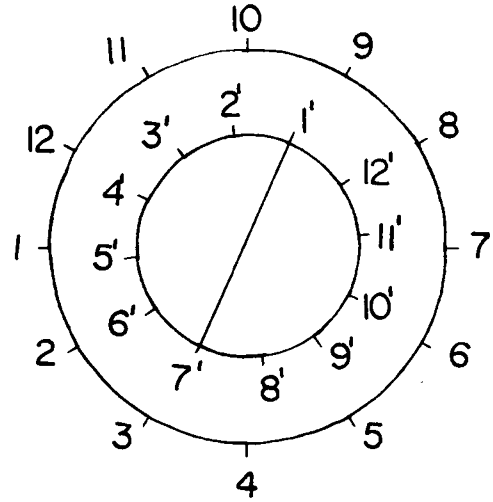


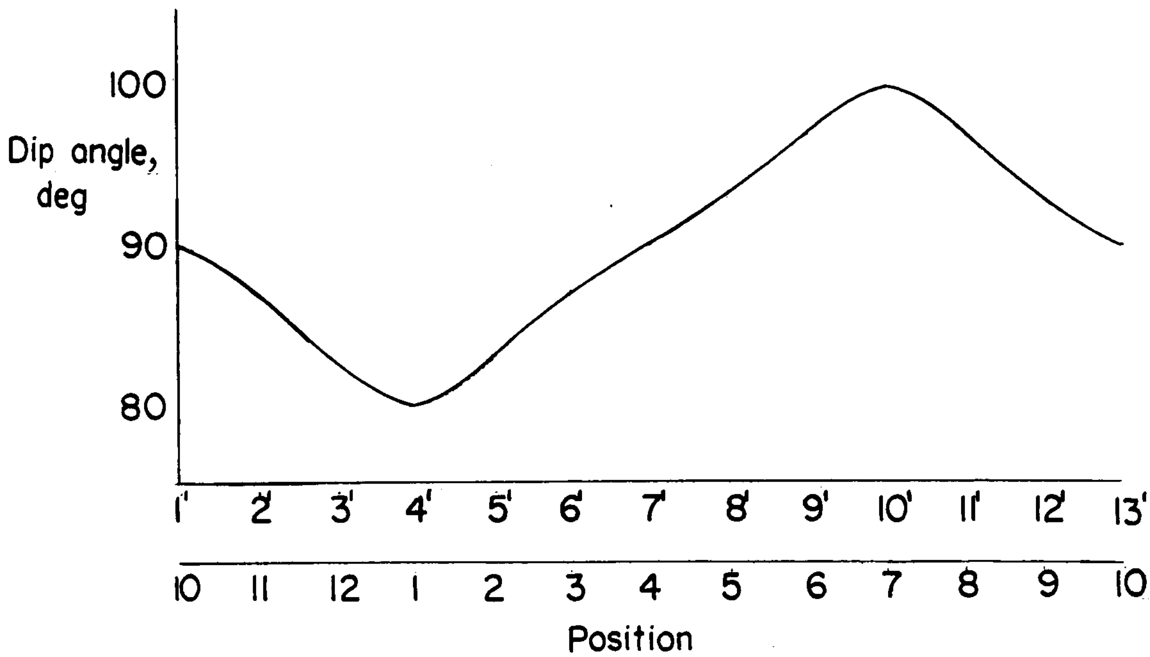
Figure 15.- Determination of aerodynamic moments.



(a) First orbit.



(b) Second orbit.



(c) Dip angle for first orbit. $\Omega = 3^{\circ}$.

Figure 16.- Determination of magnet gimbal angles.