Nonlinear Modeling and Control of a Propellant Mixer*

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Abstract

A mixing chamber used in rocket engine combustion testing at NASA Stennis Space Center is modeled by a second order nonlinear MIMO system. The mixer is used to condition the thermodynamic properties of cryogenic liquid propellant by controlled injection of the same substance in the gaseous phase. The three inputs of the mixer are the positions of the valves regulating the liquid and gas flows at the inlets, and the position of the exit valve regulating the flow of conditioned propellant. The outputs to be tracked and/or regulated are mixer internal pressure, exit mass flow, and exit temperature. The outputs must conform to test specifications dictated by the type of rocket engine or component being tested downstream of the mixer. Feedback linearization is used to achieve tracking and regulation of the outputs. It is shown that the system is minimum-phase provided certain conditions on the parameters are satisfied. The conditions are shown to have physical interpretation.

1 Introduction

The NASA John C. Stennis Space Center (SSC) conducts extensive ground-based testing and flight certification of rocket engines, in particular, of the Space Shuttle Main Engine (SSME). Combustion chambers and turbomachinery related to rocket engines are also tested at SSC. This work is part of an on-going effort to develop a software package providing flexibility in simulation and control tasks [10, 2, 3] frequently found in test operations at SSC. Test conditions require that liquid propellants, namely liquid oxigen and liquid hydrogen (LH2) be supplied to the engine or component at very precise conditions of temperature, pressure and mass flow rate. An excess or deficiency in any of these three flow parameters may result in damaged components or in a sub-optimal test. To achieve the required conditions, the delivery system includes a mixing chamber, henceforth referred to as "mixer". The mixer subsystem is depicted in Fig. 1. LH2 is stored in the run tank, which is kept at a constant pressure by an independent control loop. Gaseous hydrogen (GH2) is stored in high pressure bottles, and has a higher temperature than the LH2. One control valve, referred to as the "liquid valve" is used to manipulate the flow of LH2 into the mixer. There is also a "gas valve" and an "exit

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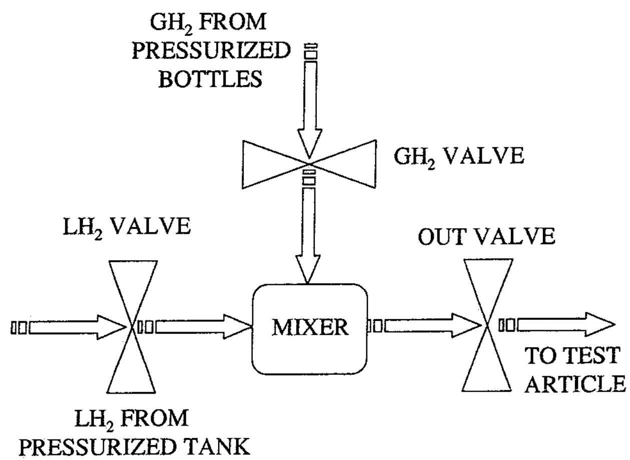

Figure 1: Mixer Subsystem

valve". The positions of the valves constitute the only control variables for the mixer subsystem. The objective of the mixer control system is to specify a MIMO controller which achieves tracking and regulation of mixer outputs to desired values. The outputs of interest are the mixer pressure, exit flow temperature, and exit mass flow. The MIMO system, thus, has three inputs and three outputs. A dynamic model and controller are intended to replace the current method of operation of the mixer, which uses only a steady-state thermodynamic model and extensive heuristics. The use of the modeling and control techniques described herein is expected to provide great flexibility and better mixer performance.

2 Transient Thermodynamics Model of the Mixer

2.1 Valve Models

The flow w_l of LH2 through the liquid valve depends on the pressure difference across the valve, the density of the source LH2 and the liquid valve opening coefficient C_{vl} according to [1]:

$$w_l = C_1 C_{vl} \sqrt{(P_l - P)\rho_l} \tag{1}$$

In English units, P_l is the source LH2 pressure and P is the mixer pressure in psia; ρ_l is the source LH2 density in lbm/ft^3, and $C_1 = 1.76 \times 10^{-2}$. For gas flow, there is a separate expression [1]:

$$w_g = \begin{cases} C_2 C_{vg} \sqrt{P_g^2 - P^2} \left(\frac{\sqrt{T_g \rho_g}}{P_g} \right) & \text{,when } P_g < 2P \\ \\ C_4 C_{vg} \sqrt{T_g} \rho & \text{,when } P_g \geq 2P \text{ (choked flow)} \end{cases} \tag{2}$$

where T_g is the source GH2 temperature in degrees Rankine ($^\circ R$), P_g is the source GH2 pressure in psia, and C_{vg} is the gas valve opening coefficient. In English units, $C_2 = 2.857 \times 10^{-2}$ and $C_4 = 2.423 \times 10^{-2}$.

2.2 Mass and Energy Balances

The set of two nonlinear differential equations which constitute the model can be derived from the conservation of mass for compressible flow and from the First Law of Thermodynamics for a control volume including transient terms [11]. The equations may be expressed in terms of internal energy and density derivatives as:

$$\dot{\rho} = \frac{1}{V}(w_l + w_g - w_e) \quad (3)$$

$$\dot{u} = \frac{1}{\rho V}\left[w_l(h_l - u) + w_g(h_g - u) - w_e(h_e - u)\right] \quad (4)$$

where V is the fixed mixer volume, h_l, h_g and h_e are the source liquid, source gas, and exit enthalpies, respectively. The mass flows w_i are functions of pressure, and are calculated according to Eqs.(1) and (2).

2.3 State Variable Model

Further manipulations are required to put the above physical model in standard state-space form. It is assumed that enthalpy is conserved across the exit valve, and the definition [11] $h = u + CP/\rho$, where C is a constant depending on the choice of units, is used. For English units, $C = 0.185$. The resulting model is compactly expressed as

$$\dot{\rho} = f_1(P(\rho,u),\rho)C_{vl} + f_2(P(\rho,u),\rho)C_{vg} + f_3(P(\rho,u),\rho)C_{ve} \quad (5)$$

$$\dot{u} = g_1(P(\rho,u),\rho,u)C_{vl} + g_2(P(\rho,u),\rho,u)C_{vg} + g_3(P(\rho,u),\rho,u)C_{ve} \quad (6)$$

where

$$\begin{aligned}
f_1 &= C_1'\sqrt{(P_l - P)\rho_l} \\
f_2 &= C_2', \text{ if } P_g \geq 2P \\
f_2 &= C_4'\sqrt{P_g^2 - P^2}, \text{ if } P_g < 2P \\
f_3 &= C_3'\sqrt{(P - P_s)\rho} \\
g_1 &= f_1\left(\frac{h_l - u}{\rho}\right) \\
g_2 &= f_2\left(\frac{h_g - u}{\rho}\right) \\
g_3 &= C_3''\sqrt{\frac{P - P_s}{\rho}\frac{P}{\rho}}
\end{aligned}$$

The constants are given by $C_1' = C_1/V$, $C_2' = C_2\sqrt{T_g}\rho_g/V$, $C_3' = -C_3/V = -C_1/V$, $C_3'' = CC_3'$ and $C_4' = C_4\sqrt{T_g}\rho_g/V$.

2.4 Output Definitions and Controls Model

Mixer operation requires the simultaneous tracking of exit temperature, exit flow, and mixer pressure. Exit temperature and mixer pressure are functions of the energy and density states. Computation of these functions requires the intervention of thermodynamic tables and interpolation algorithms which cannot be represented in closed form. However [7], to each exit temperature and mixer pressure combination in the expected range of operation there corresponds a unique value of the state $[\rho, \;\; u]^T$. Therefore, it is convenient to specify density and energy as outputs, along with exit mass flow. Desired state trajectories that correspond to desired exit temperature and mixer pressure can be either pre-computed off-line, or they can be obtained through a reference prefilter. With these output definitions, the controls model becomes

$$\begin{cases} \begin{bmatrix} \dot{\rho} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} f_1(\rho,u) & f_2(\rho,u) & f_3(\rho,u) \\ g_1(\rho,u) & g_2(\rho,u) & g_3(\rho,u) \end{bmatrix} \begin{bmatrix} C_{vl} \\ C_{vg} \\ C_{ve} \end{bmatrix} \\ y = [\rho, \;\; u, \;\; -Vf_3C_{ve}]^T \end{cases} \tag{7}$$

where $[C_{vl} \;\; C_{vg} \;\; C_{ve}]^T$ is the control vector and y is the output vector.

3 Model Properties

3.1 Uniqueness of Equilibrium

The equilibrium point indicates the steady values of the density and internal energy of the mixer when the control valves are set at fixed positions, for a given set of input flow characteristics. For given values of the input fluid properties and C_v coefficients, setting $\dot{\rho}_{cv} = 0$ establishes that $\dot{m}_e = \dot{m}_g + \dot{m}_l$ and results in an expression relating the equilibrium density $\bar{\rho}_{cv}$ to the equilibrium mixer pressure $\bar{P}$:

$$\bar{\rho}_{cv} = \rho(\bar{P}) \tag{8}$$

Setting $\dot{u}_{cv} = 0$ gives the equilibrium exit enthalpy $h_e = h_{cv}$ in terms of the input enthalpies and the mass flows:

$$\bar{h}_{cv} = \frac{\dot{m}_l h_l + \dot{m}_g h_g}{\dot{m}_l + \dot{m}_g} = \bar{h}(P) \tag{9}$$

The above enthalpy must match the thermodynamic property data at $\bar{P}$ and $\bar{\rho}_{cv}$ of Eq.(8), that is

$$\bar{h}_{cv} = h_{th}(\bar{\rho}_{cv}, \bar{P})$$

Substituting Eqs.(8) and (9) into the above equation results in a single expression which gives the equilibrium pressure:

$$\bar{h}(P) = h_{th}(\rho(\bar{P}), \bar{P}) \tag{10}$$

A graphical interpretation of the equilibrium solution is shown in Fig. 2. The curve $\rho\bar{P}$ vs. $\bar{P}$ has been drawn on the base plane. This plane curve is mapped to a space curve by the thermodynamic property function h_{th}. The equilibrium exit enthalpy $\bar{h}(P)$ is a function of pressure only and the corresponding surface has also been graphed. The point where the space curve pierces the surface is

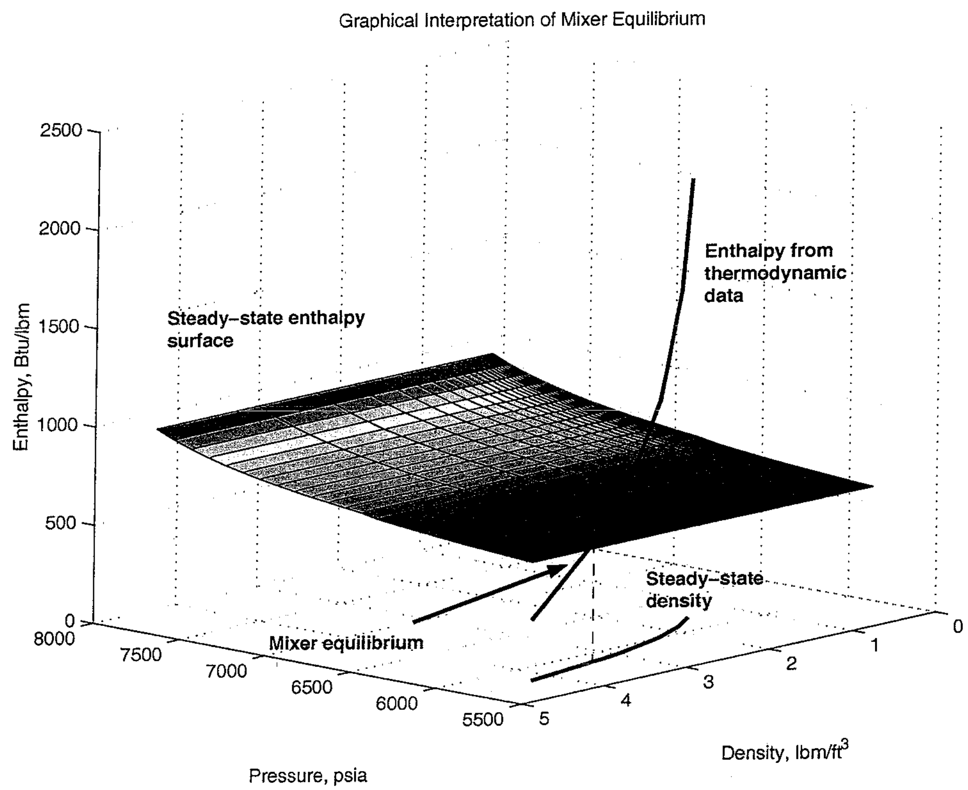

Figure 2: Mixer equilibrium

the equilibrium point of the system. The monotonicity of the curve and surface indicate that there exists only one equilibrium point in the range of interest of actual mixer operation. An iterative procedure is developed in [7] that calculates the equilibrium point, given thermodynamic parameters and fixed control inputs.

3.2 Valve Positions for a Prescribed Operating Condition (Reachability)

Having three independent controls in a two-state model allows the selection of steady values for the states, and, in addition, an extra degree of freedom is available. This degree of freedom can be used to fix the exit mass flow with the desired thermodynamic properties, as shown next. Supppose it is desired to have a given exit mass flow, with prescribed temperature (measured at the outlet of the exit valve). Let these quantities be denoted by w_e and T_e. The back pressure P_s at the outlet valve exit is assumed to be constant. The problem is to determine the valve coefficients that achieve this. Two degrees of freedom are used for w_e and T_e, and the third one is used to meet a desired mixer operating pressure, P. The back pressure P_s and T_e determine the enthalpy h, which is constant across the valve. Therefore, the enthalpy and mixer pressure at the exit valve inlet are known and, in turn, determine the mixer density from thermodynamic data. A static energy and material balance gives the required input flows that achieve a prescribed flow-exit enthalpy combination. The following formulas are straightforward to derive:

$$\begin{aligned} w_l &= \frac{w_e(h-h_g)}{h_l-h_g} \\ w_g &= \frac{w_e(h_l-h)}{h_l-h_g} \end{aligned} \tag{11}$$

where h_g and h_l are the gas and liquid supply enthalpies, respectively. Using the exit flow and the input equilibrium flows from Eq.(11), the three valve coefficients are determined from Eq.(1) and (2). This procedure is complementary to the one described in the previous section, which calculates the equilibrium point. It shows, in a rather informal way, that the system is reachable in the thermodynamic range of interest.

4 Feedback Linearization Analysis

Feedback linearization [12, 8, 5], has been succesfully applied in a variety of cases related to thermal process control; for instance in [6, 9, 4]. In this section the technique is applied to the mixer. The model of Eq.(7) has a characteristic which prevents direct application of input/output linearization theory. It is seen that the third component of y is statically related to one of the control inputs, namely C_{ve}. This implies that the partial relative degree of y_3 is zero. Conceivably, one could use a time-explicit control of the form

$$C_{ve}(t,\rho,u) = -\frac{y_{3d}(t)}{Vf_3(P(\rho,u),\rho)}$$

to attain perfect tracking of the output mass flow rate and use the two remaining controls to force the flow to have the desired pressure and temperature. This approach, however, is not robust.

Therefore a a feedforward law is not considered. One way to get around this problem altogether is to augment the exit valve channel with an integrator, that is, let:

$$\dot{C}_{ve} = v$$

where v is a new control input. Now C_{ve} is regarded as a state, and the resulting system is of third order, with three inputs and outputs. If the arguments of functions g_i and f_i are dropped from the notation, the new system equations become:

$$\begin{aligned} \dot{\rho} &= f_1 C_{vl} + f_2 C_{vg} + f_3 C_{ve} &(12)\\ \dot{u} &= g_1 C_{vl} + g_2 C_{vg} + g_3 C_{ve} &(13)\\ \dot{C}_{ve} &= v &(14)\\ y &= [\rho \quad u \quad -V f_3 C_{ve}]^T &(15) \end{aligned}$$

Upon differentiating the outputs once and rearranging, the derivatives can be written as:

$$\dot{y} = D + Ew$$

where $\dot{y} = [\dot{y}_1 \;\; \dot{y}_2 \;\; \dot{y}_3]^T$, $w = [C_{vl} \;\; C_{vg} \;\; v]^T$ and

$$D = \begin{bmatrix} f_3 C_{ve} \\ g_3 C_{ve} \\ -V C_{ve}^2 [\frac{\partial f_3}{\partial P}(\frac{\partial P}{\partial \rho} f_3 + \frac{\partial P}{\partial u} g_3) + \frac{\partial f_3}{\partial \rho} f_3] \end{bmatrix}$$

$$E = \begin{bmatrix} f_1 & f_2 & 0 \\ g_1 & g_2 & 0 \\ -V C_{ve}[\frac{\partial f_3}{\partial P}(\frac{\partial P}{\partial \rho} f_1 + \frac{\partial P}{\partial u} g_1) + \frac{\partial f_3}{\partial \rho} f_1] & -V C_{ve}[\frac{\partial f_3}{\partial P}(\frac{\partial P}{\partial \rho} f_2 + \frac{\partial P}{\partial u} g_2) + \frac{\partial f_3}{\partial \rho} f_2] & -V f_3 \end{bmatrix}$$

It is seen that the partial relative degrees are all one. Provided E is invertible in a region Ω of the state space, the feedback law

$$w = E^{-1}(\dot{y}_d - \Gamma(y - y_d) - D) \tag{16}$$

achieves exact linearization of the system, with tracking error dynamics given by

$$(\dot{y} - \dot{y}_d) + \Gamma(y - y_d) = 0$$

If Γ is chosen as a diagonal positive-definite matrix, the resulting control law is a "decoupling control", since the dynamics of the tracking errors are noninteracting [5]. If the appropriate function definitions are substituted, the form of the E matrix is as follows:

$$E = \begin{bmatrix} C_1'\sqrt{(P_l - P)\rho_l} & f_2 & 0 \\ C_1'\sqrt{(P_l - P)\rho_l}\left(\frac{h_l - u}{\rho}\right) & f_2\left(\frac{h_g - u}{\rho}\right) & 0 \\ -\frac{V C_{ve} C_1' C_3' \sqrt{(P_l - P)\rho_l}}{2\sqrt{(P - P_s)\rho}}\left[\rho \frac{\partial P}{\partial \rho} + (h_l - u)\frac{\partial P}{\partial u} + (P - P_s)\right] & -\frac{V C_{ve} f_2 C_3'}{2\sqrt{(P - P_s)\rho}}\left[\rho \frac{\partial P}{\partial \rho} + (h_g - u)\frac{\partial P}{\partial u} + (P - P_s)\right] & -V C_3'\sqrt{(P - P_s)\rho} \end{bmatrix}$$

4.1 Input-Output Linearizability of Augmented Model

The ability to construct a feedback linearization controller hinges on stable zero dynamics of the augmented system, and on the invertibility of matrix E.

4.1.1 Invertibility of E

By inspection, it is readily seen that the first two rows of E are linearly independent provided $h_g \neq h_l$. This has a direct physical interpretation: if the two fluids have the same thermal properties (i.e., enthalpies), the ability to change the thermal properties of the mixture by changing the relative flows is lost. Gas and liquid enthalpies are different for the expected operating conditions. The third row is linearly independent from the first two if $P - P_s > 0$, which is also true for the mixer. Therefore E is invertible over the whole range of expected mixer operating conditions.

4.1.2 Zero Dynamics of the Augmented Model

As it is known [5], the zero dynamics is unchanged upon augmentation with an integrator. This implies that one may examine the non-augmented model for zero dynamics and draw conclusions about the augmented model's zero dynamics. Suppose the exit flow is to be held constant at a value Y_{30}. The only way in which this can be achieved is by letting

$$C_{ve}(t) = -\frac{Y_{30}}{Vf_3(t)} \tag{17}$$

at all times. Differentiating the other two outputs and equating them to zero results in

$$C_{vl}(t) = \frac{1}{f_1(t)}\left(-f_2(t)C_{vg}(t) + \frac{Y_{30}}{V}\right) \tag{18}$$

$$C_{vg}(t) = -\frac{1}{g_2(t)}\left(g_1(t)C_{vl}(t) + \frac{g_3(t)Y_{30}}{Vf_3(t)}\right) \tag{19}$$

Upon substitution and rearrangement, it is seen that there exist three uniquely defined control inputs which hold the outputs constant. Since two of the outputs coincide with the states, it trivially follows that they are kept bounded, and therefore the system has stable zero dynamics (i.e., the system is *minimum-phase*). The inputs are given by Eq. 17 and, dropping the time notation:

$$C_{vg} = \frac{\frac{Y_{30}f_1}{V}\left(\frac{g_3}{f_3} - \frac{g_1}{f_1}\right)}{f_1g_2 - g_1f_2} \tag{20}$$

$$C_{vl} = \frac{Y_{30}}{Vf_1}\left[1 - \frac{f_1f_2}{f_1g_2 - g_1f_2}\left(\frac{g_3}{f_3} - \frac{g_1}{f_1}\right)\right] \tag{21}$$

Note that the above formulas can be used to find the valve positions at which the system has a prescribed outflow and a pair of thermodynamic properties. Therefore it produces the same results as in Section 3.2.

5 Simulation Study of the Feedback Linearization Controller

Functions that compute the pressure and its partial derivatives at each time step were implemented in Matlab. Table 1 summarizes the parameters used in the simulations. The decoupled control gains were set as 10 for the density, 10 for the energy and 5 for the exit flow. The controller has exact knowledge of all parameters, except the initial conditions. The objective in Simulation 1 is to transfer the mixer from the initial conditions to a pressure of 6000 psia, exit temperature of

Parameter	Value	Units
P_l	8500	psia
T_l	-340	$^\circ F$
P_g	13500	psia
T_g	90	$^\circ F$
P_s	5533	psia
ρ_o	3.85	lbm/ft^3
u_o	180	Btu/lbm

Table 1: Feedback Linearization Parameters

$-200^\circ F$ and exit flow of 40 lbm/s. With the known and fixed back pressure it is determined that the mixer states must be transferred to $\rho = 2.93\ lbm/ft^3$ and $u = 400.9$ Btu/lbm for the pressure and exit temperature to converge to their desired values. The reference states and flow are specified as constants, leaving the trajectories to be determined by the controller. In Simulation 2, the mixer is asked to reach the previous equilibrium point and then raise the exit temperature to $0^\circ F$ and the pressure to 7000 psia, while reducing the exit flow to 10 lbm/s. This corresponds to lowering the density to $\rho = 2.143$ and raising the energy to $u = 995.9$. This time, the states and the flow are required to track ramp functions which transfer from old to new values in 5 seconds. Figs. 3 and 4 show the results of Simulations 1 and 2, respectively. The steady values required are exactly achieved. However, the transient response of the exit flow in the simulations has some overshoot. This may be reduced by modifying the control gains, but perhaps at the expense of slowing down the response.

6 Conclusions

A mathematical model for the mixer is presented. The model consists of a second order nonlinear MIMO system having density and internal energy as states. The inputs are valve positions represented by flow coefficients, and the controlled outputs are mixer pressure, exit temperature and exit mass flow. Model reliability is ensured by the use of function calls to real thermodynamic data. It is shown that, in the expected range of operation, the mixer has a single equilibrium point for each set of fixed valve coefficients. Conversely, the reachability of operating conditions is shown by the possibility of determining a unique set of valve coefficients that achieve a desired set ot steady outputs. A feedback linearization controller is derived for the mixer system, and it is shown that the system is minimum-phase and the linearizing matrix is invertible under parametric conditions having nice physical interpretations. The feedback linearization controller gives good results. Further work in the topic includes examining other control techniques providing robustness against variations in the supply fluid properties (bottle and tank depletion) when they cannot be measured.

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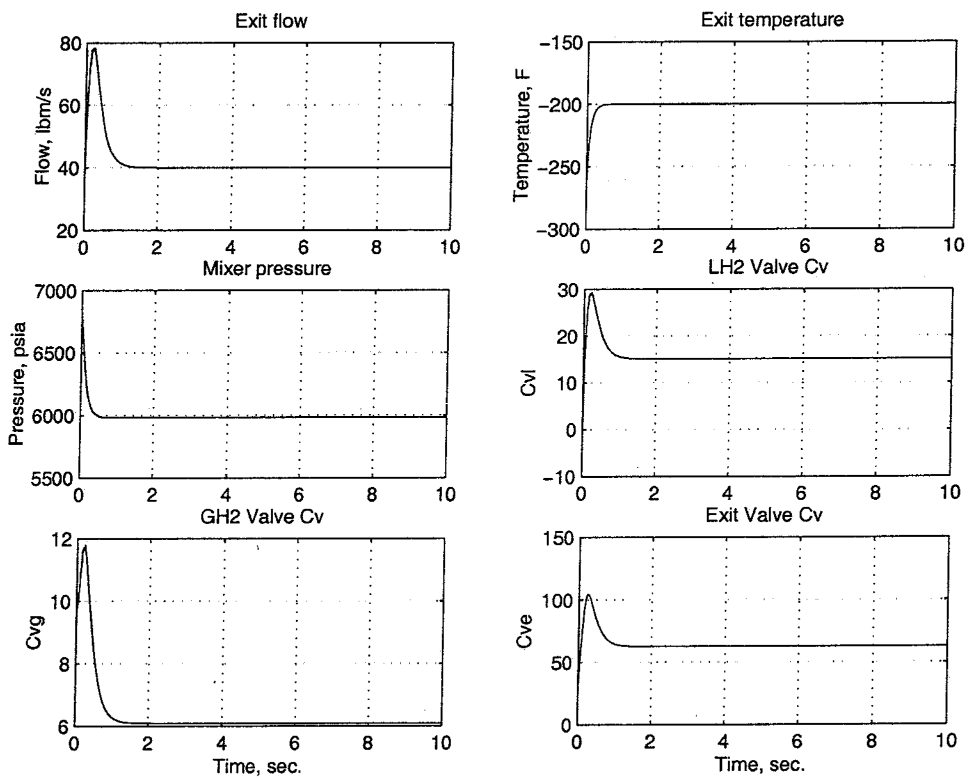

Figure 3: Simulation of Feedback Linearization Controller

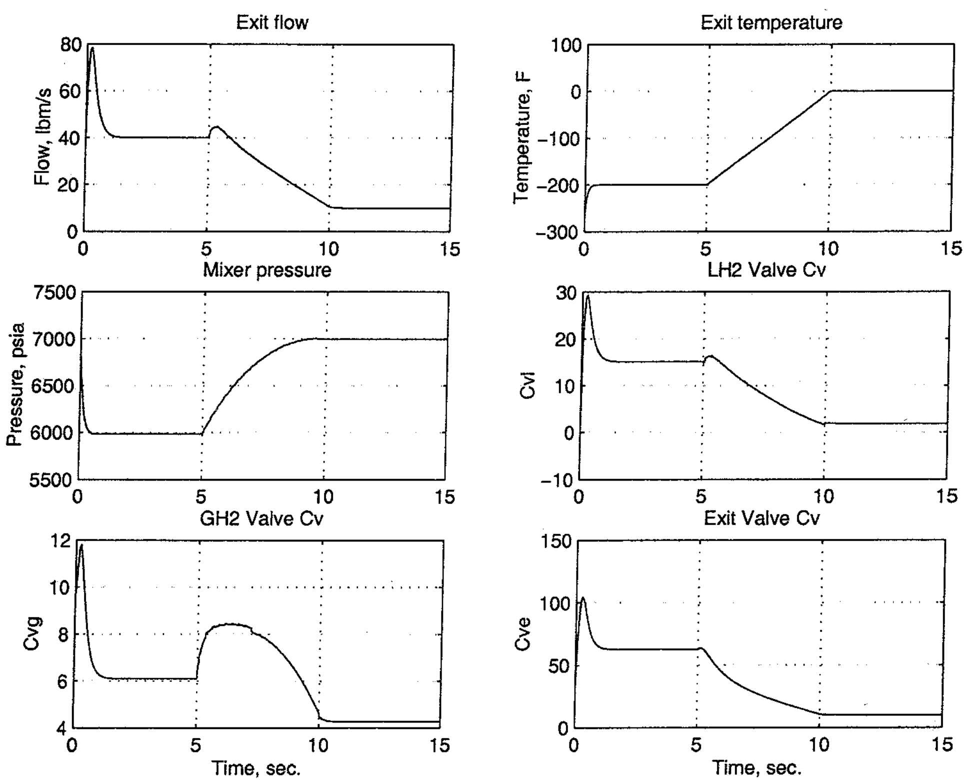

Figure 4: Simulation of Feedback Linearization Controller

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