

DRAFT PAPER

FOR REVIEW

LINEAR STATE-SPACE REPRESENTATION OF THE DYNAMICS OF RELATIVE MOTION, BASED ON RESTRICTED THREE BODY DYNAMICS

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Abstract

Precision Formation Flying is an enabling technology for a variety of proposed space-based observatories, including the Micro-Arcsecond X-ray Imaging Mission (MAXIM), the associated MAXIM pathfinder mission, Stellar Imager (SI) and the Terrestrial Planet Finder (TPF). An essential element of the technology is the control algorithm, requiring a clear understanding of the dynamics of relative motion. This paper examines the dynamics of relative motion in the context of the Restricted Three Body Problem (RTBP). The natural dynamics of relative motion are presented in their full nonlinear form. Motivated by the desire to apply linear control methods, the dynamics equations are linearized and presented in state-space form. The stability properties are explored for regions in proximity to each of the libration points in the Earth/Moon - Sun rotating frame. The dynamics of relative motion are presented in both the inertial and rotating coordinate frames.

Introduction - Precision Spacecraft Formation Flying

A Distributed Spacecraft System (DSS) is a collection of two or more spacecraft functioning to fulfill a shared or common objective. As a subset of the DSS architecture, formation flying missions add the requirement to maintain a relative

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position and/or orientation with respect to each other, or a common target. The term precision formation flying implies a requirement for continuous control (normally implemented in discrete time) to maintain the formation within the design specification. Control system designs for precision formation flying missions will vary, based on the dynamic environment. For example, the dynamic environment for low Earth orbit differs significantly from that experienced near an Earth/Moon-Sun libration point. Focused on developing a control strategy to support the mission type of TPF,^{5,10,14} MAXIM^{3,9,12} and SI,^{1,13} the dynamic environment model for this analysis is based on the dynamics of the general restricted three body problem (RTBP) with the Earth/Moon and Sun as the primary bodies. The analysis considers a simple two spacecraft formation. The spacecraft are designated Leader and Follower. The Follower spacecraft is controlled to maintain a desired trajectory with respect to the Leader spacecraft.

The well known Hill's or Clohessy-Wiltshire equations describe the relative motion of a spacecraft with respect to a circular reference orbit.² The objective is to develop a similar set of equations describing the relative motion of a Follower spacecraft with respect to the Leader (target) spacecraft, given restricted three body dynamics. Segermann and Zedd discuss the dynamics of relative motion near the Earth/Moon-Sun L_2 point.¹¹ The analysis is based on circular restricted three body problem (CRTBP), using the CRTBP rotating frame with the location of L_2 to reference the spacecraft position. In previous work the authors explored nonlinear control strategies for achieving the goal of precision formation flying.⁶⁻⁸ The analysis focused on the nonlinear equations of relative motion, expressed in inertial

coordinates, allowing application in the more general elliptical restricted three body problem. In other related work, Hamilton, et al. developed a linear control strategy for formation flying about the Earth/Moon-Sun L_2 point.,⁴ employing the dynamics matrix for the linearized equations of motion of a spacecraft in orbit about the L_2 point. In contrast, this work references the dynamics of relative motion of the Follower with respect to the Leader spacecraft without direct linkage to any specific point in the RTBP reference frame.

This work begins with a review of the equations of relative motion developed from the general nonlinear form expressed in inertial coordinates. The equations are linearized and cast in a state-space form for a time-varying linear system. The state equation is expressed in both the inertial and RTBP rotating frame. The stability properties of the equations of relative motion are examined for each of the libration points.

Restricted Three Body Problem

The Restricted Three Body Problem (RTBP) examines the behavior of an infinitesimal mass in the combined gravitational field of two finite masses orbiting their common center of mass. For spacecraft stationed near any of the Earth/Moon-Sun libration points, the orbital dynamics are governed by gravity and solar pressure plus thruster action. The principal gravitational sources are the Sun and the Earth/Moon system. The Earth/Moon system is treated as a combined mass located at the system center of mass. The spacecraft are comparably small such that their mutual gravitational interaction is insignificant.

A typical two spacecraft formation stationed near L_2 is depicted in Fig. 1. The spacecraft are designated Leader and Follower. In this scenario, the Leader spacecraft is intended to follow a ballistic trajectory with infrequent control for orbit maintenance. Control is applied to the Follower spacecraft to maintain a specified trajectory relative to the Leader spacecraft.

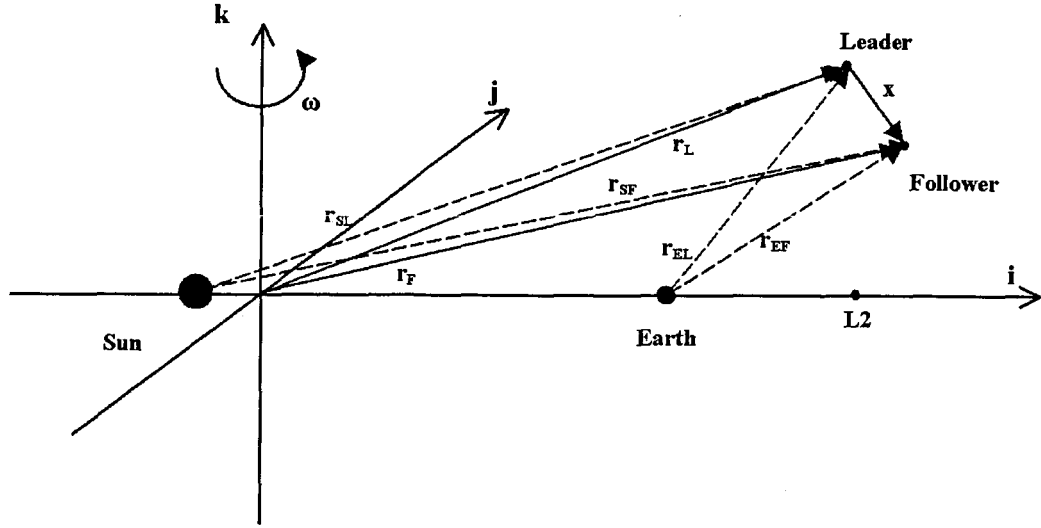


Figure 1: Two Spacecraft Orbiting in the Earth/Moon - Sun Rotating Frame

As stated, the principle environmental forces experienced by a spacecraft stationed near any Earth/Moon-Sun libration point, in this example L_2 , are gravity and solar pressure. These forces, combined with thruster action, drive the spacecraft dynamics. Based on the reference vectors shown in Figure 1, the Leader dynamics (per unit mass) are given by:

$$\ddot{\mathbf{r}}_L = -\mu_{em} \frac{\mathbf{r}_{EL}}{\|\mathbf{r}_{EL}\|_2^3} - \mu_s \frac{\mathbf{r}_{SL}}{\|\mathbf{r}_{SL}\|_2^3} + \mathbf{f}_{solar,L} + \mathbf{f}_{pert,L} + \mathbf{u}_{thrust,L} \quad (1)$$

Where:

- \mathbf{r}_* - Position vectors, depicted in Figure 1
- μ_{em} - Gravitational Parameter for Earth/Moon
- μ_s - Gravitational Parameter for Sun
- $\mathbf{f}_{solar,L}$ - Perturbing Force on Leader spacecraft due to solar pressure
- $\mathbf{f}_{pert,L}$ - Perturbing Force on Leader spacecraft due to other gravitational sources
- $\mathbf{u}_{thrust,L}$ - External control force applied to Leader spacecraft

The Follower dynamics per unit mass have the same form, given by:

$$\ddot{\mathbf{r}}_F = -\mu_{em} \frac{\mathbf{r}_{EF}}{\|\mathbf{r}_{EF}\|_2^3} - \mu_s \frac{\mathbf{r}_{SF}}{\|\mathbf{r}_{SF}\|_2^3} + \mathbf{f}_{solar,F} + \mathbf{f}_{pert,F} + \mathbf{u}_{thrust,F} \quad (2)$$

Differencing Eqs. (1) and (2) yields the relative motion of the Follower with respect to the Leader:

$$\begin{aligned} \ddot{\mathbf{x}} &= \ddot{\mathbf{r}}_F - \ddot{\mathbf{r}}_L \\ &= -\mu_{em} \frac{\mathbf{r}_{EF}}{\|\mathbf{r}_{EF}\|_2^3} - \mu_s \frac{\mathbf{r}_{SF}}{\|\mathbf{r}_{SF}\|_2^3} + \mathbf{f}_{solar,F} + \mathbf{f}_{pert,F} + \mathbf{u}_{thrust,F} \\ &\quad - \left\{ -\mu_{em} \frac{\mathbf{r}_{EL}}{\|\mathbf{r}_{EL}\|_2^3} - \mu_s \frac{\mathbf{r}_{SL}}{\|\mathbf{r}_{SL}\|_2^3} + \mathbf{f}_{solar,L} + \mathbf{f}_{pert,L} + \mathbf{u}_{thrust,L} \right\} \\ &= -\mu_{em} \left\{ \frac{\mathbf{r}_{EF}}{\|\mathbf{r}_{EF}\|_2^3} - \frac{\mathbf{r}_{EL}}{\|\mathbf{r}_{EL}\|_2^3} \right\} - \mu_s \left\{ \frac{\mathbf{r}_{SF}}{\|\mathbf{r}_{SF}\|_2^3} - \frac{\mathbf{r}_{SL}}{\|\mathbf{r}_{SL}\|_2^3} \right\} \\ &\quad + (\mathbf{f}_{solar,F} - \mathbf{f}_{solar,L}) + (\mathbf{f}_{pert,F} - \mathbf{f}_{pert,L}) + \mathbf{u}_{thrust,F} - \mathbf{u}_{thrust,L} \\ &= -\left\{ \frac{\mu_{em}}{\|\mathbf{r}_{EF}\|_2^3} + \frac{\mu_s}{\|\mathbf{r}_{SF}\|_2^3} \right\} \mathbf{x} - \mu_{em} \left\{ \frac{1}{\|\mathbf{r}_{EF}\|_2^3} - \frac{1}{\|\mathbf{r}_{EL}\|_2^3} \right\} \mathbf{r}_{EL} \\ &\quad - \mu_s \left\{ \frac{1}{\|\mathbf{r}_{SF}\|_2^3} - \frac{1}{\|\mathbf{r}_{SL}\|_2^3} \right\} \mathbf{r}_{SL} + \Delta \mathbf{f}_{solar} + \Delta \mathbf{f}_{pert} + \mathbf{u}_{thrust,F} - \mathbf{u}_{thrust,L} \end{aligned} \quad (3)$$

Eq. (3) provides an exact expression of the nonlinear dynamics of relative motion between the Follower and Leader spacecraft. The next step is to linearize the relative dynamics of the Follower with respect to the Leader.

Linearized Dynamics

Linear control design requires linearized system dynamics. Neglecting disturbances, Eq. 3 becomes:

$$\begin{aligned} \ddot{\mathbf{x}} = & - \left\{ \frac{\mu_{em}}{\|\mathbf{r}_{EF}\|_2^3} + \frac{\mu_s}{\|\mathbf{r}_{SF}\|_2^3} \right\} \mathbf{x} - \mu_{em} \left\{ \frac{1}{\|\mathbf{r}_{EF}\|_2^3} - \frac{1}{\|\mathbf{r}_{EL}\|_2^3} \right\} \mathbf{r}_{EL} \\ & - \mu_s \left\{ \frac{1}{\|\mathbf{r}_{SF}\|_2^3} - \frac{1}{\|\mathbf{r}_{SL}\|_2^3} \right\} \mathbf{r}_{SL} + \mathbf{u}_{thrust,F} - \mathbf{u}_{thrust,L} \end{aligned} \quad (4)$$

As an aside, examine the term $\left\{ \frac{1}{\|\mathbf{r}_{EF}\|_2^3} - \frac{1}{\|\mathbf{r}_{EL}\|_2^3} \right\}$

$$\begin{aligned} \left\{ \frac{1}{\|\mathbf{r}_{EF}\|_2^3} - \frac{1}{\|\mathbf{r}_{EL}\|_2^3} \right\} &= \left\{ \frac{1}{\|\mathbf{r}_{EL} + \mathbf{x}\|_2^3} - \frac{1}{\|\mathbf{r}_{EL}\|_2^3} \right\} \\ &= \left\{ \frac{1}{[(\mathbf{r}_{EL} + \mathbf{x}) \cdot (\mathbf{r}_{EL} + \mathbf{x})]^{3/2}} - \frac{1}{\|\mathbf{r}_{EL}\|_2^3} \right\} \\ &= \left\{ [(\|\mathbf{r}_{EL}\|_2^2 + \|\mathbf{x}\|_2^2) + 2(\mathbf{r}_{EL} \cdot \mathbf{x})]^{-3/2} - \|\mathbf{r}_{EL}\|_2^{-3} \right\} \\ &= \left\{ \left[\left(1 + \left(\frac{\|\mathbf{x}\|_2}{\|\mathbf{r}_{EL}\|_2} \right)^2 + 2 \frac{(\mathbf{r}_{EL} \cdot \mathbf{x})}{\|\mathbf{r}_{EL}\|_2^2} \right)^{-3/2} - 1 \right] \|\mathbf{r}_{EL}\|_2^{-3} \right\} \end{aligned} \quad (5)$$

Since $\mathbf{r}_{EL} \gg \mathbf{x}$,

$$1 + \left(\frac{\|\mathbf{x}\|_2}{\|\mathbf{r}_{EL}\|_2} \right)^2 + 2 \frac{(\mathbf{r}_{EL} \cdot \mathbf{x})}{\|\mathbf{r}_{EL}\|_2^2} \approx 1 + 2 \frac{(\mathbf{r}_{EL} \cdot \mathbf{x})}{\|\mathbf{r}_{EL}\|_2^2} \quad (6)$$

Substitute Eq. 6 in the final expression of Eq. 5, then apply binomial expansion to first order.

$$\begin{aligned}
\left\{ \frac{1}{\|\mathbf{r}_{EF}\|_2^3} - \frac{1}{\|\mathbf{r}_{EL}\|_2^3} \right\} &\approx \left\{ \left[1 + 2 \frac{(\mathbf{r}_{EL} \cdot \mathbf{x})}{\|\mathbf{r}_{EL}\|_2^2} \right]^{-3/2} - 1 \right\} \|\mathbf{r}_{EL}\|_2^{-3} \\
&= \left\{ \left[1 + \left(-\frac{3}{2}\right) \left(2 \frac{(\mathbf{r}_{EL} \cdot \mathbf{x})}{\|\mathbf{r}_{EL}\|_2^2} \right) + H.O.T. \right] - 1 \right\} \|\mathbf{r}_{EL}\|_2^{-3} \quad (7) \\
&\approx -3 (\mathbf{r}_{EL} \cdot \mathbf{x}) \|\mathbf{r}_{EL}\|_2^{-5}
\end{aligned}$$

Likewise:

$$\left\{ \frac{1}{\|\mathbf{r}_{SF}\|_2^3} - \frac{1}{\|\mathbf{r}_{SL}\|_2^3} \right\} \approx -3 (\mathbf{r}_{SL} \cdot \mathbf{x}) \|\mathbf{r}_{SL}\|_2^{-5} \quad (8)$$

Combining Eqs. 7 and 8 yields.

$$\left\{ \frac{\mu_{em}}{\|\mathbf{r}_{EF}\|_2^3} + \frac{\mu_s}{\|\mathbf{r}_{SF}\|_2^3} \right\} \approx \frac{\mu_{em}}{\|\mathbf{r}_{EL}\|_2^3} \left[1 - \frac{3 (\mathbf{r}_{EL} \cdot \mathbf{x})}{\|\mathbf{r}_{EL}\|_2^2} \right] + \frac{\mu_s}{\|\mathbf{r}_{SL}\|_2^3} \left[1 - \frac{3 (\mathbf{r}_{SL} \cdot \mathbf{x})}{\|\mathbf{r}_{SL}\|_2^2} \right] \quad (9)$$

Substituting Eqs. 7, 8 and 9 into Eq. 4.

$$\begin{aligned}
\ddot{\mathbf{x}} &\approx - \left\{ \frac{\mu_{em}}{\|\mathbf{r}_{EL}\|_2^3} \left[1 - \frac{3 (\mathbf{r}_{EL} \cdot \mathbf{x})}{\|\mathbf{r}_{EL}\|_2^2} \right] + \frac{\mu_s}{\|\mathbf{r}_{SL}\|_2^3} \left[1 - \frac{3 (\mathbf{r}_{SL} \cdot \mathbf{x})}{\|\mathbf{r}_{SL}\|_2^2} \right] \right\} \mathbf{x} \\
&\quad + \left\{ 3 \frac{\mu_{em}}{\|\mathbf{r}_{EL}\|_2^5} (\mathbf{r}_{EL} \cdot \mathbf{x}) \right\} \mathbf{r}_{EL} + \left\{ 3 \frac{\mu_s}{\|\mathbf{r}_{SL}\|_2^5} (\mathbf{r}_{SL} \cdot \mathbf{x}) \right\} \mathbf{r}_{SL} \\
&\quad + \mathbf{u}_{thrust,F} - \mathbf{u}_{thrust,L}
\end{aligned} \quad (10)$$

$$\text{Note: } (\mathbf{r}_{EL} \cdot \mathbf{x}) \mathbf{r}_{EL} = \mathbf{r}_{EL} (\mathbf{r}_{EL}^\top \mathbf{x}) = [\mathbf{r}_{EL} \mathbf{r}_{EL}^\top] \mathbf{x}$$

Rewrite Eq. 10 as:

$$\begin{aligned}
\ddot{\mathbf{x}} &\approx - \left\{ \frac{\mu_{em}}{\|\mathbf{r}_{EL}\|_2^3} \left[1 - \frac{3 (\mathbf{r}_{EL} \cdot \mathbf{x})}{\|\mathbf{r}_{EL}\|_2^2} \right] + \frac{\mu_s}{\|\mathbf{r}_{SL}\|_2^3} \left[1 - \frac{3 (\mathbf{r}_{SL} \cdot \mathbf{x})}{\|\mathbf{r}_{SL}\|_2^2} \right] \right\} \mathbf{x} \\
&\quad + 3 \frac{\mu_{em}}{\|\mathbf{r}_{EL}\|_2^5} \{ [\mathbf{r}_{EL} \mathbf{r}_{EL}^\top] \mathbf{x} \} + 3 \frac{\mu_s}{\|\mathbf{r}_{SL}\|_2^5} \{ [\mathbf{r}_{SL} \mathbf{r}_{SL}^\top] \mathbf{x} \} \\
&\quad + \mathbf{u}_{thrust,F} - \mathbf{u}_{thrust,L} \\
&= \left\{ - \left(\frac{\mu_{em}}{\|\mathbf{r}_{EL}\|_2^3} \left[1 - \frac{3 (\mathbf{r}_{EL} \cdot \mathbf{x})}{\|\mathbf{r}_{EL}\|_2^2} \right] + \frac{\mu_s}{\|\mathbf{r}_{SL}\|_2^3} \left[1 - \frac{3 (\mathbf{r}_{SL} \cdot \mathbf{x})}{\|\mathbf{r}_{SL}\|_2^2} \right] \right) \mathbf{I}_3 \right. \\
&\quad \left. + 3 \frac{\mu_{em}}{\|\mathbf{r}_{EL}\|_2^5} [\mathbf{r}_{EL} \mathbf{r}_{EL}^\top] + 3 \frac{\mu_s}{\|\mathbf{r}_{SL}\|_2^5} [\mathbf{r}_{SL} \mathbf{r}_{SL}^\top] \right\} \mathbf{x} \\
&\quad + \mathbf{u}_{thrust,F} - \mathbf{u}_{thrust,L}
\end{aligned} \quad (11)$$

In summary the linearized dynamics are expressed as:

$$\begin{aligned}
\ddot{\mathbf{x}} &= \mathbf{A}(t) \mathbf{x} + \mathbf{u}_{thrust,F} - \mathbf{u}_{thrust,L} \\
\mathbf{A}(t) &= \left\{ - \left(\frac{\mu_{em}}{\|\mathbf{r}_{EL}\|_2^3} \left[1 - \frac{3(\mathbf{r}_{EL} \cdot \mathbf{x})}{\|\mathbf{r}_{EL}\|_2^2} \right] + \frac{\mu_s}{\|\mathbf{r}_{SL}\|_2^3} \left[1 - \frac{3(\mathbf{r}_{SL} \cdot \mathbf{x})}{\|\mathbf{r}_{SL}\|_2^2} \right] \right) \mathbf{I}_3 \right. \\
&\quad \left. + 3 \frac{\mu_{em}}{\|\mathbf{r}_{EL}\|_2^5} [\mathbf{r}_{EL} \mathbf{r}_{EL}^T] + 3 \frac{\mu_s}{\|\mathbf{r}_{SL}\|_2^5} [\mathbf{r}_{SL} \mathbf{r}_{SL}^T] \right\} \mathbf{x} \\
&= \left\{ - \left(\frac{\mu_{em}}{\|\mathbf{r}_{EL}\|_2^3} \left[1 - \frac{3(\mathbf{r}_{EL} \cdot \mathbf{x})}{\|\mathbf{r}_{EL}\|_2^2} \right] + \frac{\mu_s}{\|\mathbf{r}_{SL}\|_2^3} \left[1 - \frac{3(\mathbf{r}_{SL} \cdot \mathbf{x})}{\|\mathbf{r}_{SL}\|_2^2} \right] \right) \mathbf{I}_3 \right. \\
&\quad \left. + \frac{3 \mu_{em}}{\|\mathbf{r}_{EL}\|_2^3} [\hat{\mathbf{r}}_{EL} \hat{\mathbf{r}}_{EL}^T] + \frac{3 \mu_s}{\|\mathbf{r}_{SL}\|_2^3} [\hat{\mathbf{r}}_{SL} \hat{\mathbf{r}}_{SL}^T] \right\}
\end{aligned} \tag{12}$$

Note: $\hat{\mathbf{r}}_{EL}$ and $\hat{\mathbf{r}}_{SL}$ denote unit vectors along \mathbf{r}_{EL} and \mathbf{r}_{SL} , respectively.

For missions orbiting any of the Earth/Moon - Sun libration points with $\|\mathbf{x}\|_2$ small compared to $\|\mathbf{r}_{EL}\|_2$, allows further simplification of Eq. 12

$$\begin{aligned}
\mathbf{A}(t) &= \left\{ - \left(\frac{\mu_{em}}{\|\mathbf{r}_{EL}\|_2^3} + \frac{\mu_s}{\|\mathbf{r}_{SL}\|_2^3} \right) \mathbf{I}_3 + \frac{3 \mu_{em}}{\|\mathbf{r}_{EL}\|_2^3} [\hat{\mathbf{r}}_{EL} \hat{\mathbf{r}}_{EL}^T] \right. \\
&\quad \left. + \frac{3 \mu_s}{\|\mathbf{r}_{SL}\|_2^3} [\hat{\mathbf{r}}_{SL} \hat{\mathbf{r}}_{SL}^T] \right\}
\end{aligned} \tag{13}$$

For convenience, the expression for $\mathbf{A}(t)$ is consolidated in terms of coefficients c_1 and c_2 .

$$\begin{aligned}
\mathbf{A}(t) &= \{ -(c_1 + c_2) \mathbf{I}_3 + 3 c_1 [\hat{\mathbf{r}}_{EL}(t) \hat{\mathbf{r}}_{EL}(t)^T] + 3 c_2 [\hat{\mathbf{r}}_{SL}(t) \hat{\mathbf{r}}_{SL}(t)^T] \} \\
c_1 &= \mu_{em} \|\mathbf{r}_{EL}\|_2^{-3} \\
c_2 &= \mu_s \|\mathbf{r}_{SL}\|_2^{-3}
\end{aligned} \tag{14}$$

The linearized dynamics in matrix form are:

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_3 \\ \mathbf{A}(t) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_3 \end{bmatrix} (\mathbf{u}_{thrust,F} - \mathbf{u}_{thrust,L}) \tag{15}$$

Material to be Added

The following topics will be addressed in the final paper:

- Express Eq. 15 in terms of the rotating RTBP frame.
- Characterize the stability properties at various locations within the RTBP rotating frame, including the libration points.

REFERENCES

1. Carpenter, K. G., et al., "The Stellar Imager (SI) Mission Concept," *SPIE's Astronomical and Instrumentation Conference*, Waikoloa, Hawaii, August 2002.
2. Clohessy, W. H. and Wiltshire, R. S., "Terminal Guidance System for Satellite Rendezvous," *Journal of the Aerospace Sciences*, Sept. 1960
3. Gendreau, K. C. and Cash, W. C. and Shipley, A. F. and White, N., "The MAXIM Pathfinder X-ray Interferometry Mission," *SPIE's X-Ray and Gamma-Ray Telescopes and Instruments for Astronomy Conference*, Waikoloa, Hawaii, August 2002.
4. Hamilton, N. H. and Folta, D. and Carpenter, R., "Formation Flying Satellite Control Around The L2 Sun-Earth Libration Point," *Proceedings of the AAS/AIAA Astrodynamics Specialist Conference*, August 2002
5. Lawson, P. R., "The Terrestrial Planet Finder: the Search for Life Elsewhere," *Proceedings of the 1999 IEEE Aerospace Conference*, Aspen, CO, March 2001, Vol. 4, pp. 2005-2011.
6. Luquette, R. J. and Sanner, R. M., "A Nonlinear Approach to Spacecraft Trajectory Control in the Vicinity of a Libration Point," *Proceedings of the Flight Mechanics Symposium*, Goddard Space Flight Center, June 2001.
7. Luquette, R. J. and Sanner, R. M., "A Nonlinear Approach to Spacecraft Formation Control in the Vicinity of a Collinear Libration Point," *Proceedings of the AAS/AIAA Astrodynamics Specialist Conference*, July 2001, Paper No. AAS 01-330.
8. Luquette, R. J. and Sanner, R. M., "A Nonlinear, Six-Degree of Freedom, Precision Formation Control Algorithm, Based on Restricted Three Body Dynamics," *26th Annual AAS Guidance and Control Conference*, February 2003, Paper No. AAS 03-007.
9. "Official Web Site for the MicroArcsecond X-Ray Imaging Mission (MAXIM)," <http://maxim.gsfc.nasa.gov/>.

10. Noecker, C. and Kilston, S., "Terrestrial Planet Finder: the Search for Life Elsewhere," *Proceedings of the 1999 IEEE Areospace Conference*, Aspen, CO, March 1999, Vol. 4, pp. 49-57.
11. Segerman, A. M. and Zedd, M. F., "Preliminary Planar Formation-Flight Dynamics Near Sun-Earth L_2 Point," *Proceedings of the AAS/AIAA Space Flight Mechanics Meeting*, Ponce, Puerto Rico, February 2003, Paper No. AAS 03-133.
12. Shipley, A. F. and Cash, W. C. and Gendreau, K. C. and Gallagher, D., "MAXIM Interferometer Tolerances and Tradeoffs," *SPIE's X-Ray and Gamma-Ray Telescopes and Instruments for Astronomy Conference*, Waikoloa, Hawaii, August 2002.
13. "The Stellar Imager (SI) Homepage," <http://hires.gsfc.nasa.gov/%7Esi/>.
14. "The Terrestrial Planet Finder (TPF) Homepage," http://planetquest.jpl.nasa.gov/TPF/tpf_index.html.
15. Wie, B., *Space Vehicle Dynamics and Control*, AIAA Education Series., AIAA, Inc. Virginia, 1998.