PSEUDO LINEAR ATTITUDE DETERMINATION OF SPINNING
SPACECRAFT

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This paper presents the overall mathematical model and results from pseudo linear recursive estimators of attitude and rate for a spinning spacecraft. The measurements considered are vector measurements obtained by sun-sensors, fixed head star trackers, horizon sensors, and three axis magnetometers. Two filters are proposed for estimating the attitude as well as the angular rate vector. One filter, called the q-Filter, yields the attitude estimate as a quaternion estimate, and the other filter, called the D-Filter, yields the estimated direction cosine matrix. Because the spacecraft is gyro-less, Euler’s equation of angular motion of rigid bodies is used to enable the estimation of the angular velocity. A simpler Markov model is suggested as a replacement for Euler’s equation in the case where the vector measurements are obtained at high rates relative to the spacecraft angular rate.

Extended Abstract

**q-Filter Dynamics**

The first dynamics equation we consider is the following Euler’s equation for the angular motion of a spacecraft (SC). It is [1, pp. 522, 523]

\[ \dot{\omega} = \Gamma^{-1}[(I\omega + h)\times]\omega + \Gamma^{-1}(T - \dot{h}) \]  

(1)

where I is the SC inertia matrix, ω is the angular velocity vector, h is the angular momentum of the momentum wheels, and T is the external torque acting on the SC. The symbol \([a\times]\) denotes the cross product matrix of the general vector a. Attitude is represented by the attitude quaternion whose kinematic equation is [1, p. 512]

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The unbiased white-noise vector $w_o$ accounts for the inaccuracies in the modeling of the SC angular dynamics, and $w_q$ is an unbiased white-noise vector that accounts for modeling errors in the quaternion kinematics.

When the measurements come at a relatively high frequency we may be able to replace the SC angular dynamics model in Eq. (4.a) with a simpler Markov model [2]. Consequently, Eq. (4.a) is replaced by the model

$$
\begin{bmatrix}
\dot{\omega} \\
\dot{q}
\end{bmatrix} =
\begin{bmatrix}
-I^{-1}[(I\omega + h)\times] & 0 \\
\frac{1}{2}Q & 0
\end{bmatrix}
\begin{bmatrix}
\omega \\
q
\end{bmatrix} +
\begin{bmatrix}
I^{-1}(T - h) \\
0
\end{bmatrix} +
\begin{bmatrix}
w_o \\
w_q
\end{bmatrix}
$$

(4.b)

where $[\tau]$ is a diagonal matrix whose elements are the inverse of suitable time constants.

**q-Filter Measurement Model**

$$
b_m = \begin{bmatrix}
0 & H_j(r_j, q)
\end{bmatrix}
\begin{bmatrix}
\omega \\
q
\end{bmatrix} + v_{jb}
$$

(5)

where
is the reference vector corresponding to vector sensor j, and \( v_b \) is white noise.

**D-Filter Dynamics**

Using Euler's equation and assuming the spacecraft attitude is represented as a direction cosine matrix, the dynamics take on the following form:

\[
\begin{bmatrix}
\dot{\omega} \\
\dot{d}
\end{bmatrix} =
\begin{bmatrix}
I^{-1}[(I\omega + h)\times] & 0 \\
\mathcal{D} & 0
\end{bmatrix}
\begin{bmatrix}
\omega \\
d
\end{bmatrix}
+ I^{-1}(T - \dot{h}) +
\begin{bmatrix}
w_\omega \\
w_s
\end{bmatrix}
\]

(6)

where \( d^T = [d_1^T \ d_2^T \ d_3^T] \), \( d_j^T \) is the transpose of the jth column of the direction cosine matrix, and \( \mathcal{D} = \begin{bmatrix} [d_1 \times] \\ [d_2 \times] \\ [d_3 \times] \end{bmatrix} \) where \([d_j \times]\) is the skew symmetric matrix for jth column of the direction cosine matrix.

**The D-Filter Measurement Model**

For vector measurements, \( b_{jm} = [d_{1r} \ | d_2r \ | d_3r] + v_{j,b} \) where \( r \) is the corresponding reference vector for the observation and \( d_i \) is the ith column of the direction cosine matrix. This equation can be rearranged to form the measurement model:
\[ \mathbf{b}_{jn} = \begin{bmatrix} 0 & \mathbf{R}_j \end{bmatrix} \begin{bmatrix} \mathbf{\omega} \\ \mathbf{d} \end{bmatrix} + \mathbf{v}_{jb} \] (7)

**Conclusion**

Both the q-Filter and the D-Filter will be tested against simulated data and a comparison will be made of the relative performance of each.

**References**
