PSEUDO LINEAR ATTITUDE DETERMINATION OF SPINNING SPACECRAFT

Richard R. Harman† and Itzhack Y. Bar-Itzhack*

This paper presents the overall mathematical model and results from pseudo linear recursive estimators of attitude and rate for a spinning spacecraft. The measurements considered are vector measurements obtained by sun-sensors, fixed head star trackers, horizon sensors, and three axis magnetometers. Two filters are proposed for estimating the attitude as well as the angular rate vector. One filter, called the *q-Filter*, yields the attitude estimate as a quaternion estimate, and the other filter, called the *D-Filter*, yields the estimated direction cosine matrix. Because the spacecraft is gyro-less, Euler's equation of angular motion of rigid bodies is used to enable the estimation of the angular velocity. A simpler Markov model is suggested as a replacement for Euler's equation in the case where the vector measurements are obtained at high rates relative to the spacecraft angular rate.

Extended Abstract

q-Filter Dynamics

The first dynamics equation we consider is the following Euler's equation for the angular motion of a spacecraft (SC). It is [1, pp. 522, 523]

$$\dot{\boldsymbol{\omega}} = I^{-1}[(I\boldsymbol{\omega} + \mathbf{h})\times]\boldsymbol{\omega} + I^{-1}(\mathbf{T} - \dot{\mathbf{h}}) \tag{1}$$

where I is the SC inertia matrix, $\boldsymbol{\omega}$ is the angular velocity vector, $\mathbf{h}$ is the angular momentum of the momentum wheels, and $\mathbf{T}$ is the external torque acting on the SC. The symbol $[\mathbf{a}\times]$ denotes the cross product matrix of the general vector $\mathbf{a}$. Attitude is represented by the attitude quaternion whose kinematic equation is [1, p. 512]

† Aerospace Engineer, Tel: 301-286-5125, Fax: 301-286-0369
Flight Dynamics Analysis Branch, Code 595
Mission Engineering and Systems Analysis Division
NASA-GSFC Greenbelt, MD 20771
Email: richard.r.harman@nasa.gov

* Sophie and William Shamban Professor of Aerospace Engineering
Technion-Israel Institute of Technology. Asher Space Research Institute
Haifa 32000, Israel. Tel: 972-4-829-3196, Fax: 972-4-829-2030.
Email: ibaritz@technion.ac.il

$$\dot{\mathbf{q}} = \tfrac{1}{2} Q \boldsymbol{\omega} \tag{2}$$

where

$$Q = \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix} \tag{3}$$

In the q-filter we augment Eqs. (1) and (2) to form the following dynamics equation, which includes the noise terms

$$\begin{bmatrix} \dot{\boldsymbol{\omega}} \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} I^{-1}[(\mathbf{I}\boldsymbol{\omega}+\mathbf{h})\times] & 0 \\ \tfrac{1}{2}Q & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \end{bmatrix} + \begin{bmatrix} I^{-1}(\mathbf{T} - \dot{\mathbf{h}}) \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_{\omega} \\ \mathbf{w}_{q} \end{bmatrix} \tag{4.a}$$

The unbiased white-noise vector $\mathbf{w}_{\omega}$ accounts for the inaccuracies in the modeling of the SC angular dynamics, and $\mathbf{w}_{q}$ is an unbiased white-noise vector that accounts for modeling errors in the quaternion kinematics.

When the measurements come at a relatively high frequency we may be able to replace the SC angular dynamics model in Eq. (4.a) with a simpler Markov model [2]. Consequently, Eq. (4.a) is replaced by the model

$$\begin{bmatrix} \dot{\boldsymbol{\omega}} \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} -[\tau] & 0 \\ \tfrac{1}{2}Q & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_{\omega} \\ \mathbf{w}_{q} \end{bmatrix} \tag{4.b}$$

where $[\tau]$ is a diagonal matrix whose elements are the inverse of suitable time constants.

q-Filter Measurement Model

$$\mathbf{b}_{jm} = \begin{bmatrix} \mathbf{0}_3 & H_j(\mathbf{r}_j, \mathbf{q}) \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \end{bmatrix} + \mathbf{v}_{jb} \tag{5}$$

where

$$H_j(\mathbf{r}_j,\mathbf{q})=\begin{bmatrix} q_1r_1+q_2r_2+q_3r_3 & -q_2r_1+q_1r_2-q_4r_3 & -q_3r_1+q_4r_2+q_1r_3 & q_4r_1+q_3r_2-q_2r_3 \\ q_2r_1-q_1r_2+q_4r_3 & q_1r_1+q_2r_2+q_3r_3 & -q_4r_1-q_3r_2+q_2r_3 & -q_3r_1+q_4r_2+q_1r_3 \\ q_3r_1-q_4r_2-q_1r_3 & q_4r_1+q_3r_2-q_2r_3 & q_1r_1+q_2r_2+q_3r_3 & q_2r_1-q_1r_2+q_4r_3 \end{bmatrix}_j ,$$

$\mathbf{r}_j$ is the reference vector corresponding to vector sensor j, and $\mathbf{v}_{jb}$ is white noise.

D-Filter Dynamics

Using Euler's equation and assuming the spacecraft attitude is represented as a direction cosine matrix, the dynamics take on the following form:

$$\begin{bmatrix} \dot{\boldsymbol{\omega}} \\ \dot{\mathcal{d}} \end{bmatrix}=\begin{bmatrix} I^{-1}[(I\boldsymbol{\omega}+\mathbf{h})\times] & 0 \\ \mathcal{D} & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{\omega} \\ \mathcal{d} \end{bmatrix}+\begin{bmatrix} I^{-1}(\mathbf{T}-\dot{\mathbf{h}}) \\ \mathbf{0} \end{bmatrix}+\begin{bmatrix} \mathbf{w}_{\omega} \\ \mathbf{w}_{\delta} \end{bmatrix} \qquad (6)$$

where $\mathcal{d}^T=\begin{bmatrix} \mathbf{d}_1^T & \mathbf{d}_2^T & \mathbf{d}_3^T \end{bmatrix}$, $\mathbf{d}_j^{\,T}$ is the transpose of the jth column of the direction cosine matrix, and $\mathcal{D}=\begin{bmatrix} [\mathbf{d}_1\times] \\ [\mathbf{d}_2\times] \\ [\mathbf{d}_3\times] \end{bmatrix}$ where $[\mathbf{d}_j\,x]$ is the skew symmetric matrix for jth column of the direction cosine matrix.

The D-Filter Measurement Model

For vector measurements, $\mathbf{b}_{j,m}=\left[\mathbf{d}_1r_1\,|\,\mathbf{d}_2r_2\,|\,\mathbf{d}_3r_3\right]+\mathbf{v}_{j,b}$ where $\mathbf{r}$ is the corresponding reference vector for the observation and d_i is the ith column of the direction cosine matrix. This equation can be rearranged to form the measurement model:

$$\mathbf{b}_{jm} = \begin{bmatrix} \mathbf{0}_3 & R_j \end{bmatrix} \begin{bmatrix} \omega \\ d \end{bmatrix} + \mathbf{v}_{jb} \tag{7}$$

Conclusion

Both the q-Filter and the D-Filter will be tested against simulated data and a comparison will be made of the relative performance of each.

References

[1] Wertz, J.R., (Ed.), *Spacecraft Attitude Dynamics and Control*, Reidel Publishing Co., Dordrecht, Holland, 1978.

[2] Oshman, Y., and Markley, F. L., "Sequential Attitude and Attitude-Rate Estimation Using Integrated-Rate Parameters," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 3, 1999, pp. 385–394.