

TECHNICAL NOTE

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SOME RECENT STUDIES IN STRUCTURAL DYNAMICS OF ROTOR AIRCRAFT

By George W. Brooks and Milton A. Silveira

Langley Research Center
Langley Field, Va.

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SUMMARY

This paper presents a summary of four recent studies relating to the structural-dynamics problems of rotor-powered aircraft. The first study concerns the measurement by means of dynamic models of the forces and moments at the hubs of various rotor configurations. The results of this study show that the periodic components of the forces and moments are highly dependent on both the rotor configuration and the flight condition.

The second study treats the problem of resonance amplifications of rotor-blade stress and shows that by using multiple flapping hinges or flex-joints it is possible to control the natural frequencies of the rotor blade so that conditions of resonance between the frequencies of the aerodynamic input forces and the natural frequencies of the lower blade modes are avoided for all rotor speeds.

Two studies of the stability of rotor aircraft are also discussed. One of these involves the mechanical instability or ground resonance of rotorcraft wherein the rotor support in each of two mutually perpendicular directions in the rotor plane is represented as a multiple-degree-of-freedom system in contrast to the system having a "single" degree of freedom normally used in helicopter analysis. The consideration of the rotor support system as a two-degree-of-freedom system predicts additional unstable ranges of rotor speed not predicted by former analyses. The other instability treated is propeller whirl for which the significant motions are the pitching and yawing motions of the propeller disk which are coupled together by gyroscopic forces. The effect of the significant variables on the instability is presented and shows that the speed at which the instability occurs can be increased by increasing the relative stiffness of the propeller in the pitch and yaw directions or by increasing the effective structural damping of the nacelle.

INTRODUCTION

The importance of structural-dynamics problems in the development and use of rotor-powered aircraft is well recognized, and the interest

in these problems continues as the state of the art is advanced. Although it is necessary to reevaluate the problems as major developments in performance or changes in configuration occur, the basic situation remains that these aircraft are subjected during flight operations to a variety of aerodynamic loads and load distributions and they may encounter several types of instability such as ground resonance, propeller whirl, and flutter.

In the design of these aircraft to minimize the response of the structure to the loads imposed in flight and to avoid instabilities, it is necessary to know the nature and magnitudes of the applied loads and to define and control the system characteristics such as component natural frequencies.

This paper will briefly discuss some aspects of four recent studies relating to structural-dynamics problems of rotor-powered aircraft. These studies represent parts of investigations currently in progress at the Langley Research Center for which complete results are not presently available. These include: (1) the determination of the force and moments at the hubs of various rotor configurations; (2) the use of multiple flapping hinges to control the natural frequencies of rotor blades; (3) the effect of additional degrees of freedom in the rotor support on the mechanical stability of rotors; and (4) the effect of stiffness and damping on the gyroscopic whirling stability of propellers.

SYMBOLS

a_1	radial location of first flapping hinge, ft
a_2	radial location of second flapping hinge, ft
a_3	radial location of third flapping hinge, ft
b_a	normalized imaginary part of root of characteristic equation $b_a = \frac{b}{\omega_r}$
F_{VIB}	vibratory component of axial force, lb
F_{STEADY}	steady component of axial force, lb
g	structural damping coefficient
N	integer number
R	rotor radius, ft

V	forward velocity, ft/sec
V_d	reference forward velocity, ft/sec
α	rotor angle of attack, deg
θ	collective pitch angle of rotor blades, deg
μ	tip-speed ratio, $\frac{V \cos \alpha}{\Omega R}$
ϕ	propeller pitch angle, deg
ψ	propeller yaw angle, deg
ω_n	natural frequency of nth mode of rotor blade, radians/sec
ω_r	reference frequency, radians/sec
Ω	rotational frequency, radians/sec
Ω_a	normalized rotational frequency, Ω/ω_r

RESULTS AND DISCUSSION

A Study of the Periodic Forces and Moments at the Rotor Hub

This section of the paper treats the variations of the forces and moments at the hub of various rotor configurations.

A wind-tunnel investigation has been conducted with the use of model rotors on a dynamic balance which is capable of measuring the steady and periodic components of the forces and moments about three mutually perpendicular axes through the hub as shown in figure 1. The forces and moments were measured over a range of operating conditions for four rotor configurations which included a two-, three-, and four-blade flapping rotor and a two-blade teetering rotor. All rotor configurations employed identical blades with a chord of 2.06 inches and a solidity of 0.02 per blade. The blades were scaled to possess dynamic properties representative of those in current use. The rotor diameter was 66 inches. The variables studied included collective pitch angles of 0° , 3° , and 6° ; rotor angles of attack of -10° , -5° , 0° , and 5° ; and variations of rotor speeds and tunnel speeds to encompass a range of tip-speed ratios from 0 to 0.45.

The measured loads were harmonically analyzed to determine the magnitudes and frequencies of the periodic components. Some typical samples of these data are presented in figures 2 and 3 for a collective pitch angle of 3° and a rotor angle of attack of -5° .

The Nth harmonic vibratory components of the axial force for N-blade flapping rotors are plotted as a function of tip-speed ratio in figure 2. The ordinate is the ratio of the vibratory force to the steady force measured at that tip-speed ratio, and data are presented for the two-, three-, and four-blade flapping configurations. Figure 3 presents a comparison of similar data for the two-blade flapping and teetering configurations. These initial results indicate that the vibratory forces increase slightly as the tip-speed ratio increases from zero and then decrease before rising sharply at the higher tip-speed ratios. In general, the data show that the variation of the vibratory forces with μ exhibits trends which are similar to the measured vibration trends on rotorcraft, and that the magnitudes of the vibratory forces are highly dependent on both the flight condition and rotor configuration.

The Use of Multiple Flapping Hinges or Flex-Joints to Control the Natural Frequencies of Rotor Blades

An analytic study has been conducted to investigate the control of the natural frequencies of rotor blades by means of multiple flapping hinges and this section of the paper will present some of the results of that study. A typical rotor frequency diagram for a conventional blade is shown in figure 4. The frequencies of the applied aerodynamic forces and the natural frequencies are presented as a function of rotor speed. The frequencies of the aerodynamic forces are indicated by the straight lines which radiate from the origin, and the natural frequencies of the flapping and first two elastic modes are shown by the dashed lines. For helicopter operation, the rotor speed is restricted to a rather narrow range during normal operation, and the primary problem is to design the blade so that the natural frequencies and aerodynamic input frequencies are as widely separated as possible at these rotor speeds in order to minimize resonant amplifications of blade bending stresses. For some types of V/STOL aircraft, it may be desirable to vary the rotor speed over a significant range to provide optimum performance. An examination of figure 4 shows that it is virtually impossible to vary the rotor speed of a conventional blade without encountering resonance involving some of the modes.

In order to obtain greater control over the natural frequencies and to minimize the resonance problem, a study has been made of a rotor blade which employs "rigid" segments connected by multiple flapping hinges or

flex-joints as shown in figure 5. This figure also presents the frequency diagram for such a blade. The use of multiple flapping hinges permits the natural frequencies of the lower modes at any rotor speed to be placed between the frequencies of the harmonic excitation forces as shown. A wide range of control over these frequencies is available by choice of number of hinges, hinge location, and mass distribution of the segments. For a blade having N flapping hinges, the first N natural frequencies will be multiples of the rotor speed as shown here, and thus the natural frequencies of these modes are fixed relative to the aerodynamic input frequencies at all rotor speeds. In general, the investigation is primarily concerned only with the lower harmonics of the aerodynamic loading, and there seems to be little necessity for more than three hinges. Figure 6 shows the variations in natural frequencies obtainable for a rotor of uniform mass distribution with three flapping hinges where the most inboard hinge is located at 4 percent of the radius ($a_1 = 0.04R$). The first natural frequency is independent of the outboard hinge locations. The locations of the second and third hinges may be selected so that the natural frequencies of the second and third modes of the blades may be fixed as desired relative to the rotor speed. For example, the frequencies for the case shown in figure 5, where the blade natural frequencies are 2.5 and 5.5 times the rotor speed for the second and third modes, respectively, were obtained by selecting $a_2 = 0.4R$ and $a_3 = 0.64R$.

The results of this study show that the blade natural frequencies can be widely varied or controlled by using multiple flapping hinges or flex-joints.

A Study of the Mechanical Instabilities of a Rotor on a Support Having Two Degrees of Freedom

Mechanical instability, commonly called ground resonance, first became a problem with the autogiro, and continued to be a major problem for many types of helicopters. It was found that this instability is not dependent on aerodynamic forces; the energies involved are stored in the rotor by virtue of its rotation. The designs of autogiros and helicopters are such that the balance of mass and spring forces necessary to produce the instability occurs only when the aircraft are in partial or total contact with the ground. However, when rotors are mounted on flexible wings, as in the case of some current and proposed VTOL and STOL aircraft, the instability may occur in flight and result in serious accidents. For the autogiro and the helicopter, the structure which supports the rotor may generally be considered as a system having a "single" degree of freedom in each of the two mutually perpendicular directions in the rotor plane as analyzed in references 1 to 3. For V/STOL configurations, however, the rotor support has at least two degrees of freedom in

any horizontal direction, one of which represents the mass and stiffness of the rotor mount relative to the wing, and another of which represents the mass and stiffness of the wing relative to the fuselage.

A study has been conducted to determine the mechanical instabilities of two-blade teetering rotors applicable to V/STOL aircraft such as shown in figure 7. The degrees of freedom considered in this analysis involve motions which are parallel to the plane of the rotor and include blade chordwise bending, shaft bending, and wing bending and torsion. This analysis differs from previous well-known analyses in that the rotor support is represented as a two-degree-of-freedom system, rather than by a system which has a "single" degree of freedom as treated in reference 1 and in other papers.

A typical stability diagram for the system studied is presented, in a rotating coordinate system, in figure 8. The imaginary parts of all roots of the system (where it is assumed that any characteristic motion is represented by $X = X_0 e^{\lambda_a t}$, where $\lambda_a = a_a + ib_a$) are plotted as a function of the rotor speed. Both the abscissa and ordinate are normalized by dividing through by a reference frequency. The complete system has six degrees of freedom, five of which are coupled and the other uncoupled. The uncoupled mode is that of the antisymmetric chordwise bending mode of the blades. The coupled modes involve motions of the blades, hub, and rotor support system (wing). The natural frequencies are the purely imaginary roots and are shown as the solid lines. This frequency diagram shows that there are five regions (shaded) where one or more of the roots become complex. Further analyses show that the real part a_a of one of these roots in each case is positive, indicating instability. Three of the regions, in which the instabilities are divergent oscillations, are indicated by the vertical shading. This instability is commonly referred to as ground resonance. Two other regions of instability are also indicated by the lightly shaded areas. The motions involved with these instability regions are purely divergent. Each of these two regions is bounded by two so-called critical speeds.

A comparison of the results of this study with those of previous analyses is shown in figure 9. The figure shows that the inclusion of the additional degree of freedom in the rotor support introduces an additional region of pure divergence bounded by two additional critical speeds. Two additional regions of divergent oscillations are also encountered.

All of the mechanical instabilities discussed are subject to control by the inclusion of damping in the system. Both damping of the hub relative to the wing, and of the wing relative to the fuselage, were investigated to determine the effect of damping on the mechanical

instabilities. Figure 10 shows the manner in which the regions for divergent oscillations vary with damping. The damping, in percent critical, is the same for the two uncoupled components (i.e., hub or wing) and is plotted as a function of rotor speed. As the damping is increased from zero, the width of the first and third instability regions increases, whereas the width of the second region decreases. However, it should be pointed out that the magnitudes of the unstable roots diminish in all cases. As the damping is further increased, all unstable speed ranges close and ultimately disappear.

The results of this study show that the mechanical instability problem for V/STOL aircraft may be substantially different than that for autogiros and helicopters, in that additional degrees of freedom of the system lead to additional possibilities for instability. The use of damping is effective in the elimination of these instabilities; however, from a practical standpoint, the difficulty of obtaining sufficient damping in the wing must be considered.

A Discussion of the Whirling Instability of Propellers

This section of the paper will present a brief introductory discussion of another type of instability called propeller whirl which might be of concern to V/STOL aircraft. As shown in figure 11, the propeller disk undergoes pitching and yawing motions as a result of shaft pitching and yawing relative to the wing, shaft bending, and wing torsion. Gyroscopic forces couple the pitching and yawing motions together, and when the aerodynamic forces are included, an instability can arise.

This instability was recognized many years ago (ref. 4); however, the stiffness levels of airplane engine installations were so high that the instability was of no real concern and further considerations of the problem vanished. Recently, the requirements for increased vibration isolation has led to softer engine mountings. These requirements, coupled with the larger overhangs of power-plant installations and higher flight speeds, have resulted in reduced margins for propeller whirl instability. It is also likely that the rigidity of movable rotor-wing systems such as used on some V/STOL aircraft might be relatively low, and it is therefore important that these systems be examined to be sure that they are stable in this respect.

Some results which have been obtained from studies of conventional aircraft propellers are shown in figure 12. These results give a broad insight into the phenomena and indicate some of the significant variables involved. Further details on the solution of the problem for some specific configurations are given in reference 5. Figure 12 shows the effects of damping and stiffness on the velocities at which whirl instability is encountered. The structural damping coefficient g , which represents

some mean value of the damping of the pitch and yaw motions of the propeller, is plotted as a function of the ratio of velocity to a reference velocity. For the case presented in figure 12, the ratio of damping in the pitch and yaw directions is unity and an increase in g represents an equal increase in damping of the pitch and yaw motions. Curves which separate the stable and unstable regions are presented for different values of relative effective stiffness of the shaft at the propeller hub. This stiffness also corresponds to some mean value of the stiffnesses in the pitch and yaw directions. For the case presented in figure 12 the stiffness is assumed to be equal in the pitch and yaw directions. However, for cases where the stiffness is not equal in the two directions, the trends would still be similar to those shown. In a general study the effects of the various individual parameters should be investigated.

The figure shows that, for a given stiffness level, the instability may be encountered at a low forward speed if the structural damping is low. However, with a small increase in damping, the speed at which the instability occurs is increased substantially. Conversely, for a given value of damping, represented by a horizontal line on the figure, the speed at instability may be substantially increased by increasing the stiffness. This stiffness of course reflects both the stiffness of the propeller shaft and of the wing support structure. Another variable of importance, not shown on this chart, is the effective pivot point. The stability is generally increased as the pivot point is moved rearward. From the standpoint of V/STOL applications, the effect of flapping hinges on this phenomenon may also be significant and should be examined.

CONCLUDING REMARKS

Some recent studies in structural dynamics of rotor aircraft have been reviewed in this paper. The results of these studies are as follows:

1. The results obtained during the measurement of the forces and moments at the rotor hub for various dynamic model rotor configurations indicate that the vibratory components of these forces and moments are dependent upon both the rotor configuration and on the flight condition. A few samples of data are presented which show that the levels of the vibratory forces increase as the tip-speed ratio is increased from zero, and then decrease slightly before rising sharply at the higher tip-speed ratios.

2. The natural frequencies of rotor blades can be controlled by the use of multiple flapping hinges or flex-joints. For a blade having N flapping hinges, the first N natural frequencies will be multiples of the rotor speed. By proper selection of hinge locations, the

L
1
4
3
1

frequencies of the rotor blade can be placed between the aerodynamic loading frequencies for all rotor speeds thus reducing resonant amplification of blade stresses.

3. The mechanical instability of a rotor may be substantially different from the results of former analyses when considering an additional degree of freedom in the rotor support system.

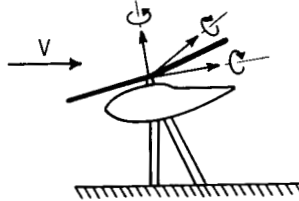
4. The whirling instability of propellers is dependent upon the structural damping and relative stiffness of the system. The speed at which the instability occurs is increased by increasing either the damping or stiffness of the system.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., November 18, 1960.

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SCOPE OF INVESTIGATION OF PERIODIC FORCES AND MOMENTS AT ROTOR HUB



ROTOR CONFIGURATIONS

4-BLADE }
3-BLADE } FLAPPING
2-BLADE }
2-BLADE TEETERING

VARIABLES

COLLECTIVE PITCH ANGLE, $\theta = 0^\circ, 3^\circ, 6^\circ$
ROTOR ANGLE OF ATTACK, $\alpha = -10^\circ, -5^\circ, 0^\circ, 5^\circ$
TUNNEL VELOCITY }
ROTOR SPEED } $\mu = 0 \text{ TO } 0.45$

Figure 1

Nth HARMONIC OF AXIAL FORCE OF N-BLADE FLAPPING ROTOR

$\theta = 3^\circ; \alpha = -5^\circ$

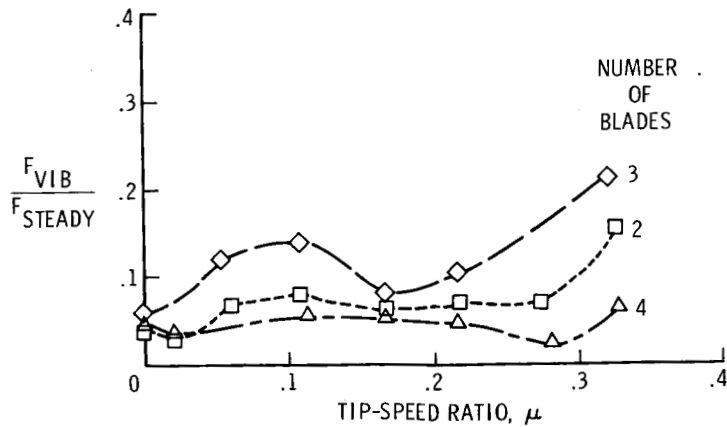


Figure 2

SECOND HARMONIC OF AXIAL FORCE FOR 2-BLADE ROTOR

$$\theta = 3^0; \alpha = -5^0$$

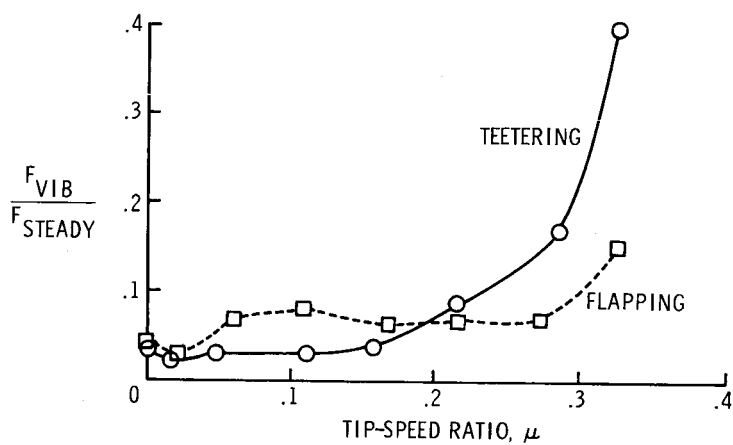


Figure 3

BLADE FREQUENCY DIAGRAM CONVENTIONAL BLADE

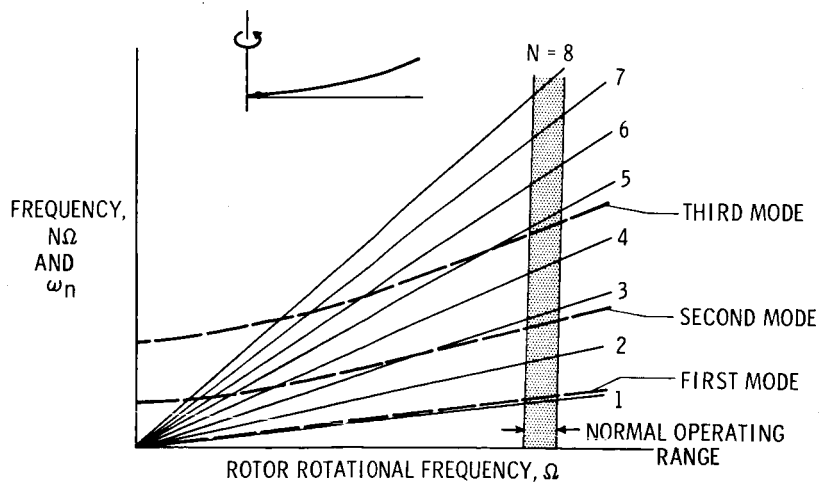


Figure 4

BLADE FREQUENCY DIAGRAM

MULTIPLE-HINGE BLADE

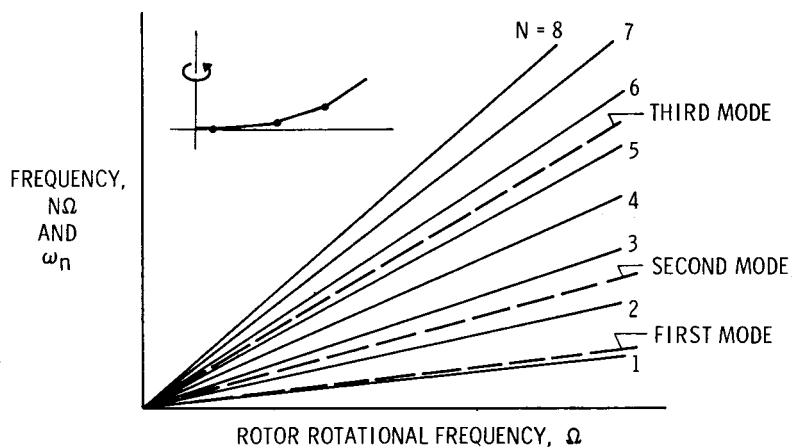


Figure 5

VARIATION OF NATURAL FREQUENCIES OF 3-HINGE RIGID-SEGMENT BLADE WITH HINGE LOCATIONS

$a_1/R = 0.04$

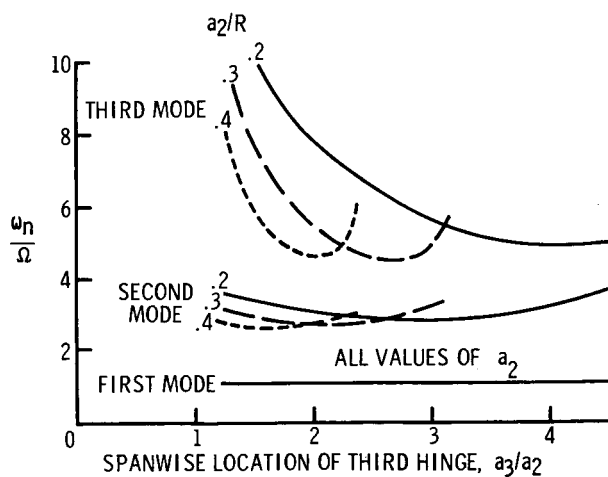
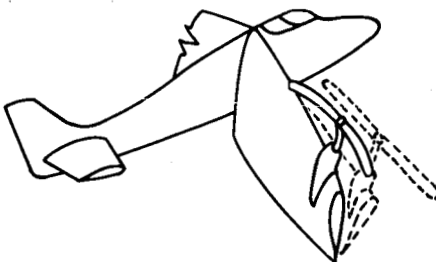


Figure 6

MECHANICAL INSTABILITY OF ROTORCRAFT

AERODYNAMIC FORCES UNNECESSARY



SIGNIFICANT MOTIONS ARE PARALLEL TO ROTOR PLANE:

1. BLADE CHORDWISE BENDING
2. SHAFT BENDING
3. WING BENDING AND TORSION

Figure 7

MECHANICAL INSTABILITIES AND NATURAL FREQUENCIES OF

2-BLADE ROTOR ON 2-DEGREE-OF-FREEDOM SUPPORT

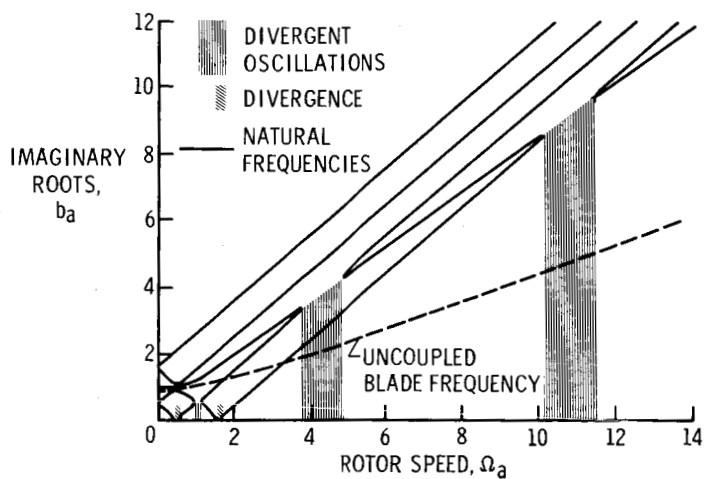


Figure 8

MECHANICAL INSTABILITY

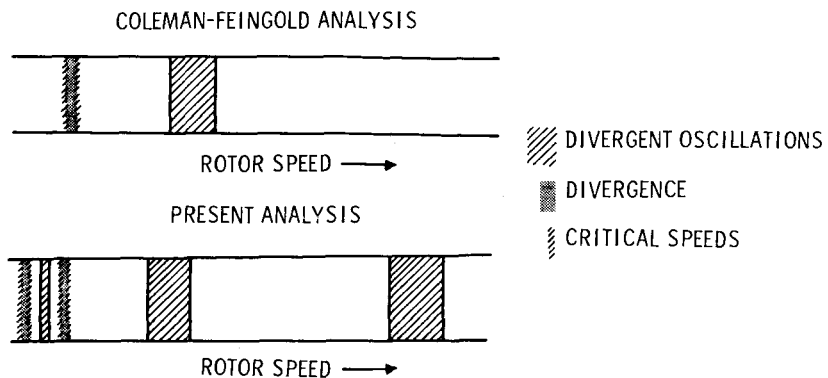


Figure 9

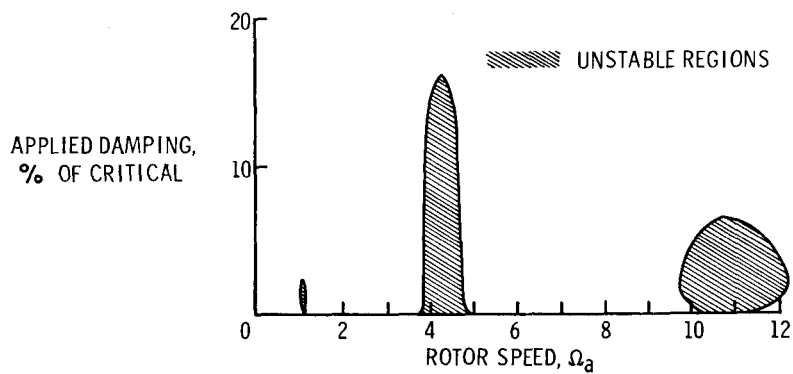
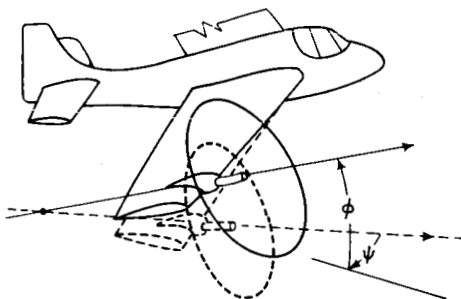
EFFECT OF DAMPING ON
DIVERGENT OSCILLATIONS

Figure 10

WHIRLING INSTABILITY OF ROTORCRAFT

AERODYNAMIC FORCES NECESSARY



SIGNIFICANT MOTIONS ARE GYROSCOPIC MOTIONS OF ROTOR:

1. SHAFT PITCHING AND YAWING
2. SHAFT BENDING
3. WING TORSION

Figure 11

WHIRLING INSTABILITY

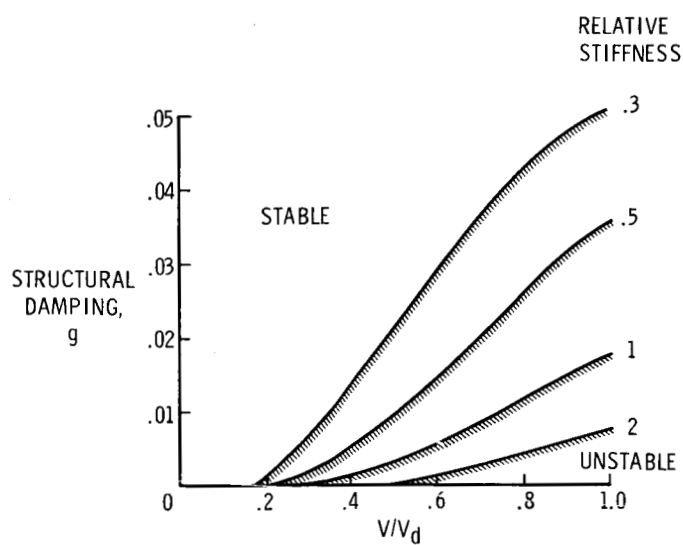


Figure 12