

# ELIMINATION OF GRAVITY INFLUENCE ON FLAME PROPAGATION VIA ENHANCEMENT OF THE SAFFMAN-TAYLOR INSTABILITY

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## INTRODUCTION

In this analytical work the influence of the Saffman-Taylor instability [1] on flame propagation is formulated for computational investigation. Specifically, it is of interest to examine the influence of this instability as a potential means of eliminating the effect of gravitational acceleration on the development of thermoacoustic instability. Earlier experimental investigations of thermoacoustic instability [2-6] employed tubes of large circular or annular cross-section, such that neither heat loss nor viscosity at the burner walls was of significant importance in influencing flame behavior. However, it has been demonstrated recently [7] that flames propagating between closely spaced walls, may be subject to long-wavelength wrinkling associated with the Saffman-Taylor instability, known to be relevant when a less-viscous fluid pushes a more-viscous fluid through a porous medium or between two closely spaced walls.

## FORMULATION

Conservation equations governing the interaction between a flame and the dynamics of the flow through which it propagates have been formulated for conditions under which the activation energy characterizing the overall chemical reaction is large and the flame thickness and characteristic chemical time are much smaller than the length and time scales characterizing temporal and spatial variations of the flow field, respectively [8]. These equations have been solved analytically for freely propagating flames with weak curvature and flow-field strain to provide jump conditions for the flow field across the flame and an evolution equation governing the motion of the flame [9-12]. Recently, the formulation has been extended to account for the presence of acoustic waves in the flow that may influence flame dynamics and result in thermoacoustic instabilities [4, 13, 14]. This formulation does not take into account, however, influences of heat loss or viscosity at the burner walls, either of which could be important if the walls confining flame propagation are sufficiently close. In the present work the formulation of Searby and Rochwerger [14] is employed to account for the presence of acoustic waves, but with the addition of a term in the momentum conservation equation accounting for the effect of viscosity at the burner walls, following the phenomenological approach of Joulin and Sivashinsky [7]. The following equations are assumed to govern fluctuations of the velocity and pressure field ( $\underline{u}$ ,  $p$ ), assumed to be small.

$$\left. \begin{aligned} \mathbf{r} \frac{\partial \underline{u}}{\partial t} + \frac{\partial \underline{u}}{\partial x} &= -\nabla p + (\nabla \mathbf{a}) \frac{d\bar{P}}{dx} + \mathbf{e} \text{Pr} s (\nabla^2 \underline{u} - f \underline{u}) \\ \frac{d\bar{P}}{dx} &= -\mathbf{r} \text{Ri} - \mathbf{e} \text{Pr} (s/\mathbf{r}) f \\ \nabla^2 p &= (\nabla^2 \mathbf{a}) \frac{d\bar{P}}{dx} \end{aligned} \right\} \quad (1)$$

The independent spatial variables appearing in Eq. (1) are measured in units of the tube diameter, annulus width or flat-plate separation distance  $d$ , while time is measured in units of  $d/U_L$ ; where  $U_L$  is the laminar burning speed for the given reactive mixture. Like the independent spatial

variables, the dependent flame-surface perturbation  $\mathbf{a}$  about the location of the mean flame-surface plane is also measured in units of  $d$  while velocity, pressure and density  $\mathbf{r}$  are measured in units of  $U_L$ ,  $\mathbf{r}_u U_L^2$  and  $\mathbf{r}_u$ , respectively; where  $\mathbf{r}_u$  is the density of the cold reactant flow. Other parameters appearing in the equation are the Prandtl and Richardson numbers defined as  $\text{Pr} \equiv \mathbf{n}/\mathbf{a}$  and  $\text{Ri} \equiv g d / U_L^2$ , where  $\mathbf{n}$  and  $\mathbf{a}$  are the kinematic viscosity and thermal diffusivity of the mixture, respectively; and the ratios  $\mathbf{e}$  and  $s$  of the flame thickness to  $d$  and of the thermal conductivity of the mixture to its value in the cold reactant flow, respectively. The parameter  $f$  is a constant friction factor accounting for the effect of viscosity at the inner walls of the tube or annulus within which, or flat plates between which, the flame propagates and is determined by the profile of the mean velocity across the flow cross-section. Note that the independent spatial variable  $x$  is measured normal to the reaction zone of the flame, having been introduced through a transformation from laboratory coordinates to a coordinate system that moves with the flame locally [8].

The motion of the flame, considered as an interface separating the cold reactants from the hot products, is dictated by its advection by the flow and its local rate of propagation into the reactants, as determined by thermodynamic and transport properties of the flow and local flow-field strain and flame-surface curvature. Small local perturbations about the mean reaction-zone location of a quasi-planar, weakly stretched flame (with small flow-field strain and flame-surface curvature) are predicted [8-10, 12, 15-18] to obey a linear evolution equation when the overall chemical activation energy is assumed to be large; namely,

$$\frac{\partial \mathbf{a}}{\partial t} = u_{(x=0^-)} + \mathbf{e} \left[ \nabla^2 \mathbf{a} - (\partial u / \partial x)_{(x=0^-)} \right], \quad (2)$$

where the velocity fluctuation along the direction of mean flame propagation  $u$  and its gradient  $\partial u / \partial x$  are evaluated in the reactant flow just upstream of the flame, as denoted. Eqs. (1) and (2) comprise a closed set of five equations to be solved for the five flame and flow-field perturbation variables ( $\mathbf{a}$ ,  $p$ ,  $\underline{\mathbf{u}}$ ). Since the flow-field equations are valid only in the incompressible regions on either side of the flame, jump conditions on the flow variables must be satisfied at the moving flame front to ensure continuity of the flow across the flame. The jump conditions are obtained in the same manner as that leading to Eq. (2), through integration of the full set of conservation equations, including those for energy and reactant species, across the reaction and preheat zones of the flame using asymptotic expansions that exploit large chemical activation energies and the disparity between the thicknesses of the reactant and preheat zones and between the overall flame thickness and the length scales over which order-unity variations of the hydrodynamic reactant and product flow occur.

Evaluation of linear stability proceeds with the assumption of harmonic flow-field and flame-surface fluctuations ( $\underline{\mathbf{u}}, p, \mathbf{a}$ ) according to  $e^{\Omega t + i \mathbf{k} \cdot \underline{\mathbf{y}}}$ , where  $\mathbf{k}$  is the wavenumber vector characterizing fluctuations in the transverse coordinate plane, measured in units of  $d^{-1}$ , and  $\Omega$  represents the growth rate (real part) and frequency of fluctuations (imaginary part) measured in units of  $U_L / d$ . Calculation of the fluctuation amplitudes by solution of Eqs. (1) and (2), subject to the jump conditions across the flame, results in a dispersion relation which governs whether the fluctuation amplitudes will grow in time or be attenuated by stabilizing influences. The dispersion relation, which takes the form

$$A(k)\Omega^2 + B(k)\Omega - C(k) = 0, \quad (3)$$

where  $A$ ,  $B$  and  $C$  are functions of the magnitude  $k$  of the wavenumber vector characterizing fluctuations in the transverse coordinate plane and the thermodynamic and transport properties of the reactant and product flows, has been derived in earlier studies of the stability of freely propagating wrinkled flames with weak curvature and flow-field strain [9-12]. Recently, corrections to the functions  $A$ ,  $B$  and  $C$  valid through order  $\mathbf{e}^2$  to account for the effect of viscosity at the

burner walls has been provided by Joulin and Sivashinsky [7]. However, in this recent work second-order influences on flame stability associated with flame stretch were neglected. In the present work, the recent corrections have been incorporated into the earlier formulation to provide the following forms of the functions  $A$ ,  $B$  and  $C$  that account for influences of both viscous damping of the mean flow that could give rise to the Saffman-Taylor instability as well as flame-stretch, which may provide attenuation of small-scale flame-surface wrinkles.

$$\left. \begin{aligned} A &= (\mathbf{s} + 1) + \mathbf{e}k(\mathbf{s} - 1)\left(M_a - \frac{\mathbf{s}}{\mathbf{s} - 1}J\right) \\ B &= 2\mathbf{e}k\mathbf{s}\left[1 + \mathbf{e}k\mathbf{s}(M_a - J)\right] + \mathbf{e}^2 \text{Pr} f(\mathbf{s}^{3/2} + \mathbf{s}) \\ C &= (\mathbf{e}k)^2 \mathbf{s}(\mathbf{s} - 1)\left[1 + (2\text{Pr} - 1)h - \mathbf{e}k\left(c_b + \frac{3\mathbf{s} - 1}{\mathbf{s} - 1}M_a - \frac{2\mathbf{s}}{\mathbf{s} - 1}J\right)\right] \\ &\quad - \mathbf{e}k(\mathbf{s} - 1)\left\{\left[1 - \mathbf{e}k\left(M_a - \frac{\mathbf{s}}{\mathbf{s} - 1}J\right)\right]\text{Ri} - \mathbf{e}^2 \text{Pr} f\mathbf{s}\left(\frac{\mathbf{s}^{3/2} - 1}{\mathbf{s} - 1}\right)\right\} \end{aligned} \right\} \quad (4)$$

The new parameters introduced in Eq. (4) are defined as follows,

$$\left. \begin{aligned} h &\equiv \int_0^1 [c_b - c(\mathbf{q})] d\mathbf{q} \\ J &\equiv (\mathbf{s} - 1) \int_0^1 \frac{c(\mathbf{q})}{1 + (\mathbf{s} - 1)\mathbf{q}} d\mathbf{q} \\ M_a &\equiv \frac{\mathbf{s}}{\mathbf{s} - 1}J - \frac{1}{2}l_e \int_0^1 \frac{c(\mathbf{q})\ln(\mathbf{q})}{1 + (\mathbf{s} - 1)\mathbf{q}} d\mathbf{q} \\ l_e &\equiv \mathbf{b}(L_e - 1), \quad \mathbf{b} \equiv \frac{E}{Rl_b} \left(\frac{\mathbf{s} - 1}{\mathbf{s}}\right) \end{aligned} \right\}, \quad (5)$$

where  $\mathbf{q}$  denotes the ratio of the departure of the temperature from that of the reactants ( $T - T_u$ ) to the temperature difference across the flame ( $T_b - T_u$ ),  $c$  is the ratio of the thermal conductivity to the specific heat normalized by the same ratio evaluated in the cold reactant mixture, and  $L_e$  is the Lewis number defined as the ratio of the thermal diffusivity to the mass diffusivity of the limiting reactant, where one reactant is considered to be present in excess. The Zel'dovich number  $\mathbf{b}$  is defined as the ratio of the activation energy  $E$  to the product of the gas constant  $R$  and burnt-gas temperature  $T_b$ , normalized by a measure of the temperature difference across the flame, and is assumed to be large. The parameter  $\mathbf{s}$  is simply the ratio of the temperature of the burnt gases to that of the cold reactant flow. The dispersion relation as defined in Eq. (3) indicates that flow-field and flame-surface perturbations will be unstable and grow exponentially in amplitude when the sign of last term of the equation,  $C(k)$ , is positive. Otherwise, the growth rate  $\Omega$  has no positive real part and such perturbations will be attenuated as a result of stabilizing influences. The new terms in the dispersion relation accounting for possible influence of the Saffman-Taylor instability appear as second-order corrections to coefficients  $A$  and  $B$  involving the product of the friction factor  $f$  and the Prandtl number. Note in particular that although the effect of gravitational acceleration acting along the mean propagation direction, exhibited through the Richardson number  $\text{Ri}$ , is stabilizing in that the value of  $C(k)$  decreases with increasing  $\text{Ri}$ , the new second-order viscous term counteracts the influence of this stabilizing effect and may cause  $C(k)$  to become positive when the tube diameter, annulus width or flat-plate separation distance  $d$  is sufficiently small.

The problem of linear stability can be posed alternatively, in terms of the same coefficients  $A$ ,  $B$  and  $C$ , by considering the dispersion relation as the characteristic equation associated with a second-order evolution equation governing a linear oscillator. Specifically, the second-order, linear ordinary differential equation

$$A(k) \frac{\partial^2 \mathbf{a}}{\partial t^2} + B(k) \frac{\partial \mathbf{a}}{\partial t} - C(k) \mathbf{a} = 0 \quad (6)$$

has solutions of the form  $\exp(\Omega t)$ , with the growth rate  $\Omega$  satisfying the dispersion relation given in Eq. (3) resulting from solution of the set of Eqs. (1) and (2) and application of the flow-field jump conditions at the flame front. Analysis of Eq. (6) can be fruitful, therefore, in obtaining predictions of the stability of flame-surface perturbations once the coefficients  $A$ ,  $B$  and  $C$  have been determined in the manner described above. Eq. (6) serves also as a point of departure for consideration of the influence of acoustic flow-field fluctuations on flame stability, following Searby and Rochwerger [14] and Bychkov [13], through the introduction an oscillating acceleration field as an additional term in the function  $C(k)$ . Specifically, replacing  $R_i$  in this function with  $R_i - U_a \mathbf{w} \cos(\mathbf{w}t)$ , where  $U_a$  and  $\mathbf{w}$  are the amplitude and frequency of axial acoustic velocity fluctuations present in the flow measured in units of  $U_L$  and  $U_L/d$ , respectively, results in the following modified form of Eq. (6).

$$\left. \begin{aligned} A(k) \frac{\partial^2 \mathbf{a}}{\partial t^2} + B(k) \frac{\partial \mathbf{a}}{\partial t} - [C(k) - D(k) U_a \mathbf{w} \cos(\mathbf{w}t)] \mathbf{a} = 0 \\ D(k) \equiv \mathbf{e}k(\mathbf{s} - 1) \left[ 1 - \mathbf{e}k \left( Ma - \frac{\mathbf{s}}{\mathbf{s} - 1} J \right) \right] \end{aligned} \right\} \quad (7)$$

Analysis of Eq. (7) allows the calculation of stability diagrams predicting acoustic-velocity amplitudes and wavenumbers for which flame propagation is unstable [13, 14]. The stability diagrams are obtained in the same manner as that described in detail by Bychkov [13]. However, predictions made with the modified formulation presented herein account for the importance of the Saffman-Taylor instability, associated with the disparity between the viscosities of the reactant and product flows, when the tube diameter, annulus width or flat-plate separation distance  $d$  is sufficiently small.

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