UNCERTAINTY ANALYSIS ON HEAT TRANSFER CORRELATIONS FOR RP-1 FUEL IN COPPER TUBING

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ABSTRACT

NASA is studying kerosene (RP-1) for application in Next Generation Launch Technology (NGLT). Accurate heat transfer correlations in narrow passages at high temperatures and pressures are needed. Hydrocarbon fuels, such as RP-1, produce carbon deposition (coke) along the inside of tube walls when heated to high temperatures. A series of tests to measure the heat transfer using RP-1 fuel and examine the coking were performed in NASA Glenn Research Center's Heated Tube Facility. The facility models regenerative cooling by flowing room temperature RP-1 through resistively heated copper tubing. A Regression analysis is performed on the data to determine the heat transfer correlation for Nusselt number as a function of Reynolds and Prandtl numbers. Each measurement and calculation is analyzed to identify sources of uncertainty, including RP-1 property variations. Monte Carlo simulation is used to determine how each uncertainty source propagates through the regression and an overall uncertainty in predicted heat transfer coefficient. The implications of these uncertainties on engine design and ways to minimize existing uncertainties are discussed.

INTRODUCTION

In the design of current and future rocket engine concepts, methods of thrust chamber cooling must be carefully considered. At high chamber pressures often desired to achieve increased performance, regenerative cooling can be particularly effective.[1,2,3] Design factors such as maximum chamber wall temperature, thermal stress, maximum heat flux, heat flux distribution, and maximum pressure drops along the channels must be considered in engine design.[4,5] Good heat transfer correlations and models are needed to determine these factors. When a hydrocarbon fuel such as RP-1 is used as a coolant, coking also becomes an important factor. Coking is the process in which a thin layer of carbon deposit forms inside the tube wall at high temperatures.[6,7]

In order to derive heat transfer correlations and better understand the thermal behavior of RP-1, a series of heated tube tests are being performed at NASA Glenn Research Center (GRC) using copper tubing. These tests track the changes in tube wall temperature over time and along the tube in order to examine the coking phenomenon. Measurements of applied voltage, applied current, fuel flow rate, wall temperature, inlet temperature and outlet temperature are made during the test, and these measurements are used along with property data to determine Nusselt (Nu), Reynolds (Re), and Prandtl (Pr) numbers for each test. A heat transfer correlation for Nu number is derived from these numbers using least squares regression analysis.[1,6,8,9,10]

Uncertainties in measurements are caused by factors such as the limitations of the instruments and random fluctuations in the test environment. For experiments involving multiple measurements, the uncertainties in each measurement contribute to uncertainty in the final experimental result. The two basic types of uncertainty are random (precision) and systematic (bias) uncertainty. Random uncertainty can be thought of as the variation or scatter in a data set. If multiple measurements of the same condition are taken using the same instrumentation, each measurement would be affected by a different random error. The systematic errors would affect the mean value of those measurements. Most experiments contain both random and systematic errors.[11]

Random uncertainties are generally analyzed using statistical methods. Since systematic errors affect all measurements in the same way, they cannot be seen in the data. Instead, sources of systematic error must be identified and the uncertainties estimated. Often this can be done using manufacturer specifications. Although the actual fixed errors in a measurement are unknown, manufacturer specifications tell the range in which the value of the error should lie. When this information is not available, systematic uncertainty must be estimated through knowledge of the equipment or other logical considerations. Random and systematic uncertainties as well as multiple uncertainty sources can be combined using the root sum square (RSS). When two different measurements are made with the same or similar equipment, the systematic errors may be the same. The uncertainties in the measurements are said to be correlated, and the effects of this correlation must be considered.[11]

This report identifies the uncertainty sources in the heated tube test measurements, property data, and those associated with the calculations themselves. Using Monte Carlo simulation, the effect of these uncertainties on the Nu number correlation is determined. Finally, the impact of uncertainty in Nu number on engine parameters is considered.

RESULTS AND DISCUSSION

EXPERIMENTAL SETUP AND UNCERTAINTIES

In order to develop a detailed uncertainty analysis of the heat transfer correlations, the uncertainties of each measurement, intermediate calculation and modeling assumption used in deriving the heat transfer correlations must be determined. The testing procedures and calculation methods must be well understood to estimate elemental uncertainties realistically.

To develop heat transfer correlations and study coking effects, engineers at NASA GRC began a series of tests to measure the heat transfer effectiveness of RP-1 fuel.[Error! Bookmark not defined.,6] The tests use small copper tubes to model the coolant passages in a rocket engine. Pressurized RP-1 flows through the copper tubing at 1000 psia, and is piped into a return tank. The flow rate is measured as it enters the tube, and heating is initiated once flow is established inside the tube. The tube is resistively heated by applying a voltage across its length. The applied voltage and current are measured. The outer wall temperature is measured with k-type thermocouples brazed along the length of the tube. The temperatures at the inlet and the outlet of the tube are also measured with k-type thermocouples. Using these measurements and available property data, calculations are made to determine the dimensionless Re, Pr, and Nu numbers at each thermocouple. The behavior of the wall temperature over the course of the tests is also monitored to investigate the effects of coke build up.

From the dimensionless parameters, a heat transfer correlation is determined in the form

$$Nu = e^c \left[1 + \frac{x}{D} \right] Re^m Pr^4 , \tag{1}$$

where x/D is the ratio of the distance along the tube to the inner diameter of the tube. The above equation is transformed into a linear form with two regression variables. The regression constants are determined using the standard linear regression equations. Further information on the correlation form can be found in Ref. 10. These calculations are made assuming a "clean" tube with no carbon deposition. For this reason, only the first five seconds of test data are used. It is assumed that up until this point no significant amount of carbon build-up has occurred. Data is sampled at a rate of 1Hz and the first five readings are averaged to constitute one data point in the regression for each thermocouple.

Several of the instruments used to make measurements during the tests pass signals through a signal amplifier. The amplifiers are a significant source of both systematic and random errors. To estimate random uncertainty in the amplifiers, a supplemental experiment was performed. A constant signal output by the millivolt reference source was passed through the amplifier and recorded by the data recording system (DRS). This test is performed on several of the amplifiers. Data from each amplifier were combined to make a general estimation of the random uncertainty of a typical amplifier. The standard deviation of the recorded signals is $S_{X_{recorded}}=0.0063$mV. For large sample size and a confidence level of 95%, random uncertainty is considered to be $2S_x$.[Error! Bookmark not defined.] The uncertainty of the recorded signals is a combination of both the uncertainty in the reference source and the uncertainty of the amplifiers. The standard deviation of the reference signal recorded directly is $S_{X_{reference}}=0.0028$mV. The uncertainty of the total recorded signal is the RSS of the uncertainty of the reference source and the amplifiers. Therefore, the random uncertainty of the amplifiers can be estimated to be

$$P_{amp} = \sqrt{\left(2 \cdot S_{X_{recorded}}\right)^2 - \left(2 \cdot S_{X_{reference}}\right)^2} = 0.011\,\text{mV} . \tag{2}$$

The amplifiers are calibrated before each run using a millivolt reference source and multimeter. The systematic error in the amplifier is from uncertainties in the reference sources and the calibration process. According to manufacturer specifications, the uncertainty on the reference source is ±(0.005% of reading +5μV), and the uncertainty in the multimeter is ± 0.0015% of reading.[12] Random fluctuations in the amplifier during calibration make this error different for each calibration. Once the amplifiers are set, the error caused by these random errors will remain the same for all measurements until it is recalibrated. The bias uncertainty in the reference sources and the bias uncertainty arising from the random uncertainty in the amplifiers are combined using the RSS rule to determine the total bias uncertainty in the amplifiers.

When voltage is applied across the tube, this affects the thermocouple measurement and introduces an additional uncertainty. The thermocouples pick up an additional voltage from the voltage that is being applied across the tube. This effect is dependent on the positioning of the individual thermocouple wire pairs relative to the tube and the applied voltage. Some of the error caused by voltage influence is removed through calibration. Preliminary runs are made in which a positive voltage is first applied to the tube and then the leads are swapped and a negative voltage is applied. In most cases, a difference of up to ten degrees is seen between measured wall temperatures at the same input power level. A correction factor is applied to make the curves for positive and negative voltage match. It is estimated that approximately 2°F uncertainty remains after the data has been corrected. This uncertainty is considered random because the effect is different from one thermocouple to another in both magnitude and direction and varies with the amount of voltage applied. For a particular thermocouple on a particular tube, the uncorrected data will be biased in one direction before it has been corrected. Once the temperature readings are corrected, each reading could be biased positive or negative depending on voltage, which fluctuates during the course of the test.

The uncertainty in each measurement is determined by combining the uncertainty sources using the RSS so that $U_x^2 = B_x^2 + P_x^2$ where B_x is the total systematic uncertainty in the measurement and P_x is the total random uncertainty in the measurement. Since the uncertainties from the amplifiers and the DRS are voltage uncertainties, the propagation equation must be used to relate these to a measurement uncertainty. The measurement, M, is a function of the voltage. Using the propagation equation, the uncertainty in M from the voltage uncertainty is

$$U_M^2 = \sum_{i=1}^{J} \theta_i^2 B_{X_i}^2 + \sum_{i=1}^{J} \theta_i^2 P_{X_i}^2 ,$$

(3)

where B_{x_i} and P_{x_i} are the systematic and random uncertainties of the voltage measurements and θ_i is the derivative of the measurement with respect to the voltage. Error! Bookmark not defined. For each measurement, the uncertainty sources and the overall uncertainty at a nominal condition are shown in Figure 1.
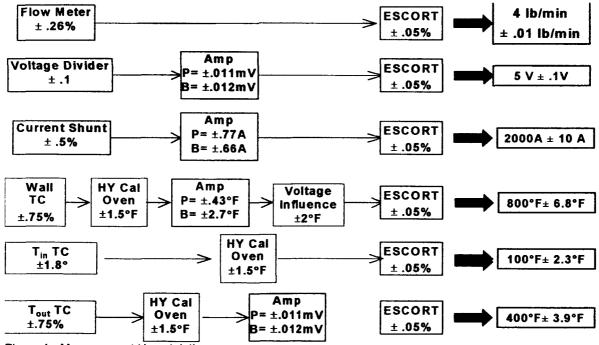

Figure 1 - Measurement Uncertainties

Density, enthalpy, thermal conductivity, and viscosity data for RP-1 fuel is used to calculate several parameters including the Re and Pr numbers used in the heat transfer correlation. Although RP-1 has been used extensively in the rocket industry, these properties have not been closely studied at temperatures and pressures above standard conditions. Data from a prediction model, which relates properties to a known reduced state based on the principal of corresponding states, is used to determine fluid properties in the data reduction.[13] Similar models and limited data are available for comparison.[14,15,16] The values predicted by these four sources are significantly different from each other. The variation between viscosity data from the different sources can be seen in Figure 2.

The variation in other properties is similar. Also, the composition of RP-1 can vary from one batch to another. Since RP-1 is generated in rather large batches, most test data is generated using the same batch of fuel.
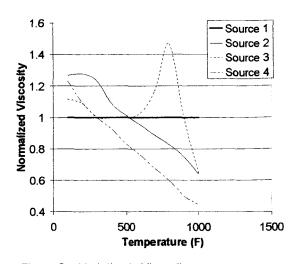

Figure 2 – Variation in Viscosity

The uncertainties in the fluid properties are determined from the standard deviation of the values given by the different sources. For each fluid property, the uncertainty is calculated over a range of relevant temperatures at 1000 psia. Large sample size cannot be assumed, and for 95% confidence and only four data points, the uncertainties are calculated as, $B=3.182S_x$.[11] Based on the range of fluid temperatures of interest, an uncertainty value for each property was determined. This uncertainty is considered systematic because the same property data is used in each test and any errors are fixed in the property relations.

The uncertainties in physical dimensions of the tube are also important. The tubes are drawn with an estimated dimensional uncertainty of ± 0.003". The significant source of uncertainty in the thermocouple location is the braze metal that surrounds the thermocouple wires at the point of attachment, which makes determining the exact location of contact difficult. The uncertainty in this measurement is estimated to be ± 0.01". The uncertainty in the distance between bus bars is estimated to be ± 0.03".

The temperature at the inner wall of the tube is calculated by modeling the resistivity of the tube, the energy input, and the outer wall temperature. The difference between the inner and outer wall temperatures is around 10°F. The uncertainty in the inner wall temperature from modeling assumptions in this calculation is estimated as ±2°F. The enthalpy at the inlet of the tube is determined from the fluid temperature and the property data. Downstream from this point, the enthalpy is calculated at the location of each thermocouple by modeling the energy input over each section of the tube. The mean fluid temperature at each location is then determined from the enthalpy using the property data. Assuming a reasonably good model, the uncertainty of this calculation is estimated to be about 3%.

REGRESSION UNCERTAINTY CALCULATION

The traditional method of calculating the uncertainty of a least-squares regression is a statistical method based on the difference between measured values and the values predicted by the regression. This statistic is called the standard error of regression and only includes random uncertainty.[11] Although only two regression constants are used in this study and the regression calculation is reduced to a simple bivariate problem, Nu and its uncertainty vary with both Re and Pr numbers. Therefore, the random uncertainty is calculated using the statistics for multivariate regression.

To deal with the additional dimensions in multivariate regression, the system is converted into matrix notation,

$$\vec{Y} = \vec{X}\beta .$$ (4)

In this expression, $\vec{Y}$ is a vector containing the predicted values of the dependent variable and β is a vector containing the regression constants. For two independent variables and N data points, $\vec{X}$ is a Nx3 matrix containing ones in the first column, the first independent variable in the second column, and the second independent variable in the final column. The standard error of regression for two independent variables is calculated by the expression,

$$S_Y = \left[\frac{\sum_{i=1}^{N}(Y_i - \hat{Y}_i)^2}{N-3} \right]^{1/2} .$$ (5)

The uncertainty on values predicted by the regression is the confidence interval for future observations and is calculated by the expression,

$$P_Y = t\sqrt{S_Y^2 \cdot \left(\vec{X}_0' \cdot \left(\vec{X}\vec{X}\right)^{-1} \vec{X}_0\right)}, \qquad (6)$$

where $\vec{X}_0' = [1, \; X_{10}, \; X_{20}]$ and X_{10} and X_{20} are the values for X_1 and X_2, respectively, at which the prediction is made.[17] It can be seen from these equations that the uncertainty depends on the values of X_1 and X_2 and must be calculated at each value of interest.

To apply these equations to the correlation expressed in Equation (1), it must first be linearized into the form,

$$\log Nu = c + m\log Re + .4\log Pr, \qquad (7)$$

where $Y_i = \log Nu_i$, $X_{1i} = \log Re_i$, and $X_{2i} = .4\log Pr_i$. The vector β contains the regression constants and is expressed as,

$$\beta = [c \quad m \quad 1]. \qquad (8)$$

Using these definitions and the equations for the standard error of regression and prediction intervals, an estimate of the prediction interval for the log Nu is determined where,

$$(\log Nu)_{true} = (\log Nu)_{predicted} \pm P_Y. \qquad (9)$$

This can then be manipulated into the equation,

$$Nu_{true} = e^{\left((\log Nu)_{predicted} \pm P_Y\right)} \qquad (10)$$

making the random uncertainty in Nu predicted by the regression

$$P_{Nu} = e^{\left((\log Nu) \pm P_Y\right)} - Nu. \qquad (11)$$

Since traditional regression analysis only addresses the random uncertainty an additional method is needed to determine the systematic uncertainty in Nu. A Monte Carlo simulation of the experiment can be used to determine the uncertainty in the result. This technique is well developed in Ref. 11for uncertainty analysis in general.

In a Monte Carlo simulation, the experiment is simulated mathematically through many iterations using a computer. Taking a measurement is simulated by perturbing nominal "true" values of the measurement by elemental errors. In each iteration an elemental error for each uncertainty source is drawn randomly from Gaussian populations where the standard deviation is determined from the uncertainty by

$$S_X = \frac{U_{X_i}}{2} \qquad (12)$$

The experimental result is calculated for each iteration, and the uncertainty of the result is determined from the standard deviation of the calculated results.

Several important factors must be considered. Each data point used in the regression is actually an average of values from a single test. Correlated errors play a significant role in the uncertainty of this regression. The errors introduced by modeling assumptions must be included during the calculation of the results. The uncertainties in Re_{new} and Pr_{new} used to predict Nu numbers must also be considered.

In the actual testing, the results of the first five seconds (readings) at steady state of each test are averaged, and the average values of Nu, Re, and Pr numbers for each test are used in the regression. Since all five readings of a given test are made with the same instrumentation at approximately the same time, all bias errors should be correlated and the readings should vary by random errors only. To account for this each nominal set of test measurements is duplicated five times and then perturbed by random errors.

For errors correlated between two measurements the actual values of the errors are unknown, but they are known to be the same for both measurements. In a Monte Carlo simulation, one elemental error will be selected from a Gaussian distribution and that single error is used to perturb all correlated measurements within that iteration.

Although the uncertainties due to the DRS are all the same, the actual errors could vary from one channel to another. Therefore, the uncertainties due to the DRS are not considered correlated between channels, and a different value is randomly drawn for each measurement. Within each channel, these errors should be the same from one test to another. Although the bias uncertainties from the amplifiers are the same, the actual errors will vary from one amplifier to another. This is because random fluctuations during calibration cause the calibrations to be different. The amplifiers are generally recalibrated before each test so all readings on a single amplifier from a single test will have the same amplifier bias error. The bias errors will be different after each recalibration. Uncertainty due to the

measuring instrument itself, such as the flow meter and current shunt, are assumed to be correlated for all tests and readings.

From calibration information, it appears that measurements made using the same channel in the reference oven have correlated uncertainties but uncertainties are not correlated between channels. Since a different wire is used for each temperature measurement along the tube wall, and new thermocouple wires are used for each test piece, uncertainties in different thermocouples are not considered to be correlated. Within a given test, there are no changes and all readings with a given thermocouple are considered correlated. Random uncertainties are generally not considered to be correlated.

For simplicity, the dimensions of the tube are assumed to be correlated. This is not an unreasonable assumption since any variations would likely arise in the manufacturing and the same manufacturing process is used for all tubes. The overall effect of uncertainty due to the tube dimensions will be small in any case. The measurements of the thermocouple and bus bar locations are not considered to be correlated. The property data curves are the same in all cases, so these uncertainties are considered correlated.

The values of each error for all uncertainties are determined in the main code. Three major calculation steps must be done in each iteration. The first step is a complex calculation to determine the Re, Pr, and Nu numbers at each of the conditions that would be tested. This step is performed by a data reduction program. The main code passes the perturbed measurements and the error values for fluid properties, dimensions, and modeling assumptions to the data reduction code. Within the data reduction code, fluid properties, dimensions and modeling assumptions are perturbed to include the uncertainties in these parameters as the values are calculated. Once Nu, Re, and Pr numbers have been calculated, their values are passed back to the main code.

The second step is to perform standard linear regression on the results. The five Nu, Re, and Pr values from each thermocouple of each test are averaged. These averaged values are used to generate a heat transfer correlation for Nu number following the form of Equation (1). The final step is to predict Nu numbers over a range of Re and Pr numbers. In making a prediction for Nu using the correlation, the Re and Pr numbers at the flow condition of interest must be determined. The uncertainty in these Re_{new} and Pr_{new} values can be significant and must be included. These uncertainties will be different for each application of the heat transfer correlation. For most cases, the effect of these uncertainties must be added to the overall uncertainty in Nu number that is reported in this paper. There are several different existing sources for RP-1 property data and the differences between them are rather large. The existing property data sources use similar methods and the uncertainties are based on the variation between them; the uncertainties in properties are the same. If more precise property data is generated in the future, the uncertainties in the property data used to calculate the Re_{new} and Pr_{new} numbers will be different.

There are three possible scenarios for the uncertainties in property data considered in this analysis. In the first case, the property data used to calculate the Re_{new} and Pr_{new} numbers could be the same as that used to derive the heat transfer correlation. For this case, the uncertainties in fluid properties will be correlated. This reduces the overall uncertainty in the Nu number prediction. In the other two scenarios, the property data used to determine Re_{new} and Pr_{new} are different from that used in deriving the heat transfer correlation, and the uncertainties are uncorrelated. If the fluid properties used to determine Re_{new} and Pr_{new} are from one of the existing property data sources, the uncertainties for fluid properties in Re_{new} and Pr_{new} and those used to derive the heat transfer correlation will have the same magnitude. This is the second case considered. In the final case, it is assumed that some new property data source is used.

In the first case, the uncertainties in Re_{new} and Pr_{new} from uncertainties in property data are considered correlated. In this case, the calculation begins with an array of appropriate bulk fluid temperatures and flow rates. Using the same method as in the data reduction, Re_{new} and Pr_{new} numbers are calculated from these values. These values are also perturbed by the same property errors that were used previously in the iteration. A Nu number prediction is made in each iteration using the perturbed Re_{new} and Pr_{new} values. The uncertainty calculated in this manner includes uncertainties in Re_{new} and Pr_{new} that result from the uncertainty in fluid properties. Any additional uncertainty sources in Re_{new} and Pr_{new} must still be added using the RSS to determine the total uncertainty in a Nu prediction. In the second case, it is assumed that similar property data sources are used and the data are uncorrelated. As with the correlated case, values for Re_{new} and Pr_{new} are calculated using the same method as in the data reduction. However, the fluid properties used to calculate Re_{new} and Pr_{new} are perturbed by different errors than the fluid properties used in the data reduction. The uncertainty calculated in this manner will include uncorrelated uncertainties in Re_{new} and Pr_{new} from the property data. Any additional uncertainty sources in Re_{new} and Pr_{new} must be added using the RSS rule. In the final case, no uncertainty is assumed in Re_{new} and Pr_{new}. The values are calculated in the same fashion as the first two cases, but are not perturbed by elemental errors. The calculated regression coefficients are applied at the same values of Re_{new} and Pr_{new} in each iteration to predict the Nu number.

The uncertainty calculated in this manner includes no uncertainty in Re_{new} and Pr_{new}. The uncertainties in these values must be added later by using the RSS rule.

The Monte Carlo simulation incorporates all of the nuances of the data regression. The correlation between measurement uncertainties is considered. The additional uncertainties that arise during the calculation of Re, Pr, and Nu numbers are accounted for. The correlation between uncertainties in property data used in generating the regression and in making predictions using the regression is considered. The results of this Monte Carlo simulation and the random uncertainty calculation are reported in the following chapter.

UNCERTAINTY ANALYSIS RESULTS

The Monte Carlo simulation produces a series of uncertainty values for a range of Re_{new} and Pr_{new} pairs. Least squares curve fits put this data into a more useful form. The equations for uncertainty as a function of Re number and Pr number are shown in Table 1. In this experiment, the correlation between uncertainties in Re and Pr numbers in the data reduction equation and the uncertainties in Re_{new} and Pr_{new} reduces the overall uncertainty in the Nu number prediction. This is because the correlation effectively removes the contribution of uncertainty due to thermal conductivity and viscosity.

Table 1 - Equations for Uncertainty in Nu Number Predictions

Case Considered	Uncertainty Equation
Systematic Uncertainty – Correlated	$U = 0.0012 * Re_{new} + 2.7 * Pr_{new} - 30.2$
Systematic Uncertainty – Uncorrelated Property Uncertainty in Re_{new} and Pr_{new}	$U = 0.0012 * Re_{new} + 3.0 * Pr_{new} - 32.7$
Systematic Uncertainty – Uncorrelated No Property Uncertainty in Re_{new} and Pr_{new}	$U = 0.0010 * Re_{new} + 3.05 * Pr_{new} - 27.4$

For the correlated case, the uncertainty over this range can be seen in .The uncertainty increases with Re and Pr numbers. The systematic uncertainty in Nu number for the uncorrelated case can be as high as 36%, while in the correlated case is never above 34%. Even though no uncertainty in the Re_{new} and Pr_{new} values is assumed, the uncertainties for the third case with no correlation are comparable to or higher than the correlated case. This is because the correlation counteracts the effect of some of property uncertainties in the regression.
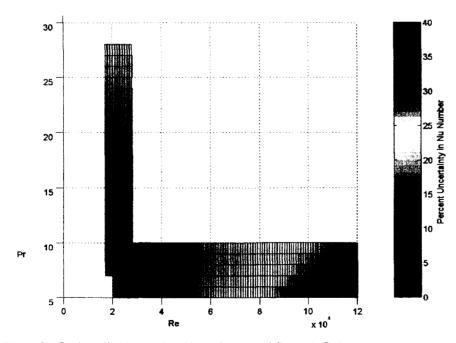

Figure 3 - Systematic Uncertainty Nu – Correlated Property Data

Although using the same property data values greatly reduces the uncertainty, the uncertainty in the correlated case is still substantial. It is also unlikely that the same property data sources will be used in most applications of the heat transfer correlation, because those property data sources will be unknown or unavailable to many who use the correlation. In those cases, the uncertainties in the uncorrelated cases must be used. The uncertainties in property data used in determining the heat transfer correlation must be reduced in order to reduce overall uncertainty further. Assuming no uncertainty in property data reduces the systematic uncertainty to less than 2%. Although the property data uncertainties could never be eliminated, this indicates how significant the uncertainties in property data are.

If the uncertainty in property data is reduced, other uncertainties could become significant. The percentage uncertainty in Nu number resulting from amplifier uncertainty is less than .1%. The percent uncertainty in Nu number from temperature measurements is less than 0.2%. The percent systematic uncertainty in Nu from uncertainties in measured parameters, voltage, current, flow rate, and temperatures, can be as high as 1.6%, which is a significant portion of the non-property uncertainty. The systematic uncertainty arising from dimensional uncertainties can be as high as .7%.

To examine which of these non-property uncertainties are most significant, the uncertainty percentage contributions (UPC) of each uncertainty source can be determined. The overall systematic uncertainty is the RSS of the uncertainty from each source, and the UPC of an uncertainty source is determined by

$$UPC = \frac{\left(\partial Nu/\partial X_i\right)^2 \left(U_{X_i}/X_i\right)^2}{\left(U_{Nu}/Nu\right)^2}.^{11} \qquad (13)$$

The entire numerator term in Equation (13) is the square of the result of the Monte Carlo simulation for that uncertainty source. The term in the denominator is the square of the overall systematic uncertainty in Nu number assuming no uncertainties in property data. Since all of the uncertainties vary with the values of Re and Pr, the UPC's will also vary with Re and Pr. The values shown in

Table 2 are the ranges in which the UPC's and are the contribution to overall systematic uncertainty in Nu number assuming no uncertainties in property data. Of these sources, the measurement uncertainties contribute the most to the uncertainty in Nu number.

Table 2 - Percent Contribution to Systematic Uncertainty in Nu Number Assuming No Uncertainty in Property Data

Uncertainty Source	UPC
Preston Amplifiers	<1%
Temperature Uncertainties	<1.5%
Measurement Uncertainties	65%-80%
Dimensional Uncertainties	5%-30%
Modeling Assumptions	10%-20%

To estimate the total uncertainty in Nu, the random uncertainty in the experiment must be included. Random uncertainty is calculated statistically and using the Monte Carlo simulation. The results of the statistical calculation reflect the total random uncertainty in the data. The Monte Carlo simulation determines the random uncertainty from identified sources. The random uncertainty calculated using the Monte Carlo simulation includes random fluctuations of the amplifiers and the influence of the applied voltage on the thermocouple measurements. The percent uncertainty in Nu from these sources is less than .05%, which shows that the random uncertainties of these parameters cause negligible uncertainty in Nu number. The random uncertainty in Nu number calculated statistically can be as large as 12%. This indicates that the random uncertainty is significant. The great difference between the two calculation methods indicates that there are additional random uncertainty sources not identified. The random uncertainty as a function of Re_{new} and Pr_{new} is determined by the expression,

$$U = 0.0003 * Re + 1.1 * Pr - 12.1. \qquad (14)$$

When property uncertainty is included, the systematic uncertainties contribute at least 85% of the overall uncertainty in Nu. If the property data is improved significantly, the random uncertainties could contribute almost all of the overall uncertainty in Nu number.

The overall uncertainty in a predicted Nu number will be a combination of random and systematic uncertainty. Additional uncertainties in Re_{new} and Pr_{new} must also be included. These uncertainties are combined using the RSS square law by,

$$U_{Nu}^2 = U_{regress-sys}^2 + U_{regress-rand}^2$$
$$+ \left(\frac{\partial Nu}{\partial Re_{new}}\right)^2 (U_{Re_{new}})^2 + \left(\frac{\partial Nu}{\partial Pr_{new}}\right)^2 (U_{Pr_{new}})^2 , \quad (15)$$

where $U_{regress-sys}$ is the systematic uncertainty, $U_{regress-rand}$ is the random uncertainty, $U_{Re_{new}}$ is the uncertainty in the Re_{new} value used, and $U_{Pr_{new}}$ is the uncertainty in the Pr_{new} value used. The values used for $U_{regress-sys}$, $U_{Re_{new}}$, and $U_{Pr_{new}}$ depend on which of the three cases for property data are appropriate. In the first case, the uncertainties in property data are correlated, and the $U_{Re_{new}}$ and $U_{Pr_{new}}$ values will not include property data uncertainties. In the second case, the uncertainties in property data for Re_{new} and Pr_{new} are uncorrelated but the same as those in the data reduction. The $U_{Re_{new}}$ and $U_{Pr_{new}}$ values will not include property data uncertainties either. For the third case, the uncertainties in property data used in calculating Re_{new} and Pr_{new} are different from those of the property data used in the regression, and have not been included. The $U_{Re_{new}}$ and $U_{Pr_{new}}$ values must include uncertainties in the property data used as well as any other uncertainties in Re_{new} and Pr_{new}.

IMPACT ON ENGINE DESIGN

When using the Nu number correlation generated through this experiment it is important to understand the impact that the uncertainty will have on engine designs. A thorough uncertainty analysis of an engine design is beyond the scope of this thesis. Instead, the general effect of Nu number uncertainty on various parameters is characterized.

Assuming nominal values of Re_{new}=80000 and Pr_{new}=9, correlated property uncertainties, and negligible uncertainties in Re_{new} and Pr_{new}, the nominal uncertainty in Nu number can be assumed to be about 24%. These Re and Pr values are in the middle of the test range. The correlation of property uncertainties and negligible uncertainties in Re_{new} and Pr_{new} are best-case assumptions for the existing property data. The values reported here are only appropriate at the conditions described and do not include any uncertainties in additional parameters. They also ignore possible uncertainty correlations between other parameters. The uncertainties in other parameters in engine design could be significant and are beyond the scope of this thesis. The uncertainties presented are included only to indicate the typical impact uncertainties in Nu number have on engine design.

The expression for determining heat flux to a coolant tube in rocket nozzles is

$$\left(\frac{q}{A}\right) = h(T_{iw} - T_{mean}), \quad (16)$$

where (q/A) is the heat flux through the wetted surface area of the tube, T_{iw} is the coolant side wall temperature, h is the convection coefficient, and T_{mean} is the mean temperature of the coolant.[5] The convection coefficient is determined from the Nu number by,

$$h = \frac{Nu \cdot k}{D}, \quad (17)$$

where k is the thermal conductivity and D is the inner diameter of the tube.[18]
Assuming no uncertainties in k or D, the propagation equation for uncertainty in h is determined by,

$$\left(\frac{U_h}{h}\right)^2 = \left(\frac{Nu}{h}\frac{\partial h}{\partial Nu}\right)^2 \left(\frac{U_{Nu}}{Nu}\right)^2 = \left(\frac{Dh/k}{h}\frac{k}{D}\right)^2 \left(\frac{U_{Nu}}{Nu}\right)^2 = \left(\frac{U_{Nu}}{Nu}\right)^2. \quad (18)$$

Therefore the uncertainty in the convection coefficient is approximately 24%. The uncertainty in heat flux resulting from an uncertainty in h, assuming all other parameters are known exactly, is determined by the propagation equation, which reduces to

$$\left(\frac{U_{q/A}}{q/A}\right) = \left(\frac{U_h}{h}\right) = \left(\frac{U_{Nu}}{Nu}\right) = 24\%. \quad (19)$$

Assuming heat flux and bulk fluid temperature are known, the coolant side wall temperature, T_{iw}, can also be determined from Equation (16). The propagation equation applied to T_{iw} is expressed as

$$\left(\frac{U_{T_{wi}}}{T_{iw}}\right)^2 = \left(\frac{q/_A}{q/_A + T_{mean}h}\right)^2 \left(\frac{U_h}{h}\right)^2 .$$

(20)

As this expression shows, the uncertainty in wall temperature is not only a function of the uncertainty in h, but also a function of the conditions at which it is evaluated. Assuming nominal values of

$$q/_A = 10\frac{BTU}{in^2 \cdot s}, \quad T_{mean} = 250°F, \text{ and } h = 0.016\frac{BTU}{in^2 \cdot s \cdot F},$$

(21)

the uncertainty in wall temperature can be estimated to be 17% or about 150°F.

Wall temperature is an important factor in determining thermal stress in a rocket nozzle. The uncertainty in Nu will therefore influence the estimation of thermal stresses as well. The tangential thermal stress, σ_{TTh}, is calculated from the expression,

$$\sigma_{TTh} = \frac{E\alpha\left(T_{wg} - T_{iw}\right)}{2(1 - \eta)}, 5$$

(22)

where E is the modulus of elasticity, α is the coefficient of thermal expansion, and η is Poisson's ratio. Assuming all other parameters are known exactly, the uncertainty in σ_{TTh} resulting from an uncertainty in T_{iw} is defined using the propagation equation by the expression,

$$\left(\frac{U_{\sigma_{TTh}}}{\sigma_{TTh}}\right)^2 = \left(\frac{T_{iw}}{T_{wg} - T_{iw}}\right)^2 \left(\frac{U_{T_{wi}}}{T_{iw}}\right)^2 ,$$

(0.23)

where T_{wg} is the gas side wall temperature. Assuming T_{wg} = 1500°F and T_{iw} = 875°F, the uncertainty in the tangential thermal stress will be approximately 24%. This could have significant impact on engine design because increased margins of safety can lead to increased weight.

The uncertainties calculated for σ_{TTh}, T_{iw}, (q/A), and h are representative of the uncertainties that will result from the uncertainties in Nu number discussed in this paper. This demonstrates how important identifying the uncertainties becomes. If the uncertainties in these parameters are understood, they can be used to consider worst-case design scenarios and required margins.

SUMMARY AND CONCLUSIONS

This thesis identifies the experimental and theoretical uncertainties that exist in determining a heat transfer correlation for Nu number using heated tube data. The propagation of these uncertainties in regression analysis is considered. It is shown that the overall systematic uncertainty in a predicted Nu number can be as high as 35%. The random uncertainty in the heat transfer correlation is also significant and can be as high as 12%. The overall uncertainty in Nu number is the RSS of the random and systematic components and could be as high as 36%. The impact on engine design parameters is also discussed.

The magnitude of the uncertainty values and their impact on engine design parameters highlights how important it is to reduce the uncertainties in Nu number. Several steps can be taken to achieve this. By using the same property values in design as those used in the reduction of test data, the uncertainty in predicted Nu number can be reduced by about 15% of the uncorrelated systematic uncertainty. However, the most important thing that can be done is to obtain more accurate property data at the conditions of the engine-operating environment. This will have a dramatic effect on the uncertainty. Once the property data is improved significantly, random uncertainties will dominate. At this point, steps should be taken to reduce random variation in the data by improved process control or identifying additional factors that influence Nu number.

In addition to the quantitative uncertainties listed here, several additional factors affect the accuracy of the data and should be considered. In an actual rocket nozzle, only one side of the tube is heated. The tubes are also not straight, but conform to the shape of the rocket nozzle. The effects of asymmetric heating and the additional geometric complexity must be accounted for. The heat transfer correlation discussed here is derived for a clean tube. When considering an engine over its operating life, the effects of coking must be included. Also, the variations in the composition of RP-1 from one batch to another should be considered.

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