

**Atmospheric Rotational Effects on Mars Based on the
NASA Ames General Circulation Model: Angular
Momentum Approach.**

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Popular Summary

The objective of the investigation is to determine the motion of the rotational axis of Mars as a result of mass variations in the atmosphere and condensation and sublimation of CO_2 ice on the polar caps.

A planet experiences this type of motion if it has an atmosphere, which is changing its mass distribution with respect to the solid body of the planet and/or it is asymmetrically changing the amount of ice at the polar caps. The physical principle involved is the conservation of angular momentum, one can get a feeling for it by sitting on a well oiled swivel chair holding a rotating wheel on a horizontal direction and then changing the rotation axis of the wheel to a vertical direction. The person holding the wheel and the chair would begin to rotate in opposite direction to the rotation of the wheel.

The motions of Mars' atmosphere and the ice caps variations are obtained from a mathematical model developed at the NASA Ames Research Center. The model produces outputs for a time span of one Martian year, which is equivalent to 687 Earth days.

The results indicate that Mars' axis of rotation moves in a spiral with respect to a reference point on the surface of the planet. It can move as far away as 35.3 cm from the initial location as a result of both mass variations in the atmosphere and asymmetric ice variations at the polar caps. Furthermore the pole performs close to two revolutions around the reference point during a Martian year. This motion is a combination of two motions, one produced by the atmospheric mass variations and another due to the variations in the ice caps. The motion due to the atmospheric variations is a spiral performing about two and a half revolutions around the reference point during which the pole can move as far as 40.9 cm. The motion due to variations in the ice caps is a spiral performing almost three revolutions during which the pole can move as far as 32.8 cm.

Abstract.

The NASA-Ames general circulation model has been used to compute time series for atmospheric inertia and relative angular momentum terms. Model outputs were used also to compute time series representing the inertia terms due to CO_2 condensation and sublimation on the surface of Mars. Some of these terms were used to generate time series representing the forcing functions for the equatorial components of the linearized Liouville equations of rotational motion. These equations were then solved numerically for a period of a Martian year (669 sols) to obtain a time series for the position of the rotation pole on the surface of Mars.

The results of the investigation indicate that mass variation in the atmosphere is as important as the formation and sublimation of ice caps on the surface of the planet. Numerical integration of the equations of rotational motion yields pole displacements as large as 32.8 cm (ice caps solution), 40.9 cm (atmospheric effects), or 35.3 cm (both effects combined). Fourier analysis of the time series corresponding to the equatorial components of pole displacement for the ice caps solution as well as the atmospheric effects solution shows that the (1/3)-annual harmonic has the largest coefficient in three cases, with magnitudes in the 8-10 cm range. Fourier analysis of the equatorial components of polar motion for the combined solution yields main harmonics of 5.66 cm (x), (1/3)-annual and 7.86 cm (y), annual.

1. Introduction.

The rotational variations of a planet can be analyzed into axial and equatorial components. The axial variations (along the z-axis, which is the rotation axis) are reflected in changes in the length of day (LOD). The equatorial variations (x, y) produce changes in the orientation of the axis of rotation (polar motion). The solution of Liouville's equations provides the changes in planetary rotation, i.e., changes in LOD and polar motion.

The methodology of planetary rotational investigations can follow the angular momentum approach or the torque approach. The chosen methodology determines the boundaries of the appropriate control volume. The angular momentum method involves the computation of terms containing the products of inertia of the atmosphere ("mass terms") and their time derivatives, as well as relative angular momentum terms ("motion terms") and their derivatives.

The objective of this investigation is to use the angular momentum methodology to compute and analyze how the atmosphere affects the equatorial components of the rotation of Mars, based on outputs from the NASA Ames General Circulation Model (GCM). The model provides values of wind velocity, density and pressure, which serve as inputs to the calculation of the terms which appear in Liouville's equations of rotational motion.

The torque approach was used in a previous investigation by Sanchez et al. (2003), referred below as Paper I, to compute polar motion and LOD variations. The LOD variations were computed also using the angular momentum methodology. Mars' solid body was modeled as rigid, with the Love number $k_2 = 0$.

The studies of atmospheric effects on the rotation of Mars constitute a growing body of scientific literature. Paper I provides a number of references. Many of these works were concerned only with seasonal variations in the rotation rate (LOD). Those which computed polar motion variations were concerned with effects due to products of inertia produced by condensation and sublimation of CO₂ on Mars' surface. They are cited below.

Chao and Rubincam (1990) estimated that a 1° ice cap offset from the rotation axis would excite polar motion with amplitude of 13 milliarcseconds (mas) or a 21-cm polar shift at the surface.

Yoder and Standish (1997) estimated the spatial orientation of the Martian pole of rotation and axial rotation parameters for January 1, 1980 (midpoint in the Viking epoch). They used a model for the seasonal mass exchange between the ice caps and atmosphere to obtain estimates of polar motion due to asymmetric ice cap changes. These variations range from 10 to 20 mas at 1, 1/2, and 1/3 year, and 5 mas or less for 1/4 year. The relation between amplitude and period is associated with resonance effects caused by a wobble period estimated in the 193-212 day range.

Defraigne et al. (2000) computed Mars' rotational variations using the output of a global circulation model for the Martian atmosphere developed by Forget et al. (1999). Their polar motion results are 11 mas (annual) and 3 mas (semi-annual), which yield a displacement similar to that obtained by Chao and Rubincam.

Van den Acker et al. (2002) obtained total amplitudes of 10 cm on the surface for both the annual and semiannual polar motion excited by the atmosphere and ice caps.

The NASA Ames GCM is a finite difference model based on the primitive equations of meteorology expressed in spherical sigma coordinates (σ = pressure at height / surface pressure). The resolution is 7.5° (latitudinal) by 9° (longitudinal). The version of the model used here has 30 vertical layers extending from the surface to 100 km. The output files are written every 1.5 hours of simulated time, 16 times per sol. There are 10704 records in the data file, representing a time span of 669 sols (687 Earth days) which is the length of the Martian year.

Kinetic energy is dissipated in the model by frictional interaction with the surface and a "sponge" layer at the model top. Surface friction is parameterized using an adaptation of the bulk boundary layer scheme of Deardorff (1972). The sponge layer exists in the top three layers and is based on a simple Rayleigh friction scheme.

The outputs from the NASA Ames GCM provide a 1.5 hour sampling rate, therefore the results for the forcing functions are not limited to annual and semi-annual components.

The model used here for the body of Mars does not include a liquid core or a solid inner core, therefore associated near-diurnal and other possible resonances are excluded. We hope to study them in a future work. Mars' solid body is considered as elastic with a value of the Love number $k_2 = 0.153$, as determined by Yoder et al.

(2003) from analysis of Mars Global Surveyor radio tracking. Mars' deformation due to attraction and loading by the polar caps has not been included. Defraigne et al. (2000) have estimated that these effects amount to a few percent of those obtained for the inertia of the caps. These rheological effects are functions of Love numbers, which are presently subject to uncertainties in their values. These effects are of smaller magnitudes than those of effects produced by uncertainties in present day atmospheric model parameters.

The paper presents the results of Fourier analysis of time series representing various quantities. For each pair of terms, $a_k \cos kt + b_k \sin kt = (a_k^2 + b_k^2)^{1/2} \cos(kt - \phi)$, we refer to the quantity $(a_k^2 + b_k^2)^{1/2}$ as the "power at frequency k ". A plot of this quantity as a function of k is called the power spectrum, as shown by Hamming (1986, page 515). Note that this convention makes the units of power the same as those of the particular time series under consideration. When reference is made to "total power", it is meant the sum over the entire frequency range. Excitation or forcing function magnitudes are given in Hadley units. One Hadley (H) is equal to 10^{18} Newton-meters.

The structure of the paper is as follows. The Liouville equations of rotational motion are introduced in Section 2. Time series and analysis of the equatorial excitation functions are treated in Section 3. The computation of polar motion constitutes the subject of Section 4. Summary and conclusions appear in Section 5.

2. Liouville Equations.

Liouville gave the basic equations of motion in 1858. A linearized form of the equatorial components is given below.

$$\begin{aligned} A (\omega_x)' + (C - B) \Omega \omega_y &= - (h_x - \Omega I_{xz})' + \Omega (h_y - \Omega I_{yz}) + L_x \\ B (\omega_y)' - (C - A) \Omega \omega_x &= - (h_y - \Omega I_{yz})' - \Omega (h_x - \Omega I_{xz}) + L_y \end{aligned} \quad (1)$$

Where A and B are the equatorial moments of inertia, C is the polar moment of inertia, I_{yz} and I_{xz} are the products of inertia, h_x and h_y are the relative angular momentum terms. Ω is the mean angular speed of rotation, ω_x and ω_y are the equatorial components of

angular velocity. L_x and L_y are the equatorial components of external torque. The primes denote derivatives with respect to time.

The time varying parts of the products of inertia can be separated into three components. A part due to the rotational deformation of the solid body of the planet. A part due to CO_2 and H_2O ices condensing and sublimating on the surface of the planet, and a part due to mass redistribution in the atmosphere. The expressions for the products of inertia can then be written as follows,

$$\begin{aligned} I_{yz} &= -[k_2 R^5 \Omega / (3 G)] \omega_y + (\Delta I_{yz})_a + (\Delta I_{yz})_s \\ I_{xz} &= -[k_2 R^5 \Omega / (3 G)] \omega_x + (\Delta I_{xz})_a + (\Delta I_{xz})_s \end{aligned} \quad (2)$$

The first term in equations (2) is the rotational deformation part. The second term denotes the atmospheric part. The contribution due to surface ice is indicated by the third term. With respect to the rotational expression, k_2 is the second-degree tidal effective Love number, R is the radius of the planet, and G is the gravitational constant.

Substitution of equations (2) into equations (1) and solving for ω_x and ω_y yields,

$$\mathbf{Y}' - \mathbf{N} \mathbf{Y} = \mathbf{F}_a + \mathbf{F}_s + \mathbf{L} \quad (3)$$

$$\mathbf{Y} = [c_1 \omega_x, c_2 \omega_y]^T \quad (4)$$

$$\mathbf{N} = \begin{bmatrix} 0 & -d_1 \\ d_2 & 0 \end{bmatrix} \quad (5)$$

$$\mathbf{F}_a = [F_{ax}, F_{ay}]^T \quad (6)$$

$$F_{ax} = - (h_x - \Omega \Delta I_{xz})' + \Omega (h_y - \Omega \Delta I_{yz}) \quad (7)$$

$$F_{ay} = - (h_y - \Omega \Delta I_{yz})' - \Omega (h_x - \Omega \Delta I_{xz})$$

$$\mathbf{F}_s = [F_{sx}, F_{sy}]^T \quad (8)$$

$$F_{sx} = \Omega \Delta I_{xz}' - \Omega^2 \Delta I_{yz}$$

$$F_{sy} = \Omega \Delta I_{yz}' + \Omega^2 \Delta I_{xz} \quad (9)$$

$$\mathbf{L} = [L_x, L_y]^T \quad (10)$$

$$c_1 = A + k_2 R^5 \Omega^2 / (3 G) \quad (11)$$

$$c_2 = B + k_2 R^5 \Omega^2 / (3 G) \quad (12)$$

$$d_1 = \Omega [(C - B) - k_2 R^5 \Omega^2 / (3 G)] \quad (13)$$

$$d_2 = \Omega [(C - A) - k_2 R^5 \Omega^2 / (3 G)] \quad (14)$$

where the superscript "T" denotes transposed.

The solution of equation (3) provides the trajectory of the point of intersection of the axis of rotation with the surface of the planet by the well - known relations

$$\begin{aligned} x &= (\omega_x / \Omega) R \\ y &= (\omega_y / \Omega) R \end{aligned} \quad (15)$$

The value of the polar moment of inertia C used in the calculations was obtained from,

$$C = 0.366 M R^2 \quad (16)$$

Where $M = 6.4185 \times 10^{23}$ kg and $R = 3389920$ m. The values of the equatorial moments of inertia A and B follow from the values of the gravitational coefficients C_{20} and C_{22} and the value of C,

$$A = M R^2 (C_{20} + 0.366 - 2. C_{22}) \quad (17)$$

$$B = M R^2 (C_{20} + 0.366 + 2. C_{22}) \quad (18)$$

The values for the gravitational coefficients are those given by Lemoine et al. (2001).

By setting the forcing functions equal to zero in equation (3), the eigenvalues of matrix N yield the associated free period of oscillation,

$$P = 2 \pi / [(d_1 d_2) / (c_1 c_2)]^{1/2} \quad (19)$$

Letting $A = 2.68594 (10^{36}) \text{ kg-m}^2$, $B = 2.68433 (10^{36}) \text{ kg-m}^2$, $C = 2.69956 (10^{36}) \text{ kg-m}^2$, and $k_2 = 0.153$ (Yoder et al., 2003), the following values for the Martian Chandler period are obtained, as shown in Table 1 below.

It is seen that triaxiality, with $B < A$, decreases the Chandler period by 10 days for a rigid Mars, for an elastic Mars, the reduction is 14 days. Elasticity increases the Chandler period, 29 days for an equatorial symmetric Mars, 25 days for a triaxial Mars.

3. Excitation functions.

The condensation and sublimation of CO_2 on the surface and in the atmosphere of Mars produce changes in the atmospheric mass distribution as well as changes in the polar caps. From the standpoint of rotational dynamics these changes are manifested in time variations in the moments and products of inertia. Another source of rotational variations is due to the atmospheric winds, which contribute to the relative angular momentum terms.

Figure 1 exhibits the time series for mass variation in the atmosphere and the ice caps. Note that they are negatives of each other, which indicates mass conservation is satisfied. The atmospheric variation occurs with respect to a mean value of $236 (10^{14}) \text{ kg}$, the ice mass variation has a mean of $35 (10^{14}) \text{ kg}$. Total power is $39 (10^{14}) \text{ kg}$ for each series. Main harmonics are annual, semiannual, (1/3)-annual, and (1/4)-annual.

The products of inertia and relative angular momentum terms are computed from the following expressions,

$$I_{xz} = \int_V \rho r^2 \sin\phi \cos\phi \cos\lambda dV \quad (20)$$

$$I_{yz} = \int_V \rho r^2 \sin\phi \cos\phi \sin\lambda dV \quad (21)$$

$$h_x = \int_V \rho r (v \sin\lambda - u \sin\phi \cos\lambda) dV \quad (22)$$

$$h_y = \int_V \rho r (-v \cos\lambda - u \sin\phi \sin\lambda) dV \quad (23)$$

where ρ is the atmospheric density, r is the distance from the center of planet, ϕ is latitude, λ is east longitude, u is the eastward velocity component, v is the northward velocity component, and V stands for the volume of the atmosphere.

Terms involving time derivatives of the equatorial relative angular momentum terms (h_x , h_y) as well as time derivatives of the products of inertia I_{xz} and I_{yx} appear in the linearized Liouville equations (Eq. 1). The following difference approximation formulas, as given by Greenspan (1974), have been used to compute the first derivatives. For the first point in the time series,

$$q'(t) \sim [-3 q(t) + 4 q(t+h) - q(t+2h)]/(2h) \quad (24)$$

For the interior points,

$$q'(t) \sim [q(t+h) - q(t-h)]/(2h) \quad (25)$$

For the last point,

$$q'(t) \sim [3 q(t) - 4 q(t-h) + q(t-2h)]/(2h) \quad (26)$$

The variable "q" in Equations 24 – 26 above denotes either relative angular momentum or product of inertia, "t" is time, and "h" is the time step in the series, specifically $h = (1/16)$ of a Martian day. Primes signify time derivatives.

The amplitudes of products of inertia, relative angular momentum, and corresponding time derivatives range over several orders of magnitude. However these terms appear in the equations of motion multiplied by Ω raised to various powers, therefore they all have to be considered in combinations referred to as excitation or forcing functions.

The linearized Liouville equations, in the form of Equation (3), show forcing functions on the right side due to atmospheric wind and mass changes and to ice formation and sublimation. If the angular momentum approach is taken, these forcing functions will drive the forced solution and no torque due to the atmosphere will appear in the control volume. Equations (7) and (9) give the atmospheric and ice contributions respectively.

Figures 2 and 3 present the time series and power spectra for the equatorial components of the atmospheric forcing functions.

Figures 4 and 5 portray the results for the ice caps excitation functions. Tabulated results based on power spectrum analysis are given in Table 2. Note that the main harmonics for the ice caps excitation functions are all long periodic: annual, semiannual, (1/3)-annual, etc. The excitation functions based on atmospheric effects exhibit daily and sub-daily harmonics among the five most powerful. Figures 6 and 7 show time series and power spectra for the equatorial components corresponding to the sum of ice caps and atmospheric effects. Tabulated results appear in Table 3. As expected, annual, semiannual, (1/3)-annual, daily and sub-daily harmonics rank among the most powerful. The long period harmonics are associated with seasonal effects, involving the revolution of Mars around the Sun and the inclination of its rotation axis with respect to the plane of the orbit. Daily and sub-daily periods are associated with the daily rotational motion and with variations in the products of inertia, which correspond to tesseral geographic mass variations, and with variations in the relative angular momentum terms.

The magnitude $= (x^2 + y^2)^{1/2}$, of forcing due to ice caps condensation and sublimation reaches its maximum (2.36 H) at the end of the northern hemisphere summer (320 sols). This is approximately the time when total mass variations reach a maximum in the ice caps and in the atmosphere (313 sols). The forcing magnitude associated with atmospheric effects reaches its maximum (7.29 H) at the very end of the fall (505 sols), coincident with the maximum variation in atmospheric products of inertia: ΔI_{xz} (505 sols) and ΔI_{yz} (507 sols). The combined effect (ice plus atmosphere) reaches a maximum of 7.26 H at 505 sols.

4. Computation of polar motion.

The trajectory of the point of intersection of the rotation axis with the surface of Mars is obtained from equations (15), which require the solution of equation (3). The solution was obtained by means of a numerical integration package using the Runge-Kutta-Fehlberg (4,5) method with step size control.

Equation (3) was solved separately for effects due to ice condensation and sublimation on the surface of the planet and for

effects due to mass redistribution in the atmosphere. The total effect is obtained by addition, due to the linearity of the equations.

Table 2 presents Fourier analysis results for the time series corresponding to the equatorial components of pole displacement for the ice caps solution as well as the atmospheric effects solution. The (1/3)-annual harmonic has the largest coefficient in three cases, with magnitudes in the 8-10 cm range. The coefficients of the main harmonics are of similar magnitude for the ice caps and for the atmospheric effects.

It is possible to consider the (x, y) components of the polar displacement as components of a vector characterized by magnitude $= (x^2 + y^2)^{1/2}$ and phase $= \arctan (y/x)$. Time series of magnitude and phase, as well as power spectrum results for the polar displacement due to atmospheric variations are shown in Figure 8. Corresponding results for ice condensation and sublimation appear in Figure 9. Results for the combination of both are given in Figures 10 - 12.

The maximum displacement due to ice is 32.8 cm, occurring at 548 sols. Atmospheric effects yield a maximum of 40.9 cm at 478 sols. The combined effect has a maximum of 35.3 cm at 616 sols.

The phase angle time series for the solution due to ice condensation and sublimation indicates a spiral motion performing three revolutions about the origin. The solution based on atmospheric mass variations and the combined solution both portray a spiral motion performing two revolutions during the Martian year.

Table 3 lists the main harmonics for the x and y components of the combined solution. Compare with the main harmonics associated with the forcing functions, which appear in Table 3 also. Note the (1/3)-annual harmonic, which is magnified by the proximity of its period (223 sols) to the natural period of 211 sols. The daily and sub-daily harmonics appearing in the forcing functions are not very powerful in the pole displacement spectra.

Table 4 presents power spectrum results for the time series associated with the magnitude of the displacement of the pole, $(x^2 + y^2)^{1/2}$, corresponding to the solution due to ice caps variations, the solution based on atmospheric variations, and the combined solution based on ice caps and atmospheric variations.

5. Summary and conclusions.

The NASA-Ames general circulation model has been used to compute time series for atmospheric inertia and relative angular momentum terms. Model outputs were used also to compute time series representing the inertia terms due to CO_2 condensation and sublimation on the surface of Mars. Some of these terms were used to generate time series representing the forcing functions for the equatorial components of the linearized Liouville equations of rotational motion. These equations were then solved numerically to obtain a time series for the position of the rotation pole on the surface of Mars.

The amount of atmospheric mass involved in condensation and sublimation is equal to 16.52% of the total atmospheric mass.

The annual, semiannual and (1/3)-annual harmonics appear among the five most powerful in the equatorial power spectra for both the ice caps and atmospheric effects, daily and sub-daily harmonics are also prominent in the atmospheric forcing but not in the ice caps forcing.

The results of the investigation indicate that mass variation in the atmosphere is as important as the formation and sublimation of ice caps on the surface of the planet. Numerical integration of the equations of rotational motion yields pole displacements as large as 32.8 cm (ice caps solution), 40.9 cm (atmospheric effects), or 35.3 cm (both effects combined). Fourier analysis of the equatorial components of polar motion for the combined solution yields main harmonics of 5.66 cm (x), (1/3)-annual and 7.86 cm (y), annual.

The Mars' model used in the investigation corresponds to a tri-axial, elastic solid body without a fluid core. For the adopted parameter values, the associated natural or Chandler period is 211.6 sols. When forced by ice caps and atmospheric variations the response of the model is a function of the separation between the forcing frequency and the natural frequency. Consequently, the (1/3)-annual harmonic of the forcing is magnified in the spectra corresponding to the equatorial components of polar motion. Inversely, the daily and sub-daily harmonics of forcing are minimized. However, if an ellipsoidal fluid core were incorporated in the Mars' model, a near-diurnal natural frequency would appear which might magnify the response to the daily and sub-daily

harmonics present in the forcing functions. Additional resonances might appear if a solid inner core is introduced in the model.

The methodology of planetary rotational investigations can follow the angular momentum approach or the torque approach. The torque approach was used in Paper I to compute polar motion and LOD variations although no ice caps effects were included in the polar motion calculation. Discrepancies between the two approaches have previously appeared in investigations concerned with atmospheric effects on the rotation of the Earth and Mars. In theory the results from the torque approach and the results from the angular momentum approach should be identical. It is possible that the spatial grids used in the models are not sufficiently fine to achieve the necessary numerical accuracy conducive to a convergence of results for the two methodologies. Some authors have expressed their preference for the angular momentum approach, i.e., Defraigne et al. (2000). At the present time the quantity and quality of Mars' rotational data is not sufficient to ascertain which methodology is producing more realistic results. However, the amplitudes of the main harmonics as obtained in this investigation are certainly within the range of detection of future geodetic missions to Mars, such as the planned and postponed NetLander Ionospheric and Geodesic Experiment (NEIGE).

Future investigations will benefit from the refinement of the atmospheric models to be expected as more and better data becomes available, as well as from more realistic models of Mars' inner structure.

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Table 1. Chandler period as function of triaxiality and elasticity.
 $A = 2.68594 (10^{36}) \text{ kg-m}^2$, $B = 2.68433 (10^{36}) \text{ kg-m}^2$, $C = 2.69956 (10^{36}) \text{ kg-m}^2$. Chandler period in sols.

Moments of Inertia	k_2	Chandler Period
A, A, C	0.	196.8
A, B, C	0.	186.1
A, A, C	0.153	225.4
A, B, C	0.153	211.6

Table 2. Main harmonics of forcing functions and equatorial polar motion displacements due to ice caps and atmospheric variations. Frequency in cycles per year. Amplitudes of forcing functions in Hadleys. Amplitudes of polar motion displacements in cm.

X - component

Ice Caps Variations				Atmospheric Variations			
Forcing Function		Pole Displacement		Forcing Function		Pole Displacement	
Frequency	Amplitude	Frequency	Amplitude	Frequency	Amplitude	Frequency	Amplitude
1	0.70	3	9.56	1	1.83	3	8.03
2	0.41	1	4.35	669	0.57	1	4.45
3	0.15	2	2.29	668	0.20	2	4.12
5	0.07	4	2.13	670	0.19	4	2.61
4	0.05	5	1.66	3	0.18	5	0.96

Y - component

Ice Caps Variations				Atmospheric Variations			
Forcing Function		Pole Displacement		Forcing Function		Pole Displacement	
Frequency	Amplitude	Frequency	Amplitude	Frequency	Amplitude	Frequency	Amplitude
1	0.73	3	9.05	1	0.55	1	9.89
2	0.14	2	4.32	669	0.49	3	6.98
3	0.07	1	3.17	2	0.26	4	2.89
5	0.04	4	1.63	671	0.16	2	2.32
6	0.03	5	1.12	667	0.15	5	1.30

Table 3. Main harmonics of the excitation function due to ice and atmospheric variations combined and associated pole displacement. Frequency in cycles per year. Forcing function amplitude in Hadleys. Pole displacement amplitude in cm.

X – component

Forcing Function		Pole Displacement	
Frequency	Amplitude	Frequency	Amplitude
1	1.74	3	5.66
669	0.57	1	3.86
2	0.37	2	3.40
668	0.20	4	2.43
3	0.20	5	1.58

Y – component

Forcing Function		Pole Displacement	
Frequency	Amplitude	Frequency	Amplitude
1	0.89	1	7.86
669	0.49	3	6.25
2	0.31	2	2.77
671	0.16	4	2.48
667	0.15	5	1.53

Table 4. Main harmonics of polar motion displacement ($\sqrt{x^2 + y^2}$).
Solution based on ice caps variations. Solution based on
atmospheric variations. Solution based on ice caps and atmospheric
variations combined. Frequency in cycles per year. Amplitude in cm.

Ice Caps Variations		Atmospheric Variations		Combined Solution	
Frequency	Amplitude	Frequency	Amplitude	Frequency	Amplitude
1	4.30	2	5.51	4	3.96
2	2.76	4	3.09	3	3.03
5	1.52	3	2.77	1	2.52
4	1.47	1	2.65	2	0.79
3	1.04	5	1.94	5	0.61

Figure Captions.

Figure 1. Mass variation time series.

b) Atmosphere.

c) Ice.

Figure 2. Atmospheric variation forcing function. X - component.
Amplitude in Hadleys. One Hadley is equal to 10^{18} Newton-meters.

a) Time series.

b) Power Spectrum.

Figure 3. Atmospheric variation forcing function. Y - component.

a) Time series.

b) Power Spectrum.

Figure 4. Ice caps variation forcing function. X - component.

a) Time series.

b) Power Spectrum.

Figure 5. Ice caps variation forcing function. Y - component.

a) Time series.

b) Power Spectrum.

Figure 6. Atmospheric variation plus ice caps variation forcing function. X - component.

a) Time series.

b) Power Spectrum.

Figure 7. Atmospheric variation plus ice caps variation forcing function. Y - component.

a) Time series.

b) Power Spectrum.

Figure 8. Polar motion due to atmospheric variations.

a) Displacement = $(x^2 + y^2)^{1/2}$.

b) Phase = $\arctan (y/x)$.

c) Power Spectrum.

Figure 9. Polar motion due to ice caps variation.

- a) Displacement = $(x^2 + y^2)^{1/2}$.
- b) Phase = $\arctan (y/x)$.
- c) Power Spectrum.

Figure 10. Polar motion due to atmospheric and ice caps variations combined. X - component.

- a) Amplitude.
- b) Power Spectrum.

Figure 11. Polar motion due to atmospheric and ice caps variations combined. Y - component.

- a) Amplitude.
- b) Power Spectrum.

Figure 12. Polar motion due to atmospheric and ice caps variations combined.

- a) Displacement = $(x^2 + y^2)^{1/2}$.
- b) Phase = $\arctan (y/x)$.
- c) Power Spectrum.























